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SINGLE DEGREE OF FREEDOM SYSTEM IDENTIFICATION USING LEAST SQUARES, SUBSPACE AND ERA-OKID IDENTIFICATION ALGORITHMS

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SUMMARY

This paper presents the identification of a time invariant single degree of freedom linear system, acting as a shear building subjected to base excitation, using three different methodologies. One is the least squares method with two different algorithms. First, the off-line algorithm is used in order to make the system identification by means of a modal parameter estimation, and then the recursive algorithm is implemented for the observation of the parameter time variation during the excitation. The second and third methods are the off-line subspace system identification algorithm and the ERA-OKID identification algorithm, which allow to identify the system modal parameters. A comparison of the estimated response is made between the three methodologies and a better agreement could be observed with the off-line subspace system identification estimated response. However, the off-line subspace system identification algorithm and the ERA-OKID identification algorithm are not totally appropriate if a physical parameter estimation is required. On the other hand, by means of the least squares recursive algorithm it is possible to observe a sudden stiffness variation similar to a structural failure during the peak acceleration of the excitation. An example is presented for two earthquake excitations and the influence of the forgetting factor and the covariance matrix initial values for the recursive least squares method could be observed.

INTRODUCTION

Health monitoring of civil structures during seismic events is a field of research that has developed at recent years. The negative effects that strong motions produce in structures, such as buildings or bridges, is well known. For that reason, those kind of structures have been instrumented in order to register the accelerations, velocities and displacements produced by earthquakes. This information can be used to review the structural safety by means of structural analysis and design tools, that allow to identify potentially damaged resistant elements. However, in most cases this work takes several days of continuous analysis and the results can not be used in order to give an immediate evaluation of the structure. An alternative method is the observation of changes in structural parameters such as stiffness and modal values.

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In any case, several algorithms have been proposed in order to carry out the structural identification. Those algorithms can be of two types: a) state-space identification algorithms Juang [1], Juang [2], Van Overschee [3], and b) second order systems identification algorithms Worden [4]. Recently, many researches, like Bernal [5], Bernal [6] and Smyth [7] have developed and used some of these algorithms.

In the present paper the Least Squares, ERA-OKID and Subspace Identification algorithms are used to obtain the modal parameters of a time invariant single degree of freedom linear system, acting as a shear building subjected to base excitation. Two Matlab [8] simulations of the system's response are used. One of these simulations considers the El Centro earthquake (1940) as the system excitation, while the second one considers the SCT Mexico City earthquake (1985) (SCTEO) as the input of the system. For both cases, the original system parameters are chosen in order to obtain the maximum responses depending on the excitation. Three different noise levels are also considered.

IDENTIFICATION ALGORITHMS

Least Squares Method (LSM)

The least squares method allows to identify physical parameters of civil structures when they are modeled as second order systems. These parameters are basically the mass, stiffness and damping. The formulation of the recursive least squares algorithm used in this paper is shown in equations (1) to (3).

$$\left\{\hat{\beta}\right\}_{n+1} = \left\{\hat{\beta}\right\}_{n} + \left\{K\right\}_{n+1}\left(y_{n+1} - \left\{\phi\right\}_{n+1}^{T}\left\{\hat{\beta}\right\}_{n}\right) \quad (1)$$

$$\left\{K\right\}_{i+1} = \frac{\left[P\right]_{i}\left\{\phi\right\}_{i+1}}{\lambda + \left\{\phi\right\}_{i+1}^{T}\left[P\right]_{i}\left\{\phi\right\}_{i+1}} \quad (2)$$

$$\left[P\right]_{i+1} = \frac{1}{\lambda}\left(1 - \left\{K\right\}_{i+1}\left\{\phi\right\}_{i+1}^{T}\right)\left[P\right]_{i} \quad (3)$$

When the identification algorithm is applied to a second order system it is necessary to have a previous knowledge of the system's structure, i.e., it is not enough to have the time-history inputs and outputs, and furthermore, it is required to know the relationship between the different parts of the system. If this requirement is accomplished, then the identification algorithm allows to obtain the physical parameters, whereas with state-space algorithms, the modal parameters are obtained directly.

ERA-OKID algorithm

The first state-space algorithm presented is the ERA-OKID Juang [1], Juang[2]. This methodology is a two-step procedure. First the OKID algorithm Juang [1] is used to obtain the Markov parameters from the input and output signal records in accordance with equation (4).

$$\overset{mxl}{y} = \underset{mx\left[(r+m)p+r\right]}{\bar{Y}} \; \overset{\left[(r+m)p+r\right]xl}{V} \quad (4)$$

where

$$
\begin{aligned}
y &= \begin{bmatrix} y(p) & y(p+1) & \dots & y(l-1) \end{bmatrix} \\
\bar{Y} &= \begin{bmatrix} D & C\bar{B} & C\bar{A}\bar{B} & \dots & C\bar{A}^{p-1}\bar{B} \end{bmatrix} \qquad (5) \\
V &= \begin{bmatrix} u(p) & u(p+1) & \dots & u(l-1) \\ v(p-1) & v(p) & \dots & v(l-2) \\ v(p-2) & v(p-1) & \dots & v(l-3) \\ \vdots & \vdots & \ddots & \vdots \\ v(0) & v(1) & \cdots & v(l-p-1) \end{bmatrix}
\end{aligned}
$$

and

$$
\begin{aligned}
\bar{A} &= A + GC \\
\bar{B} &= \begin{bmatrix} B + GD & -G \end{bmatrix} \qquad (6) \\
v(k) &= \begin{bmatrix} u(k) \\ y(k) \end{bmatrix}
\end{aligned}
$$

The matrix y is an mxl output data matrix where m is the number of outputs and l is the number of data samples. The matrix u is an rxl input matrix where r is the number of inputs. The matrix $\bar{Y}$, of dimensions mx$[(r+m)p+r]$ with p an integer, sufficiently large, such that $C\bar{A}^k\bar{B} \approx 0$ for $k \geq p$, contains all the system and observer Markov parameters. G is an nxm arbitrary matrix chosen to make the matrix $\bar{A}$ as stable as desired.

Once the Markov parameters are calculated, it is possible to use them in the ERA algorithm Juang [1] to obtain the system state-space matrices, in agreement with equations (7) and (8).

$$
\begin{aligned}
\hat{A} &= \Sigma_n^{-\frac{1}{2}} R_n H(1) S_n \Sigma_n^{-\frac{1}{2}} \\
\hat{B} &= \Sigma_n^{\frac{1}{2}} S_n^T E_r \qquad (7) \\
\hat{C} &= E_m^T R_n \Sigma_n^{\frac{1}{2}}
\end{aligned}
$$

and

$$
H(k-1) = \begin{bmatrix} Y_k & Y_{k+1} & \dots & Y_{k+\beta-1} \\ Y_{k+1} & Y_{k+2} & \dots & Y_{k+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{k+\alpha-1} & Y_{k+\alpha} & \dots & Y_{k+\alpha+\beta-2} \end{bmatrix} \qquad (8)
$$

Equation (8) is the Hankel matrix, R_n and S_n are the matrices formed by the first n columns of R and S respectively, and R and S are orthonormal such that

$$
H(0) = R\Sigma S^T \qquad (9)
$$

and

$$
\begin{aligned}
E_m^T &= \begin{bmatrix} I_m & O_m & \cdots & O_m \end{bmatrix} \\
E_r^T &= \begin{bmatrix} I_r & O_r & \cdots & O_r \end{bmatrix}
\end{aligned}
\tag{10}
$$

O_i is defined as a null matrix of order i and I_i is an identity matrix of order i.

From the minimum order realization $\left[\hat{A},\ \hat{B},\ \hat{C}\right]$, it is possible to find the eigensolution of the realized state matrix and transform the realized model to modal coordinates. From equation (11), then calculate the system's damping and frequencies.

$$
\begin{aligned}
\hat{A} &= \hat{\Psi}\hat{\Lambda}\hat{\Psi}^{-1} \\
\hat{B}_m &= \hat{\Psi}^{-1}\hat{B} \\
\hat{C}_m &= \hat{C}\hat{\Psi}
\end{aligned}
\tag{11}
$$

Subspace Identification algorithm

The robust combined subspace algorithm proposed by Van Overschee [3] was used. This method required different oblique and orthogonal projections of the row space of the output block Hankel matrices on the row space of the block Hankel matrices consisting of inputs and outputs, see equation (12). The order of the system is obtained from the singular value decomposition of the weighted oblique projection and the state-space system matrices can be obtained using equations (13) and (14).

$$
\begin{aligned}
\vartheta_i &= Y_f /_{U_f} W_p \\
Z_i &= Y_f / \begin{pmatrix} W_p \\ U_f \end{pmatrix} \\
Z_{i+1} &= Y_f^- / \begin{pmatrix} W_p^+ \\ U_f^- \end{pmatrix}
\end{aligned}
\tag{12}
$$

$$
\begin{pmatrix} \Gamma_{i-1}^t . Z_{i+1} \\ \overline{Y_{i|i}} \end{pmatrix} = \begin{pmatrix} A \\ C \end{pmatrix} . \Gamma_i^t . Z_i + \mathrm{K} . U_f + \begin{pmatrix} \rho_w \\ \overline{\rho_v} \end{pmatrix}
\tag{13}
$$

$$
\begin{aligned}
B, D = \arg\min_{B,D} n \, \| & \begin{pmatrix} \Gamma_{i-1}^t . Z_{i+1} \\ Y_{i|i} \end{pmatrix} - \begin{pmatrix} A \\ C \end{pmatrix} . \Gamma_i^t . Z_i \\
& - \mathrm{K}\left(B, D\right) . U_f \, \|_F^2
\end{aligned}
\tag{14}
$$

where

$$
\begin{aligned}
\Gamma_i &= U_1 S_1^{1/2} \\
\Gamma_{i-1} &= \underline{\Gamma_i}
\end{aligned}
\tag{15}
$$

IDENTIFICATION RESULTS

Modal parameters identification

The time invariant single degree of freedom linear system's properties, such as mass, stiffness and damping, are chosen in order to obtain the maximum responses when the El Centro and SCTEO earthquakes are respectively applied.

The system's properties for the El Centro earthquake are shown in equation (16).

$$
\begin{aligned}
m &= 0.01428 \frac{kg \sec^2}{cm} \\
c &= 0.033 \frac{kg \sec}{cm} \\
k &= 16.00 \frac{kg}{cm} \\
T &= 0.1877 \sec \\
\varsigma &= 0.05
\end{aligned}
\qquad (16)
$$

On the other hand, the system's properties when the SCTEO earthquake is applied are shown in equation (17).

$$
\begin{aligned}
m &= 0.01428 \frac{kg \sec^2}{cm} \\
c &= 0.033 \frac{kg \sec}{cm} \\
k &= 0.14 \frac{kg}{cm} \\
T &= 2.00 \sec \\
\varsigma &= 0.05
\end{aligned}
\qquad (17)
$$

When the three algorithms are applied to the time-invariant single degree of freedom linear system, the following results, showed in Table 1, are obtained for the natural period of the system *T*.

Table 1. Identified natural period of the system *T*

Earthquake	Algorithm	0% Noise Level [rad/s]	2% Noise Level [rad/s]	5% Noise Level [rad/s]
El Centro	LSM	0.1877	0.1877	0.1878
El Centro	ERA-OKID	0.1880	0.1880	0.1880
El Centro	Subspace	0.1880	0.1880	0.1880
SCTEO	LSM	2.00	2.01	2.00
SCTEO	ERA-OKID	2.01	1.99	0.04
SCTEO	Subspace	2.01	2.01	2.01

The results for the damping factor, when the identification algorithms are applied to the system, are showed in Table 2.

Table 2. Identified damping factor ζ

Earthquake	Algorithm	0% Noise Level	2% Noise Level	5% Noise Level
El Centro	LSM	0.0500	0.0506	0.0514
El Centro	ERA-OKID	0.0491	0.0492	0.0493
El Centro	Subspace	0.0499	0.0502	0.0504
SCTEO	LSM	0.0500	0.0516	0.0584
SCTEO	ERA-OKID	0.0494	0.0210	0.0282
SCTEO	Subspace	0.0501	0.0501	0.0503

Estimated system response

Once the modal parameters are identified, the estimated system response was obtained in order to compare the effectiveness of the three algorithms. The estimated accelerations, velocities and displacements were calculated, and used to obtain the error defined by equation (18)

$$e = \frac{\sqrt{\left(\sum_{i=1}^{n}\left(y-\hat{y}\right)^{2}\right)}}{n} \qquad (18)$$

In Tables 3, 4 and 5 are the error values for the estimated accelerations, velocities and displacements respectively.

Table 3. Calculated error for estimated accelerations

Earthquake	Algorithm	0% Noise Level	2% Noise Level	5% Noise Level
El Centro	LSM	1.2404	1.2478	1.2601
El Centro	ERA-OKID	0.0371	0.0639	0.1203
El Centro	Subspace	0.0080	0.0518	0.1138
SCTEO	LSM	0.0661	0.0717	0.1598
SCTEO	ERA-OKID	0.0171	1.4531	2.1719
SCTEO	Subspace	0.0005	0.0299	0.0676

Table 4. Calculated error for estimated velocities

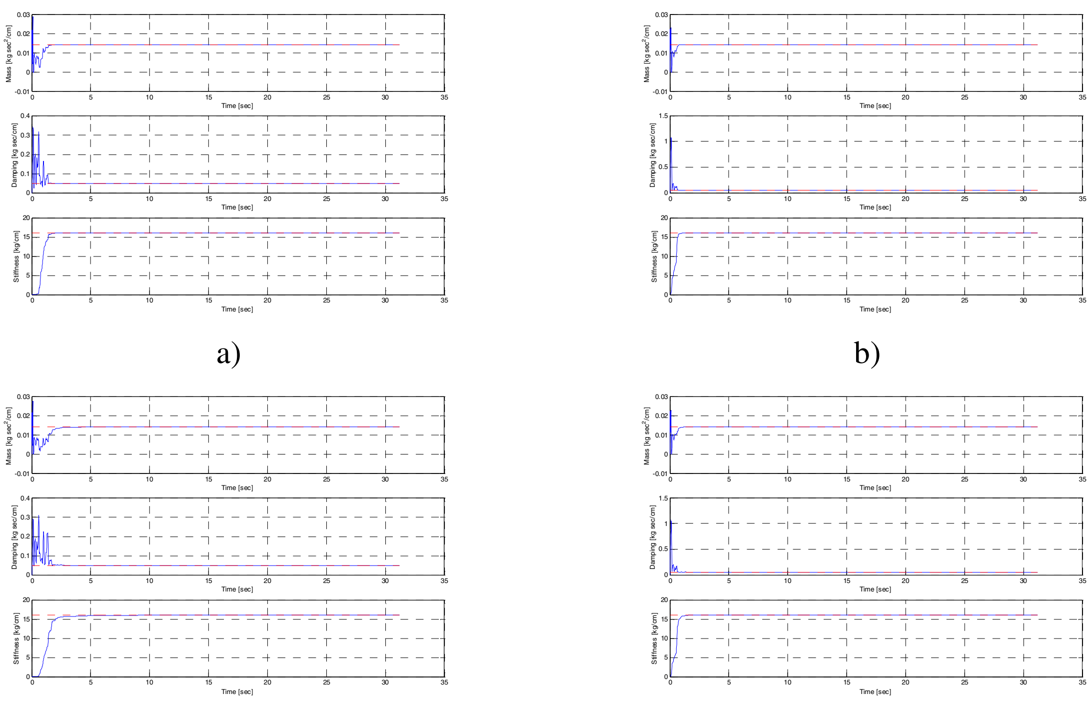
Earthquake	Algorithm	0% Noise Level	2% Noise Level	5% Noise Level
El Centro	LSM	0.0356	0.0355	0.0355
El Centro	ERA-OKID	0.0044	0.0048	0.0058
El Centro	Subspace	0.0022	0.0028	0.0042
SCTEO	LSM	0.0210	0.0268	0.0609
SCTEO	ERA-OKID	0.0046	0.4758	0.6644
SCTEO	Subspace	0.0002	0.0096	0.0217

Table 5. Calculated error for estimated displacements

Earthquake	Algorithm	0% Noise Level	2% Noise Level	5% Noise Level
El Centro	LSM	0.0000	0.0001	0.0001
El Centro	ERA-OKID	$3.5520x10^{-5}$	$5.9398x10^{-5}$	$1.1221x10^{-4}$
El Centro	Subspace	$5.5146x10^{-6}$	$4.7749x10^{-5}$	$1.0657x10^{-4}$
SCTEO	LSM	$5.4963x10^{-14}$	$4.5957x10^{-3}$	$1.7379x10^{-2}$
SCTEO	ERA-OKID	$1.7195x10^{-3}$	$1.4747x10^{-1}$	$2.1942x10^{-1}$
SCTEO	Subspace	$4.5946x10^{-5}$	$3.0803x10^{-3}$	$6.8926x10^{-3}$

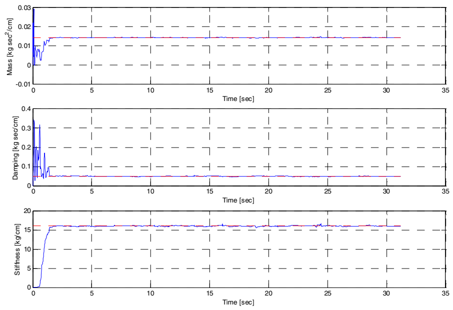
On-line parameter identification

Figures 1, 2 and 3 show the time variation of the identified structural parameters for the SDOF system using El Centro earthquake. Three different noise levels and four combinations of the forgetting factor (λ), and covariance matrix initial values (V), for the recursive least squares method were used. The theoretical values are shown in dashed red line, while the identified values are shown in continuous blue line.



a) b) c) d)

Figure 1. Structural parameters time variation for 0% noise level. a)λ = 0.95, V = 10. b)λ = 0.95, V = 1000. c)λ = 0.99, V = 10. d)λ = 0.99, V = 1000.



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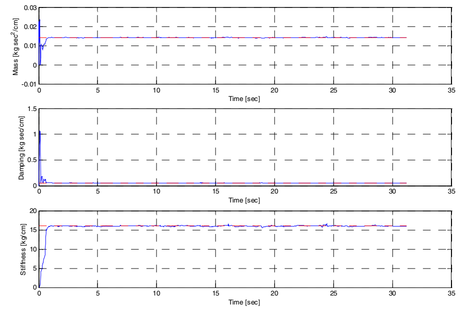

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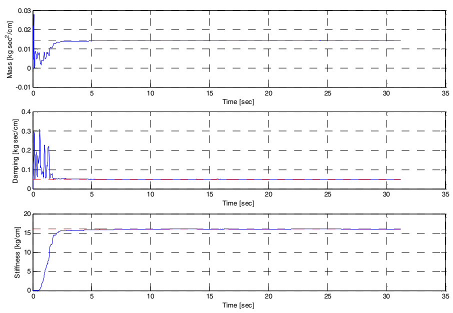

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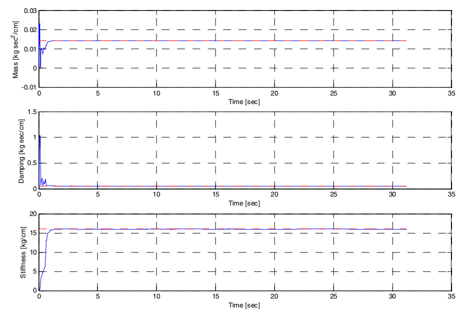
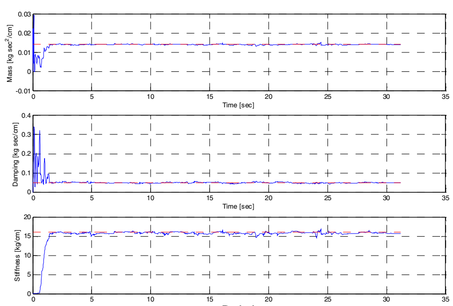


d)

Figure 2. Structural parameters time variation for 2% noise level. a)$\lambda = 0.95$, $V = 10$. b)$\lambda = 0.95$, $V = 1000$. c)$\lambda = 0.99$, $V = 10$. d)$\lambda = 0.99$, $V = 1000$.



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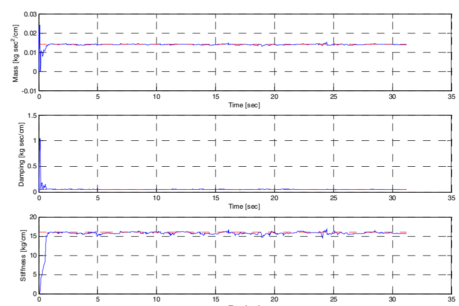

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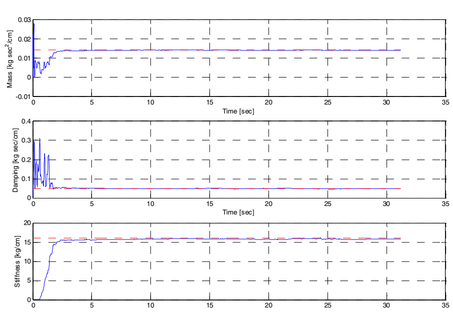

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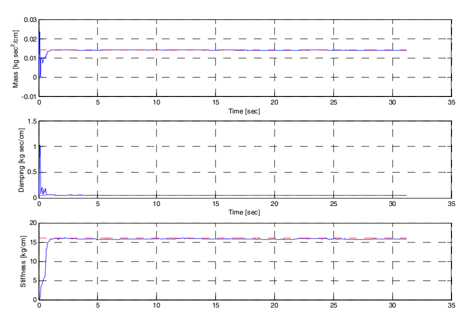
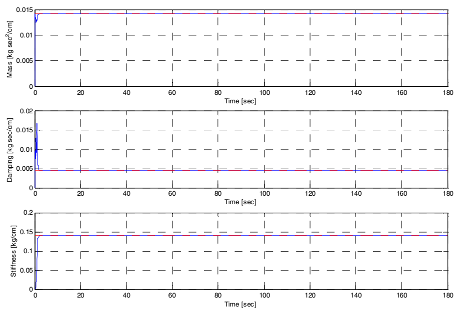


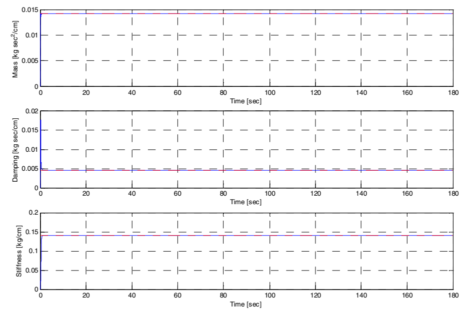
d)

Figure 3. Structural parameters time variation for 5% noise level. a)$\lambda = 0.95$, $V = 10$. b)$\lambda = 0.95$, $V = 1000$. c)$\lambda = 0.99$, $V = 10$. d)$\lambda = 0.99$, $V = 1000$.

Figures 4, 5 and 6 show the time variation of the identified structural parameters for the SDOF system using SCTEO earthquake. Three different noise levels and four combinations of the forgetting factor (λ), and covariance matrix initial values (V), for the recursive least squares method were used. The theoretical values are shown in dashed red line, while the identified values are shown in continuous blue line.



a)



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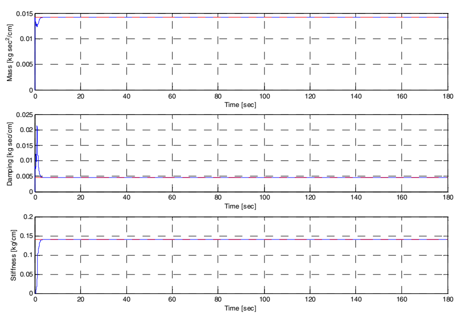

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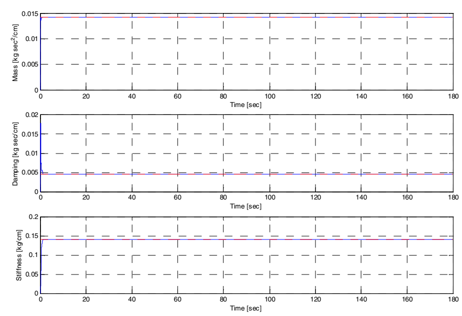


d)

Figure 4. Structural parameters time variation for 0% noise level. a)λ = 0.95, V = 10. b)λ = 0.95, V = 1000. c)λ = 0.99, V = 10. d)λ = 0.99, V = 1000.

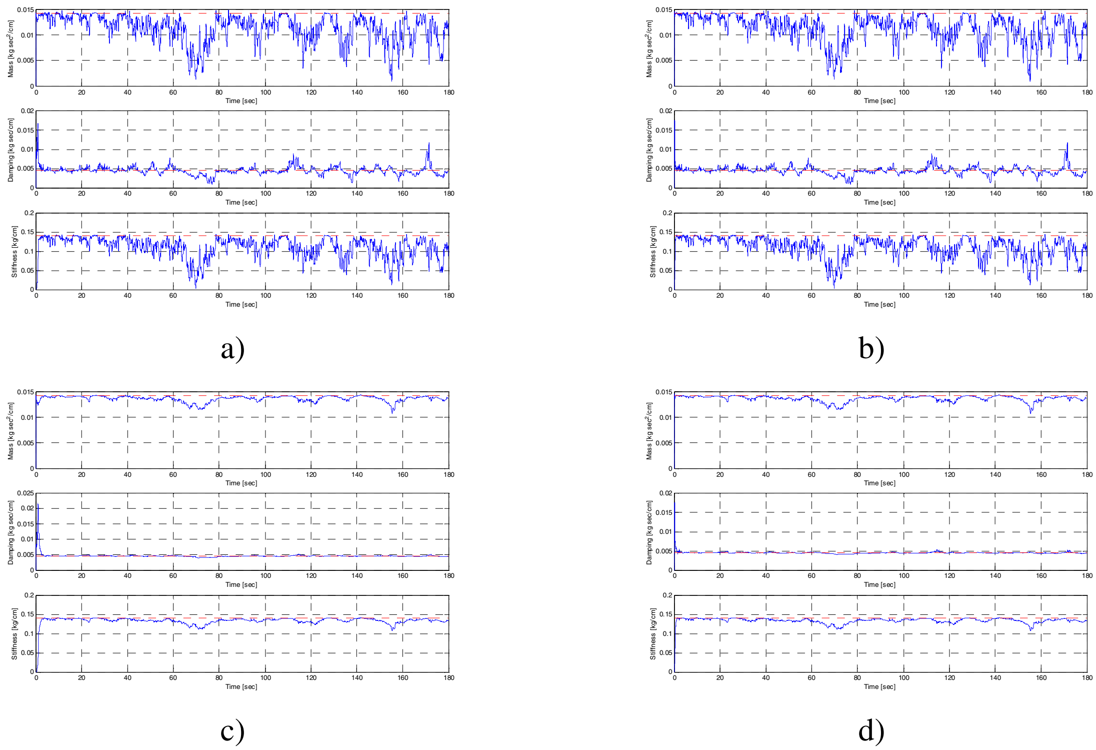


Figure 5. Structural parameters time variation for 2% noise level. a)$\lambda = 0.95$, $V = 10$. b)$\lambda = 0.95$, $V = 1000$. c)$\lambda = 0.99$, $V = 10$. d)$\lambda = 0.99$, $V = 1000$.

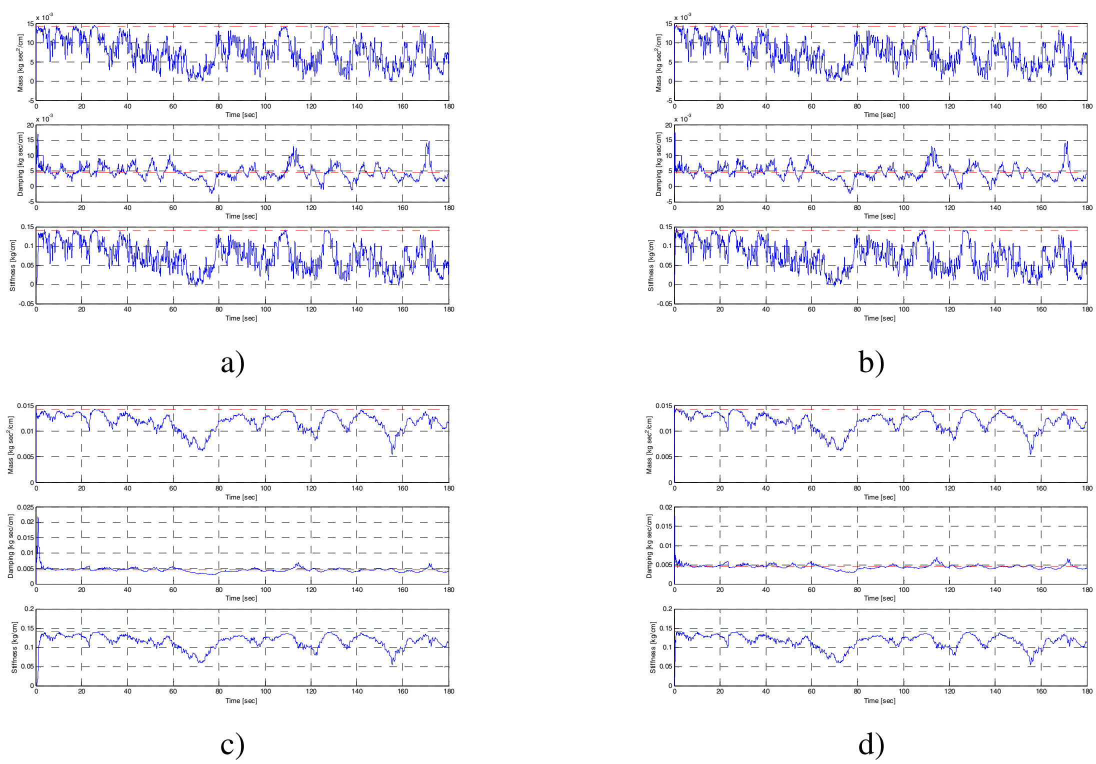
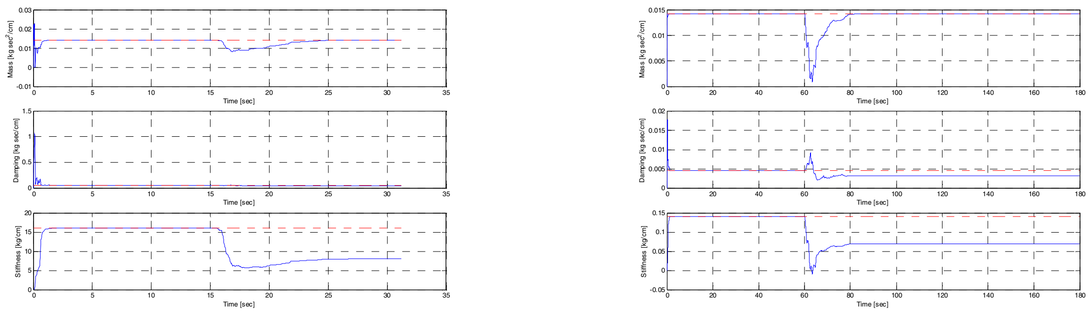


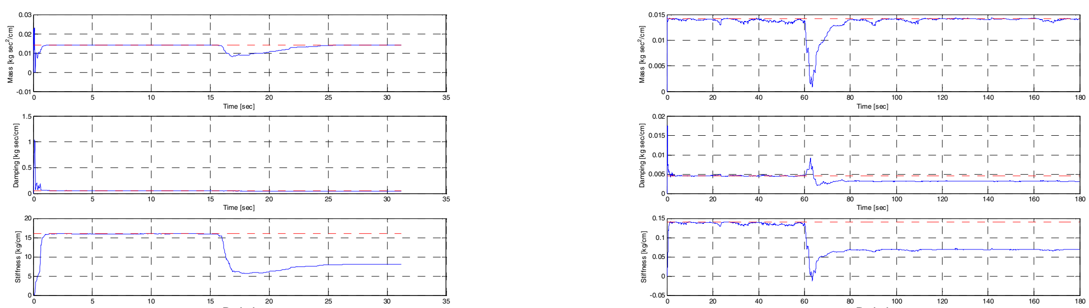
Figure 6. Structural parameters time variation for 5% noise level. a)$\lambda = 0.95$, $V = 10$. b)$\lambda = 0.95$, $V = 1000$. c)$\lambda = 0.99$, $V = 10$. d)$\lambda = 0.99$, $V = 1000$.

It is of special interest to observe the time variation of the structural parameters when an abrupt stiffness degradation is present, during the high intensity acceleration record. For both earthquakes, it is assumed that a stiffness degradation of 50% has occurred. Figures 7, 8 and 9 show the time variation of the identified structural parameters for the SDOF system using El Cetro and SCTEO earthquakes. Again, three different noise levels with $\lambda = 0.99$ and $V = 1000$ were used. The theoretical values are shown in dashed red line, while the identified values are shown in continuous blue line.



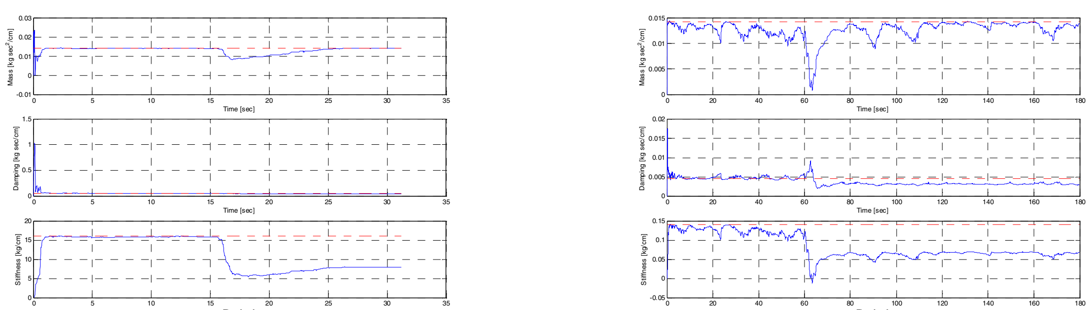
a) b)

Figure 7. Structural parameters time variation for 0% noise level and stiffness degradation. a)EL Centro. b)SCTEO.



a) b)

Figure 8. Structural parameters time variation for 2% noise level and stiffness degradation. a)EL Centro. b)SCTEO.



a) b)

Figure 9. Structural parameters time variation for 5% noise level and stiffness degradation. a)EL Centro. b)SCTEO.

RESULT ANALYSIS

From the simulations it could be observed that the identification of modal parameters could be done with any of the three algorithms if the input excitation has a broad frequency range, such as the El Centro earthquake, however, with an excitation of the kind of the SCTEO earthquake, with a narrow frequency range, the identification for this parameters is poor with noise levels from 2% and higher.

When the estimated system responses are compared among the three algorithms, it is clear that the subspace algorithm allows to obtain the smallest error for almost every case. In that sense, even with the highest noise level, the subspace algorithm represents a good option in order to estimate the system response.

On the other hand, the ERA-OKID algorithm gives a higher error when used with the SCTEO earthquake as the base excitation. This might probably be due to the harmonic nature of the earthquake signal.

From Figures 1 to 9, it can be observed that the best combination for the forgetting factor λ and the covariance matrix initial value V was λ = 0.99 and V=1000. However, it is important to note that the smaller the λ value, the faster the algorithm can respond to changes. Yet, if it is chosen too small, the results become very sensitive to noise.

CONCLUSIONS

The present research shows that good results can be obtained with the three identification algorithms when applied to a noisy system and the excitation has a high frequency content. On the other hand, when the excitation is a harmonic signal, the subspace algorithm gives better estimations. This kind of state-space identification algorithm could allow to obtain reliable modal parameters, in order to establish a fault detection criteria through the variation of the fundamental natural periods with very harmonic earthquakes, as is the case in Mexico City.

Nevertheless, it is important to consider the possibility to improve the implementation of the recursive Least Squares Method algorithm in civil structures to obtain the structural parameters, such as stiffness and damping, that would allow to evaluate the structures behavior or fault detection.

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