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A SEMI-EMPIRICAL FORMULA FOR ESTIMATING RESIDUAL STRAIN OF COHESIVE SOILS UNDER RANDOM EARTHQUAKE LOADS

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SUMMARY

A new semi-empirical formula for estimating the residual strain of cohesive soils under random earthquake loads is presented in the paper. Using the incremental method of cycle-by-cycle the new formula suitable for the irregular loading is obtained by taking the first partial derivative of the existing residual strain formula for the constant amplitude loading with respect to cyclic number. When the incident loading is uniform, the calculated results by the new formula are nearly the same as those by the existing formula. For excitation of the random earthquake loading, the calculated results by the new formula are compared to the results by the dynamic triaxial tests on the cohesive soils. In the earthquake wave input triaxial tests, five typical earthquake waves including shock and vibration types are used. Meanwhile, CM test and EM test, separately representing the maximum peak of the seismic stress history appears when the triaxial loading piston reaches the lowest position and the highest position, as well as RM test, representing the seismic stress history is reversed in time to form the back stress history, are conducted. The comparison results between calculation and tests indicate that the presented formula here can efficiently and practically describe the time-dependent process and final values of the soil residual stains under the actual seismic loads and also can illustrate the effects of the different types of seismic waves, CM and EM tests and load order on the development and difference of the residual stains of soils.
Keywords: irregular earthquake loading; cohesive soils; residual strain; semi-empirical formula; dynamic triaxial tests

INTRODUCTION

The evaluation for permanent deformation of soils under seismic loading become more and more important subject considering the damage phenomena concerned with subsoil and geotechnical structures during earthquakes as well as the recent need for the performance-based aseismic design in engineering. Many researches (Ishihara *et al*, 1973, 1984; Xie, 1987; Nagase *et al*, 1987; Wang, 2000;Yuan *et al*, 2003) have shown that the effects of the load asymmetry and irregularity of seismic waves are significant on the dynamic behavior of soils, especially the permanent deformation process of soft cohesive soils and saturated sands. However, most of the existing models and formulae used in engineering for estimating

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the residual strain of the soil elements are naturally suitable for the uniform cyclic loading of equiamplitude (Serff *et al*, 1976; Haldar *et al*, 1988). When actual seismic waves are applied to soils, the random waves are generally transformed into the sinusoidal loading with a certain cyclic number according to method of Seed (Seed *et al*, 1971). If the irregular seismic waves are turned into the sinusoidal loading, some important effects, such as asymmetry and irregularity of the loading, different properties of soils, transverse non-uniform distribution of soil layer and non uniform distribution of building weight, on the permanent deformation of soils and differential settlements of subsoil due to earthquake shaking can not be considered. Also, the real process of soil deformation can not be obtained and the irregular effects of the time-varying seismic loads on soil deformation can not be included in the analyses as well. As a result, the definition of equivalent cyclic number of equiamplitude loading perhaps is not reliable for evaluating the soil deformation in many cases, especially for describing the earthquake-induced difference of permanent deformation in the soil layer.

MODEL

The permanent deformation of soil elements due to a constant sinusoidal stress load can be evaluated using the dynamic triaxial tests and the formula in Ref.10 is written here

$$\bar{\varepsilon}_p = 10\left[\frac{1}{c_5}\cdot\frac{\bar{\sigma}_d}{\sigma_3}\right]^{\frac{1}{s_5}}\left(\frac{N}{10}\right)^{-\frac{s_1}{s_5}} \tag{1}$$

where

$\bar{\varepsilon}_p$: residual strain potential of soil elements;

σ_3: confining stress surrounding soil elements;

N: the number of constant amplitude cycles;

$\bar{\sigma}_d$: the equivalent constant amplitude of the random stress wave;

$c_5 = c_6 + s_6(k_c\text{-}1)$;

$s_5 = c_7 + s_7(k_c\text{-}1)$;

k_c: consolidation ratio of the soil element;

s_1, c_6, c_5, c_7, s_7: the coefficients related to the type and property of soil element as shown in Ref.10.

Using the dynamic triaxial tests, Eqn.1 has been checked and proved to be correct for the uniform cyclic loads.

To obtain the residual strain under the random cyclic loading, the irregular earthquake loading is first turned into a series of cyclic loads with different amplitude. For a complex earthquake wave, the small wave in the big cycle has little contribution to the developing of residual strain and can be neglected in the calculation. The irregular earthquake loading is then divided into a series of cyclic loads with different amplitude as shown in Fig.1.

In this paper, one of key points in formulation of the residual strain potential of soil elements under excitation of the random loading is to use the following incremental method

$$\varepsilon_p^i = \varepsilon_p^{i-1} + \Delta\varepsilon_p^i \qquad i=1 \text{ to } M \tag{2}$$

Where

ε_p^i: the accumulative residual strain after the ith stress cycle;

ε_p^{i-1} : the accumulative residual strain after the i-1th stress cycle, and $\varepsilon_p^0 = 0$;

M: the total cyclic number of the random stress loading;
$\Delta\varepsilon_p^i$: the residual strain produced by the ith stress cycle.

Another key point in the paper is to set up the increment model of the residual strain $\Delta\varepsilon_p^i$. When the cyclic number becomes i from i-1 and the amplitude of the stress loading becomes σ_d^i from σ_d^{i-1}, the known relations of $\varepsilon_p - \sigma_d^i$ and $\varepsilon_p - \sigma_d^{i-1}$ are exhibited in Fig.1. From physical point of view, the increment of the residual strain $\Delta\varepsilon_p^i$ caused by σ_d^i will be separated from trace of σ_d^{i-1} and move to the tangent direction of trace of σ_d^i. Then $\Delta\varepsilon_p^i$ caused by σ_d^i can be expressed as the first partial derivative of with respect to N in Eqn.1, which represents the increment of the residual stain due to σ_d^i from i-1th stress cycle to ith stress cycle as well as means the movement of residual strain from the point A to the point B as shown in Fig.2. Letting N=i-1 in Eqn.1, $\Delta\varepsilon_p^i$ can be formed as

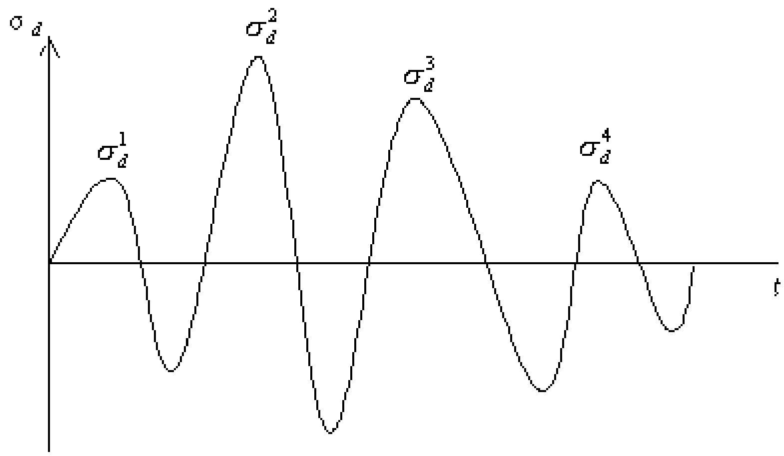


Fig.1 A series of cyclic loads with different amplitudes

$$\Delta\varepsilon_p^i = \left(\frac{\sigma_d^i}{\sigma_3 c_5}\right)^{\frac{1}{s_5}} \left(-\frac{s_1}{s_5}\right)\left(\frac{i-1}{10}\right)^{\left(-\frac{s_1}{s_5}-1\right)} \qquad i=2 \text{ to } M \tag{3}$$

and the first increment of the residual strain $\Delta\varepsilon_p^1$ can be expressed as

$$\Delta\varepsilon_p^1 = 10\left[\frac{1}{c_5}\cdot\frac{\sigma_d^1}{\sigma_3}\right]^{\frac{1}{s_5}}\left(\frac{1}{10}\right)^{-\frac{s_1}{s_5}} \tag{4}$$

In Eqns. 1 and 2, the effect of the frequency of the stress loading is not taken into account, because there are a lot of results proving that the effect of the incident frequency in the range of usual seismic waves can be negligible generally (Matsuda *et al*, 1988).

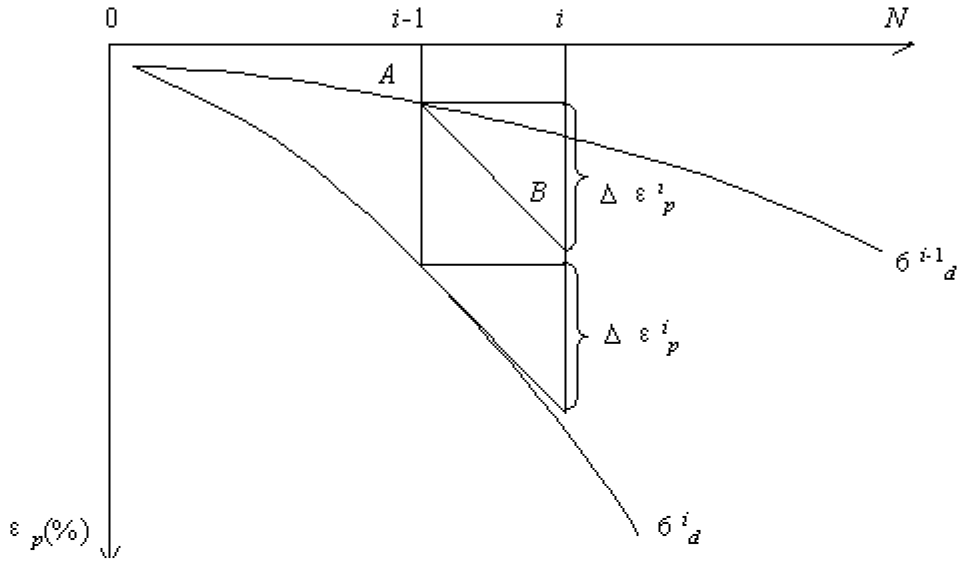


Fig.2 The increment of the residual strain caused by σ_d^i

In the actual seismic stress history, the stress amplitudes on two sides of each cycle generally are not symmetrical. Considering that the major part of residual strain is controlled by the compressive stress pulse in the dynamic triaxial tests (Ishihara *et al*, 1984; Yuan *et al*, 2003) as well as in the shaking table tests (Meng *et al*, 2002), the stress amplitude σ_d^i in Eqn.3 can be taken as the compression stress peaks. For extensional stress, the rebound strain is calculated by similar principle to that of Martin *et al*. However, many experimental results show that the extensional stress has little contribution to the permanent strain of soils and as a result, the extensional stress here is considered in a simple way. Actually, from view of the permanent displacement of soils and getting more simplicity, the extensional stress can be removed from the calculation.

VALIDATION OF THE NEW FORMULA

Comparison with the former formula for uniform loading

When the uniform sinusoidal loads are applied to Eqn.2, the residual deformations by Eqn.2 are compared with the results by Eqn.1. The comparison results are illustrated in Table 1, in which former and present represent the results by Eqn.1 and Eqn.2, respectively.

The comparison of results indicates that the errors between the former and the new formulae are small enough and the maximum error is less than 5% as shown in the Table 1.

Comparison with testing results for random earthquake loading

For the actual earthquake loading, the validation of Eqn.2 can be checked using the results by the dynamic triaxial tests. The tests on soil specimens are conducted by the auto earthquake-wave-input triaxial apparatus, newly developed at the Institute Engineering Mechanics, CSB, China. The new apparatus has the high accuracy and the wide frequency range of 0-20 Hz by a closed-circuit control of both force and displacement (Sun *et al*, 2002). Using the apparatus the seismic stress history can be applied exactly to the soil specimens.

The cases of the dynamic triaxial tests are listed in Table 2. Five actual earthquake records are employed in the tests, respectively being the acceleration record at Beijing Hotel during the Tangshan Earthquake of China in 1976 (simply named as Tangshan wave), the acceleration record at the Tianjin Hospital in the

Ninghe Earthquake of China in 1976 (Tianjin wave), the acceleration record at Ninghe during the Ninghe Earthquake of China in 1976 (Ninghe wave), the acceleration record at the Qianan during aftershock of the Tangshan Earthquake of China in 1976 (Qianan wave) and the EL-Centro wave recorded in the Imperical Valley Earthquake of the Unite State of America in 1940 (El-Centro wave).

Table 1 Comparison of the calculated residual strains between former and present formulae under uniform sinusoidal loading (S_1=-0.128, S_5=0.186, C_5=0.714, K_c=1.5,σ_3=200kPa)

Cyclic number N	Formulae and error	Amplitude of stress (kPa)					
		10	20	30	40	90	110
5	Former	3.84e-6	1.60e-4	1.41e-3	6.63e-3	0.52	1.53
	Present	4.03e-6	1.68e-4	1.48e-3	6.96e-3	0.54	1.60
	Error %	4.95	5.00	4.96	4.98	3.85	4.58
10	Former	6.19e-6	2.57e-4	2.27e-3	1.07e-2	0.84	2.46
	Present	6.44e-6	2.67e-4	2.36e-3	1.11e-2	0.87	2.56
	Error %	4.04	3.89	3.96	3.74	3.57	4.07
50	Former	1.87e-5	7.88e-4	6.89e-3	3.23e-2	2.53	7.44
	Present	1.91e-5	7.92e-4	7.01e-3	3.29e-2	2.57	7.57
	Error %	2.14	0.51	1.74	1.86	1.58	1.75
100	Former	3.02e-5	1.25e-3	1.11e-2	5.20e-2	4.08	11.9
	Present	3.06e-5	1.27e-3	1.12e-2	5.27e-2	4.12	12.1
	Error %	1.32	1.60	0.90	1.35	0.98	1.17

Table 2 Cases of the dynamic triaxial tests

Case No.	Soil type	Inputting wave	Consolidation ratio	Confining Stress(kPa)	Stress peak amplitude (kPa)
1	Muck	Tianjin (EM)	1.73	200	150
2		Tianjin (CM)	1.73	200	150
3		Tianjin (RE)	1.73	200	150
4	Mucky clay	EL-Centro (EM)	1.5	200	150
5		EL-Centro (CM)	1.5	200	150
6	Mucky clay	Qianan (EM)	2.3	100	100
7		Qianan (CM)	2.3	100	100
8	Mucky soil	Ninghe (EM)	1.3	200	100
9		Ninghe (CM)	1.3	200	100
10	Mucky soil	Tangshan (EM)	1.5	130	50
11		Tangshan (CM)	1.5	130	50
12	Clay	El-Centro (EM)	1.5	50	60
13		El-Centro (CM)	1.5	50	60

The undisturbed soil specimens from Tanggu area and Hangzhou City in China are used in the tests, and different confining stress σ_3, different consolidation ratio k_c and different peak amplitude of dynamic stress σ_d are taken for the tests. The soil properties and the related calculation parameters determined by the soil properties are listed in Table 3.

In the testing, CM test and EM test, are carried out separately to the two same undisturbed soil specimens. As proposed by Ishihara, CM test means that the maximum peak of the seismic stress history appears when the triaxial loading piston reaches the lowest position. EM test means that the maximum peak stress is oriented so that the maximum peak is obtained at the highest position of the loading piston in the tests. Before the dynamic tests the two specimens suffer from the same confining stress and the same initial axial static stress. RE test in Table 2 means that the seismic stress history is reversed in time to form the back stress history.

The comparison results of the final residual strains by the formula and test for all of the cases are listed in Table 4. The results In Table 4 indicate the errors between the calculation and test for the 13 events are about 4%-40% and the average error is 15%.

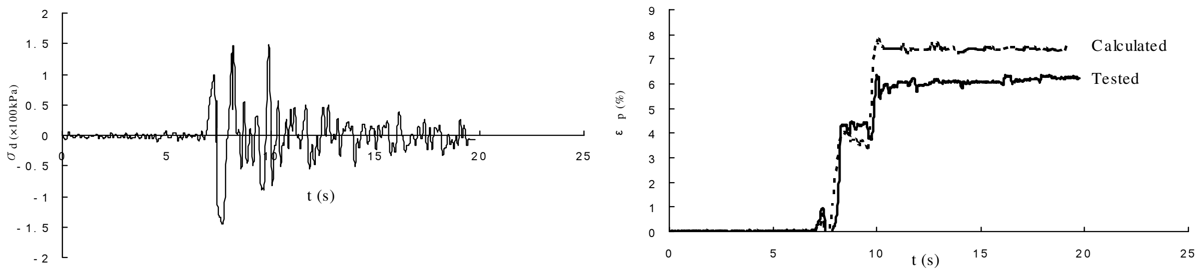
Table 3 Soil properties and parameters used in numerical calculation

No	Water content (%)	Density (g/cm^3)	S_1	C_6	S_6	C_7	S_7
1¡¢2¡¢3	46.6	1.80	-0.17	0.48	0.25	0.15	0.0
4¡¢5	39.5	1.82	-0.16	0.60	0.20	0.17	0.0
6¡¢7	30.7	1.96	-0.15	0.65	0.21	0.18	0.0
8¡¢9	39.5	1.82	-0.16	0.58	0.22	0.16	0.0
10¡¢11	40.8	1.82	-0.16	0.59	0.21	0.16	0.0
12¡¢13	24.7	1.85	-0.13	0.80	0.30	0.18	0.0

Table 4 Comparison of the final residual strains between test and formula

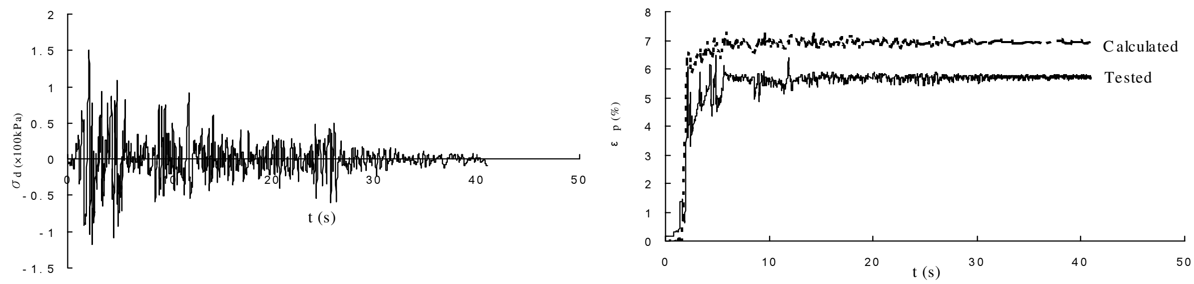
Case No	Soil type	Inputting wave	Tested (%)	Calculated (%)	|Error| (%)
1	Muck	Tianjin (EM)	6.20	7.43	19.8
2					
3					
		Tianjin (CM)	3.95	5.10	29.1
		Tianjin (RE)	3.18	3.06	3.8
4	Mucky clay	EL-Centro (EM)	2.13	2.35	10.3
5					
		EL-Centro (CM)	5.78	6.94	20.1
6	Mucky clay	Qianan (EM)	0.93	0.77	17.2
7					
		Qianan (CM)	1.45	1.32	9.0
8	Mucky soil	Ningke (EM)	1.55	1.93	24.5
9					
		Ninghe (CM)	3.25	3.74	15.1
10	Mucky soil	Tangshan (EM)	0.32	0.37	15.6
11					
		Tangshan (CM)	0.83	1.15	38.6
12	clay	EL-Centro (EM)	0.98	0.87	11.2
13					
		EL-Centro (CM)	1.54	1.35	12.3

To check the validation of the new formula in simulating the characteristics of soil residual deformation under actual seismic loading, the time histories of residual strains by the formula and by the test are exhibited in Figs.3-7, respectively for the five inputs of seismic waves, i.e. No.1, No.5, No.7, No.9 and No.11 cases.



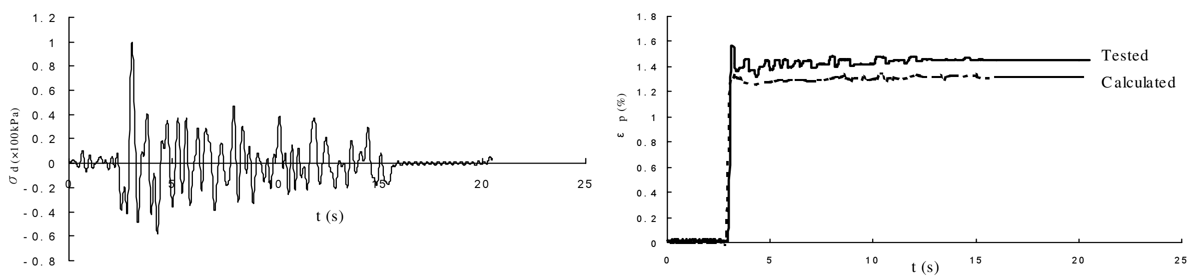
(a) (b)

Fig. 3 Comparison of residual strain histories between formula and test for specimen No.1



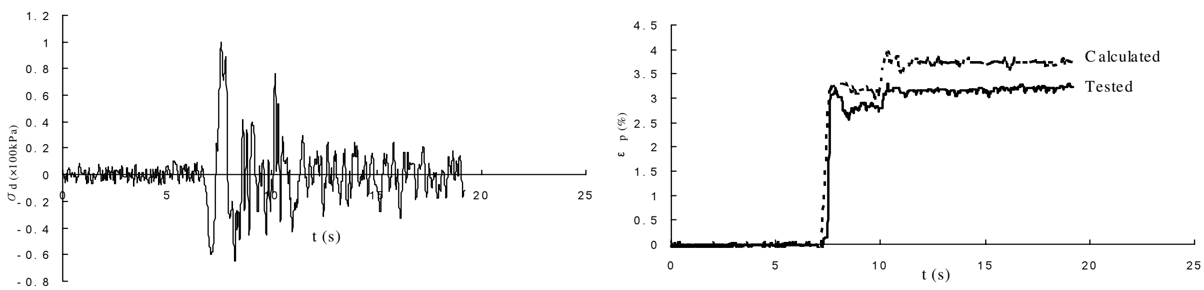
(a) (b)

Fig. 4 Comparison of residual strain histories between formula and test for specimen No.5



(a) (b)

Fig. 5 Comparison of residual strain histories between formula and test for specimen No.7

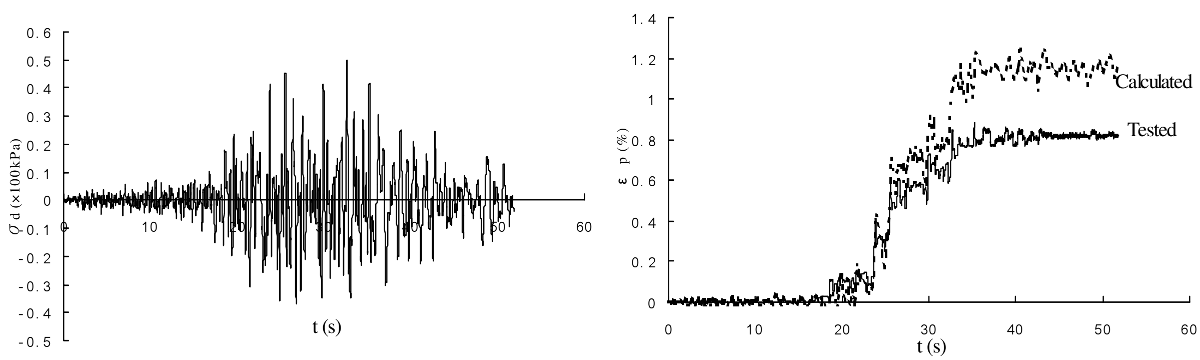


(a) (b)

Fig. 6 Comparison of residual strain histories between formula and test for specimen No.9

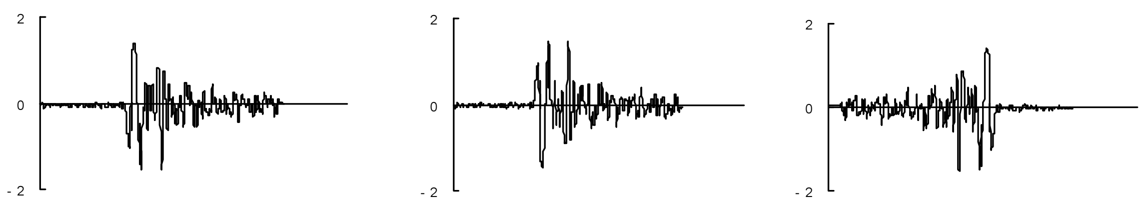
For the different types of incident waves such as the shock type of Tianjin wave, EL-Centro wave, Qianan wave and Ninghe wave, as well as the vibration type of Tangshan wave, the results in Figs.3-7 show that the residual strain histories calculated by the new formula are agreeable with the testing results in time process of the residual strain development.

Furthermore, to identify the validation of the new formula in simulating the effect of order of the seismic loading on development of residual strain, three earthquake loads are taken as the axial dynamic stresses to apply three identical soil specimens, respectively, i.e. No.1, No.2 and No.3 cases. Three earthquake loads naturally come from one seismic record and they are named as CM wave, EM wave and RM wave as shown in Fig. 8, respectively. It can be seen from Fig.8 that the order of the irregular loading has an obvious effect both on the process of the residual strain and on the final value of the residual strain. The formula can simulate the basic characteristics of the effect of the load order on the residual histories because the calculated results by the formula are quite similar to the testing results. Also, it can be got from Fig.8 that the new formula can present a good simulation in the stain difference under excitation of CM and EM waves because the differences obtained by the formula and by the test, one being 46% and another 56%, are near. For more cases, it can be seen from Table 4 that the simulation in the stain difference of CM and EM wave inputs by the new formula is successful as well.

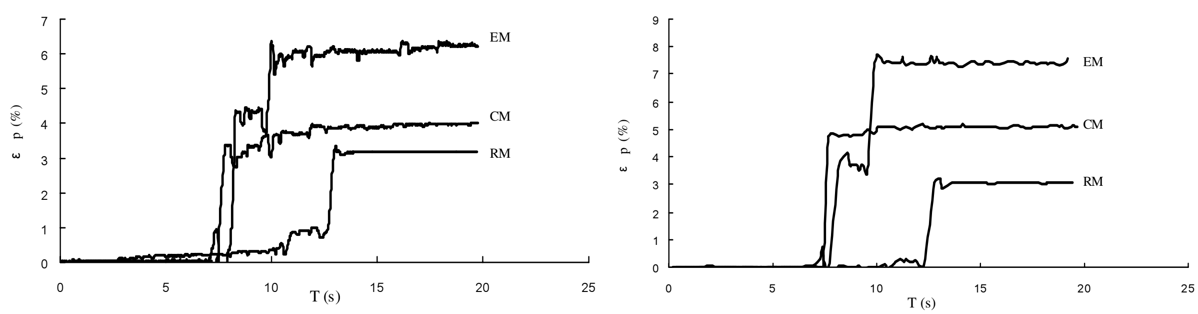


(a) (b)

Fig. 7 Comparison of residual strain histories between formula and test for specimen No.11



CM wave EM wave RM wave



Tested strain histories Calculated strain histories

Fig. 8 Effect of load order on residual strain histories by formula and test for specimens of No.1-3

Conclusion

The new semi-empirical formula suitable for evaluating the residual strain of cohesive soils under irregular dynamic loading is presented in the paper by using the incremental method. For the uniform cyclic loading of equiamplitude, results by the new formula are quite agreeable to those by the existing formula. When the actual seismic waves are excited, the validation of the new formula has been carefully checked by the dynamic triaxial tests. The comparison results between the formula and tests indicate that the presented formula can efficiently and practically describe the time-dependent process and final values of the soil residual stains under the actual seismic loads and also can illustrate the effects of the different types of seismic waves, CM and EM tests and load order on the development and difference of the residual stains of soils.

The technique on the deformation analysis will be applied more and more in aseismic design of foundation and geotechnical structures. Because the transform from the irregular seismic to the sinusoidal loading of equiamplitude is removed and the residual strain is obtained directly from the actual random seismic loading, the new semi-empirical formula here can results in the permanent deformation in a more convenient way and may give more reasonable evaluation of soil deformation. Moreover, the new semi-empirical formula here can simulate the differential deformation and the differential settlement of subsoil and geotechnical structures resulting from the un-uniform inertia, transverse non-uniform distribution of soil layer and non-uniform distribution of building weight due to seismic shaking, while the exiting method may have limitation in the aspect.

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