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EFFECT OF EQUIVALENT LINEARIZATION IN DIRECT DISPLACEMENT BASED SEISMIC DESIGN OF BRIDGE COLUMNS

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SUMMARY

Displacement based design is a seismic design methodology that uses displacements as the basis for the design procedure. In displacement based design procedure, displacement demand of inelastic Single Degree of Freedom (SDOF) system is represented by the first (elastic) mode of vibration of equivalent linear system. Many approximate linearization methods to estimate maximum inelastic displacement in single degree of freedom systems are available. Using Iwan's, Kowalsky's, Chopra's and Gulkan's equivalent linear systems four bridge columns are designed. These SDOF systems were then subjected to nonlinear timehistory analyses using five artificial ground motions. The plastic rotation of the hinge at the base of the bridge column is estimated then compared with the plastic rotation used in direct displacement based design

INTRODUCTION

Displacement based design is a seismic design methodology that uses displacements as the basis for the design procedure.[1,2,3,4,5] In displacement based design procedure, displacement demand of inelastic Single Degree of Freedom (SDOF) system is represented by the first (elastic) mode of vibration of equivalent linear system. Many approximate linearization methods to estimate maximum inelastic displacement in single degree of freedom systems are available. In this study, the effects of four equivalent linearization methods to direct displacement based design of SDOF bridge columns are investigated.

The idealized SDOF bridge column is shown in Figure 1

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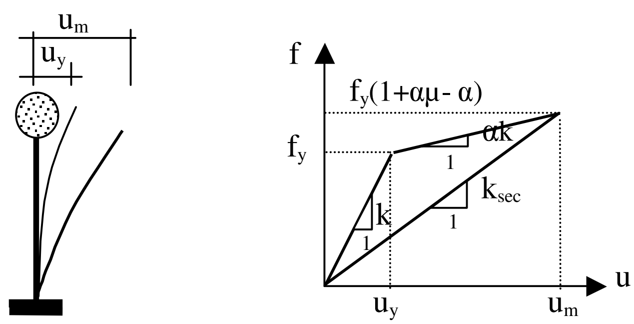

Figure 1 Idealized SDOF bridge column and force displacement relationship

EQUIVALENT LINEAR SYSTEMS

The concept of equivalent linear systems to assess the inelastic response was first proposed by Jacobsen[6]. Equivalent viscous damping was analytically defined by Jacobsen by assuming a sinusoidal earthquake response in the equation of motion and integrating the resulting expression over one cycle of response and equating that to the area of a rigid perfectly plastic hysteretic response. Subsequently to Jacobsen's work various other definitions of equivalent systems were proposed as reviewed by Jennings [7].

Gulkan and Sozen [8], and more recently Bonacci [9] utilized experimental results to asses the equivalent damping of non-linear systems. In this method, ξ_{eq} is defined by an equivalent linear viscous dashpot system that dissipates all of the input energy of a particular ground motion. The basic formulation behind this philosophy is given by

$$\xi_{eq}\left[2m\omega_0\int\dot{u}^2dt\right] = -m\int\ddot{v}_g\dot{u}dt \tag{1}$$

In the above equation, the left hand side represents the energy dissipated by the fictitious linear viscous dashpot system that has mass m and fundamental circular frequency ω_0. The right hand side is the input energy of excitation $\ddot{v}_g$. The term $\dot{u}$ represents the relative velocity of the SDOF system. A suite of experiment conducted with SDOF systems under different excitation types supplied the necessary information to derive the following substitute damping relation is given by:

$$\xi_{eq} = \xi_o + 0.2\left(1 - \frac{1}{\sqrt{\mu}}\right) \tag{2}$$

In this equation ξ_0 is the initial viscous damping and μ is the ductility of the SDOF system.

Iwan [10] proposed equations for the equivalent damping ratio based on optimizing ξ_{eq} in order to minimize the root mean square of errors between maximum displacement from the inelastic and the equivalent linear systems. Iwan carried out time history analysis for a number of SDOF hysteretic systems in the period range of 0.4-4.0 sec. subjected to 12 earthquakes. Based on the empirically derived optimal values of ξ_{eq}, the following formula for the equivalent linear systems damping relation is given by;

$$\xi_{eq} = \xi_o + 0.0587(\mu - 1)^{0.371} \qquad 3$$

More recently Kowalsky [1] used the secant stiffness at maximum deformation for defining the period shift together with the Takeda hysteretic model [11] to derive an equation for the equivalent viscous damping ratio. For unloading stiffness factor of 0.5 and a post yield to initial stiffness ratio, α, the equivalent damping ratio is given by;

$$\xi_{eq} = \xi_o + \frac{1}{\pi}\left(1 - \frac{1-\alpha}{\sqrt{\mu}} - \alpha\sqrt{\mu}\right) \qquad 4$$

Chopra [12] is also defined the effective damping concept through the equivalent viscous damping obtained by the equating the energy dissipated in a vibration cycle of the structure and an equivalent viscous system. Based on this statement the equivalent damping ratio is given by;

$$\xi_{eq} = \frac{1}{4\pi}\frac{E_D}{E_S} \qquad 5$$

E_D= the actual energy dissipate in a full cycle of loading-unloading-reloading
E_S=elastic strain energy

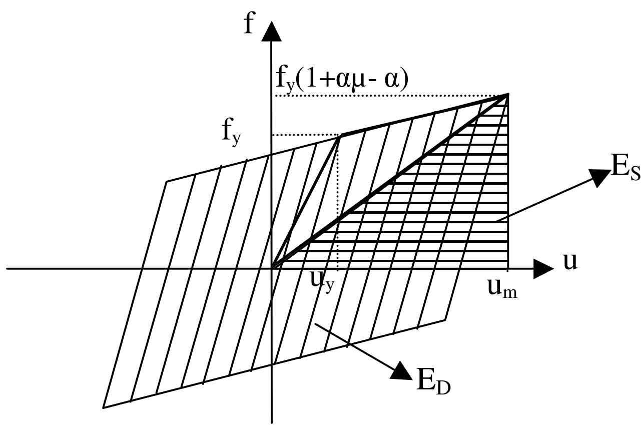

Figure 2. Graphical representations of E_D and E_S for a bilinear system

For the bilinear system of Figure 2 E_D,E_S and equivalent damping ratio is given by

$$E_D = 4\left(f_y u_m - u_y f_y (1 + \alpha\mu - \alpha)\right) \qquad 6$$

$$E_S = \frac{1}{2} f_y (1 + \alpha\mu - \alpha) u_m \qquad 7$$

$$\xi_{eq} = \frac{2}{\pi}\frac{(\mu - 1)(1 - \alpha)}{\mu(1 + \alpha\mu - \alpha)} \qquad 8$$

A comparison of equivalent damping ratio of the various method s with $\xi_0 = 0.05$ and $\alpha = 0.05$, is shown in Figure 3

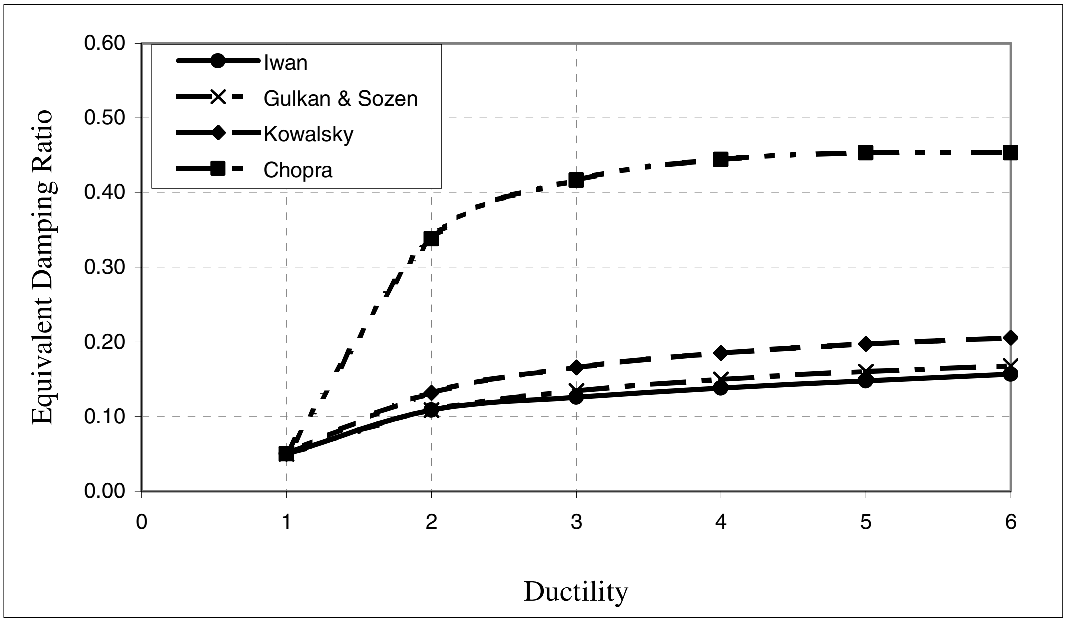

Figure 3 A comparison of equivalent damping ratios in four approximate methods

DIRECT DISPLACEMENT BASED DESIGN

Chopra [13] adapted a direct displacement-based design procedure for bilinear SDF systems (Figure 1) using elastic design spectra from Priestley and Calvi [5]. Sequence of steps is outlined below;

1. Estimate the yield deformation u_y for the system.

2. Determine acceptable plastic rotation θ_p of the hinge at the base. An acceptable plastic rotation θ_p=0.02 of the hinge at the base is determined.

3. Determine design displacement u_m

$$u_m = u_y + h\theta_p$$
$$u_m = u_y + 0.02\ h \qquad (9)$$

and design ductility factor

$$\mu = \frac{u_m}{u_y} \qquad (10)$$

4. Estimate the total equivalent viscous damping for the design ductility factor from Equation 2 for Gulkan's equivalent linear system (GELS), Equation 3 for Iwan's equivalent linear system (IELS),

Equations 4 for Kowalsky's equivalent linear system (CELS) and Equations 5 for Chopra's equivalent linear system (KELS) or from Figure 3.
5. Enter the deformation design spectrum for elastic systems with known u_m and ξ_{eq} to read T_{eq} (Figure 4). Determine the secant stiffness;

$$k_{sec} = \frac{4\pi^2}{T_{eq}^2} m \qquad 11$$

where *m* is the mass of the system.

6. Determine the required yield strength f_y from Figure 1:

$$f_y = \frac{k_{sec} u_m}{1+\alpha\mu-\alpha} \qquad 12$$

7. Estimate member sizes and detailing (reinforcement in R/C structures, connections in steel structures) to provide f_y . Calculate initial elastic stiffness *k* and

$$u_y = \frac{f_y}{k} \qquad 13$$

8. Repeat steps 3 to 7 until a satisfactory solution is obtained.

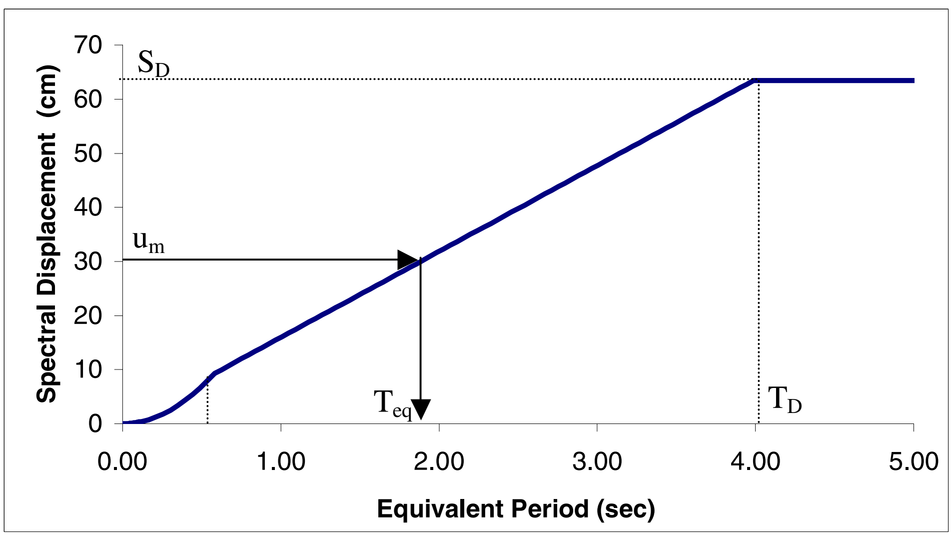

Figure 4 Determination of T_{eg} using maximum displacement

EXAMPLES

Four bridge column with height h=5,8,10,12m were designed using direct displacement based design. Iwan's (IELS), Kolwalsky's (KELS), Chopra's (CELS) ang Gulkan's (GELS) equivalent linear system used in each bridge column design.

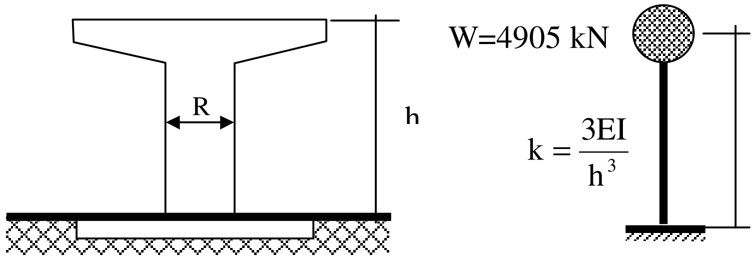

Figure 5 Designed bridge column with different height

Each column has a weight of 4905 kN on the top of the circular column. Columns are idealized as a SDOF system as shown in Figure 5

For the transverse ground motion, the bridge column can be idealized as an SDF system (Figure5) with its lateral stiffness computed from;

$$k = \frac{3EI}{h^3} \tag{14}$$

where *E* is the elastic modulus of concrete, *I* is the effective moment of inertia of the reinforced-concrete cross section, and *h* is the column height. Based on the American Concrete Institute design provisions ACI 318-95[14], the effective EI for circular columns subjected to lateral load is given by MacGregor [15],

$$EI = E_c I_g \left(0.2 + 2\rho_t \gamma^2 \frac{E_s}{E_c} \right) \tag{15}$$

where I_g is the second moment of inertia of the gross section, E_c and E_s are the elastic module of concrete and reinforcing steel, ρ_t is the longitudinal reinforcement ratio, and γ is the ratio of the distances from the center of the column to the center of the outermost reinforcing bars and to the column edge. The system properties selected are: concrete strength = 27.6 MPa , steel strength =413 MPa and γ = 0.9.

The displacement response spectra given in Appendix I, Part B SEAOC Blue Book [16] is used in direct displacement based design. The displacement spectra is constructed for soil type D in zone 4 sites for Earthquake III (EQ-III) which represents a rare event and is two thirds of maximum considered event defined in SEAOC Blue Book. EQ-III is such an earthquake that has an annual probability of exceedance ranging between 0.12 percent and 0.4 percent (mean recurrence interval of approximately 250 to 800 years). As shown in Figure 4

Table 1 The radius of designed circular bridge columns

	IELS	**KELS**	**CELS**	**GESL**
5 m.	1.5 m.	1.5 m.	1.1 m.	1.5 m.
8 m.	1.5 m.	1.5 m.	1.1 m.	1.5 m.
10 m.	1.5 m.	1.5 m.	1.1 m.	1.5 m.
12 m.	1.5 m.	1.5 m.	Column not designed	1.5 m.

Circular columns are designed using direct displacement based design and the radiuses of the columns are given in Table 1. 12 m circular column can't design using CELS because the maximum displacement for determining the effective period because the maximum displacement exceeds the maximum displacement in the displacement response spectra. Direct displacement based design is an iterative procedure as given before. The design iterations of 5m, 8m, 10m, and 12m columns are given in Table 2,3,4 and 5

Table 2 Iterative design results for 5m column

	İtr. No	**u_y (cm)**	**u_m (cm)**	**μ**	**ξ_{eq} (%)**	**T_{eq} (sn)**	**k_{sec} kN/cm**	**f_y (kN)**	**ρ_t (%)**	**Design f_y (kN)**	**Design k kN/cm**	**u_y (cm)**
IELS	1	2.5	12.5	5	16	1.10	162.97	1697.6	0.0232	2000	744.21	2.69
	2	2.69	12.69	4.72	15	1.10	162.97	1743.3	0.0241	2046.20	761.8	2.69
KELS	1	2.5	12.5	5	20	1.20	136.94	1426.4	0.0177	1693	638.7	2.65
	2	2.65	12.65	4.77	20	1.21	134.68	1433.4	0.0177	169.3	638.7	2.65
CELS	1	2.5	12.5	5	45	1.73	65.89	686.32	0.0208	737.20	201.91	3.65
	2	3.65	13.65	3.74	44	1.86	57.00	684.37	0.0208	737.20	201.91	3.65
GELS	1	2.5	12.5	5	21	1.22	132.49	1656.1	0.0232	1953.20	726.63	2.69
	2	2.69	12.69	4.72	21	1.24	128.25	1627.2	0.0232	1953.20	726.63	2.69

Table 3 Iterative design results for 8m column

	İtr. No	**u_y (cm)**	**u_m (cm)**	**μ**	**ξ_{eq} (%)**	**T_{eq} (sn)**	**k_{sec} kN/cm**	**f_y (kN)**	**ρ_t (%)**	**Design f_y (kN)**	**Design k kN/cm**	**u_y (cm)**
IELS	1	4	20	5	16	1.76	63.66	1061.0	0.0232	1250	181.69	6.88
	2	6.88	22.88	3.33	14	1.93	52.94	1085.1	0.0241	1278.9	186.0	6.88
KELS	1	4	20	5	20	1.93	52.94	882.31	0.0177	1058.13	155.94	6.79
	2	6.79	22.79	3.36	17	2.06	46.47	947.13	0.0196	1120.63	164.5	6.81
	3	6.81	22.81	3.35	17	2.06	46.47	948.58	0.0196	1120.63	164.5	6.81
CELS	1	4	20	5	45	2.77	25.70	428.33	0.0208	460.75	49.29	9.35
	2	9.35	25.35	2.71	40	3.3	18.11	422.78	0.0208	460.75	49.29	9.35
GELS	1	4	20	5	21	1.95	51.86	1037.2	0.0232	1220.75	177.40	6.88
	2	6.88	21.88	3.33	19	2.15	42.66	976.10	0.02	1120.63	166.7	6.72
	3	6.72	22.72	3.38	19	2.13	43.46	987.66	0.021	1162.62	170.96	6.80
	4	6.80	22.80	3.35	19	2.14	43.06	981.76	0.021	1162.62	170.96	6.80

Table 4 Iterative design results for 10m column

	İtr. No	**u_y (cm)**	**u_m (cm)**	μ	ξ_{eq} (%)	**T_{eq} (sn)**	**k_{sec} kN/cm**	**f_y (kN)**	ρ_t (%)	**Design f_y (kN)**	**Design k kN/cm**	**u_y (cm)**
IELS	1	5	25	5	16	2.21	40.37	841.13	0.0232	1000	93.03	10.75
	2	10.75	30.75	2.86	14	2.60	29.17	820.64	0.0232	976.60	90.8	10.75
KELS	1	5	25	5	20	2.40	34.23	713.22	0.0177	846.5	79.84	10.60
	2	10.60	30.60	2.89	16	2.70	27.05	756.43	0.0196	896.50	84.24	10.64
	3	10.64	30.64	2.88	16	2.70	27.05	757.68	0.0196	896.5	84.24	10.64
CELS	1	5	25	5	45	3.45	16.57	345.16	0.0216	368.60	25.76	14.31
	2	14.31	34.31	2.40	38	4.34	10.47	335.73	0.0245	398.2	27.8	14.31
GELS	1	5	25	5	21	2.44	33.12	828.04	0.0223	976.60	90.83	10.75
	2	10.75	307.5	2.86	18	2.83	24.62	757.17	0.0196	896.5	27.8	10.64
	3	10.64	30.64	2.88	18	2.87	23.94	733.59	0.0196	896.5	84.24	10.64

Table 5 Iterative design results for 12m column

	İtr. No	**u_y (cm)**	**u_m (cm)**	μ	ξ_{eq} (%)	**T_{eq} (sn)**	**k_{sec} kN/cm**	**f_y (kN)**	ρ_t (%)	**Design f_y (kN)**	**Design k kN/cm**	**u_y (cm)**
IELS	1	6	30	5	16	2.65	28.08	702.00	0.0232	833.33	.53.83	15.48
	2	15.48	39.48	2.55	13	3.25	18.67	648.02	0.0232	813.83	52.60	15.48
KELS	1	6	30	5	20	2.87	23.94	598.50	0.0177	705.42	46.20	15.27
	2	15.27	29.27	2.57	15	3.40	17.06	621.01	0.0196	705.42	48.7	14.47
	3	14.47	38.47	2.66	16	3.40	17.06	605.99	0.0182	677.92	46.84	14.47
CELS	Column not designed											
GELS	1	6	30	5	21	2.93	22.97	689.09	0.0223	813.83	52.56	15.48
	2	15.48	39.48	2.55	17	3.56	15.56	614.33	0.0187	698.0	47.5	14.70
	3	14.70	38.70	2.63	17	3.49	16.19	626.57	0.0191	707.08	48.11	14.70

Table 6 Reinforcement details of designed bridge columns

	IELS	KELS	CELS	GELS
5 m.	53ϕ32	39ϕ32	28ϕ30	49ϕ32
8 m.	53ϕ32	43ϕ32	28ϕ30	46ϕ32
10 m.	49ϕ32	43ϕ32	33ϕ30	43ϕ32
12 m.	49ϕ32	40ϕ32	Column not designed	42ϕ32

DYNAMIC EARTHQUAKE ANALYSES

Four bridge columns designed using four different equivalent linear systems (IELS, KELS,CELS and GELS). The columns were then subjected to nonlinear time history analyses using artificial ground motions to control the plastic rotation used in direct displacement based design of bridge columns. For the nonlinear time history analyses RAM Perform2D [17] software is used.

Creation of artificial ground motions

Five artificial ground motions were created using the program SIMQKE [18-19], which is the part of the Ruaumoko program package [20]. All motions were scaled to the acceleration response spectrum of a zone 4 earthquake in soil type D, from SEOAC Blue Book. The displacement response spectra of these ground motions as well as the target design spectrum are shown in Figure 6

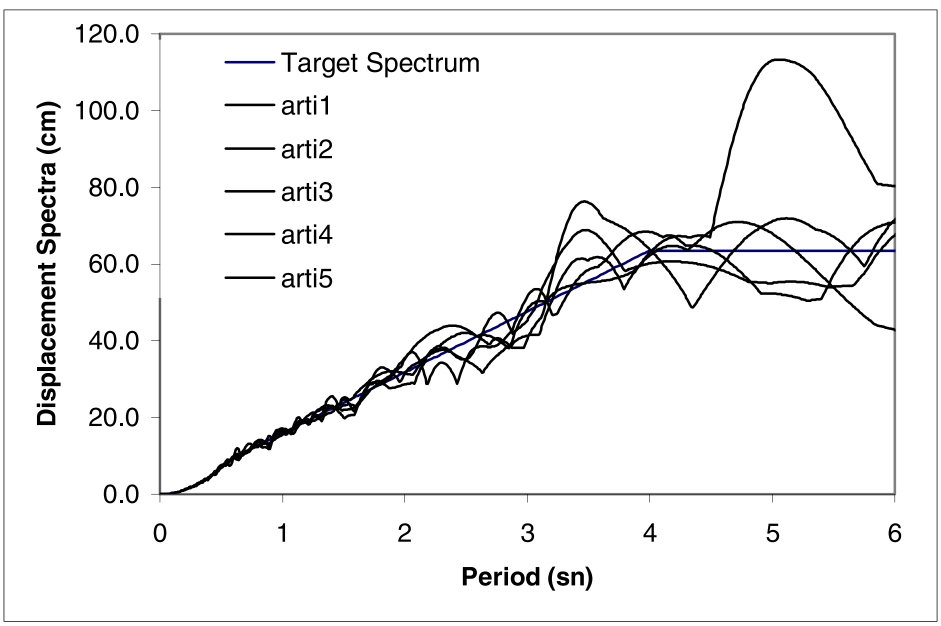

Figure 6 Displacement response spectra of generated earthquake records

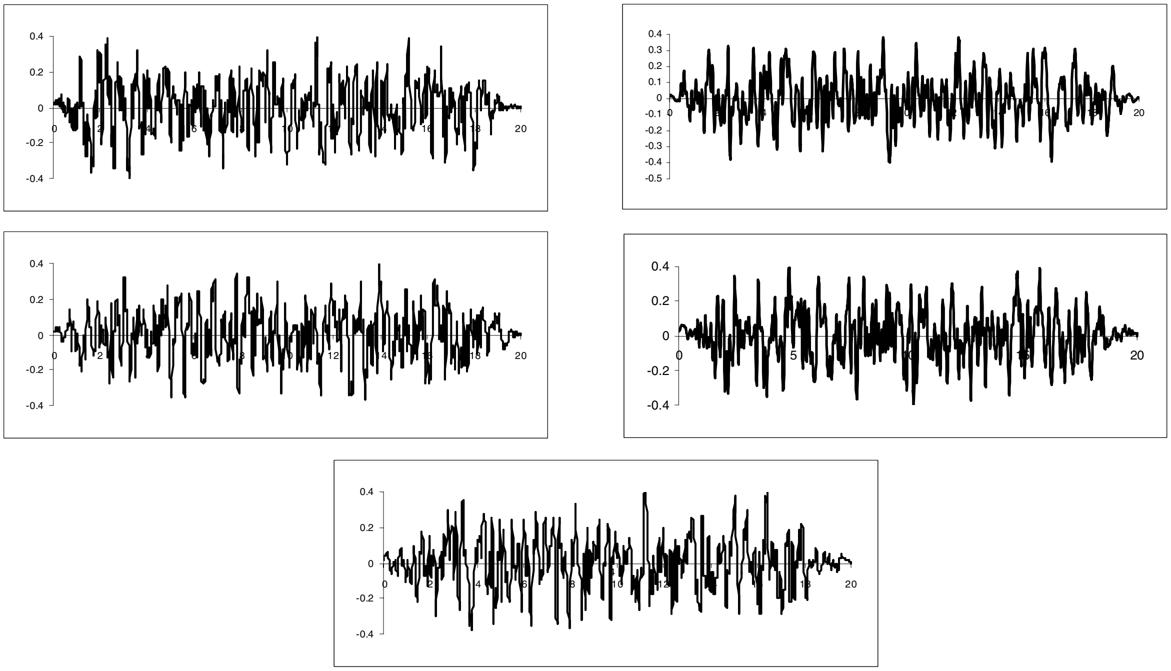

Figure 7 Created artificial ground motions

In RAM Perform2D a value for the Curvature Stiffness ratio α_χ is required in the input. Therefore, the Displacement stiffness ratio α must be converted into the corresponding Curvature stiffness ratio. Method is given by Kowalsky [1]

$$\alpha_\chi = \frac{3L_p}{L\left(\frac{1}{\alpha} - 1\right)} \qquad 16$$

Plastic hinge length is given as [21]

$$L_p = 0.081 + 0.022 d_b f_y \qquad 17$$

where l is the column height, d_b is the radius of the reinforcement and f_y is the strength of reinforcement. Plastic hinge length and the curvature stiffness ratio in given in Table 7

Table 7 plastic hinge lengths and the curvature stiffness ratio of columns

	IELS			KELS			CELS			GELS		
	d_b	Lp	α_χ	d_b	Lp	α_χ	d_b	Lp	α_χ	d_b	Lp	α_χ
5 m	0.032	0.69	0.0218	0.032	0.69	0.0218	0.03	0.67	0.0212	0.032	0.69	0.00
8 m	0.032	0.93	0.0184	0.032	0.93	0.0184	0.03	0.91	0.0180	0.032	0.93	0.00
10 m	0.032	1.09	0.0172	0.032	1.09	0.0172	0.03	1.07	0.0169	0.032	1.09	0.00
12 m	0.032	1.25	0.0165	0.032	1.25	0.0165	-	-	-	0.032	1.25	0.00

NLTHA plastic hinge rotation and maximum displacement of designed columns are given in Table 8

Column Height	Artificial Record	IELS				KELS				CELS				GELS			
		NLTHA θ_P (rad)	**Design θ_P (rad)**	**NLTHA u_m (cm)**	**Design u_m** (cm)	**NLTHA θ_P (rad**	**Design θ_P (rad)**	**NLTHA u_m (cm)**	**Design u_m (cm)**	**NLTH θ_P (rad)**	**Design θ_P (rad)**	**NLTH u_m (cm)**	**Design u_m** (cm)	**NLTH θ_P (rad)**	**Design θ_P (rad)**	**NLTH u_m (cm)**	**Design u_m** (cm)
5 m.	Arti1	0.0117	0.02	8.98	12.69	0.0111	0.02	8.64	12.65	0.0202	0.02	14.19	13.65	0.0202	0.02	12.87	12.69
	Arti2	0.0053	0.02	5.62	12.69	0.0085	0.02	7.26	12.65	0.0219	0.02	15.09	13.65	0.0079	0.02	6.74	12.69
	Arti3	0.1047	0.02	8.32	12.69	0.0171	0.02	11.74	12.65	0.0275	0.02	17.99	13.65	0.0140	0.02	9.77	12.69
	Arti4	0.0067	0.02	6.36	12.69	0.0074	0.02	6.71	12.65	0.0303	0.02	19.43	13.65	0.0078	0.02	6.66	12.69
	Arti5	0.0093	0.02	7.73	12.69	0.0099	0.02	8.02	12.65	0.0205	0.02	14.34	13.65	0.0149	0.02	10.25	12.69
8 m.	Arti1	0.0136	0.02	18.37	22.88	0.0152	0.02	20.54	22.81	0.0212	0.02	27.28	25.35	0.0160	0.02	19.72	22.80
	Arti2	0.0087	0.02	14.30	22.88	0.0098	0.02	16.00	22.81	0.0201	0.02	26.38	25.35	0.0111	0.02	15.78	22.80
	Arti3	0.0123	0.02	17.22	22.88	0.0145	0.02	19.94	22.81	0.0216	0.02	27.59	25.35	0.0174	0.02	20.86	22.80
	Arti4	0.0861	0.02	14.17	22.88	0.0115	0.02	17.45	22.81	0.0220	0.02	27.97	25.35	0.0167	0.02	20.27	22.80
	Arti5	0.0122	0.02	17.17	22.88	0.0113	0.02	17.28	22.81	0.0179	0.02	24.51	25.35	0.0126	0.02	17.01	22.80
10 m.	Arti1	0.0141	0.02	25.76	30.75	0.0143	0.02	25.94	30.64	0.0178	0.02	24.02	34.31	0.0153	0.02	26.13	30.64
	Arti2	0.0124	0.02	23.92	30.75	0.0119	0.02	23.40	30.64	0.0193	0.02	25.34	34.31	0.0148	0.02	25.58	30.64
	Arti3	0.0125	0.02	24.10	30.75	0.0082	0.02	19.46	30.64	0.0223	0.02	27.75	34.31	0.0084	0.02	19.12	30.64
	Arti4	0.0123	0.02	23.85	30.75	0.0098	0.02	21.17	30.64	0.0246	0.02	29.78	34.31	0.0120	0.02	22.80	30.64
	Arti5	0.0899	0.02	20.37	30.75	0.0111	0.02	22.59	30.64	0.0185	0.02	24.61	34.31	0.0122	0.02	23.06	30.64
12 m.	Arti1	0.0114	0.02	30.21	39.48	0.0121	0.02	30.04	38.47					0.0146	0.02	32.37	38.70
	Arti2	0.0067	0.02	24.19	39.48	0.0132	0.02	31.45	38.47					0.0138	0.02	31.38	38.70
	Arti3	0.0104	0.02	28.95	39.48	0.0057	0.02	23.55	38.47	**Column not designed**				0.0063	0.02	22.75	38.70
	Arti4	0.0175	0.02	37.92	39.48	0.0192	0.02	39.27	38.47					0.0200	0.02	40.28	38.70
	Arti5	0.0071	0.02	24.67	39.48	0.0080	0.02	24.92	38.47					0.0067	0.02	22.93	38.70

Table 8 NLTHA and design Plastic hinge rotation and maximum displacement of designed columns

CONCULATIONS

Displacement based design is a seismic design methodology that uses displacements as the basis for the design procedure. One of the key steps of the procedure is to determine the effective damping of the system. The inelastic response for the SDOF system is defined by an elastic system with higher damping ratio. Four equivalent linear systems (IELS, KELS, CELS and GELS) used for determining the effective damping. 5 m, 8 m, 10 m and 12 m height columns are designed using direct displacement based design.

12 m column can not designed using CELS because the maximum displacement demand (u_m) exceeds the S_D (constant displacement of the spectrum) in Figure 4. This means that the effective period is larger than the constant displacement-starting period of displacement spectrum. Also using all the equivalent linear systems, columns higher than 15 m cannot designed because of the same reason.

Designed columns subjected to nonlinear timehistory analyses using five artificial ground motions and the columns base hinge rotations are compared. In DDBD the plastic rotations of the base hinge of the columns is assumed θ_p=0.02 rad. When we used CELS the designed columns plastic hinge rotations exceeds 0.02 rad in most of the NLTHA. But when IELS, KELS or GELS is used the plastic rotaion is lower than 0.02 rad.

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