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SEISMIC ANALYSIS OF STEEL FRAMES WITH A VISCOELASTIC MODEL OF SEMI-RIGID CONNECTIONS

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SUMMARY

To mitigate the seismic response of partially restrained steel frames, elastomeric materials can be placed at the joints. Unfortunately, some inconsistencies affect the available computational methods. In the paper, the dynamic stiffness matrix of a flexible beam with viscoelastic hinges at both ends is derived in a consistent form. The associated integro-differential equations of motion are turned in a set of differential equations, of greater order but easier to solve. The formulation is used to build the state-space model of a SDoF frame with viscoelastic connections. A parametric study on the elastic response spectra is also included.

INTRODUCTION

In order to simplify the calculations, conventional analysis and design of steel frames are carried out under the assumption that beam-to-column joints are either perfectly rigid (fully restrained frames) or ideally pinned (flexible or simple frames). For practical purposes: (i) if the amount of moment that can be transmitted is negligibly small, the joints are assumed as pinned; (ii) if sufficiently large, the joints are assumed as rigid. In real life, however, these idealizations are not meet, since all joints are semi-rigid, in that a rotational discontinuity exists between the connected members.

During the past decade, a number of analytical and experimental researches have been conducted on Partially Restrained (PR) steel frames, i.e. steel frames with semi-rigid connections, which demonstrate the improved accuracy when the effects of the joint flexibility are taken into account. Among the structural properties, the dynamic behaviour of a PR frame may considerably differ from that of a frame made of the same members, but in which joints are rigid or pinned. As an example, it is well know that the eigenproperties (natural frequencies and modal shapes) may drastically change when the pinned beams are converted to fixed beams. If rotational springs are ideally attached at the ends of the beams, then, any intermediate configuration can be achieved by varying the spring stiffness from zero (pinned ends) to infinity (fixed ends). It follows that a reliable assessment of the joint fixity is required in predicting the

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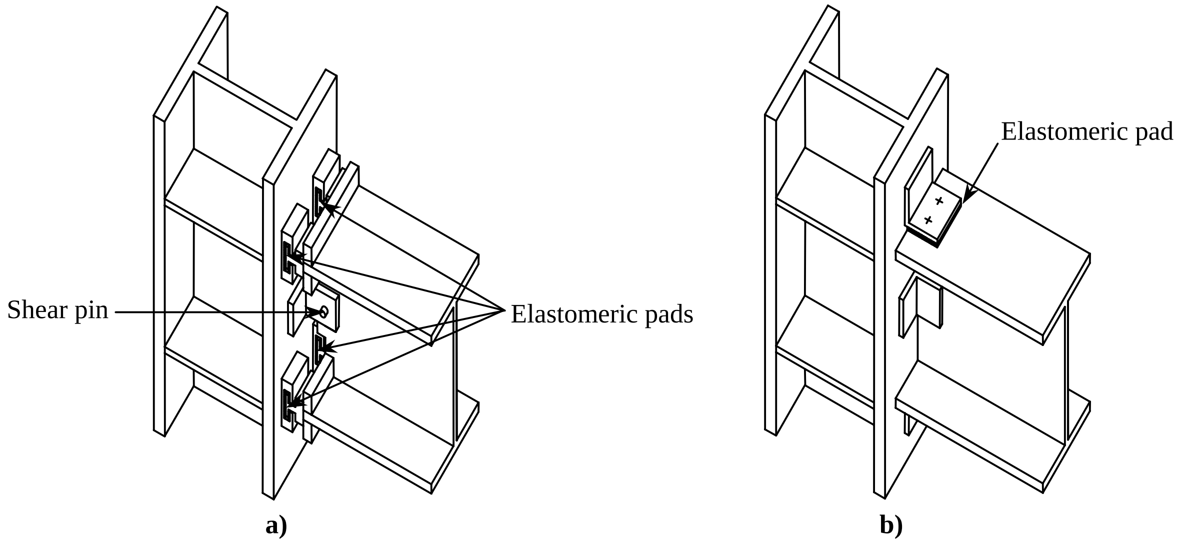


Fig. 1 – 3D schemes of alternatives viscoelastic connections

seismic response of PR frames. In particular, numerical investigations showed that the joint flexibility tends to increase the interstory drifts, and to reduce base shears and base overturning moments [1, 2].

Since the actual behaviour of the connections in a steel frame is semi-rigid, Hsu and Fafitis [3] proposed an Energy Dissipation Device (EDD) which takes advantage of the rotational discontinuity existing between the beam and the column. The connection damper (Fig. 1a) is made of elastomeric pads symmetrically placed with respect to a shear pin: so doing, the latter transmits the shear force, and each pad is subjected only to axial force. As an alternative, a simpler EDD (Fig. 1b) is considered in [4], by placing elastomeric materials between angles and beam flanges: so doing, the classical semi-rigid connection remains unchanged.

By virtue of its simplicity, the Kelvin-Voigt (KV) model, made of an elastic spring in parallel with a viscous dashpot, has been used in modelling these connection dampers, while more refined models, e.g. the Generalized Maxwell (GM) model, have been ignored. Kawashima and Fujimoto [5] derived in explicit form the dynamic stiffness matrix of a flexible beam with KV hinges at both ends. As the latter is a function of the vibration frequency, an expansion was used to obtain frequency-independent inertia, damping and stiffness matrices for the system. Recently, Xu and Zhang [4] used this model to investigate how stiffness and damping of the connections affect the seismic performances of the frame. Afterwards, Cacciola, Colajanni and Muscolino [6] carried out sensitivity analyses with respect to their nominal values. The studies elucidated that: (i) there are optimal values of stiffness and damping of the EDDs by which the seismic response of a PR frame can be significantly reduced; (ii) special cautions are required in the design procedure, as the seismic response may be highly sensitive with respect to stiffness and damping of the EDDs.

Since the KV model is only a crude approximation of the true behaviour of the viscoelastic EDDs, the latter findings suggest that a more realistic model has to be used in the dynamic analysis of PR frames with connection dampers; otherwise, the inaccuracy in modelling the EDD may affect the design procedure. The KV model, in fact, is often inappropriate to describe the dynamic stiffness of elastomeric materials or devices in which the nature of the damping is viscoelastic rather than viscous, and the inaccuracy arising when a somehow equivalent viscous damping is used may be intolerable for engineering purposes [7]. The inappropriateness of the KV model in this circumstance is implicitly confirmed in the work of Hsu and Fafitis [3]. In their KV-type model, in fact, the viscosity coefficient of the dashpot varies inversely with the vibration frequency: as a consequence, the energy dissipated in a sinusoidal cycle becomes independent of the vibration frequency. This dissipation is referred in the literature as linear hysteretic, and it can be viewed just as a particular case of the viscoelastic dissipation [8]. Moreover, in the formulation of Kawashima and Fujimoto [5] the further inconsistency exists that the matrix of inertia depends on stiffness and damping of the joints, being this dependence physically unjustified.

To overcome these theoretical and practical shortcomings, in the paper, a consistent viscoelastic model is used for the connection dampers, whose parameters can be directly evaluated from the relaxation spectrum of the elastomeric materials placed at the joints. In a first stage, the dynamic stiffness matrix of a homogeneous beam in bending with viscoelastic hinges at both ends is derived in the frequency domain. The proposed formulation leads in the time domain to integro-differential equations of motion, quite cumbersome to solve. In order to reduce the computational effort, the state of any viscoelastic system can be enlarged with a number of internal variables, that bear the information about the deformation history of the viscoelastic components; so doing, the equations of motion can be turned in a set of differential equations, easier to solve [9]. In a second stage, then, the latter approach is applied to derive the state-space model of a Single-Degree-of-Freedom (SDoF) portal frame with viscoelastic joints. In a third stage, finally, the proposed model is used to evaluate the elastic response spectra for a recorded ground motion (the 1976 Friuli earthquake), and the elastic response spectra consistent with the elastic design spectrum proposed for rigid soils in the Italian seismic code [10].

FREQUENCY-DEPENDENT DYNAMIC STIFFNESS MATRIX OF A FLEXIBLE BEAM WITH VISCOELASTIC HINGES

Let us considerer the system depicted in Fig. 2a, made of an Euler-Bernoulli beam with connection dampers at both ends. The following assumptions are made: (i) the beam is homogeneous and linear, fully defined through the elastic modulus E, the moment of inertia I_{b}, and the length ℓ; (ii) the connection dampers are rotational springs featuring a linear viscoelastic behaviour, so that the i-th one is fully defined through the relaxation functions $\phi_i(t)$, i.e. the time history of the moment due to an unit rotation suddenly applied for $t \geq 0$.

Modelling the dynamic stiffness of the connection dampers

By using the Boltzman superposition principle, the moment $M_i(t)$ experienced by the i-th EDD is given by:

$$M_i(t) = \int_0^t \phi_i(t-\tau)\left[\dot{\vartheta}_i(\tau) - \dot{\vartheta}'_i(\tau)\right]\mathrm{d}\tau \qquad (1)$$

where $\vartheta_i(t)$ and $\vartheta'_i(t)$ are the rotations at the global node i, and at the internal node i', respectively.
The KV model (Fig. 3a), i.e. an elastic spring in parallel with a viscous dashpot, has been widely used in the literature by virtue of its simplicity. The relaxation function $\phi_i(t)$ takes the expression:

$$\phi_i^{\mathrm{KV}}(t) = c_i\,\delta(t) + k_i\,\mathcal{U}(t) \qquad (2)$$

where c_i and k_i are the viscosity coefficient of the dashpot and the elastic stiffness of the spring, respectively, $\delta(\cdot)$ is the Dirac's delta function, and $\mathcal{U}(\cdot)$ is the unit step function.

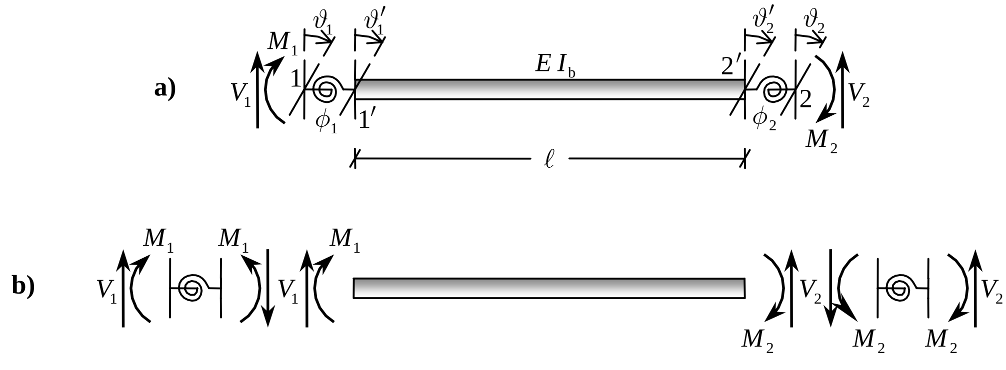


Fig. 2 – Elastic beam with viscoelastic hinges: a) global scheme; b) disassembled components

After some algebra, substitution of Eq. (2) into Eq. (1) brings:

$$M_i(t) = k_i\left[\vartheta_i(t) - \vartheta_i'(t)\right] + c_i\left[\dot{\vartheta}_i(t) - \dot{\vartheta}_i'(t)\right]$$

that is, the EDD is without memory, as the knowledge of rotation and angular velocity of nodes i and i' at time t allows computing the moment $M_i(t)$.

Since elastomeric materials to be used in practical applications may exhibit a non-negligible viscoelastic memory, the latter can be effectively approximated through the GM model (Fig. 3b), made of an elastic spring in parallel with a number of Maxwell elements, each one given by an elastic spring in series with a viscous dashpot. The relaxation function $\phi_i(t)$ takes the expression:

$$\phi_i^{\mathrm{GM}}(t) = \left(k_{i,0} + \sum\nolimits_r k_{i,r}\,\mathrm{e}^{-t/\tau_{i,r}}\right)\mathcal{U}(t) \tag{3}$$

where the equilibrium modulus $k_{i,0}$ is the long-term elastic stiffness, and the pairs $(\tau_{i,r}, k_{i,r})$ define the discrete relaxation spectrum, i.e. relaxation time and stiffness of the Maxwell elements. Usually, few Maxwell elements are sufficient to accurately model the true behaviour of viscoelastic materials. When only a single Maxwell element is used, the GM model is properly termed Standard Linear Solid (SLS) model: the latter is able to capture the viscoelastic behaviour of materials with a predominant relaxation time. After some algebra, substitution of Eq. (3) into Eq. (1) brings:

$$M_i(t) = k_{i,0}\left[\vartheta_i(t) - \vartheta_i'(t)\right] + \sum\nolimits_r k_{i,r}\int_0^t \mathrm{e}^{-(t-\tau)/\tau_{i,r}}\left[\dot{\vartheta}_i(\tau) - \dot{\vartheta}_i'(\tau)\right]\mathrm{d}\tau$$

In this case, then, the moment $M_i(t)$ experienced by the i-th EDD depends in principle on the whole time history of the relative rotation between nodes i and i'.

In order to highlight the dependence of the moment $M_i(t)$ on the vibration frequency ω, Eq. (1) can be rewritten in a mixed time-frequency domain as:

$$M_i(t) = \mathrm{j}\omega\,\mathcal{F}\left\langle\phi_i(t)\right\rangle\left[\vartheta_i(t) - \vartheta_i'(t)\right] \tag{4}$$

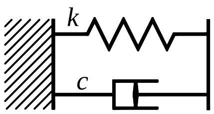
in which $\mathrm{j}=\sqrt{-1}$ is the imaginary unit, and $\mathcal{F}\langle\cdot\rangle$ stands for the Fourier transform operator. Even if formally not rigorous, Eq. (4) expresses that the complex-valued dynamic stiffness of the i-th viscoelastic hinge is:

$$k_i(\omega) = \mathrm{j}\omega\,\mathcal{F}\left\langle\phi_i(t)\right\rangle = k_i'(\omega) + \mathrm{j}k_i''(\omega) \tag{5}$$

where the real-valued storage modulus $k_i'(\omega) = \mathrm{Re}\left[k_i(\omega)\right]$ and loss modulus $k_i''(\omega) = \mathrm{Im}\left[k_i(\omega)\right]$ are even and odd functions of the vibration frequency, respectively: the former is proportional to the maximum energy stored in the hinge during a sinusoidal cycle, while the latter is proportional to the energy dissipated. Alternatively, the dynamic stiffness can be written as:

$$k_i(\omega) = k_i'(\omega)\left[1 + \mathrm{j}\eta_i(\omega)\right]$$

where the loss factor $\eta_i(\omega) = k_i''(\omega)/k_i'(\omega)$ measures the damping capability of the i-th hinge as a function of the vibration frequency: the equivalent viscous damping ratio at $\omega = \omega_0$, in fact, is $\zeta_0 = \eta(\omega_0)/2$.



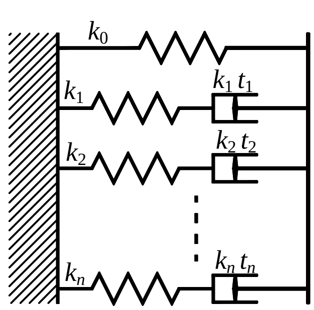


Fig. 3 – Spring-dashpot models: a) Kelvin-Voigt (KV) model; b) Generalized Maxwell (GM) model

For the KV and the GM models the dynamic stiffness is known in closed form:

$$k_i^{\mathrm{KV}}(\omega) = k_i + \mathrm{j}\omega c_i \quad ; \quad k_i^{\mathrm{GM}}(\omega) = k_{i,0} + \sum_r \frac{k_{i,r}\,\tau_{i,r}\,\omega}{\tau_{i,r}\,\omega - \mathrm{j}} \tag{6}$$

From the first of Eqs. (6) one can see that in the KV model the dissipation of energy increases proportionally with the vibration frequency. This behaviour is not meet in elastomeric materials: as a consequence, the KV approximation may fictitiously over-damps high-frequency vibrations. Oppositely, the GM model, with an adequate number of Maxwell elements, is able to approximate the actual damping of any viscoelastic EDD.

Modelling the dynamic stiffness matrix of a beam with viscoelastic hinges

After that an expedient model has been selected for the EDDs, e.g. the GM model with n Maxwell elements, Eq. (4) and (5), for $i = 1,2$, can be arranged in a matrix form as:

$$\mathbf{m}(t) = \mathbf{K}_{\mathrm{d}}(\omega)\left[\boldsymbol{\vartheta}(t) - \boldsymbol{\vartheta}'(t)\right] \tag{7}$$

where the arrays $\mathbf{m}(t) = \left[M_1(t) \quad M_2(t)\right]^{\mathrm{T}}$, $\boldsymbol{\vartheta}(t) = \left[\vartheta_1(t) \quad \vartheta_2(t)\right]^{\mathrm{T}}$, and $\boldsymbol{\vartheta}'(t) = \left[\vartheta_1'(t) \quad \vartheta_2'(t)\right]^{\mathrm{T}}$ list moments, global rotations and internal rotations at the ends of the beam, respectively, and $\mathbf{K}_{\mathrm{d}}(\omega)$ is the dynamic stiffness matrix of the viscoelastic dampers:

$$\mathbf{K}_{\mathrm{d}}(\omega) = \begin{bmatrix} k_1(\omega) & 0 \\ 0 & k_2(\omega) \end{bmatrix} = \mathrm{j}\omega \begin{bmatrix} \mathcal{F}\left\langle \phi_1(t) \right\rangle & 0 \\ 0 & \mathcal{F}\left\langle \phi_2(t) \right\rangle \end{bmatrix}$$

Since no external moments are directly applied to the internal nodes $1'$ and $2'$, $M_1(t)$ and $M_2(t)$ are just the moments acting on the beam ends (Fig. 2b). As a consequence, internal rotations and external moments are related as:

$$\mathbf{K}_{\mathrm{e}}\,\boldsymbol{\vartheta}'(t) = \mathbf{m}(t) \tag{8}$$

where $\mathbf{K}_{\mathrm{e}}$ is the stiffness matrix of the elastic beam for the rotations:

$$\mathbf{K}_{\mathrm{e}} = \frac{4\,E\,I_{\mathrm{b}}}{\ell}\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}$$

The long term stiffness of the i-th joint, i.e. $k_i(0)$, can be defined in comparison with the rotational stiffness of the beam:

$$k_i(0) = \frac{4\,E\,I_{\mathrm{b}}}{\ell}\,\frac{\nu_i}{1-\nu_i}$$

where ν_i is the associated fixity factor:

$$\nu_i = \frac{k_i(0)}{4\,E\,I_{\mathrm{b}}/\ell + k_i(0)}$$

The value of this dimensionless quantity, then, varies from $\nu_i = 0$ for an ideally pinned joint ($k_i \equiv 0$), to $\nu_i = 1$ for a perfectly rigid joint ($k_i \equiv \infty$).

Substitution of Eq. (7) into Eq. (8) brings:

$$\mathbf{K}_{\mathrm{e}}\,\boldsymbol{\vartheta}'(t) = \mathbf{K}_{\mathrm{d}}(\omega)\left[\boldsymbol{\vartheta}(t) - \boldsymbol{\vartheta}'(t)\right]$$

from which the relationship between global and internal rotations can be derived:

$$\boldsymbol{\vartheta}'(t) = \left[\mathbf{K}_{\mathrm{e}} + \mathbf{K}_{\mathrm{d}}(\omega)\right]^{-1}\mathbf{K}_{\mathrm{d}}(\omega)\,\boldsymbol{\vartheta}(t) \tag{9}$$

where the matrix $\left[\mathbf{K}_{\mathrm{e}} + \mathbf{K}_{\mathrm{d}}(\omega)\right]$ is non-singular. Upon substitution of Eq. (9) into Eq. (7), and after some algebra, the dynamic stiffness matrix of the beam with viscoelastic hinges is then obtained:

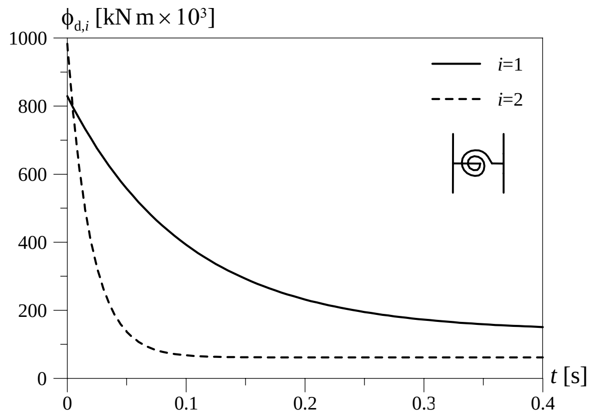


Fig. 4 – Relaxation functions of the viscoelastic hinges at the beam ends

$$\mathbf{m}(t)=\mathbf{K}_\mathrm{b}(\omega)\boldsymbol{\vartheta}(t) \quad ; \quad \mathbf{K}_\mathrm{b}(\omega)=\mathbf{K}_\mathrm{d}(\omega)\left\{\mathbf{I}_2-\left[\mathbf{K}_\mathrm{e}+\mathbf{K}_\mathrm{d}(\omega)\right]^{-1}\mathbf{K}_\mathrm{d}(\omega)\right\} \tag{10}$$

in which $\mathbf{I}_2$ stands for the 2×2 identity matrix. Consistently, in the time domain the first of Eqs. (10) becomes:

$$\mathbf{m}(t)=\int_0^t \boldsymbol{\Phi}_\mathrm{b}(t-\tau)\,\dot{\boldsymbol{\vartheta}}(\tau)\,\mathrm{d}\tau \quad ; \quad \boldsymbol{\Phi}_\mathrm{b}(t)=\mathcal{F}^{-1}\left\langle \frac{\mathbf{K}_\mathrm{b}(\omega)}{\mathrm{j}\omega}\right\rangle$$

in which $\mathcal{F}^{-1}\langle\cdot\rangle$ is the inverse Fourier transform operator, and $\boldsymbol{\Phi}_\mathrm{b}(t)$ is relaxation function matrix of the model depicted in Fig. 2a: that is, the i -th column lists the moments $M_1(t)$ and $M_2(t)$ due to a rotation $\vartheta_i(t)=1$ suddenly applied at the i -th global node for $t\geq 0$.

Some considerations by means of a simple application

The proposed procedure was implemented in order to investigate the effects of the viscoelastic properties of the EDDs on the global response of a realistic semi-rigid beam with elastomeric connections. The following parameters were selected for the beam: $E=206$ kN/mm^2 , $I_\mathrm{b}=67120\,\mathrm{cm}^4$ (corresponding to the Italian "IPE 550×106" section), and $\ell=6.00$ m; the bending stiffness, then, is: $4EI_\mathrm{b}/\ell=92{,}200\,\mathrm{kN\,m}$. The SLS model was used for the EDDs: in the first end ($i=1$), fixity factor $\nu_1=0.6$ ($\Rightarrow$ equilibrium modulus $k_{1,0}=138{,}300\,\mathrm{kN\,m}$), stiffness and relaxation time of the single Maxwell element $k_{1,1}=5\times k_{1,0}=691{,}300\,\mathrm{kN\,m}$ and $\tau_{1,1}=0.10\,\mathrm{s}$, respectively; in the second end ($i=2$), $\nu_2=0.4$ ($\Rightarrow k_{2,0}=61{,}500\,\mathrm{kN\,m}$), $k_{2,1}=15\times k_{2,0}=921{,}800\,\mathrm{kN\,m}$, and $\tau_{2,1}=0.02\,\mathrm{s}$.

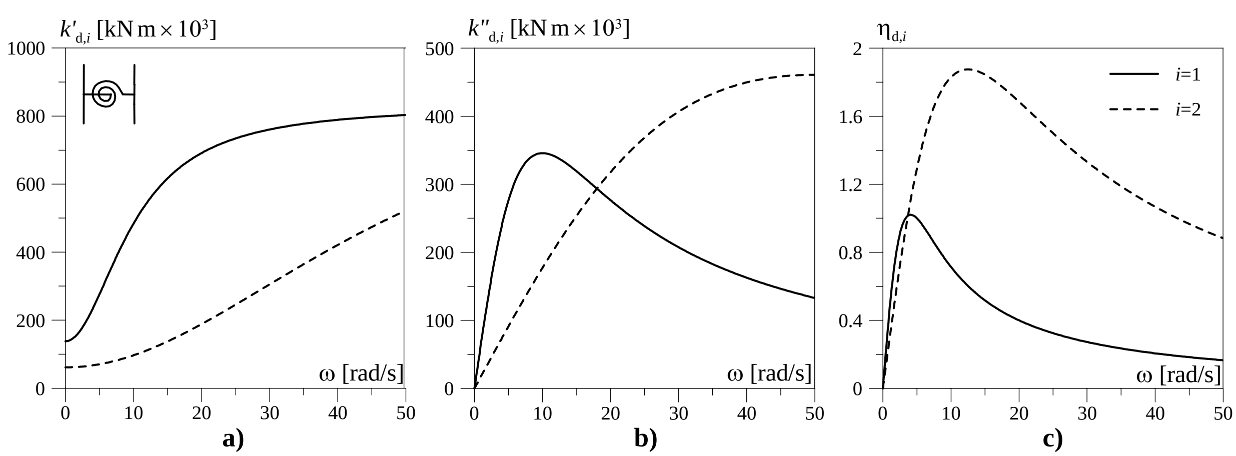


Fig. 5 – Storage modulus (a), loss modulus (b), and loss factor (c) of the viscoelastic hinges

The relaxation functions of the connection dampers are compared in Fig. 4: the dissipation is higher in the second hinge (dashed line), as the relaxation modulus is reached more quickly; oppositely, the first hinge (solid line) is stiffer, since its relaxation function decades more slowly. Fig. 5 confirms these differences; in fact: (i) the storage modulus of the first hinge, which is related to the effective stiffness, takes higher values in the frequency interval of interest (Fig. 5a), (ii) the loss modulus of the second hinge, which is related to the dissipation of energy, upcrosses the loss modulus of the first one at $\omega \cong 18$ rad/s (Fig. 5b), and (iii) the loss factor of the second hinge, which is related to the equivalent damping ratio, is much higher for $\omega > 5$ rad/s (Fig. 5c).
After that the matrices $\mathbf{K}_\mathrm{e}$ and $\mathbf{K}_\mathrm{d}(\omega)$ were defined:

$$\mathbf{K}_\mathrm{e} = \begin{bmatrix} 92.2 & 46.1 \\ 46.1 & 92.2 \end{bmatrix} \times 10^3\,\mathrm{kN\,m} \quad ; \quad \mathbf{K}_\mathrm{d}(\omega) = \begin{bmatrix} 138. + \dfrac{69.1\omega}{0.1\omega - \mathrm{j}} & 0 \\ 0 & 61.5 + \dfrac{18.4\omega}{0.02\omega - \mathrm{j}} \end{bmatrix} \times 10^3\,\mathrm{kN\,m}$$

the dynamic stiffness matrix of the beam with connection dampers was evaluated through the second of Eq. (10). The real part (storage modulus, solid line) and the imaginary part (loss modulus, dashed line) of each element $K_{\mathrm{b},i,j}(\omega)$ are depicted in Fig. 6. Since the EDD at the second end is more dissipative, the imaginary part of the element $K_{\mathrm{b},2,2}(\omega)$ takes higher values with respect to the element $K_{\mathrm{b},1,1}(\omega)$: that is, identical rotations at the ends are associated with different dissipations of energy.

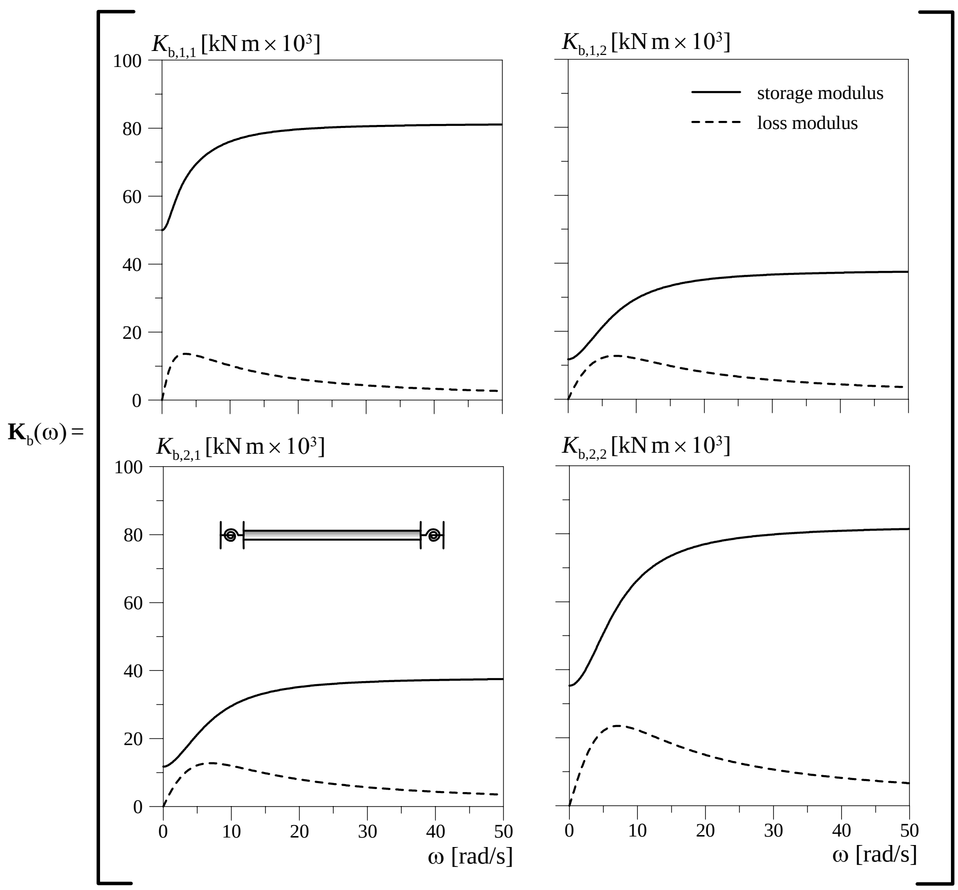


Fig. 6 – Dynamic stiffness matrix of the beam with viscoelastic hinges

CONSISTENT STATE-SPACE MODEL OF A SDOF FRAME WITH VISCOELASTIC JOINTS

Let us considerer the single-story single-bay portal frame depicted in Fig. 7, in which the beam-to-column joints are viscoelastic. The behaviour of the columns is linear elastic, while the beam with EDDs at the ends is modelled in the frequency domain through the dynamic stiffness matrix $\mathbf{K}_\mathrm{b}(\omega)$, given by the second of Eq. (10). Beam and columns are inextensible, and the degrees of freedom of the portal frame are the horizontal translation $x(t)$ and the rotations $\vartheta_1(t)$ and $\vartheta_2(t)$.

After the assemblage, the dynamic stiffness matrix of the frame can be partitioned as:

$$\mathbf{K}_\mathrm{f}(\omega)=\left[\begin{array}{c:c} k_\mathrm{f,t,t} & \mathbf{k}_\mathrm{f,r,t}^\mathrm{T} \\ \hdashline \mathbf{k}_\mathrm{f,r,t} & \mathbf{K}_\mathrm{f,r,r}(\omega)\end{array}\right]=\left[\begin{array}{c:cc} \dfrac{24\,E\,I_\mathrm{c}}{h^3} & -\dfrac{6\,E\,I_\mathrm{c}}{h^2} & -\dfrac{6\,E\,I_\mathrm{c}}{h^2} \\ \hdashline -\dfrac{6\,E\,I_\mathrm{c}}{h^2} & \dfrac{4\,E\,I_\mathrm{c}}{h}+K_\mathrm{b,1,1}(\omega) & K_\mathrm{b,1,2}(\omega) \\ -\dfrac{6\,E\,I_\mathrm{c}}{h^2} & K_\mathrm{b,2,1}(\omega) & \dfrac{4\,E\,I_\mathrm{c}}{h}+K_\mathrm{b,2,2}(\omega)\end{array}\right]$$

where $k_\mathrm{f,t,t}$ is the translation (shear-type) stiffness of the frame, $\mathbf{K}_\mathrm{f,r,r}(\omega)$ is the 2×2 dynamic stiffness matrix associated with the rotations at nodes 1 and 2, $\mathbf{k}_\mathrm{f,r,t}$ is the 2×1 array of the stiffness coefficients that couple rotations and translation, I_c and h are the moment of inertia and the height of the columns, respectively, and $K_{\mathrm{b},i,j}(\omega)$ is the (i,j) element of the 2×2 dynamic stiffness matrix $\mathbf{K}_\mathrm{b}(\omega)$. Under the assumption that the mass is lumped, the dynamic stiffness of the frame can be condensed in the form:

$$k_\mathrm{f}(\omega)=k_\mathrm{f,t,t}-\mathbf{k}_\mathrm{f,r,t}^\mathrm{T}\,\mathbf{K}_\mathrm{f,r,r}^{-1}(\omega)\,\mathbf{k}_\mathrm{f,r,t} \tag{11}$$

In a mixed time-frequency domain, the equation of the seismic motion can be written as:

$$\ddot{x}(t)+2\,\zeta_0\,\omega_0\,\dot{x}(t)+\frac{1}{m}k_\mathrm{f}(\omega)\,x(t)=-\ddot{x}_\mathrm{g}(t) \tag{12}$$

where m is the mass of the frame, $\omega_0=\sqrt{k_\mathrm{f}(0)/m}$ is its undamped natural circular frequency, the viscous damping ratio ζ_0 accounts for the dissipation of the frame without EDDs, and $\ddot{x}_\mathrm{g}(t)$ is the time history of the ground acceleration. The solution of Eq. (12) can be directly evaluated in the frequency domain as:

$$\mathcal{F}\left\langle x(t)\right\rangle=-H_\mathrm{f}(\omega)\,\mathcal{F}\left\langle \ddot{x}_\mathrm{g}(t)\right\rangle \quad ; \quad H_\mathrm{f}(\omega)=\frac{1}{k_\mathrm{f}(\omega)/m-\omega^2+2\,\mathrm{j}\zeta_0\omega_0\,\omega} \tag{13}$$

where $H_\mathrm{f}(\omega)$ is the Frequency Response Function (FRF) of the frame.

In the time domain, the equation of the seismic motion can be properly posed in an integro-differential form:

$$\ddot{x}(t)+2\,\zeta_0\,\omega_0\,\dot{x}(t)+\frac{1}{m}\int_0^t \phi_\mathrm{f}(t-\tau)\,\dot{x}(\tau)\,\mathrm{d}\tau=-\ddot{x}_\mathrm{g}(t) \tag{14}$$

where:

$$\phi_\mathrm{f}(t)=\mathcal{F}^{-1}\left\langle\frac{k_\mathrm{f}(\omega)}{\mathrm{j}\omega}\right\rangle \tag{15}$$

is the relaxation function of the frame, i.e. the time history of the shear force due to a unit displacement $x(t)=1$ suddenly applied for $t\geq 0$, from which the impulsive term $2\,\zeta_0\,\omega_0\,\delta(t)\times m$ associated with the inherent damping has been preventively removed, and separately considered.

Unfortunately, the solution of Eq. (14) is not an easy task, as standard techniques are not available. The computational effort, however, can be reduced by approximating the relaxation function with the GM model:

$$\phi_\mathrm{f}^\mathrm{GM}(t)=\left(k_0+\sum_{r=1}^{n}k_r\,\mathrm{e}^{-t/t_r}\right)\mathcal{U}(t) \tag{16}$$

in which $k_0 \equiv k_\mathrm{f}(0)$ is the equilibrium modulus of the frame, i.e. the stiffness under static loads, and n Maxwell elements, of stiffness k_r and relaxation time τ_r ($r = 1, \cdots, n$), are used to capture the time-varying portion of the relaxation function.

Substitution of Eq. (16) into Eq. (14) leads the convolution integral to be replaced with a linear combination of the horizontal translation $x(t)$ together with n additional internal variables $\lambda_r(t)$, defined as the internal strains of the n Maxwell elements:

$$\int_0^t \phi_\mathrm{f}^\mathrm{GM}(t-\tau)\,\dot{x}(\tau)\,\mathrm{d}\tau = k_0\,x(t) + \sum_{i=1}^{n} k_r\,\lambda_r(t) \tag{17}$$

where:

$$\lambda_r(t) = \int_0^t \mathrm{e}^{-\frac{t-\tau}{\tau_r}}\,\dot{x}(\tau)\,\mathrm{d}\tau \tag{18}$$

The time derivative of Eq. (18) brings [9]:

$$\dot{\lambda}_r(t) = \dot{x}(t) - \frac{\lambda_r(t)}{\tau_r} \tag{19}$$

Eqs. (17) and (19) allow to turn the integro-differential equation of motion (Eq. (14)) in a set of differential equation of order $n+2$, which governs the evolution of the complete state $\mathbf{y}(t)$:

$$\dot{\mathbf{y}}(t) = \mathbf{D}\,\mathbf{y}(t) - \mathbf{v}\,\ddot{x}_g(t) \tag{20}$$

where:

$$\mathbf{y}(t) = \left[\begin{array}{c} x(t) \\ \dot{x}(t) \\ \hdashline \lambda_1(t) \\ \vdots \\ \lambda_n(t) \end{array}\right] \quad ; \quad \mathbf{D} = \left[\begin{array}{cc:ccc} 0 & 1 & 0 & 0 & 0 \\ -\omega_0^2 & -2\zeta_0\,\omega_0 & k_1/m & \cdots & k_n/m \\ \hdashline 0 & 1 & -1/\tau_1 & & \\ \vdots & \vdots & & \ddots & \\ 0 & 1 & & & -1/\tau_n \end{array}\right] \quad ; \quad \mathbf{v} = \left[\begin{array}{c} 0 \\ 1 \\ \hdashline 0 \\ \vdots \\ 0 \end{array}\right] \tag{21}$$

So doing, the memory of the dynamic system is conveniently accounted for with the state variables $\lambda_r(t)$, and the tools of the linear system theory can be used in the analysis.

In the frequency domain, the solution of Eq. (20) is:

$$\mathcal{F}\left\langle \mathbf{y}(t) \right\rangle = -\mathbf{h}(\omega)\,\mathcal{F}\left\langle \ddot{x}_g(t) \right\rangle \quad ; \quad \mathbf{h}(\omega) = \left[\mathrm{j}\omega\,\mathbf{I}_{n+2} - \mathbf{D}\right]^{-1}\mathbf{v}$$

where the first component of the complex-valued array $\mathbf{h}(\omega)$ is the approximated FRF of the frame.

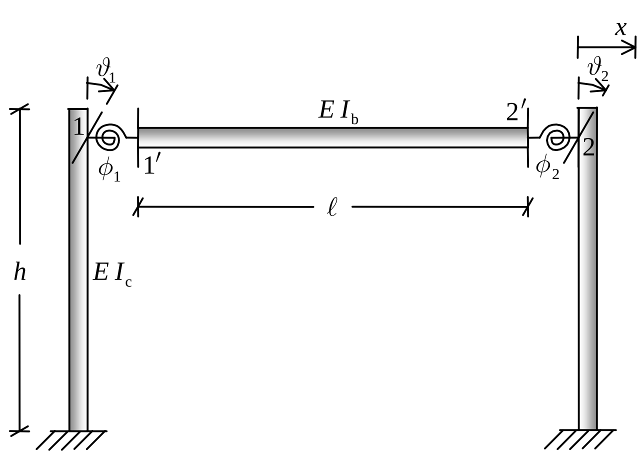


Fig. 7 – Single-story single-bay portal frame with viscoelastic beam-to-column connections

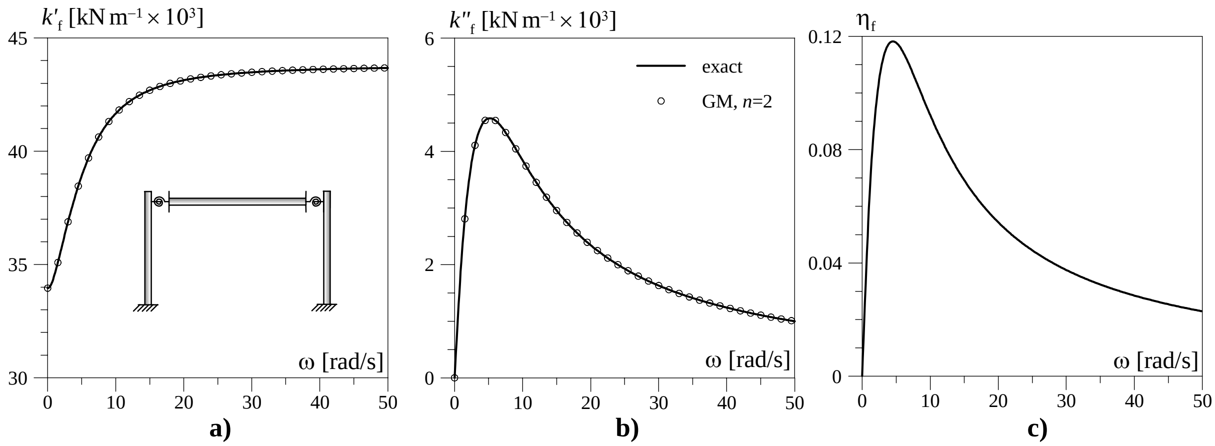


Fig. 8 – Storage modulus (a), loss modulus (b), and loss factor (c) of the portal frame with viscoelastic joints

Finally, under the assumption that the excitation is piecewise linear, an incremental solution of Eq. (20) can be obtained in the time domain through the unconditionally stable step-by-step scheme:

$$\mathbf{y}(t+\Delta t) = \mathbf{\Theta}(\Delta t)\,\mathbf{y}(t) - \boldsymbol{\gamma}_0(\Delta t)\,\ddot{x}_g(t) - \boldsymbol{\gamma}_1(\Delta t)\,\ddot{x}_g(t+\Delta t) \tag{22}$$

where Δt is the time step, $\mathbf{\Theta}(\Delta t) = e^{\mathbf{D}\Delta t}$ is the transition matrix, $\boldsymbol{\gamma}_0(\Delta t) = \left[\mathbf{\Theta}(\Delta t) - \frac{1}{\Delta t}\mathbf{L}(\Delta t)\right]\mathbf{D}^{-1}\mathbf{v}$ and $\boldsymbol{\gamma}_1(\Delta t) = \left[\frac{1}{\Delta t}\mathbf{L}(\Delta t) - \mathbf{I}_{n+2}\right]\mathbf{D}^{-1}\mathbf{v}$ are the load vectors, being $\mathbf{L}(\Delta t) = \left[\mathbf{\Theta}(\Delta t) - \mathbf{I}_{n+2}\right]\mathbf{D}^{-1}$.

Numerical validation

In order to validate the proposed approach, a SDoF PR portal frame was considered. The following parameters were selected for the columns: $I_c = 111900\ \text{cm}^4$ (corresponding to the Italian "HEA 550×166" section), and $h = 4.00$ m; the bending stiffness, then, is: $4EI_c/h = 230{,}000$ kN m. The parameters of the beam and of the viscoelastic EDDs are known, and are those of the previous section.

In a first stage, a regression analysis, with $n = 2$, was used to assess the parameters of the GM model. The computed values are: $k_0 = 33.96\times10^3$ kN/m; $\tau_1 = 0.168$ s, and $k_1 = 7.75\times10^3$ kN/m; $\tau_2 = 0.457$ s, and $k_2 = 2.08\times10^3$ kN/m. The comparison between the exact dynamic stiffness (Eq. (11), solid line) and the corresponding GM approximation (circles) is shown in Fig. 8a (real part) and 8b (imaginary part). The good agreement confirms the accuracy of the proposed formulation. In Fig. 8c the loss factor of the portal frame due to the viscoelastic joints is depicted: it is worth noting that the damping capability of the assembled frame is only a small portion of the damping capability of the EDDs. This reduction of the damping, from the viscoelastic connections to the frame, is similar to the reduction of ductility in elastoplastic structures.

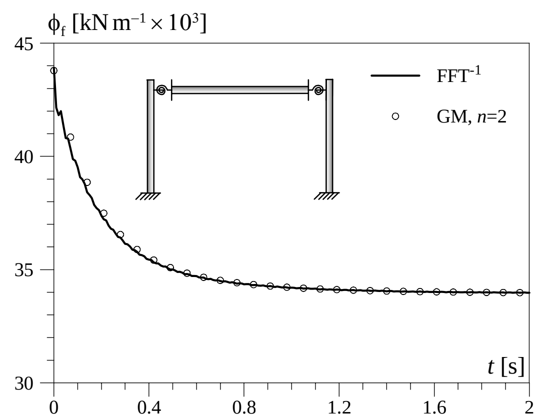


Fig. 9 – Relaxation function of the portal frame with viscoelastic joints

In a second stage, Eq. (15) and the Inverse Fast Fourier Transform (FFT^{-1}) algorithm were used to evaluate the relaxation function of the frame (solid line). The comparison with the GM approximation (circles), given by Eq. (16), is shown in Fig. 9. Also in this case the agreement is very good.
In a third stage, the FRF of the portal frame was evaluated, with $\zeta_0 = 0.02$ and $m = q\,\ell / g = 48.9 \times 10^3$ kg, where $q = 80.0$ kN/m is the vertical load uniformly distributed over the beam, and $g = 9.81$ m/s^2. Four cases were considered: (i) the exact FRF (Eq. (13), solid line); (ii) the FRF of the KV approximation, in which stiffness and damping are assumed to be independent of the vibration frequency (dot-dashed line); (iii) the FRF of the GM approximation, with $n = 2$ (circles); and (iv) the FRF of the frame without EDDs (dashed line). The comparisons in terms of logarithm of the modulus and argument are shown in Fig. 10. It appears that: (i) the GM approximation is in good agreement with the exact model; (ii) the KV model is able to capture the resonant peak of the exact model, but an important discrepancy emerges when the vibration frequency goes to zero, because of the loss of memory associated with the KV model; and (iii) the resonant peak of the frame without EDDs is higher and occurs at a lower frequency: i.e., the EDDs increases the global damping of the frame, together with its stiffness at non-zero frequencies.

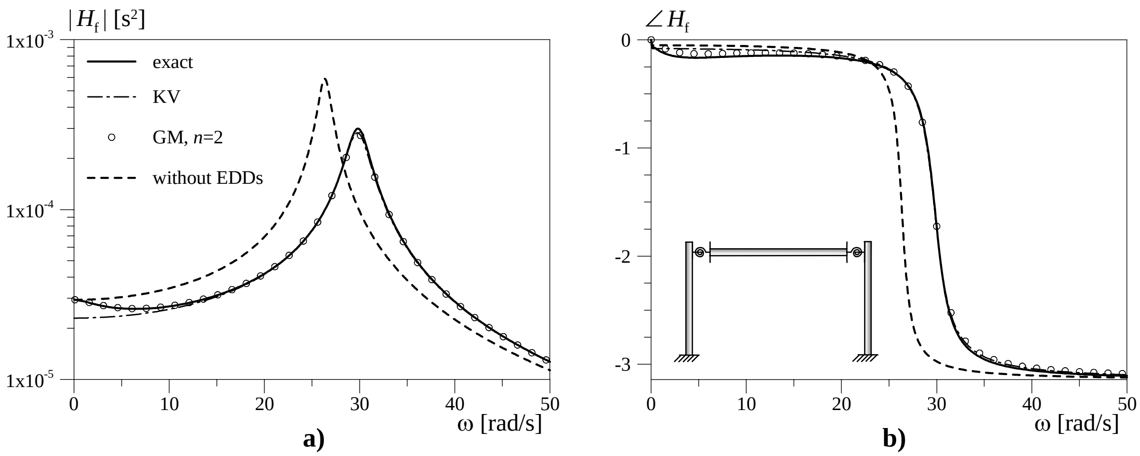


Fig. 10 – Frequency Response Function of the portal frame with viscoelastic joints: a) modulus; b) argument

EARTHQUAKE SPECTRA

In the previous sections, a viscoelastic model of dissipative semi-rigid connections has been presented and numerically validated. In order to investigate the effects of different viscoelastic joints on the seismic response of a SDoF PR portal frame, in this section, the elastic response spectra are evaluated for (i) the ground motion recorded at Tolmezzo (Italy) during the 1976 Friuli earthquake, and for (ii) a number of artificial ground motions, consistent with the elastic design spectrum given by the Italian seismic code [10] for rigid soils.
In the applications: (i) columns and beam are the same of the previous examples; (ii) the inherent viscous damping ratio is $\zeta_0 = 0.02$; (iii) by virtue of its simplicity, the SLS model (a spring in parallel with a single Maxwell element) is used to describe the EDDs at the ends of the beam; (iv) six fixity factors are considered, in order to simulate different connections flexibilities: $\nu = 0.01$ (quasi-pinned), 0.10, 0.30, 0.50, 0.70, 0.90 (quasi-rigid); (v) three relaxation times are considered, in order to simulate different elastomeric materials: $\tau = 0.001,\ 0.01,\ 0.1\,\text{s}$. Altogether, then, $6 \times 3 = 18$ viscoelastic systems are investigated. In each one, the dead load q increases up to 160 kN/m, so that the undamped natural period $T_0 = 2\pi / \omega_0$ varies from 0.02 s to 0.42 s. Finally, the ratio $\alpha = [\phi_f(0) - \phi_f(\infty)] / \phi_f(\infty)$, between the initial value of the time-varying portion of the relaxation function and the equilibrium modulus, is kept constant: $\alpha = 10$.
In a first stage, for each couple of ν and τ, the state-space model of Eqs. (20) and (21) was built, the numerical scheme of Eq. (22) was applied to obtain the dynamic response of the system under the 1976

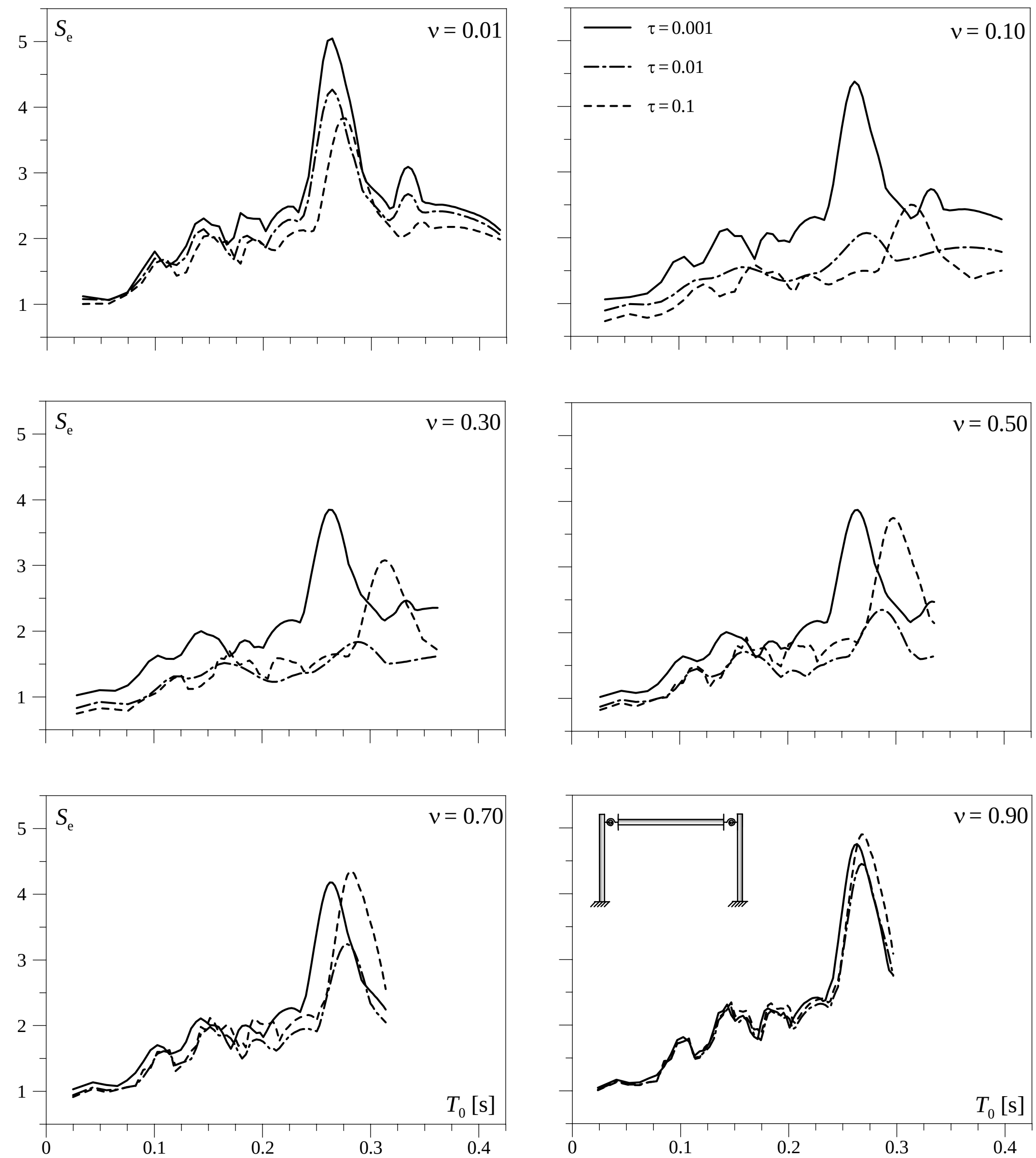


Fig. 11 – Response spectra of a SDoF frame with viscoelastic joints for the 1976 Friuli earthquake

Friuli earthquake, and the maximum absolute value of the seismic response, $\max\{|x(t)|\}$, was evaluated. In Fig. 11, the calculations were used to plot the response spectra in terms of the dimensionless pseudo-accelerations, $S_e(T_0;\nu,\tau) = \omega_0^2 \max\{|x(t)|\}/\max\{|\ddot{x}_g(t)|\}$, as functions of T_0.

The graphs tell that semi-rigid connections allow to reduce the seismic forces on the frame, since the maximum peak of the pseudo-acceleration is about $S_e = 5$ for both quasi-pinned ($\nu = 0.01$) and quasi-rigid ($\nu = 0.90$) beam-to-column joints, while the maximum peak decreases to about $S_e = 4$ for the intermediate values of the fixity factor ($\nu = 0.30,\ 0.50$).

Moreover, among the relaxation times considered in the analyses, the larger mitigation of the seismic forces is generally associated with the intermediate value $\tau = 0.01\,\mathrm{s}$ (dot-dashed lines). It is worth noting that for $\tau = 0.001\,\mathrm{s}$ (solid lines) the damping of the joints is nearly viscous, since the relaxation time is much less than T_0, and the damping force is almost impulsive. On the contrary, for $\tau = 0.1\,\mathrm{s}$ (dashed lines) the relaxation time is of the same order of T_0, and the main effect of the viscoelastic materials at the joints is to increase their fixity under dynamic actions.

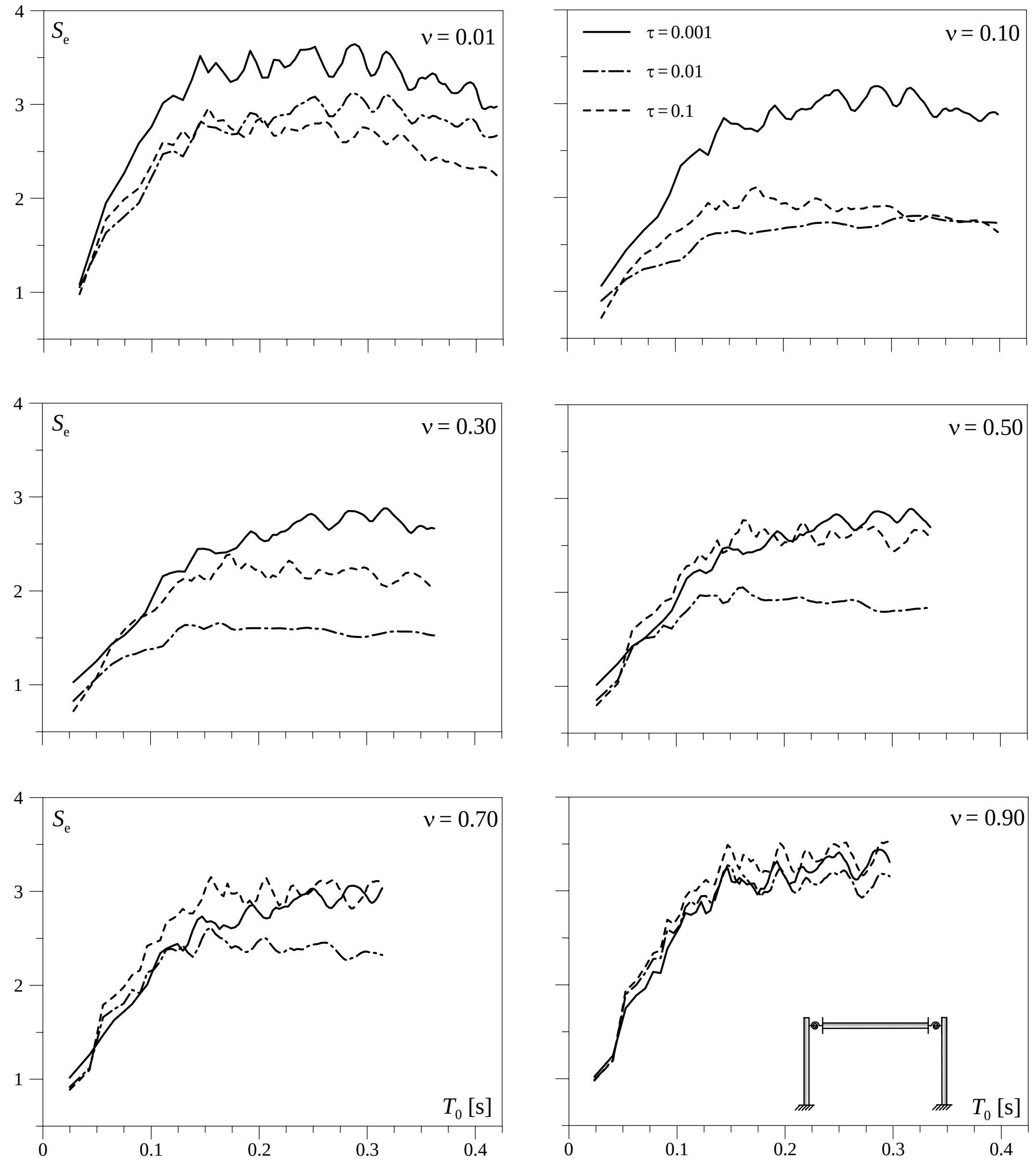


Fig. 12 – Response spectra of a SDoF frame with viscoelastic joints consistent with the design elastic spectrum for rigid soils given by the Italian seismic code

Finally, one can see that the differences among the response spectra obtained with different relaxation times is negligible in the case of quasi-rigid connections ($\nu = 0.90$): in this case, in fact, the stiffness of the beam-to-column joints is so high that the relative rotations are too small to dissipate energy.

In a second stage, by using the procedure proposed in [11], ten synthetic time histories of ground acceleration were generated, consistently with the seismic forces given in [10] for rigid soils. For each sample the elastic response spectra $S_e(T_0;\nu,\tau)$ were computed. The mean spectra are depicted in Fig. 12 as functions of T_0, showing the same tendencies highlighted for the recorded ground motion. In particular, it emerges that special cautions have to be used in selecting the values of flexibility and relaxation time of the viscoelastic joints: the ordinates of the response spectrum, in fact, may significantly change with them.

CONCLUSIONS

In this paper, a consistent model to evaluate, in both frequency and time domain, the seismic response of steel frames with viscoelastic semi-rigid joints has been proposed and validated with numerical examples.
As an application, the dynamic stiffness and the relaxation function of a SDoF system have been computed, and the improvement associated with the use of the Generalized Maxwell (GM) model, rather than an equivalent Kelvin-Voigt (KV) model, has been shown.
Finally, a parametric study on the response spectra for recorded and artificial ground motions has been presented. In both cases, the seismic forces on the frame depend on the long-term stiffness of the joints, and on the relaxation time of the elastomeric material placed therein. These parameters, then, have to be accurately selected in the design procedure, and accurately checked in the execution phase. Further analytical and experimental works are required to confirm and extend these findings.

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