**13th World Conference on Earthquake Engineering**
**Vancouver, B.C., Canada**
**August 1-6, 2004**
**Paper No. 2684**

# VALIDATION OF SHALLOW S-WAVE VELOCITY STRUCTURE INVERSED BY MICROTREMORS ARRAY OBSERVATION

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## SUMMARY

The shallow S-wave velocity structure of a site in Xiamen, China, is inversed by the hybrid method of genetic algorithm (GA) and Simplex Algorithm (SA) from the surface wave dispersion curve. The latter is inferred by the Spatial Auto-correlation method (SAC) from microtremors array observation at the site.
The inversed velocity profile is compared with that from measurement in borehole at the same site. The average velocity obtained by SAC method and Frequency Wave-number method (F-K) is almost the same, nevertheless F-K method only gives an average velocity for the upper 50-meter. The relative error of each layer is about 20% in average.
In order to validate if this accuracy acceptable for earthquake engineer, even it is comparable with that of Spectral Analysis of Surface Wave method (SASW), seismic responses of 3 site models from 15 simulated ground motion time histories are carried out. The results of 1D equivalent linearized analysis shows that the difference between the response spectra from inversed velocity structure and measured in borehole is really acceptable, whereas the response spectra from a simplified single layer model with the average velocity is quite different.
It suggests that the shallow velocity structure of an engineering site can be inversed by microtremors array observation very well, and it is not a good idea in seismic design to characterize site condition just by the average velocity alone.

## INTRODUCTION

It is well recognized in the Earthquake Engineering field that the local site condition has a great effect on ground motion and earthquake damage. From the view of wave propagation, site dynamic behavior is governed by the thickness of soil, impedance ratio $\rho V$ ( $\rho$ is density and V is velocity) and the distribution of different soil. By earthquake damage investigation and numerical analysis, researchers studied the effect of the thickness of sediment and the distribution of different soil in vertical direction, especially the location and thickness of soft layers, on the site transfer function and drew some valuable conclusions. The researches show that sediment thickness and S-wave velocity structure are the two most important factors. The variety of sediment thickness will change the shape of response spectral. The

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variety of S-wave velocity with the depth, namely S-wave velocity structure, which reflects indirectly the stiffness of sediment, impedance ratio and distribution of different soil and has a great effect on the estimation of site soil dynamic behavior, is the absolutely necessarily basic data for rating site condition effect.

But in some researches and applications, the important effect of shallow S-wave velocity structure is greatly under-estimated. Based on some general conclusions obtained in theoretical studies, some researchers bring forward some simplified method for calculating site transfer function based on average equivalent S-wave velocity [2]. The common point of these researches is substitution of detailed S-wave velocity by the average equivalent S-wave velocity of sediment. This substitution is mainly based on three considerations: 1 the error for estimation of site dynamic property induced by this substitution is acceptable. 2 the numerical calculation of site transfer function and dynamic property is complex and time-consuming. After this substitution, the calculation will be simpler. 3 it is very expensive to explore the detailed S-wave velocity structure. There is no field-testing data in most cases.

It is the basis for all the simplified methods that the error induced by this substitution is acceptable. But there are few works to verify this presupposition. With the rapidly development of computation techniques, the calculation efficiency is greatly improved, not only for the transfer function of 1-D soil model but also for soil seismic response analysis. The simplification brought by this substitution is negligible. In the simplified methods, the average equivalent S-wave velocity is always derived from the detailed S-wave velocity structure. So the velocity testing is never avoided. It is the key problem that velocity structure exploration by borehole method is too expensive. The way to solve this problem should be to develop more convenience, more economical and less environmental intrusion techniques for site S-wave velocity structure exploration.

The development of microtremors studies in recent years provided a new way for this problem. Since 80s, great breakthroughs have been made in inversion of site S-wave velocity structure by array observation of long period microtremors [4]. In many countries, especially in Japan, the microtremors array method has gained many applications. Due to different motivation, most researches on microtremors array method devote to infer the soil velocity structure down to the depth of several kilometers by long period microtremors observation and only provide the average velocity of the shallow (20-30 meter) soil layers that however is most crucial for engineering structure. Few researches focus on the detailed shallow soil S-wave velocity structure. Long period microtremors reflect the soil structure down to a large depth, whereas short period microtremors is related to the shallower soil structure. The frequency component contained in microtremors may reach to above 10Hz. If the soil velocity of the depth of several hundred meters even less than one hundred meter can be inferred from the high frequency component contained in the microtremors, it will provide a new way for geotechnical engineering exploration and will be more significant.

In this paper, basing on the vertical component of microtremors array records observed in one engineering site in Xiamen, China, we extract the Rayleigh wave dispersion curve by the Spatial Autocorrelation Method (SAC) and further inverted the shallow site soil velocity structure by hybrid method of Genetic Algorithm (GA) and Simplex Algorithm (SA). By comparison and contrast the inverted results with that from borehole testing and that from Frequency Wave-number method (F-K), the accuracy of the microtremors array observation and analysis method in this paper is analyzed. Further in order to validate if this accuracy acceptable for earthquake engineer, we calculated site seismic response of 3 site models from the view of rating of site dynamic behavior. At the same time, we also saw about the effect of the substitution of detailed shallow S-wave velocity by the average equivalent S-wave velocity of sediment on the rating of site soil dynamic behavior.

## MICROTREMORS ARRAY OBSERVATION AND ANALYSIS

### Array designing and observation

It is stricter to observe microtremors by array than just by one station. The observation at all the stations must be made absolute simultaneously. The configuration of array and distance between observation stations are very important. The configuration of array is related to analysis method. The accuracy for determining the relative position of all observation stations has a great effect on final results. We discussed the principle for array designing in another paper [5]. Here, we just briefly introduce the microtremors array observation made in one engineering site in Xiamen, China.

Larger array deployed, more deeper velocity structure can be detected and lower accuracy for shallow structure can be reached. Focused on shallow structure, arrays with small size are designed. Figure 1 shows two different size arrays, which are respectively composed of stations 1-2-4-6 and stations 1-9-11-13. The radiuses of the two arrays are respectively 15 meter and 30 meter. The stations of the two arrays were located by transit instrument. The totally 7 stations of the two arrays are observed simultaneously and only vertical component is recorded except at center stations. The center stations are observed by three-component seismometers. The sampling frequency and recording time are respectively set to 100Hz and 30 minutes.

### Dispersion curve extraction

The first and chiefly task is to extract dispersion curve of Rayleigh wave from microtremors array records. The frequency-wavenumber method (FK) and the Spatial Auto-correlation method (SAC) are the two methods that are usually used to analyze the dispersion curve of Rayleigh wave. By SAC method, the configuration of the array is limited to circle and only 4-6 observation stations are needed [7]. By FK method, the configuration of the array is flexible, but more observation stations are needed and the frequency range obtained is narrower using same size array. Devoted to inversion of shallow velocity structure and wishing gain dispersion curve for wider frequency range, we select SAC method to analyze microtremors records.

The SAC method was first developed by Aki[1]. In this method, isotropic waves that come from random directions are assumed. If waves have dispersion characteristics like surface wave, spatial auto-correlation coefficients among waves must be a function of phase velocity and frequency. Further, Spatial auto-correlation coefficient $\rho(r,\omega_0)$ with angular frequency $\omega_0$ and distance r can be related to Bessel function of the first kind of zero order $J_0(x)$ as follow.

$$\rho(r,\omega_0) = J_0(k_0 r) = J_0(\frac{\omega_0}{c(\omega_0)} r) \tag{1}$$

Here, $C(\omega_0)$ is the Rayleigh wave phase velocity, $\frac{\omega_0}{C(\omega_0)} = k$ is wave-number. We consider the vertical microtremors as Rayleigh wave. The correlation coefficients between the center station O(0, 0) and other stations $B_i(r, \theta_i)$ can be calculated by the following formula.

$$\rho_{0B_i}(r,\omega_0) = \frac{\overline{f_o f_{B_i}}}{(\overline{f_o^2} \cdot \overline{f_{B_i}^2})^{0.5}}, \quad i=1, 2, m \tag{2}$$

The coefficients between different stations can be averaged and by formula (1) we can obtain following formula.

$$\overline{\rho(r,\omega_0)} = \frac{1}{N}\sum_{i=1}^{N} \rho_{oB_i}(r,\omega_0) = J_0(\frac{\omega_0 r}{C(\omega_0)}) \tag{3}$$

First, the records can be filtered by very narrow band-pass filter with a series of center frequency and then the spatial auto-correlation coefficients are calculated. Thus the Rayleigh wave phase velocity can be obtained by above formula.

We used 819.2-second data for the SAC analysis. First all the records are divided into 81.92-sec data without heavy artificial noises. The application of the SAC analysis to these array data generated spatial auto-correlation coefficients for all the frequency interested. The coefficients from all the data set are averaged to determine the final coefficients. And then the dispersion curves are derived. Figure 2 shows the dispersion curves inferred from the microtremors records of the two arrays. Theoretically, the frequency range of the two dispersion curves from two different size arrays should be different. The difference between the radiuses of the two arrays is not so much. So the frequency range of the two dispersion curves are also not so different. The two dispersion curves are synthesized to one as the final result. The frequency of the final dispersion curve ranges from 2.5Hz to 15Hz. The depth and the resolution that can be detected are correlative to this frequency range. By the relationship between frequency, wavelength and velocity, the wavelength that can be detected range from about 12 meter to 110 meter or so. So it is possible to invert the site velocity structure shallower than one hundred meter.

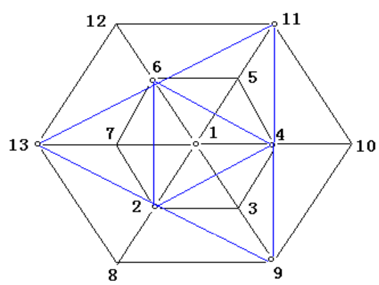

Figure1 microtremors observation array

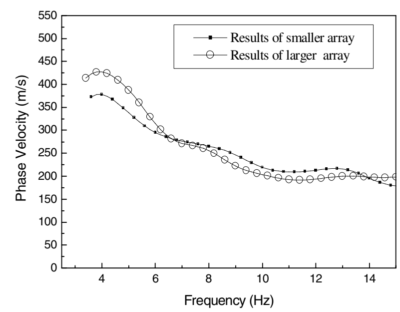

Figure 2 Dispersion Curve from microtremors analysis

## S-wave velocity structure inversion

S-wave velocity structure inversion is another key task for microtremors array method. In this paper, the above final dispersion curve from microtremors analysis is considered as objective curve for inversion. So the inversion comes down to an optimization problem for searching the minimum of one objective function. Here the algorithm that combine the genetic algorithm (GA) and Simplex Algorithm is applied to invert the soil shear-wave velocity structure from surface-wave dispersion curve. The form of the objective function used is as follow.

$$\phi_j = \sum_{i=1}^{M} [q_0(i) - q_{cj}(i)]^2 \tag{4}$$

Here, $q_o(i)$ and $q_{cj}(i)$ are respectively the dispersion curve derived from microtremors and from numerical modeling.

It is difficult to invert all the soil parameter at one time. So only the simplest case is considered here. Density of each layer is pre-assumed and P-wave velocity is connected to S-wave velocity using the relation $V_p = V_s \cdot \sqrt{\frac{\mu - 1}{\mu - 0.5}}$. Maybe there are some boreholes on the site and the number of layers and thickness of each layer are known. So only the shear wave velocity of each layer need to be determined.

Table 1 shows the S-wave velocity structure measured by borehole method. Figure 3 shows our inversion result. The solid black line and the dash one show the structure measured by borehole methods and inverted from microtremors respectively. The average relative inaccuracy of each layer is 20%.

Yamanaka from Tokyo Technology University also analyzes the same microtremors array records by F-K method and inverted the S-wave velocity structure. Table 2 shows his results. The average velocity of upper 20 meter calculated from borehole method is 252 m/s. our result is 254 m/s and result of Yamanaka is 223 m/s. It shows a good agreement.

The Spectral Analysis of Surface Wave (SASW) is another famous and widely used non-intrusive method for S-wave velocity structure exploration. This precision of our results is comparable with that of SASW [8]. But microtremors array method is more economical. And no special vibration source needed is another advantage of microtremors method.

Table 1 result of f-k method

| Layer No. | S-wave velocity | Thickness | Density |
|---|---|---|---|
| 1 | 223 | 20 | 1.5 |
| 2 | 497 | 102 | 1.8 |
| 3 | 1466 | 351 | 2.0 |
| 4 | 3200 | --- | 2.5 |

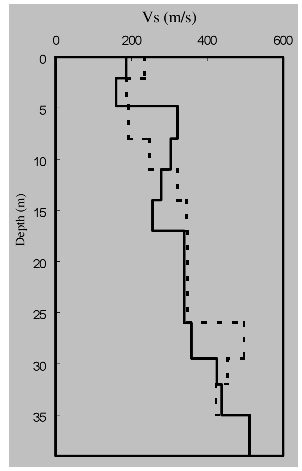


Figure 3 Comparison of our results and borehole results

## SOIL SEISMIC RESPONSE ANALYSIS

The theoretical basis of microtremors method and borehole method are different. That the microtremors array method tested is the average velocity structure of the soil under the observation array. Whereas that the borehole method tested is the velocity structure of the soil between the boreholes. So there must be some difference between results of the two methods. But the main goal of the microtremors testing is not to reflect the exact composition of the site soils but to understand the variation of the site soil stiffness with depth by S-wave velocity exploration and then to rate the site dynamic behavior. Up to now, the rating for site dynamic behavior is usually be done by first establishing the 1-D, 2-D or even 3-D analysis model based on the testing soil dynamic parameters in field and then calculating the site linear or non-linear response under different intensity input by numerical method. The seismic response is the most direct characterization of the site dynamic behavior. The 1-D site analysis model can be established respectively based on the testing results by the microtremors array method and the borehole method. By comparing the seismic response, the precision of the microtremors array method can be evaluated more directly and more objectively. In this paper, to validate if this accuracy of microtremors array method acceptable for earthquake engineer, we first generate a group of different intensity artificial ground motion and then calculate the seismic responses of the two kind of 1-D site soil models under same ground motion input.

The Trigonometric Series method is applied to generate ground motion. Its basic consideration is first constructing a stationary Gauss process with trigonometric series and then multiplying a stationary wrapping function to form a non-stationary time history. The form is given as formula (5).

$$a(t) = f(t)X(t) = f(t)\sum_{k=0}^{n} A_k \cos(\omega_k t + \psi_k) \qquad (5)$$

First start with an initial time history and then revise its amplitude spectral according to the difference between its response spectral and the object spectral. This process is repeated until the difference decrease to the given accept value. The object response spectral is determined according to the China Building Aseismic Code. The form of wrapping function is a subsection form. Its form and the coefficient contained can be found in literature [6]. Three intensity, as Intensity 6, 7, 8 are considered. Two different earthquake magnitude and source distance are considered respectively for intensity 6 and 7. One earthquake magnitude and one source distance is considered respectively for intensity 8. Three acceleration time-histories are generated for the same earthquake magnitude and source distance. The correlative coefficient is less than 10%. 5 group of time history, totally 15 acceleration records, are generated. After reduced their amplitudes by half, the 15 records are used as the input motion for the site bedrock.

There are many methods for 1-D site seismic response analysis, for example time domain linear elastic method, frequency domain linear elastic method, equivalent linearization method and time domain direct integration method. The simple and most used method is equivalent linearization method [3]. This method turns the nonlinear seismic response analysis to non-elastic linear soil seismic response analysis by following way. First give a set of initial shear modulus and damping ratio in the soil non-linear stress-strain relationship and then calculate the soil average equivalent dynamic shear stress. And in turn revise the shear modulus and damping ratio according to this stress. This process is repeated until the incorrectness decrease to the given acceptable error. Here the equivalent linearization method is applied to calculate the response spectral. The average of the results for the 3 records of one group is considered as the surface response spectral under the ground motion of this intensity and magnitude.

To see about the effect of the substitution of detailed shallow S-wave velocity by the average equivalent S-wave velocity of sediment on the rating of site soil dynamic behavior, we first simplify the microtremors observation site respectively as one horizontal multi-layered model (BM model) and one single-layered model (BS model) based on the S-wave velocity structures tested by borehole method and one horizontal multi-layered model (IM model) based on the inversed S-wave velocity structures by microtremors array method. The average equivalent S-wave velocity for BS model is calculated from the S-wave velocity structures tested by borehole method. The total thickness for BS model is 35 meter and the average equivalent S-wave velocity is 293 m/s. Table 2 shows the soils non-linear dynamic parameters referenced from the Xiamen seismic micro-zonation report.

Table 2 soils non-linear dynamic parameters

| Soil | Modulus | Shear stress | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Damping ratio | 0.000005 | 0.00001 | 0.00005 | 0.0001 | 0.0005 | 0.001 | 0.005 | 0.01 |
| Filling | G/Gm | 0.96 | 0.95 | 0.8 | 0.7 | 0.3 | 0.2 | 0.15 | 0.1 |
| | λ | 0.025 | 0.028 | 0.03 | 0.035 | 0.08 | 0.1 | 0.11 | 0.12 |
| Remnant | G/Gm | 0.951 | 0.907 | 0.662 | 0.494 | 0.164 | 0.089 | 0.019 | 0.01 |
| | λ | 0.033 | 0.039 | 0.055 | 0.061 | 0.069 | 0.07 | 0.072 | 0.072 |
| Bedrock | G/Gm | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| | λ | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |

The responses spectral for the IM model and the BS model are compared with that for BM model. Figure 4 shows comparison of the response spectral, the black solid line for result of BM model, the blue dotted line for result of IM model and the red dashed line for result of BS model. Figure 4 suggests that the response spectral of BS model is quite different from that of BM model. The responses spectral of two multi-layered models are in satisfactory agreement for all intensities. Table 3 shows the relative error of the response spectral for BS model and IM model compared with BM model in average. The relative error of the response spectral for IM model is always less than 10% and for

Intensity 8 only a little larger. Whereas for BS model, it is two times larger than that of the response spectra for IM model.

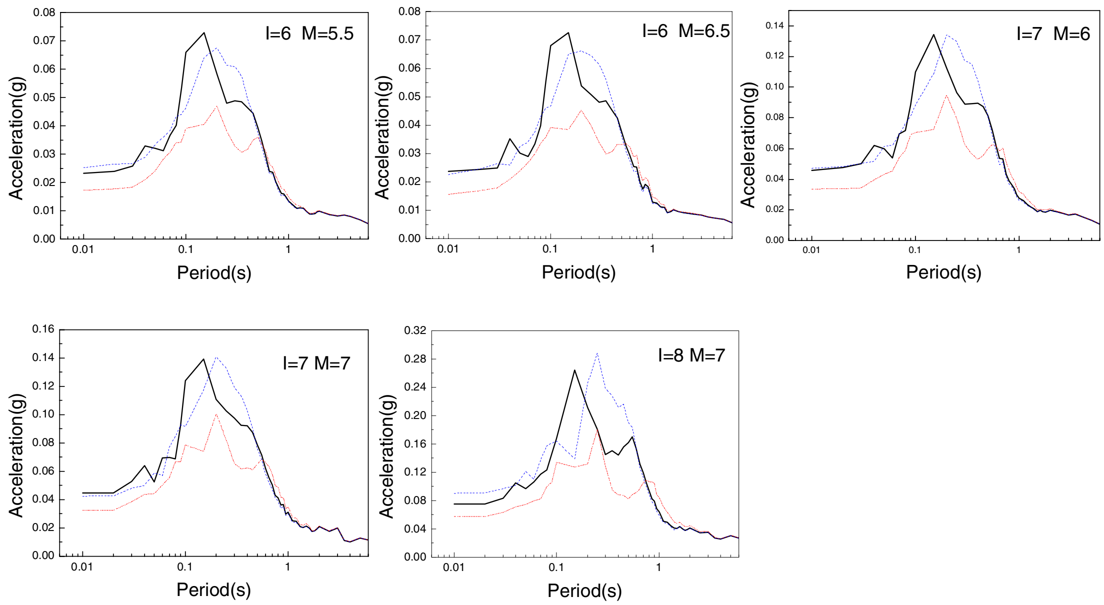


Figure 4 Comparison of response spectral

Table 3 Relative error of response spectral

| Intensity | Magnitude | Relative error of response spectral for BS model (%) | Relative error of response spectral for IM model (%) |
|---|---|---|---|
| 6 | 5.5 | 12.85 | 5.71 |
| 6 | 6.5 | 13.9 | 6.57 |
| 7 | 6 | 15.54 | 6.14 |
| 7 | 7 | 13.85 | 6.88 |
| 8 | 7 | 20.75 | 11.7 |

## CONCLUSIONS

By observation and analysis in one engineering site of Xiamen, the microtremors array method for shallow S-wave velocity structure exploration are studied in this paper. It shows that the precision of the shallow S-wave velocity structure, which inverted with the hybrid algorithm of GA and SA by the dispersion curve extracted from microtremors array records by SAC method, is comparable with other non-intrusive method such as SASW. The seismic response analysis shows that the responses spectral of IM model are in good agreement with that of BM model for all intensities. The relative error of the response spectral for IM model is always less than 10%. So it can be say that the accuracy of microtremors array method is acceptable for earthquake engineering.

In this paper, the Xiamen microtremors array observation site is also simplified as one single-layered model based on the S-wave velocity structure tested by borehole method. And the corresponding response spectral under different intensity input is calculated. The comparison with that of BM model shows that the detailed shallow S-wave velocity structure is very important for evaluated the site dynamic behavior and the error brought by the substitution of detailed shallow S-wave velocity with the average equivalent S-wave velocity is much larger than anticipated in some researches. So it is not a good idea.

Microtremors array method is convenience and economical. But microtremors is also very complex, especially when it comes to shallow velocity structure or short period component. It calls for further improvement of observation instrument system, array deployment configuration, analysis method and inversion method. The conclusion is just based on microtremors observation conducted on one site and observations conducted on more sites are needed to validate this conclusion.

## Acknowledgement

This research is sponsored by the National Nature Science Foundation under Grant No.50378032 and Joint Earthquake Science Foundation under Grant No.604034.

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