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SOIL STRUCTURE INTERACTION EFFECTS ON INELASTIC RESPONSE OF R/C STACK-LIKE STRUCTURES

Amir M. HALABIAN[1], Shabnam KABIRI[2]

SUMMARY

Reinforced concrete R/C stack-like structures such as chimneys are often analyzed using elastic analyses as fixed base cantilever beams ignoring the effect of soil-structure interaction. To investigate the effect of foundation flexibility on the response of structures deforming into their inelastic range, a method is presented to quantify the inelastic seismic response of flexible-supported R/C stack-like structures by non-linear earthquake analysis. The deformed configuration of stack-like structure is idealized as an assemblage of beam elements, while a linear sway-rocking model is implemented to model the supporting soil. The effect of concrete cracking and reinforcement yielding on the stiffness is taken into account using a non-linear moment-curvature (M-ϕ) relation. Using a practical stack-like structure and an actual ground motion as excitation, the elastic and inelastic response of structure supporting on flexible soil are calculated and compared. It is shown that indiscriminate use of presently popular ductility capacity factors may lead to erroneous conclusions in the predictions seismic performance of flexibly-supported chimneys.

INTRODUCTION

Foundation flexibility is recognized to have a significant effect on the dynamic behavior of the structures. Many researchers have taken into account the effect of soil-structure interaction on the dynamic behavior of structures where the structure exhibits linear behavior. However, in the structural dynamics field, a few researches have been conducted to investigate the effect of foundation flexibility on the response of structures such as R/C buildings deforming into their inelastic range. Darbre [1] performed the seismic analysis of a non-linearly base-isolated reactor building. In this study, the influence of the frequency dependence of the foundation stiffness coefficients on the nonlinear seismic response of the base-isolated reactor building was found to be negligible for all practical purposes. The applications of this study were limited to the seismic soil-structure interaction systems with a single nonlinearity. To address the limitation of this study, Darbre [2] examined the effect of dynamic *SSI* on the seismic behaviour of a 6-storey shear building with hysteretic elasto-plastic storey characteristics at each floor to extend the area of

[1] Assistant Professor, Faculty of Civil Engineering, Isfahan University of Technology, Isfahan, 84156 – 83111, Iran,
mahdi@cc.iut.ac.ir
[2] Graduate Student, Faculty of Civil Engineering, Isfahan University of Technology, Isfahan, 84156 – 83111, Iran

application of nonlinear *SSI*, so as to encompass dynamic soil-structure interacting systems with multiple nonlinearities. These studies and the results of the other analyses by Miranda & Bertero [3] and Priestley and Park [4] showed that the additional flexibility of an elastoplastic structure due to the foundation compliance may decrease or increase the ductility capacity of the system that in the former case would be an apparently detrimental consequence of *SSI*.

Lack of research on ductile behavior of flexible-base reinforced concrete stack-like structures, such as towers and chimneys, and on the collapse threshold for such structures subjected to strong motions, precludes reliable evaluation of their actual seismic resistance. The objective of the present study is to investigate the seismic response of R/C stack-like structures interacting with the supporting soil where the structure exhibits nonlinear behavior. The study is highlighting the effect of foundation flexibility on the seismic linear and non-linear behavior of these types of structures, which is addressed in the design codes. In the current study, a shear beam moment-curvature relation in MDOF structures has been considered in the analysis of the seismic response of chimneys interacting with the soil. The effects of changes in the stiffness of concrete members due to cracking and yielding, as well as the foundation flexibility for a practical stack-like structure are examined. As a stack-like structure, an existing model of chimney is used to illustrate the *SSI* and cracking effects in reducing or increasing the response to earthquake loading.

SYSTEM MODEL AND EQUATIONS MOTION

The system considered in this study consists of a tapered reinforced concrete chimney with hollow circular cross-section supporting on flexible soil. In practice, the response of stack-like structures is commonly analyzed using the stick modeling concept. In the lumped mass (stick) model, the structure investigated is idealized as an assemblage of sufficiently large number of beam elements and the mass of the system is considered to be concentrated at the different levels as shown in Fig. 1. The centroidal axis of beam elements has been selected as reference axis in the finite element formulation as described below. The base chimney is assumed to be rigid circular disc footing with no embedment attached to the surface of linearly elastic halfspace. To account for *SSI,* the substructure system could be represented by a sway-rocking foundation system with mass and mass moment of inertia equal to m_0 and I_0, respectively. This representation leads to a system of n+2 DOF's (n=the number of lumped masses of the superstructure): one horizontal translation (relative to the foundation) for each mass, x_i, where i=1,….,n; the horizontal translation of the foundation, x_0, and the rotation of the system, θ. Assuming small displacements, the equations of motion for the structure-foundation model illustrated in Fig. 1 can be expressed as

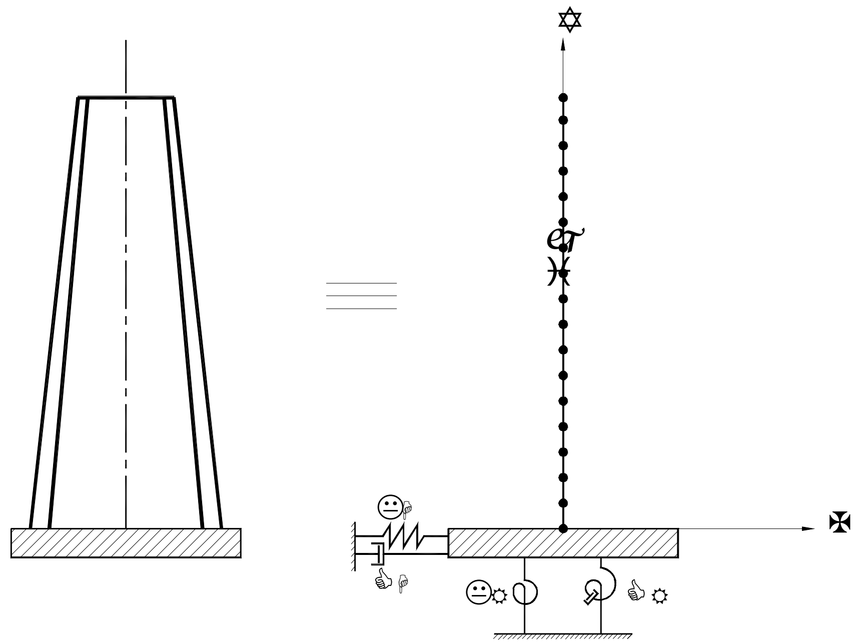
$$m_i(\ddot{x}_0 + \ddot{x}_i + h_i\ddot{\theta} + \ddot{x}_g) + \sum_{j=1}^{n} c_{ij}\dot{x}_j + \sum_{j=1}^{n} k_{ij}x_j = 0 \qquad (i=1,\ldots,n) \tag{1}$$

$$\left(\sum_{j=1}^{n} m_j + m_0\right)(\ddot{x}_0 + \ddot{x}_g) + \left(\sum_{j=1}^{n} m_j h_j\right)\ddot{\theta} + \sum_{j=1}^{n} m_j\ddot{x}_j + c_{HH}\dot{x}_0 + c_{HR}\dot{\theta} + k_{HH}x_0 + k_{HR}\theta = 0 \tag{2}$$

$$\sum_{j=1}^{n}\left(m_j h_j\left(\ddot{x}_0 + \ddot{x}_j + h_j\ddot{\theta} + \ddot{x}_g\right)\right) + \left(I_0 + \sum_{j=1}^{n} I_j\right)\ddot{\theta} + c_{RH}\dot{x}_0 + c_{RR}\dot{\theta} + k_{RH}x_0 + k_{RR}\theta = 0 \tag{3}$$

where $\ddot{x}_g$ is the free field ground motion, $k_{HH}, k_{HR}, k_{RH}, k_{RR}$ are stiffness coefficients of the foundation and $c_{HH}, c_{HR}, c_{RH}, c_{RR}$ are its damping coefficients. Equation 1 describes the dynamic equilibrium of the horizontal forces acting on the structural masses, whereas Eqs.2 and 3 express the equilibrium for the whole structure-foundation system in translation and rotation, respectively. In a matrix format Eqs.1 to 3 can be further written as

$$[M]\{\ddot{X}(t)\}+[C]\{\dot{X}(t)\}+[K]\{X(t)\}=-[M_F]\{I\}\ddot{x}_g \tag{4}$$



(a)　　　　　　(b)

Figure 1. Typical reinforced concrete stack-like structure: a) chimney b)structural model

in which:
$\{I\}$ is the unit vector and:

$$[M]=\begin{bmatrix} [m] & \vdots & \{m\} & \{mh\} \\ \cdots\cdots & \vdots & \cdots & \cdots \\ \{m\}^T & \vdots & m_0+\sum_{i=1}^{n} m_i & \sum_{i=1}^{n} m_i h_i \\ \{mh\}^T & \vdots & \sum_{i=1}^{n} m_i h_i & I_0+\sum_{i=1}^{n}(I_i+m_i h_i^2) \end{bmatrix} \qquad [K]=\begin{bmatrix} [k] & \vdots & \{0\} & \{0\} \\ \cdots & & \cdots & \cdots \\ \{0\}^T & \vdots & k_{HH} & k_{HR} \\ \{0\}^T & \vdots & k_{RH} & k_{RR} \end{bmatrix}$$

$$[C] = \begin{bmatrix} [c] & \vdots\{0\} & \{0\} \\ \cdots & \cdots & \cdots \\ \{0\}^T & \vdots\ c_{HH} & c_{HR} \\ \{0\}^T & \vdots\ c_{RH} & c_{RR} \end{bmatrix} \qquad [M_F] = \begin{bmatrix} \{m\} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ m_0 + \sum_{i=1}^{n} m_i \\ I_0 + \sum_{i=1}^{n} I_i + m_i h_i \end{bmatrix} \tag{5}$$

and the displacement vector $\{X\} = [x_1\ x_2 \ldots\ldots x_n \vdots x_0\ \theta]^T$ is the vector of structural and foundation displacements relative to foundation. In Eqs.5 $[m]$ is a diagonal mass matrix listing lumped masses m_i at each foundation-to-mass heights, h_i, $[k]$ is the stiffness matrix and $[c]$ is the viscous damping matrix of the superstructure; and $\{m\} = \{m_1\ m_2 \ldots\ldots\ m_n\}^T$; $\{mh\} = \{m_1h_1\ m_2h_2 \ldots\ldots\ m_nh_n\}$. Also $I_0 + \sum_{i=1}^{n} I_i + m_i h_i^2$ is the total moment of inertia of the structure and foundation with respect to the central axis of the foundation.

The determination of proper values for the dynamic stiffness and damping of the foundation plays an important role in obtaining an accurate seismic response of the structures. The variation of these values with dynamic soil properties is usually remarkable and consequently their effect on the superstructure behavior is important. The foundation impedance functions can be obtained from the analytical and numerical approaches of the solution of mixed boundary-value problem in elastodynamics and are generally functions of soil properties, foundation type and size, and exciting frequency. For linear footing-soil systems with two planes of symmetry, the foundation vibration problem uncouples into two SDOF problems for vertical and torsional motions and a two DOF problem for swaying and rocking and steady-state harmonic vibration at frequency ω the interaction forces acting on the footing can be expressed as

$$\begin{bmatrix} K_{VV}(\omega) & 0 & 0 & 0 \\ 0 & K_{HH}(\omega) & K_{HR}(\omega) & 0 \\ 0 & K_{RH}(\omega) & K_{RR}(\omega) & 0 \\ 0 & 0 & 0 & K_{TT}(\omega) \end{bmatrix} \begin{Bmatrix} z_0 \\ x_0 \\ \theta \\ \phi \end{Bmatrix} = \begin{Bmatrix} V \\ H \\ M \\ T \end{Bmatrix} \tag{6}$$

where z_0, x_0 are the complex amplitudes of vertical and horizontal displacements, respectively and θ, ϕ are the rocking and torsional rotation amplitudes. *V, H, M, T* are the corresponding vertical and horizontal force, moment and torque amplitudes with which the footing acts on the ground. A number of analytical and numerical approaches are available to calculate the impedance functions for both shallow and deep foundation systems that are mostly based on the assumption of elastic or viscoelastic soil continuum. For a circular foundation on the surface of a viscoelastic halfspace, the frequency-dependent impedance functions as complex values have been tabulated by Veletsos and Wei [5], Verbic and Veletsos [6] and Veletsos and Verbic [7]. These complex stiffness functions can be expressed as

$$\begin{aligned} K_{VV}(\omega) &= (k_{VV}(\omega) + ia_0 c_{VV}(\omega))\overline{K}_{VV} \\ K_{HH}(\omega) &= (k_{HH}(\omega) + ia_0 c_{HH}(\omega))\overline{K}_{HH} \\ K_{RR}(\omega) &= (k_{RR}(\omega) + ia_0 c_{RR}(\omega))\overline{K}_{RR} \\ K_{HR}(\omega) &= (k_{HR}(\omega) + ia_0 c_{HR}(\omega))\overline{K}_{HR} \\ K_{TT}(\omega) &= (k_{TT}(\omega) + ia_0 c_{TT}(\omega))\overline{K}_{TT} \end{aligned} \tag{7}$$

in which k_{ss} and c_{ss} are dimensionless coefficients depending on poisson's ratio ν and the dimensionless frequency parameter $a_0 = \omega\ r/V_s$, where r is the radius of the footing, and V_s is the shear wave velocity in the halfspace. The parameters of $\overline{K}_{VV}$, $\overline{K}_{HH}$, $\overline{K}_{RR}$, $\overline{K}_{TT}$ are the static stiffnesses corresponding to vertical, horizontal displacements and rocking , torsional rotations, respectively.

Frequency-dependent foundation impedance functions may be used. But for the horizontal and rocking response of shear building-foundation-soil systems, the frequency dependence of foundation compliance is not profound. This may be due to the fact that the first mode frequencies of most fixed-base buildings are often fairly low. In the lower frequency range, the foundation impedances are less frequency dependent than in a much higher frequency range. Thus the frequency-dependent nature of the foundation compliance in this case can usually be accounted for by evaluating the foundation compliance at the fundamental frequency of the *SS* system (Wolf [8]; Zhao [9], Darbre [1], [2]). The higher modes of the fixed-base building would be influenced by the frequency dependence of foundation compliance, but their contributions to the total response of the *SS* system are small because of their small participation factors. Therefore, in this investigation the foundation stiffnesses were calculated at the frequency equal to the first fundamental frequency of the structure obtained from the analysis of the fixed base case.

NONLINEAR MODELLING OF CONCRETE

A concrete structure softens when deformed to somewhere near its limit state of resistance so that the forces created are considerably weaker than what would be expected for a more elastic system. This effect starts with the development of cracks in the portions of the structure, which are exposed to elongation in the tension steel and by cracking and spalling of concrete. During the earthquake damages in the reinforced concrete structures are occurred at localized levels which are deformed into non-linear zone in form of microcracking and crushing of concrete, yielding of the reinforcement bars, bond deterioration, etc. The moment-curvature relation method is one of the specialty approaches used to express the real behavior of concrete structures in the non-linear zone. This method could widely served all non-standard shapes of the cross-sections such as hollow circular sections for chimneys.

Recalling the concepts of the rational method in analysis of concrete sections, the plane section theory can be used to predict the behavior of beam elements in combined axial load and flexure. The longitudinal concrete stresses are found from the longitudinal concrete strains by using the appropriate concrete stress-strain relationship in compression. Thus for a particular axial load, the values of M can be calculated for different strain distributions, ϕ. Having the calculations for a range of values of ϕ leads to the M-ϕ diagram for the specific section and the corresponding axial load (Response 2000 [10]). In a non-linear analysis the M-ϕ relations can be simplified by different backbone curves such as bi-linear, tri-linear and etc. In the current study the M-ϕ relations in monotonic loading (backbone curve) are taken tri-linear (Fig. 2a), with a post-yield hardening ratios computed by Response-2000 [10]. The ultimate moment M_u is also computed on the basis of principles of limit state of resistance theory.

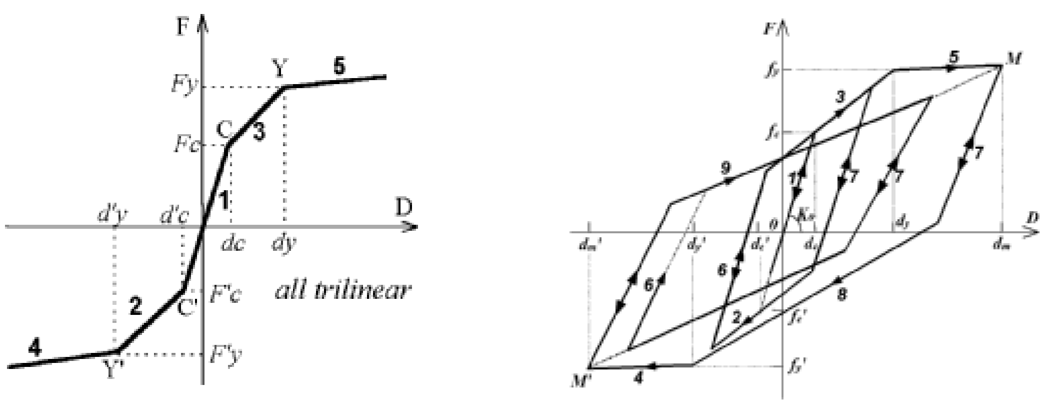
Principally, the moment-curvature relation model is used in non-linear analysis of frame element models. Moment-curvature relation model can also consider distributed non-linearity as well. In this investigation as the gravity loads are constant at different level of stack-like structures, by specifying the relation of moment and curvature at critical sections, no interaction among bending and axial forces is taken into account. Therefore, the flexibility matrix at element-end sections can be expressed by:

$$[f_j] = \begin{bmatrix} f_{yy}^{j} & 0 & 0 \\ 0 & f_{xx}^{j} & 0 \\ 0 & 0 & f_0^{j} \end{bmatrix} \tag{8}$$

where f_{yy}^{j} , f_{xx}^{j} and f_{o}^{j} are the flexibilities of the element at the corresponding element-end cross-section related to two rotational degrees of freedom and one axial deformation, respectively. Assuming the linear-distributed flexibility then the element flexural flexibility matrix is expressed as

$$[f]=\frac{l}{12}\begin{bmatrix} 3f_{yy}^{i}+f_{yy}^{j} & -f_{yy}^{i}-f_{yy}^{j} & 0 & 0 & 0 \\ & f_{yy}^{i}+3f_{yy}^{j} & 0 & 0 & 0 \\ & & 3f_{xx}^{i}+f_{xx}^{j} & -f_{xx}^{i}-f_{xx}^{j} & 0 \\ & & & f_{xx}^{i}+3f_{xx}^{j} & 0 \\ & & & & 6f_{0}^{i}+6f_{0}^{j} \end{bmatrix} \tag{9}$$

in which i and j represents the element cross-sections. Having this assumption that there is no interaction between axial force as well as shear forces and biaxial bending moments, the uniaxial models can be used to simulate the inelastic behavior of uniaxial bending deformations. All essential characteristics of the hysteretic behavior of reinforced concrete members including stiffness degradation, pinching and strength deterioration, are explicitly taken into account. The cyclic unloading-reloading relation is assumed to be degrading tri-linear, as shown in Fig. 3b. The element was implemented in the non-linear finite element program CANNY [11]. The hysteretic rules are shown in Fig. 3b and can be found in detailed in Ref. [11].



a) b)

Figure 2. Non-linear behavior of R/C structural elements: a) backbone curve b) Hysteretic loop

NON-LINEAR EARTHQUAKE ANALYSIS

For non-linear analysis, the beam element can be idealized by elastoplastic uniaxial spring, two rotational spring at each end of element and shear and axial springs located in mid span. The properties of rotational spring and the axial torsional spring are described by the moment-rotation relationship, while the shear and axial spring are specified by the force-displacement relation. A numerical integration procedure known as Newmark' β method implemented in CANNY program is used to solve the equations of motion. In the this method, the linearized equations of motions (Eq. 4) at the end of each time step is solved for incremental displacements, velocities and accelerations by direct time integration using

Newmark's average acceleration scheme with β=0.25 and γ=0.5 (The parameters β and γ define the variation of acceleration over a time step (Chopra [12]). To satisfy the equilibrium at each time step, the iterations are performed using the Newton-Raphson scheme within each time step. The time increment is assumed 0.002 sec. The Rayliegh's damping is used to model the structural damping and having taken into account the other type of damping such as soil dapmers the total damping including non-classical damping is expressed by:

$$[C] = a_m[M] + a_0[K_0] + a_k[K] + [C_v] \qquad (10)$$

The coefficients a_m, a_0 and a_k are damping factor proportional to mass matrix, initial stiffness and time-varying stiffness matrix computed from first two modes. [C_v] is also damping matrix contributed by damping elements resulted from soil impedance functions which includes soil radiation damping and soil material damping as well. Geometrical non-linearity is not taken into account. Therefore, the analysis is limited to small deformations. Masses that cause inertia loads is considered to be concentrated at structural joints.

NUMERICAL EXAMPLE

As an illustrative example of stack-like structures, a chimney located in west of Iran, whose geometrical and geological data were made available to the authors, is used in this investigation. The chimney is 194 m height with varying hollow circular sections along the height. It has a flexible shallow foundation with 29 m diameter. For the purpose of the analysis, the soil is assumed to be homogenous viscoelastic halfspace. The shear wave velocity of the soil was assumed to be 200 m/sec in the analysis to represent the supporting soil. The other material properties of the supporting soil are: mass density = 1850 kg/m^3, Poisson's ratio = 1/3. The E-W component of the 1940 El-Centro earthquake ground motion is selected as the excitation. The maximum amplitude of this ground motion is 0.2 g.
To calculate the moment-curvature relationships the stack cross sections are assumed to have two rings of longitudinal reinforcements and cages of circumferential reinforcement placed near both the internal and external faces of the section (Fig. 3). It is assumed that the bar sizes and spacing of circumferential bars are appropriate to prevent premature buckling of the longitudinal reinforcement and to provide sufficient shear strength so that the crushing of concrete due to stresses rising from axial and bending loads could be cause of the collapse of the stack. The axial forces induced into the chimneys's cross-sections are assumed

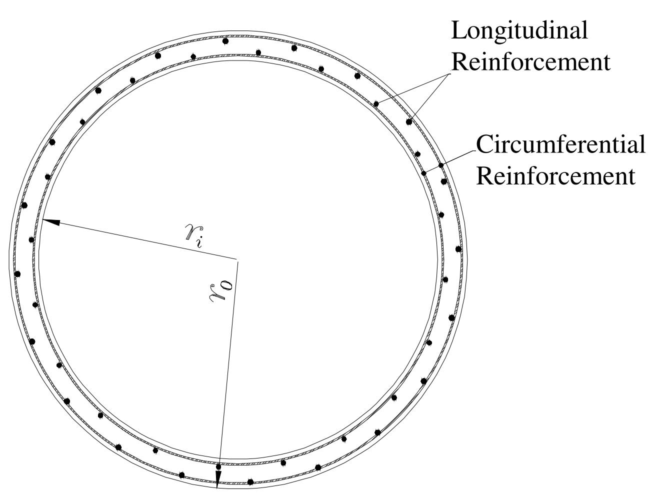
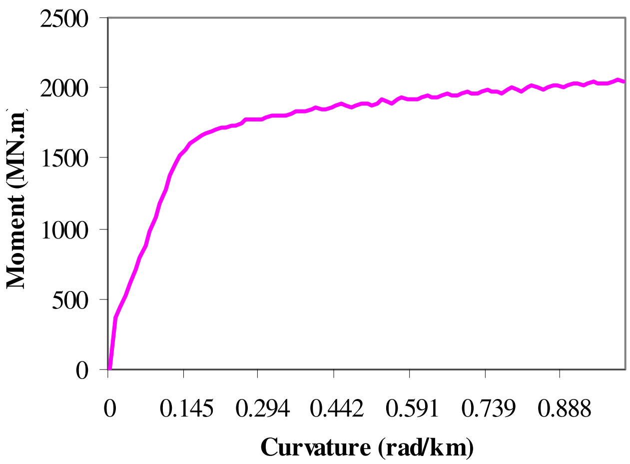
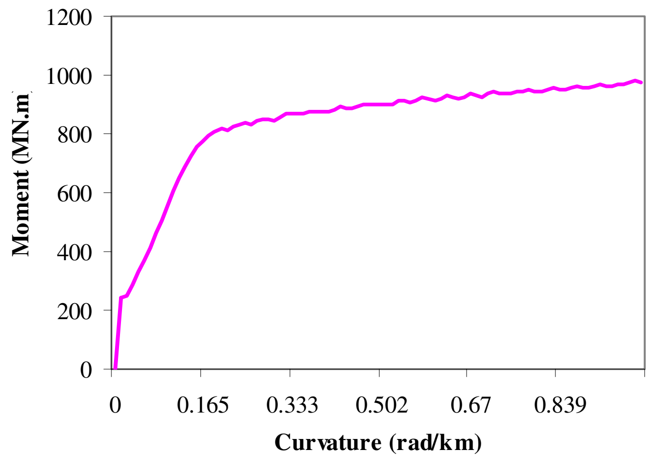


Figure 3. Typical cross section of chimney

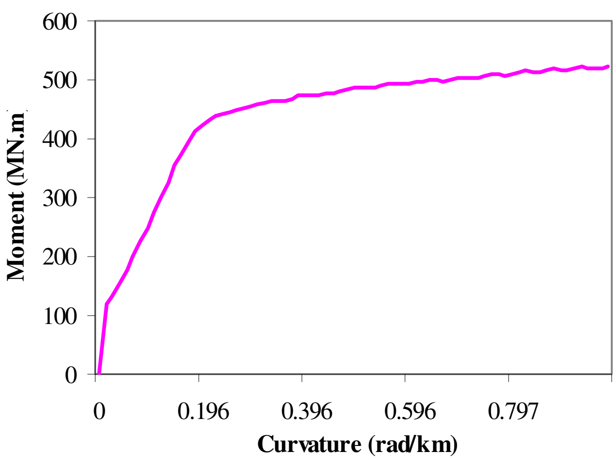
to be mainly gravitational loads, as the variation in the axial force level at a section due to horizontal ground excitation is usually small. Using Response-2000, the moment-curvature for all different sections of the stack are obtained. Figure 4 shows M-φ diagrams for several cross-sections of the stack. It can be seen quite clearly that the flexural stiffness of the sections manifested by the slope of the M-φ curve stays virtually unchanged for a certain range of bending, due to the action of axial compression, only to dramatically drop to a fraction of its base value when bending moments exceed the values necessary to cause cracking of concrete.



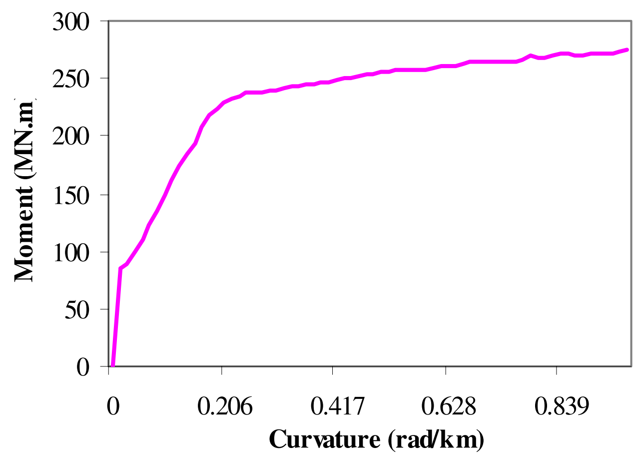
a) Base cross section



b) Cross section at El. 40 m



c) Cross section at El. 110 m



d) Cross section at El. 135 m

Figure 4. Moment-curvature relation for different cross sections

The seismic response of the chimney is determined assuming supporting soil behaves linearly and also structure itself experiences inelastic deformation during the excitation. The displacement time history at the top of the chimney is shown in Fig. 5. Response time histories for the fixed base and the flexible base chimney, assuming elastic and inelastic behavior of the structure, have also been plotted. It can be noted from Fig. 5 that the non-linear behavior of structure resulted in an increase in the top displacement of the chimney and also changed the frequency content of the response.

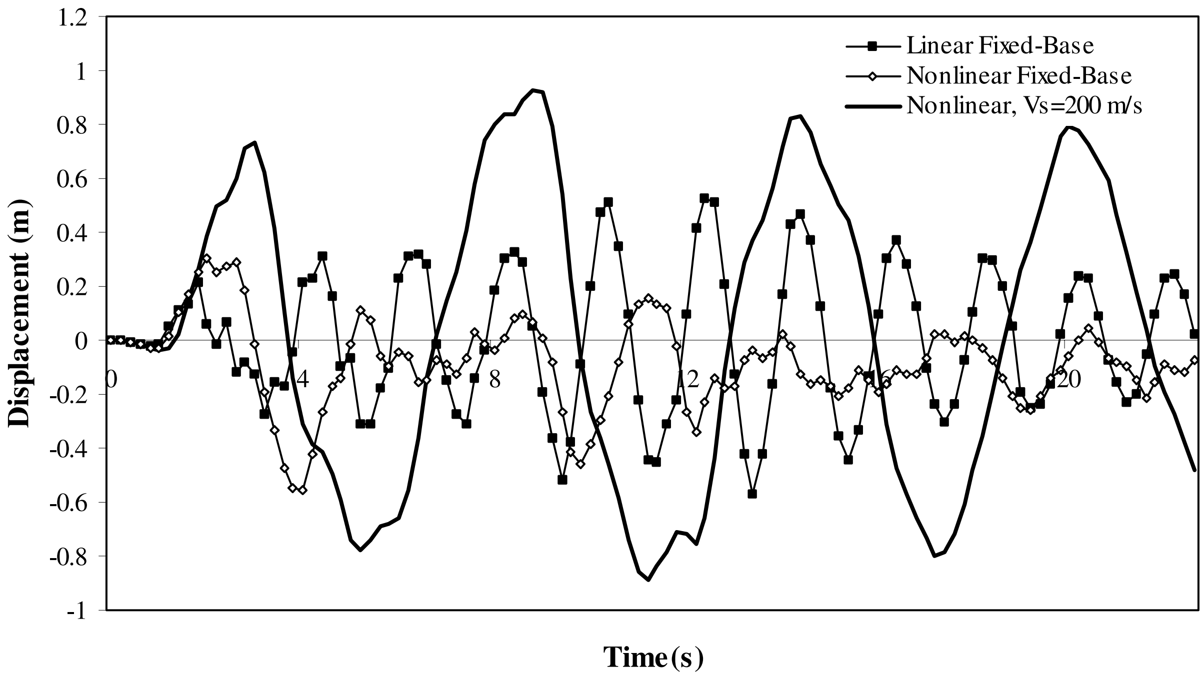


Figure 5. Top displacement time histories

Fig. 6 shows the base shear time histories of the chimney for the fixed base and the flexible base cases, assuming linear and nonlinear behavior of the structure. Fig. 6 shows that, in general, the foundation flexibility decreases the base shear induced at the base of the tower. However, the SSI alters the frequency content of the base shear time history when the structure deformed into inelastic zone. The calculated time histories of the base bending moment of the chimney for the fixed-base and the flexible base models, assuming linear and non-linear behavior of the structure are also shown in Fig. 7. It can be noted from Fig. 7 that, in general foundation flexibility decreases the bending moment induced at the base of the structure. It can also be noted that the base bending moment for the case that the structure behaves non-linearly is different than for the linear case. These results indicate the importance of the foundation flexibility on non-linear seismic behavior of the stack-like structures.

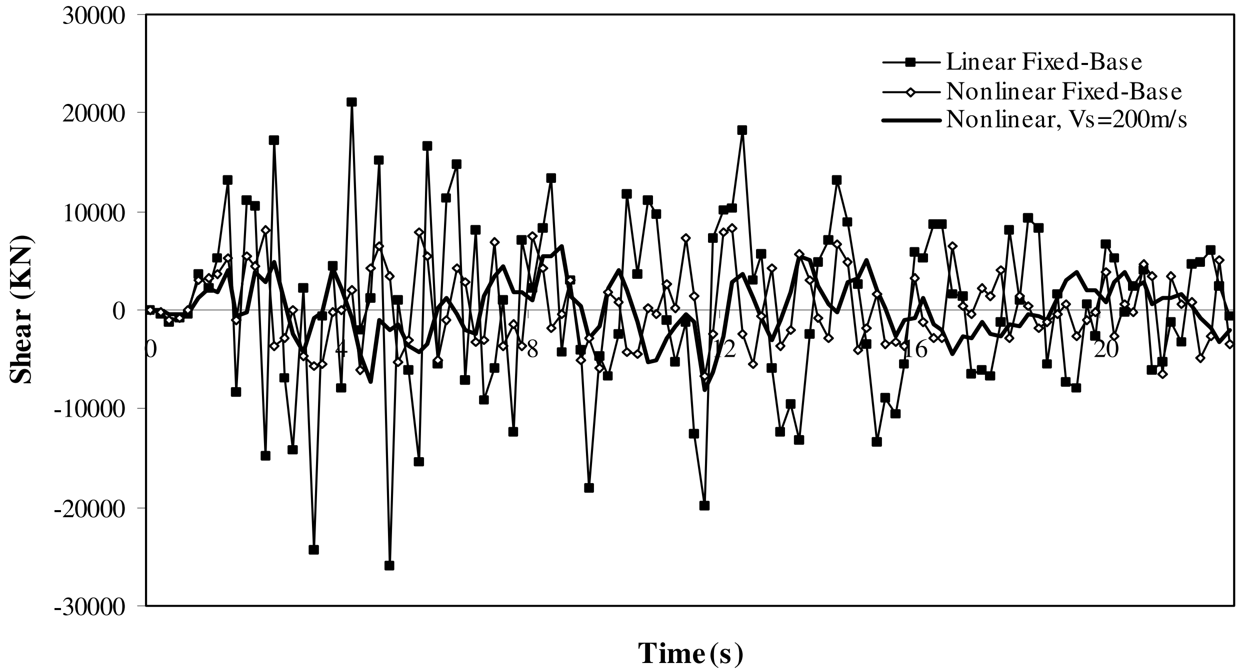


Figure 6. Base shear force time histories

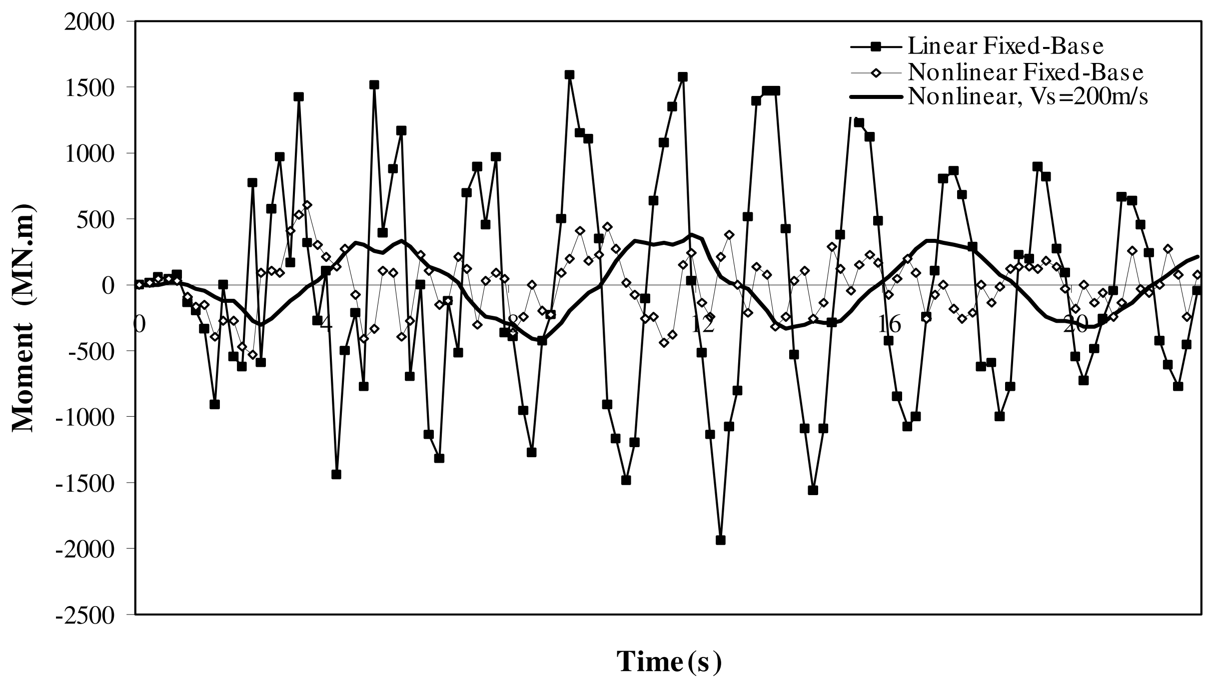


Figure 7. Base bending moment time histories

CONCLUSIONS

To gain further insight on the importance of SSI on the performance of the stack-like systems an analytical method is presented. The inelastic seismic resistance of reinforced concrete stack-like structures supporting on flexible soils by non-linear seismic analyses is evaluated. The seismic response of a practical chimney as an example of R/C stack-like chimney subjected to an earthquake excitation is calculated including soil- the linear and nonlinear structure interaction using the proposed method in this study. The following conclusions are drawn:

- The inelastic behavior of structure caused an increase in the lateral displacements of the structure. Therefore, this increase should be considered in the analysis of this type of structures, especially when the P-Δ effect is taken into account.
- The inelastic behavior of structure altered the base bending moment as well as base shear compared to that obtained from the linear structure fixed-base and also non-linear structure fixed base models. This shows that the soil-structure interaction when considered in the real case, could reduce or increase the base forces as it is implied in most design codes.

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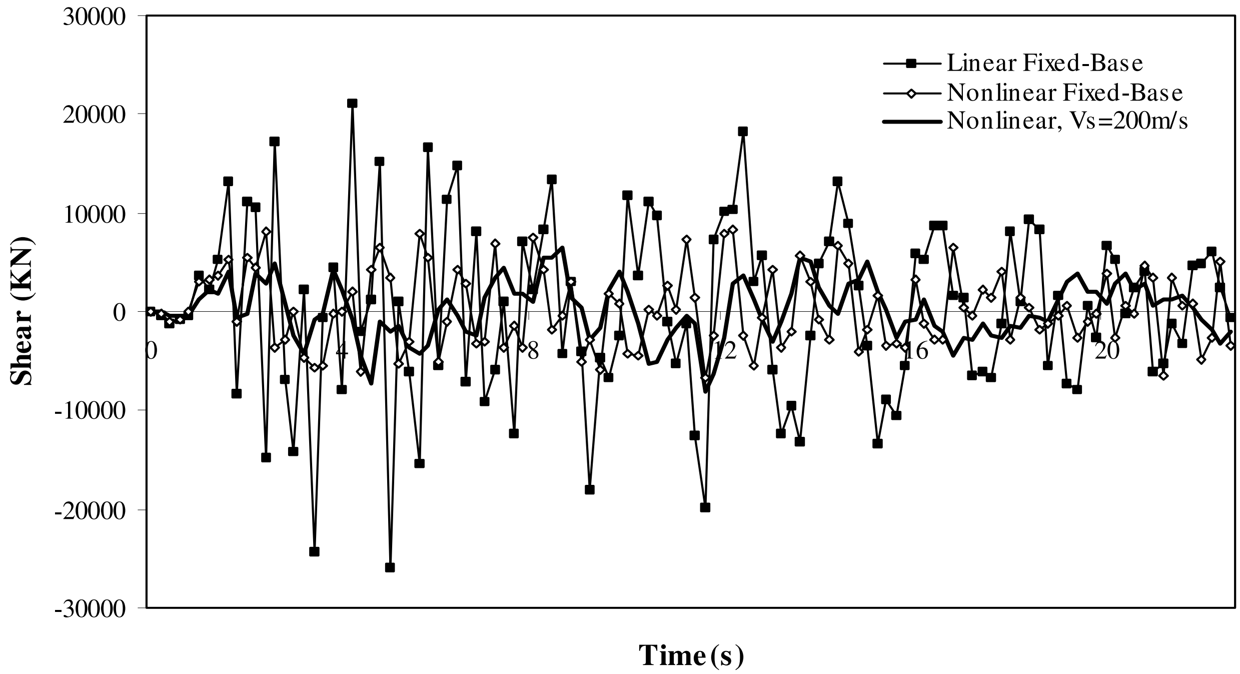

30000
20000
10000
0
-10000
-20000
-30000
Shear (KN)
0
4
8
12
16
20
Time(s)
Linear Fixed-Base
Nonlinear Fixed-Base
Nonlinear, Vs=200m/s