

NUMERICAL SOLUTIONS FOR DYNAMIC IMPEDANCE MATRIX OF RECTANGULAR FOUNDATIONS AND THEIR APPLICATIONS IN DYNAMIC FOUNDATION RESPONSE ANALYSIS

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SUMMARY

The numerical solutions for dynamic impedance matrix of embedded rectangular rigid foundation are presented based on Scaled Boundary Finite-Element Method (SBFM). The results fit well at lower frequency band with the boundary element method solutions. As an engineering application, Dynamic response of a shaking table foundation is calculated in the frequency domain based on dynamic impedance matrix solutions obtained in this paper. According to Code for Design of Dynamic Machine Foundation in China, the frequency-independent mass-spring-damping model is employed to calculate dynamic machine foundation response. The dynamic impedance matrix represented by this kind of model is constant in the whole frequency band. Dynamic impedance coefficients of rigid foundation, however, are frequency-dependent. Analysis method presented in this paper improves the precision of dynamic foundation response in the frequency domain.

INTRODUCTION

Analytical solutions for dynamic impedance matrix of embedded rectangular foundation do not exist for its geometrical complicacy. Generally, analytical solutions are only available for quite geometrical regular foundations. Aprel and Luco^[1] have formulated the series solution for the torsion impedance of semi-circular foundation on the elastic half-space. Rectangular foundations, however, only numerical solutions are available and most of works are based on boundary element method at present.

Veletsos and Wei^[2] presented the numerical solutions for dynamic impedance of rigid circular foundation in the frequency domain in 1971. Because the dynamic response of circular and strip foundations embedded in the soil involving mixed-value problems which can be solved numerically based on

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Fredholm equation, an approximate approach for rectangular foundation is translating it into an equivalent circular foundation or postulating the stress distribution between the foundation and the soil, which converts the boundary integral equation into a Fredholm equation. Thomason and Kobori^[3] obtained the dynamic flexibility solutions of the rectangular foundations based on this idea.

Wong and Luco^[4], follows Lysmer's initial idea^[5], adopted the following method to solve the dynamic impedance of arbitrary surface foundations: the dynamic flexibility matrix for the ground is obtained through discretization of integral equations by dividing the contact region between the ground and the foundation into small rectangular sub-regions and by assuming that the contact stresses are uniformly distributed within each sub-region. The dynamic stiffness matrix for the foundation, defined at the intersections of the sub-regions, is obtained by inversion of the dynamic flexibility matrix. Combining the stiffness matrices of the foundation and ground leads to a set of linear algebraic equations for the soil-foundation system in terms of the nodal displacements. Once the nodal displacements are obtained, the contact stresses for each sub-region may be easily evaluated. Uniting all of the stresses leads to the dynamic impedance functions. Wong presented the dynamic impedance matrix of the surface rectangular foundations based on direct boundary-element method, as well as Mita and Luco^[8] obtained the embedded square foundations based on mixed method.

A new approach, scaled boundary finite-element method^[9], is a general procedure to solve linear partial differential equations semi-analytically applied to hyperbolic problems for unbounded media and obtains the dynamic stiffness matrix numerically. The method combines the advantages of the finite-element method and the boundary-element method: Reduction of spatial dimension by one; No fundamental solution required; Radiation condition satisfied exactly for unbounded medium; No singular integrals to be evaluated.

As an engineering application of the SBFM solutions, dynamic response of a shaking table foundation is calculated in the frequency domain based on dynamic impedance matrix solutions obtained in this paper.

DYNAMIC IMPEDANCE MATRIX OF RIGID EMBEDDED RECTANGULAR FOUNDATIONS

Fig1. illustrates the rectangular foundation embedded in elastic half-space. Only a quarter of the foundation is given considering the geometric symmetry. Supposing the length of the foundation are 2a and 2b (a > b), the embedded depth is e. Also, defining the material density of the half-space is ρ , shear modulus is G and the Poisson's ration is υ . And assuming the harmonica responses of the foundation are $(u_1, u_2, u_3)e^{i\omega t}$ and $(\Phi_1, \Phi_2, \Phi_3)e^{i\omega t}$ under steady-state loads (F₁, F₂, F₃) $e^{i\omega t}$ and (M₁, M₂, M₃) $e^{i\omega}$ ^t, where ω is the angle-frequency of the loads.

Defining the general force vector and general displacement vector as:

$$\mathbf{F} = \{F_1, M_2/b, F_2, M_1/b, F_3, M_3/b\}^T$$
(1)

$$\mathbf{U} = \{ U_1, b\Phi_2, U_2, b\Phi_1, U_3, b\Phi_3 \}^T$$
(2)

Where b is the characteristic length of the foundation.

The force-displacement relationship with the corresponding amplitudes formulated in the degree of the rigid foundation is written as

$$\mathbf{F}(\boldsymbol{\omega}) = \mathbf{S}(\boldsymbol{\omega})\mathbf{U}(\boldsymbol{\omega}) \tag{3}$$



Fig. 1 Rectangular foundation embedded in the half-space

 $S(\omega)$ represents the dynamic impedance matrix of the unbounded medium, which is a 6×6 matrix based on the space-symmetry and the definition of the load and displacement above. $S(\omega)$ can be formulated as

$$\mathbf{S}(\boldsymbol{\omega}) = \begin{bmatrix} S_{11} & S_{12} & 0 & 0 & 0 & 0 \\ S_{21} & S_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & S_{33} & S_{34} & 0 & 0 \\ 0 & 0 & S_{43} & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix}$$
(4)

Where S_{11} is the horizontal impedance coefficient along *x*-axis, S_{22} is the rocking coefficient around *y*-axis, S_{12} is the coupled dynamic impedance coefficients between the translation alone *x*-axis and the rotation of *y*-axis. And analogously we can define S_{33} , S_{34} and S_{44} . S_{55} and S_{66} are the translation stiffness and torsion stiffness around *z*-axis respectively.

Each element of the matrix can be written as the following uniform expression;

$$S(\omega) = K\overline{S} \tag{5}$$

Where K is the static impedance and $\overline{S} = \overline{S}^r + i\omega\overline{S}^i$ is the corresponding dimensionless stiffness coefficient.

The dynamic impedance matrix of arbitrary foundation can be derived based on scaled boundary finite-element method, which is a full matrix with respect to the degrees of freedom on the foundation-medium interface. From which, the corresponding dynamic impedance of the rigid foundation can be derived according to its definition: Fourier amplitude of the steady-state load exerts on the rigid foundation if a unit amplitude steady-state displacement occurs along any degree-of-freedom direction at a special frequency^[10, 11].

The displacement-force relationship of the boundary nodes is

$$\mathbf{f} = \mathbf{S}^{\infty}(\boldsymbol{\omega})\mathbf{u} \tag{6}$$

Where,

$$\mathbf{f} = (f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23}, \cdots, f_{n1}, f_{n2}, f_{n3})^T$$
(7)

$$\mathbf{u} = (u_{11}, u_{12}, u_{13}, u_{21}, u_{22}, u_{23}, \cdots, u_{n1}, u_{n2}, u_{n3})^{T}$$
(8)

n is the total number of boundary nodes, f_{pq} and u_{pq} is the force and the displacement component of the pth ($p = 1, 2, \dots, n$) node along the x_q -direction. x_q (q = 1, 2, 3) is the Cartesian coordinate defined in Fig. 1.

Introducing displacement transfer matrix A:

$$\mathbf{u} = \mathbf{A}\mathbf{U} \tag{9}$$

Where **u** defined in Equation (8) and **U** in (2).

$$\mathbf{A} = \begin{bmatrix} \mathbf{I} & \mathbf{x}_{3}/b & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{x}_{2}/b \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & -\mathbf{x}_{3}/b & \mathbf{0} & \mathbf{x}_{1}/b \\ \mathbf{0} & -\mathbf{x}_{1}/b & \mathbf{0} & \mathbf{x}_{2}/b & \mathbf{I} & \mathbf{0} \end{bmatrix}$$
(10)

A is a $3n \times 6$ matrix and

$$\mathbf{x}_q = (x_{1q}, x_{2q}, \cdots, x_{3q})^T, \quad q = 1, 2, 3$$
 (11)

$$\mathbf{I} = (1, 1, \cdots, 1)^T \tag{12}$$

$$\mathbf{0} = (0, 0, \cdots, 0)^T \tag{13}$$

Also introducing the force transfer matrix **B**

$$\mathbf{F} = \mathbf{B}\mathbf{f} \tag{14}$$

B is a $6 \times 3n$ matrix and can be expressed as

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 1 & 0 & 0 \\ x_{13}/b & 0 & -x_{11}/b & \cdots & x_{n3}/b & 0 & -x_{n1}/b \\ 0 & 1 & 0 & \cdots & 0 & 1 & 0 \\ 0 & -x_{13}/b & x_{12}/b & \cdots & 0 & -x_{n3}/b & x_{n2}/b \\ 0 & 0 & 1 & \cdots & 0 & 0 & 1 \\ -x_{12}/b & x_{11}/b & 0 & \cdots & -x_{n2}/b & x_{n1}/b & 0 \end{bmatrix}$$

Substituting Equation (6) and Equation (9) in Equation (14), the dynamic impedance matrix S is formulated:

$$\mathbf{S} = \mathbf{B}\mathbf{S}^{\infty}(\boldsymbol{\omega})\mathbf{A}$$
(15)

NUMERICAL SOLUTIONS OF EMBEDDED RECTANGULAR FOUNDATION

The frequency-domain solutions for dynamic impedance matrix of the embedded rectangular foundation are the function of foundation shape, material property and the load frequency property. It has no analytical expression and only numerical solutions can be obtained.

Numerical solutions based on scaled boundary finite-element method are presented in this section. The interface of the rectangular foundation and the soil is divided by finite elements (Fig.2). Four 8-node elements are introduced on each surface considering the shape symmetry of the foundation. To ensure the accuracy, for large aspect ration or depth ratio, appropriate more elements are introduced.



Fig.2 Finite-element model for embedded rectangular foundation

An example is presented to verify the accuracy of the method. Defining the lengths of the foundation are both 2 meters, that is, a=b=1m. The shear modulus of the soil is $G = 1N/m^2$, density $\rho = 1kg/m^3$

and the embedded depth is 1m also. Dimensionless frequency $a_0 = \frac{\omega b}{c_s} = 1$. Additionally, Gb is

chosen as the dimensionless coefficient but not the static impedance coefficient. Comparisons between the scaled boundary finite-element method and the mixed-method are shown in Fig.3, in which the depth ratio is respectively 0.5, 1.0 and 1.5 and the Poisson's ration is 1/3. Only vertical, horizontal, rocking and torsion impedance coefficients are compared. All of the solutions consider the material-damping ration in Mita's mixed method. Correspondence principle is introduced to eliminate the difference between scaled boundary finite-element method and the mixed method. Results show that the two methods fit well at lower frequency band. Errors still exist, especially for the torsion and rocking impedance when the depth ration is larger. As a fact, errors between the two methods are inevitable because the idea and finite-element discretization differences.



c Rocking



Fig.3 Comparison between scaled boundary finite-element method and mixed method

DYNAMIC RESPONSE ANALYSIS OF A SHAKING TABLE FOUNDATION

A foundation of $6 \times 6m$ three-dimensional earthquake simulation shaking table is RC entity with size of 24 $\times 15 \times 6m$, and its mass is about 4,960.5 ton. The simulation table is about 40 ton. So the total mass is about 5,000 ton. The acceleration values of some selected points at some selected frequency are measured (see next section)^[11, 12].



Fig. 4 The comparison of foundation dynamic response with difference methods

Adopting the mass-spring-damping model prescribed in the Code for Design of Dynamic Machine Foundation, we calculated the dynamic response of the shaking table foundation under horizontal and vertical loads. The amplitude of the harmonica force is about 120 ton. The dynamic acceleration responses of the shaking table foundation are shown in Figure 4, where the dots are the measured value to the corresponding harmonic input load at the same frequency. It is worth mentioning that the dimension of the frequency-axis is 1/s, but not *rad/s*.

Figure 4 shows that the SBFM solutions are closer with the measured values than the frequency-independent spring and damping model.

CONCLUSIONS

The numerical solutions for dynamic impedance matrix of embedded rectangular rigid foundation are presented based on Scaled Boundary Finite-Element Method (SBFM). The results fit well at lower frequency band with the boundary element method solutions. The dynamic response of the shaking table foundation in frequency domain is analysed using the numerical solution of the dynamic stiffness matrix in this paper. Traditionally, the dynamic design of the dynamic machine foundation is based on the frequency-independent mass-spring-damping model. The dynamic impedance represented by this kind of model is constant in the whole frequency band. The real dynamic impedance of the rigid foundation, however, is frequency-dependent. This kind of model may introduce error in the foundation design, and in some frequency band, the error maybe very large.

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