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COHERENCY FUNCTION MODEL OF GROUND MOTION AT BASE-ROCK

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SUMMARY

At present, the method to study spatial variation of ground motions is statistic analysis based on dense array records such as SMART-1 array, etc, to get coherency function of ground motion, which would be used to depict the variation with space and frequency. But there is no coherency function model of base-rock. In this paper, spatial variation of stochastic ground motions at base-rock, which considering the factors of rupture velocity, numbers of sub-source, depth of epicenter and propagation velocity, is studied from near-field ground motions which are simulated by elastic half-space model with dislocation source of fault. In the case of lack of statistic coherency function of base-rock, the results of this paper are useful and could be referenced.

INTRODUCTION

Earthquake damage survey and research results have demonstrated that spatial distribution difference of ground motion is one of the important reasons which caused long structure (e.g. long span bridge, underground pipe) to destroy. That how to provide a reasonable input of ground motion field for seismic design of long structure is a urgent problem in earthquake engineering field. At present, the method to study spatial variation of ground motions is adopting statistic analysis based on dense array records such as SMART-1 array, etc, to get coherency function of ground motion[1,2,6,8], which would be used to depict the variation of ground motion with space and frequency. Because spatial variation of ground motion is influenced by earthquake source, propagation path and site condition, the coherency function models from seismic records of different array or different seismic records of same array are very different. However, it is very difficult for engineers to make sure which one is more appropriate for designs. Qu Tiejun, et al[9] have presented a practice model which is a average result of multiple models, but this is also depended earthquake records impliedly. On the other hand, it is unreasonable to act the coherency function models from records of soil surface as that of structure basement. There are many researchers have recognized the problem, for example, Somerville et al[10,11] have thought that the spatial variation

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of ground motion was influenced by 1) earthquake source, 2) propagation path and site condition. Kiureghian[3] thought the spatial variation of ground motion could be attributed to following four mechanisms:1) "wave passage effect", 2) "incoherence effect", 3) "attenuation effect", 4) "site response effect". But also, it is very difficult to distinguish the influence of each factor quantificationally from records. Some researchers have tried to analyze the spatial coherence theoretically. Luco and Wong [7] have given an analytical model for the coherence function developed from the analysis of wave propagation through random media. Zerva and Shinozuka[14] analyzed the effect of random variability in the source parameter on the differential ground motion.

In this paper, spatial variation of stochastic ground motions at base-rock, which considering the factors of rupture velocity, numbers of sub-source, depth of epicenter and propagation velocity, is studied. Firstly, Taking elastic half-space model with dislocation source of fault, near-field ground motions may be calculated by numerical simulation method for every stochastic sample. Then, a coherency model for base-rock site would be proposed by statistic method. Because seismic wave propagate in rock may be assumed similar at different region, in the case of lack of statistic coherency function of base-rock, the results of this paper are useful and could be referenced.

ANALYSIS MODEL AND METHOD

Analysis model of ground motions

Since elastic dislocation theory has been applied in seismology, two main kinds of methods have been developed to simulate strong ground motions. One is analytical method that is based on continuous medium, and the other is numerical method that is based on discrete medium. Only simple source and medium model could be used in analytical method. And heterogeneous fault rupture process and more complex crust and site structure could be considered in numerical method to synthesize theoretical seismogram, which has been developed quickly. In this paper, we use simple fault rupture and elastic half-space model to simulate near-field ground motions by numerical method.

A numerical method for point dislocation source model in elastic half space has been used to calculate seismograms [12]. Multiple transmitting boundaries [4, 5] is used on exterior boundaries of an analysis model of an infinite domain. The coordinate system of source is presented in Fig.1. Taking θ as the dip angle of the fault, D(t) = D_0 F(t), where D(t) is the dislocation function, D_0 is the maximum dislocation, F(t) is the source time function, the angle between the dislocation vector and the strike direction of the fault is Ψ. In elastic space, the ground motions generated by point dislocation source can be represented in cylindrical coordinates as follow:

$$\mathbf{u} = -\mathbf{u}^{I}(r,\varphi,z)\sin\psi\cos 2\theta + \mathbf{u}^{I}(r,\varphi-\frac{\pi}{2},z)\cos\psi\cos\theta + \mathbf{u}^{II}(r,\varphi,z)\cos\psi\sin\theta$$
$$+\mathbf{u}^{III}(r,\varphi,z)\sin\psi\sin 2\theta + \mathbf{u}^{IV}(r,\varphi,z)\sin\psi\sin 2\theta \qquad (1)$$

where $\mathbf{u}$ is the displacement, velocity or acceleration vector at an arbitrary point in space which depends on the dimension of source time function. The superscripts I, II, III, IV represent four types point source respectively. Source I is for strike-slip of vertical fault, source II is for dip-slip of vertical fault. Source III and source IV are related to the tension crack of horizontal fault in vertical direction and of vertical fault in horizontal direction, both are introduced for synthesize motions generated by fault with arbitrary dip angle. In this paper, we only study the case of vertical strike-slip with θ=90°, Ψ=0° and dip-slip θ=90°, Ψ=90°.

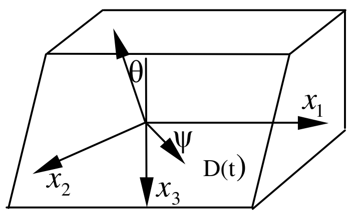

Fig.1 Coordinate system of source

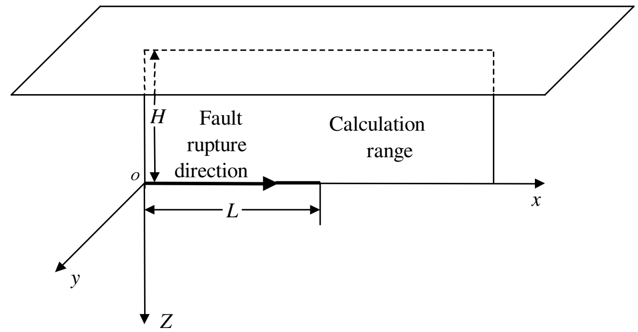

Fig.2 Sketch of a vertical fault model with unilateral rupture

Strong ground motion generated by four kinds of dislocation source of fault can be calculated by the finite difference scheme in cylindrical coordinates [13]. Elementary solution of ground motion of arbitrary point source can be synthesized according to Equation (1). If fault rupture process, size and shape are given, time history of ground motion can be synthesized by elementary solution. Then coherence function of ground motions could be calculated.

The investigation of some large earthquake (e.g. Kobe earthquake and Chi-Chi earthquake) and research results [13] show: 1) Ground motions of surface are controlled by the sub-faults which are closest surface. 2) The spatial distribution of near source field motion with inclined (thrust) fault is dissymmetry, which is different from that with vertical fault. And all of the coherency of ground motions at a linear field paralleling fault is similar. Here we mainly study vertical fault. 3) “Directivity effect” with unilateral rupture is most clear, and spatial distribution of ground is also regular. So, in this paper line source model of a vertical fault model with unilateral rupture is adopted.

Fig.2 shows the sketch of a vertical fault model with unilateral rupture. Monte Carlo method should be performed to identify the variation of fault parameters. Then the effect of randomness in the source mechanism on the strong ground motions could be examined. Because calculation time is too long with deterministic numerical simulation method for every sample, we select a statistic method with 48 samples. The parameters that used in the deterministic fault model are:

Fault length:	10000 m
Fault depth:	H=1000, 3000, 5000 m, respectively
Rupture velocity:	$v_r =$ 1800, 2000, 2200, 2500 m/s, respectively
Velocity of shear wave:	$v_s = 3000$ m/s
Velocity of compressional wave:	$v_p = \sqrt{3}v_s$
Numbers of sub-fault:	20 and 40

The computation region is H×20000 m, having coordinate x_1 (0, 0, 0), x_2 (20000 m, 0, 0), x_3 (20000 m, 0, -H), x_4 (0, 0, -H). Element size of finite difference is 20 m. Ground motions with 48 samples corresponding to different source parameters are simulated, which are used to analyze spatial coherency.

Because the deterministic source time function (especially for acceleration) is difficult to get, We use a numerical pulse as source time function, through which we can calculate coherency function, as well as track seismic phase and judge its correctness conveniently.

Expression of coherency function

The spatial coherency of two smooth stochastic processes $a_i(t)$ and $a_j(t)$, which have been parameterized by a coherency function, is defined by the relation

$$\left|\rho_{ij}(\omega)\right| = \frac{S_{ij}(\omega)}{\sqrt{S_{ii}(\omega)S_{jj}(\omega)}} \tag{2}$$

in which $S_{ii}(\omega)$, $S_{jj}(\omega)$ and $S_{ij}(\omega)$ are the auto-power spectral and cross-power spectral density function of $a_i(t)$, $a_j(t)$ at locations i and j respectively. For lacking of large numbers of earthquake records, auto-power spectral and cross-power spectral density function of ground motions could not be got directly by ensemble average method. Usually $S_{ii}(\omega)$, $S_{jj}(\omega)$ and $S_{ij}(\omega)$ must be smoothed. In this paper, all of the auto-power spectral and cross-power spectral density functions were smoothed. Meanwhile, $\left|\rho_{ij}(\omega)\right|$ is also defined as lagged coherency. In this paper, coherency function is lagged coherency.

NUMERICAL RESULTS

The observation points have coordinates x_1(10000 m, 0, 0) and x_2(12000 m, 0, 0) at surface. Coherency functions of horizontal component along the surface of base-rock corresponding to different space distance d are calculated according to equation (2). The results are plotted in Fig.3.

Coherency function of ground motions that decreases with frequency f and with spatial separation d of observation points is assumed for mathematical convenience to have the form:

$$\left|\rho(f,d)\right| = \boldsymbol{exp}[-(a+bf^2)d] \tag{3}$$

where a and b are coefficient. Formula (4) and (5) are coherency functions models of y-direction horizontal component with strike-slip and dip-slip fault respectively. The fitted values of coherency functions are plotted in Fig.4. The compare of coherency function of base-rock suggested here and that of soil surface presented by Qu Tiejun[9] are shown in Fig.5.

$$\left|\rho_{strike-slip}(f,d)\right| = \exp[-(1.3\times10^{-6} + 2.38\times10^{-5} f^2)d] \tag{4}$$

$$\left|\rho_{dip-slip}(f,d)\right| = \exp[-(2.38\times10^{-7} + 7.00\times10^{-6} f^2)d] \tag{5}$$

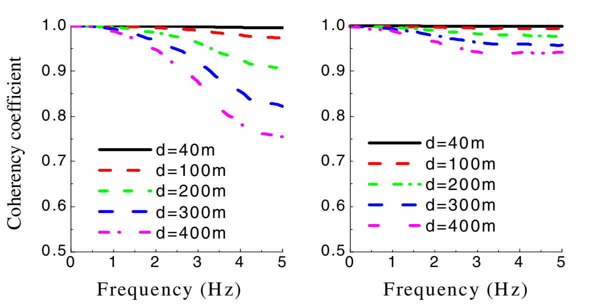

Fig.3 Coherency function of base-rock calculated in this paper

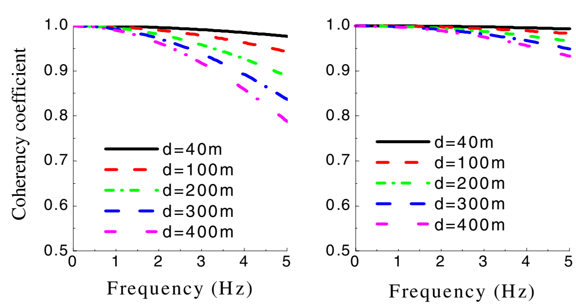

Fig.4 Coherency function of base-rock suggested in this paper

Taking one example (fault depth $H = 5000$ m, rupture velocity $v_r = 2500$ m/s, numbers of sub-fault are 20) with strike-slip, y-direction horizontal component coherency function at surface and beneath surface 1km are plotted in Fig.6

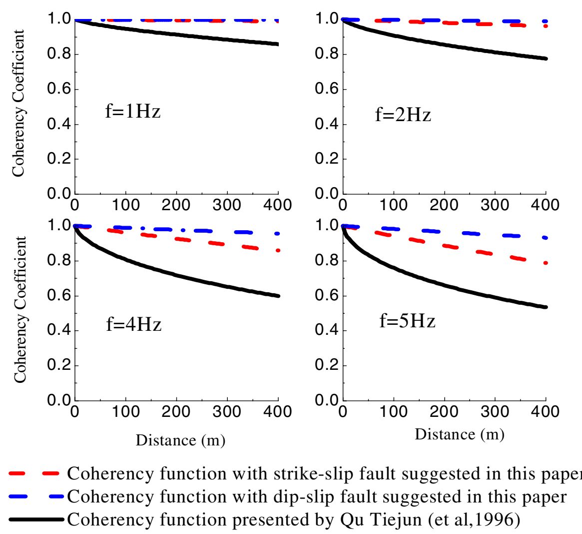

Fig.5 Compare of coherency function of base-rock with that of soil statistic results

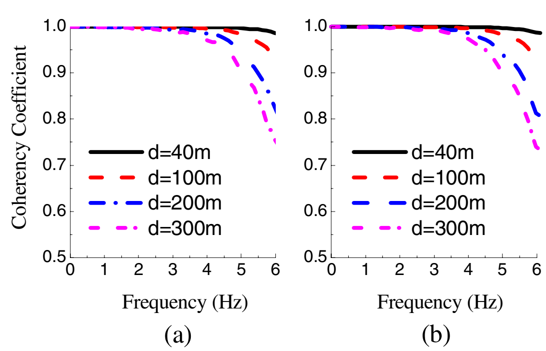

Fig.6 Coherency function of surface (a) and beneath surface 1km (b) with strike-slip fault

From above results, we have following findings:

(1) The value of coherency function at surface of base-rock calculated in this paper is much higher than that at surface of soil calculated from earthquake records, at least within 5 Hz, which suggest that the spatial coherence measured on soil may not provide a good description of the spatial coherence at base-rock.

(2) The spatial coherence at surface of base-rock with vertical strike-slip fault is better than that with vertical dip-slip fault.

(3) Sites are very complex, which include surface geology, undulation of underground base-rock (e.g. basin) and variation of soil characters. From the results we can find that the value of coherency function at surface of base-rock is the same as that under base-rock (Fig.6). The coherency function at surface of base-rock can be used as that at base-rock under soils when we consider the spatial coherence of base-rock under soils.

CONCLUSIONS AND DISCUSSION

In this paper, a method to simulate stochastic ground motion and coherency function of base-rock corresponding to strike-slip and dip-slip fault is presented. Because not all of the factors, such as site condition, are considered, the results are very different from that of the statistic analysis based on dense array records. Although the variation of earthquake source parameters (e.g. depth of epicenter, rupture velocity, and numbers of sub-source, et al.) are considered, the samples are not taken enough and the coherency function model of base-rock should be validated by more calculation and observation results. For lack of information of ground motions at base-rock, there are no statistic coherency function models of base-rock. The coherency function proposed here may be used as the referenced model for near-field ground motions of base-rock site with strike-slip fault. Next, we will intend to extend the calculating range and consider the effect of azimuth. We will adopt the results of 3-D finite element simulation of ground motion.

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