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A SIMPLIFIED METHOD FOR NONLINEAR CYCLIC ANALYSIS OF REINFORCED CONCRETE STRUCTURES: DIRECT AND ENERGY BASED FORMULATIONS

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SUMMARY

This paper presents a simplified method based on direct and energy formulations for nonlinear analysis of reinforced concrete frame structures. In the simplified method, a reinforced concrete member is modeled by an equivalent member of homogeneous nonlinear material with a derived stress-strain relationship, which satisfies the requirement that the equivalent member has the same moment-curvature behaviour as the original member. One advantage of the simplified method is its simplicity which can be easily implemented in most structural analysis computer programs with nonlinear modeling capabilities. The developed model can accurately predict the nonlinear hysteretic responses of reinforced concrete structures with frame members of arbitrary shapes and reinforcing details under severe earthquake excitations. Numerical examples of a cantilever beam and a new bridge pier system of double-skinned concrete filled tubular (DSCFT) members, recently developed in Taiwan for seismic construction, are analyzed under monotonic and cyclic loadings. The effects of concrete confinement, steel hardening, stiffness degradation, strength deterioration, and pinching are simulated in the developed simplified method. Modules of the method are developed for implementation in the Open System for Earthquake Engineering Simulation framework (OpenSees). Results from the simplified method agree well with finite element and fibre layer models, as well as with experimental testing results.

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INTRODUCTION

Many existing older structures are inadequate in their earthquake resistance and hence require retrofitting or rehabilitation to conform to the current seismic design requirements. Seismic retrofitting of existing structures is a complex problem. To properly design the retrofit scheme and minimize the cost, it is important to determine accurately the resistance and behaviour of the structures up to failure. The difficulty of analyzing reinforced concrete structural members arises from many factors such as cracking and crushing of concrete, and yielding, strain hardening, slippage and buckling of reinforcing or composite steel. When load reversals are considered, phenomena such as pinching of the hysteresis loops due to shear, bond deterioration, Bauschinger effects, and other factors also become important (Lee et al. [1]; Kwan et al. [2]). Presently, various numerical models based on fibre layer models and finite elements are available for analyzing the ultimate behaviour of reinforced concrete structures under severe loading conditions. Although the existing methods of analysis can give accurate results, they often require significant amount of computation efforts or the results are difficult to interpret. Therefore, there is still the need for efficient modeling and analysis tools that do not require excessive amount of time and effort in the preparation of modeling input data and are easy to use.

The aim of the present study is to develop a simple modeling and flexible analysis procedure for the evaluation of nonlinear behaviour of reinforced concrete structural members up to ultimate failure. The approach of the simplified model is to represent the nonlinear behavior of a complex reinforced concrete member by an equivalent member with a derived material stress-strain relationship, which gives the same moment-curvature response as the original member and can be easily implemented in most structural analysis computer progress with nonlinear modeling capabilities. Two formulations of the simplified method by direct equilibrium approach and strain energy approach are presented in this paper. The simplified method developed in the present study accounts for the nonlinear material behaviour of reinforcing steel and concrete under compression. The nonlinear behaviour of concrete after cracking and the softening behaviour after peak stress are considered. The effect of confinement on the concrete behaviour is taken into account in the simplified model. The elasto-plastic behaviour of reinforcing steel with strain hardening, and the interaction between concrete and reinforcing steel materials are modeled.

SIMPLIFIED ANALYSIS MODEL

The basic approach of the simplified analysis model is similar to the fibre layer model, except that only distinct states in the behaviour of the modeling reinforced concrete member are considered in the formulation. For flexure behaviour, the following assumptions are considered in the formulation of the simplified analysis procedure:

1. Plane sections of the member remain plane when a member is subjected to deformation. Consequently, the longitudinal strain in the steel and concrete materials at a plane section is directly proportional to the distance from the neutral axis.
2. The stress-strain relationship for the steel is modeled by a trilinear model.
3. The tensile strength of concrete is assumed to be negligible.
4. The stress-strain relationships of confined and unconfined concrete under compression are modeled by a modified Kent and Park concrete model.

In the simplified method, the original structural member section is modeled by an equivalent member section of homogeneous nonlinear material with a specified stress-strain relationship, which is derived based on the requirement that the equivalent section has the same moment-curvature behaviour as the original section.

There are two approaches in the formulation of the simplified method, namely, direct formulation and energy based formulation. In the direct formulation, selected states in the behavior of the reinforced concrete members are analyzed to establish directly points on the specified stress-strain relationship of the nonlinear homogeneous material model of the equivalent member by strain compatibility and equilibrium of internal resultant forces. For the energy based formulation, similar procedures are applied on selected states in the behavior of the reinforced concrete member, but instead considers strain energy and strain compatibility in the derivation of the specified stress-strain relationship of the nonlinear equivalent member material. Preliminary study (Deng et al. [3]) indicates that the derived stress-strain relationship of the equivalent model determined from the direct formulation in some cases can become unstable after the elastic limit is exceeded due to difficulty in convergence. The energy approach based on the consideration of equivalent strain energy can eliminate this problem and give better prediction in the behavior of the equivalent simplified model.

The material models adopted for representation of the behaviour of concrete and steel are briefly described here. The constitutive model for concrete developed by Kent and Park for confined and unconfined concrete is implemented in the simplified analysis procedure. The behaviour of reinforcing steel materials is approximated by a trilinear stress-strain relationship considering the effect of strain hardening

Modified Kent and Park Concrete Model

Many studies have been conducted on the stress-strain relationship of concrete confined by transverse reinforcement under compression. Observations and laboratory tests have shown that if the compression zone of a concrete beam or column is confined by closely-spaced stirrup ties, hoops or spirals, the ductility of concrete is significantly enhanced and the member can sustain deformations of large curvature (Kent et al. [4]; Park et al. [5]; Ozcebe et al. [6]). In the simplified method, the modified Kent and Park concrete model is adopted for modeling the material behaviour of concrete under compression. The formulations of the stress-strain relations of confined and unconfined concrete of the model are summarized here.

The constitutive model consists of an ascending branch represented by a second-degree parabolic curve and a descending linear part, as shown in Figure 1.

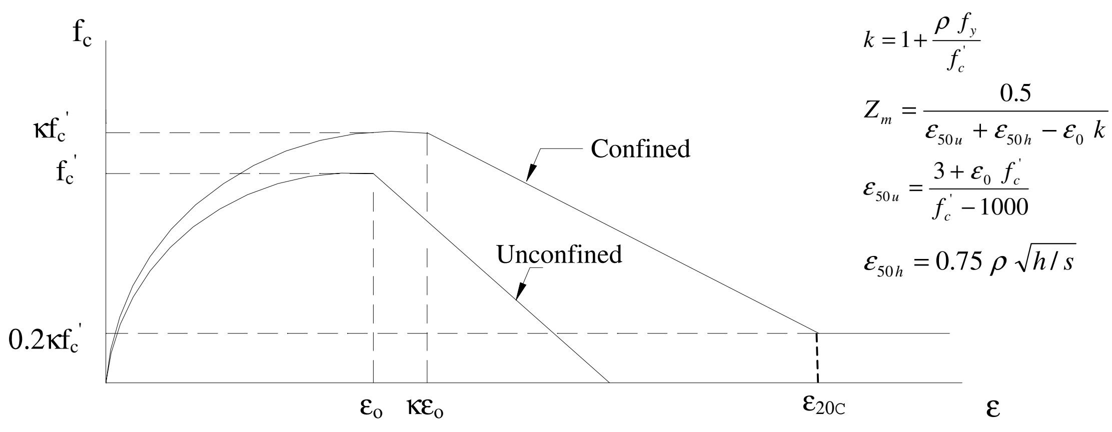


Figure 1 Modified Kent and Park model for unconfined and confined concrete

The ascending parabola is expressed by Equation (1)

$$f_c = k\ f_c^{'} \left[\frac{2\ \varepsilon_c}{\varepsilon_0\ k} - \left(\frac{\varepsilon_c}{\varepsilon_0\ k} \right)^2 \right] \qquad \varepsilon_c \leq k\ \varepsilon_0 \qquad (1)$$

where ε_c is the longitudinal concrete strain, $f_c^{'}$ is the compressive strength of concrete, ε_o is the strain of unconfined concrete corresponding to $f_c^{'}$, and k is a confinement coefficient. For unconfined concrete, the parameter k is equal to one. A commonly accepted assumption for unconfined concrete is ε_o = 0.002 (CSA [7]).

For strain greater than the corresponding strain at peak stress, the softening branch of the stress-strain relationship is approximated by a straight line presented in Equation (2)

$$f_c = k\ f_c^{'} \left[1 - Z_m \left(\varepsilon_c - \varepsilon_0\ k \right) \right] \geq\ 0.2\ k\ f_c^{'} \qquad \varepsilon_c > k\ \varepsilon_0 \qquad (2)$$

where Z_m is the stress declining ratio for the confined concrete after peak stress.

For concrete under confinement, a perfectly plastic residual behaviour is assumed at high strain level to account for the load carrying capacity of crushed concrete still effectively confined by the transverse steel. The confinement effect on the strength of concrete represented by the confinement coefficient k increases the concrete peak stress from $f_c^{'}$ to $kf_c^{'}$. It is assumed that confined concrete can sustain a constant stress of 0.2k $f_c^{'}$ at strains greater than ε_{20c} .

Trilinear Steel Model

A trilinear stress-strain relationship shown in Figure 2 is adopted to model the behaviour of reinforcing steel under tension or compression. The possibility of buckling of the compression steel is ignored at this stage of the study.

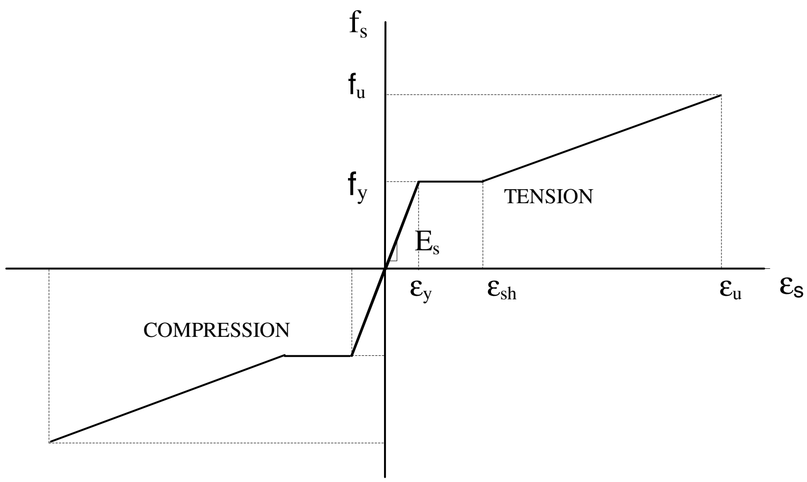
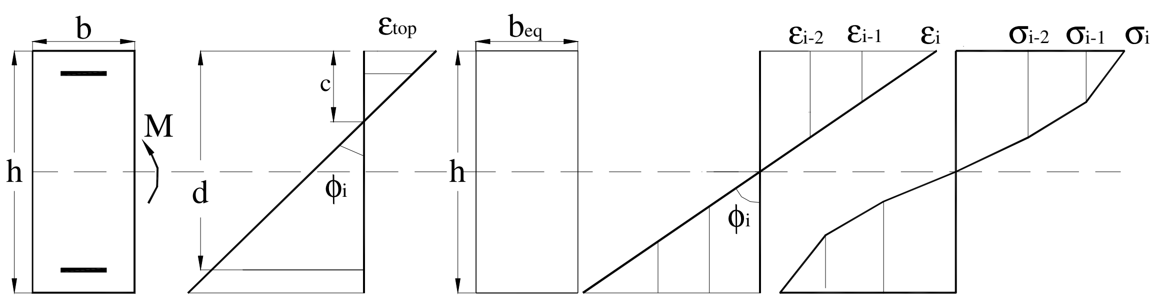


Figure 2 Steel trilinear model

DIRECT FORMULATION

Figures 3 and 4 illustrate the basic steps in the derivation of the specified stress-strain relationship of the nonlinear equivalent member material by strain compatibility and direct equilibrium of internal resultant forces. To illustrate the procedure for the derivation of the stress-strain relationship of the equivalent member section, a rectangular reinforced concrete beam member is analyzed as an example here. The procedure can be extended to the analysis of any reinforced concrete or concrete-steel composite structural members of arbitrary shapes with different reinforcing details based on the same approach.



(a) Original section (b) Equivalent section

Figure 3 Equivalent strain and stress profiles

Assuming the strain at the top surface of the rectangular member section at a particular state is ε_{top}, and the neutral axis is located at a depth of c measured from the top of the member section as shown in Figure 3, the curvature ϕ of the section can be expressed by Equation (3)

$$\phi = \frac{\varepsilon_{top}}{h-c} \tag{3}$$

where h is the member section height

Using the concrete constitutive model by the modified Kent and Park and the steel material model as shown in Figure 4, the neutral axis is determined by an iterative process based on equilibrium of the internal resultant forces of the member section. The internal force F and moment M of the section, can be obtained by integrating the stress distributions on concrete and steel over the member section area A as follows

$$F = \int \sigma dA \tag{4}$$

$$M = \int \sigma y dA \tag{5}$$

where y is the moment arm of the stress in the member section.

As discussed earlier, an equivalent member section shown in Figure 4 (a) is assumed to be composed of a uniform homogeneous material. The characteristic of this material is that the stress-strain relationship of the equivalent section, as shown in Figure 4(f), should lead to the same moment-curvature behaviour as the original member section, as shown in Figure 4(d) and 4(g). In the formulation adopted here, the equivalent section is defined to have the same total area of the original member section. In the derivation, the equivalent section is derived by having the same curvature as the original member section. The homogeneous material is assumed to have the same stress-strain behaviour in both tension and compression in the derivation of the equivalent section as shown in Figure 3(b).

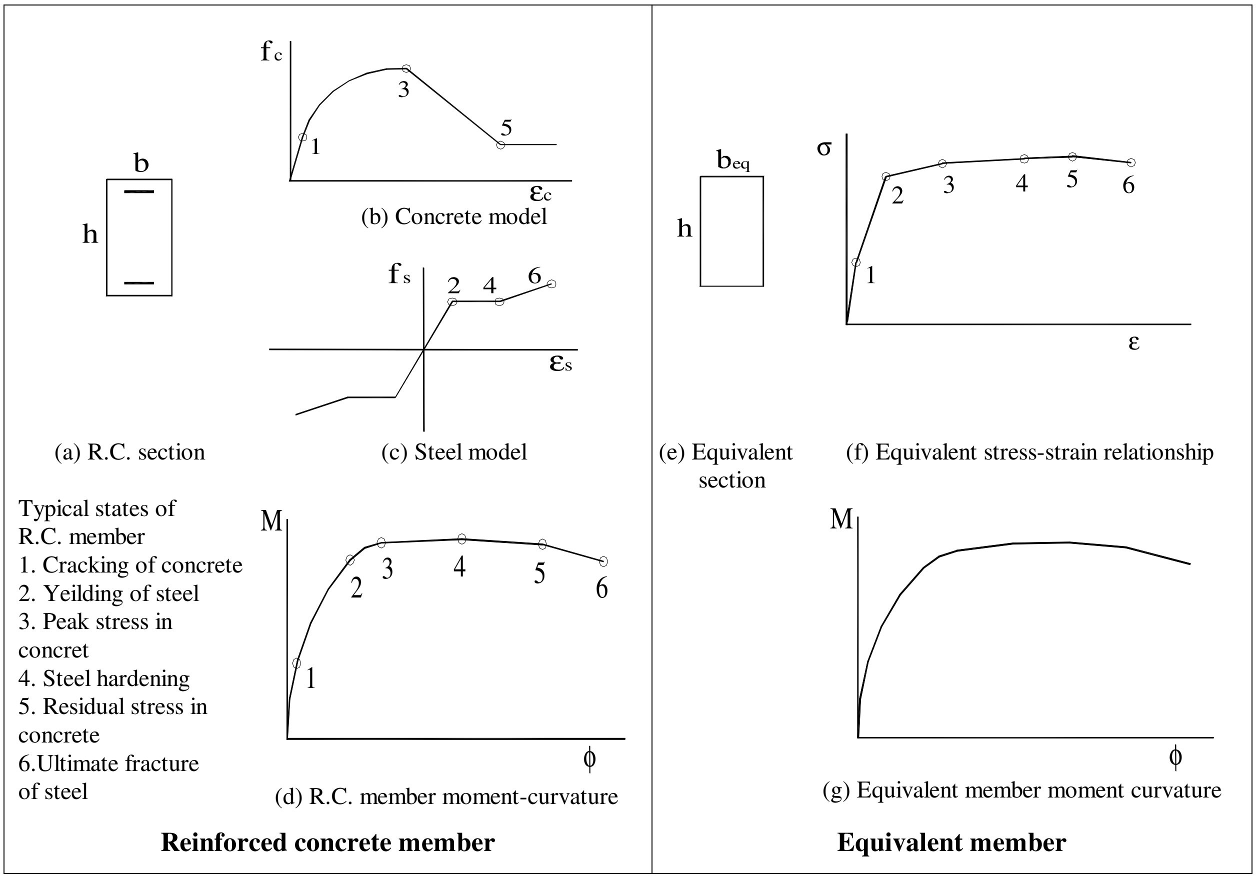


Figure 4 Simplified analysis model of reinforced concrete member section

Typical states of the reinforced concrete member section considered in the formulation are cracking of concrete, yielding of reinforcing steel, peak stress in concret, reinforcing steel hardening, residual stress in confined concrete, and ultimate fracture of reforcing steel, as illustrated in Figure 4. To obtain a more accurate and complete equivalent stress-strain relationship for the analyzed reinforced structural member, additional states in the behaviour of the reinforced structural member can be analyzed.

From the linear strain variation of the equivalent member section with the same curvature as the original member, the maximum equivalent strain at the extreme fiber of the equivalent member section is given by Equation (6)

$$\varepsilon_{eq} = \phi_{eq}\left(\frac{h}{2}\right) \tag{6}$$

where $\phi_{eq} = \phi$ and h is the height of the reinforced concrete member section.

Considering the first critical state of initial cracking of concrete, the equivalent cracking stress of the equivalent member section required to give the same moment-curvature behaviour as the original analyzed reinforced concrete section is determined as follows

$$\sigma_{(eq)cr} = \frac{M_{cr}c_{eq}}{I_{eq}} \tag{7}$$

where M_{cr} is the cracking moment of the reinforced concrete structural member section, the equivalent neutral axis depth $c_{eq} = \frac{h}{2}$, and I_{eq} is the moment of inertial of the equivalent member section.

The derivation procedure can be extended to analyze the yielding and other nonlinear states of the original reinforced concrete member. For each state, the moment can be expressed as follows

$$M_i = \sum[\Delta\sigma_{i-1}A_{i-1}d_{i-1}] + \frac{1}{2}\Delta\sigma_i A_i d_i \tag{8}$$

where M_i is the moment at the i^{th} state, $\Delta\sigma_i$ is the change of stress from the $(i-1)^{th}$ to the i^{th} state, A_i is the sectional area at the i^{th} state, and d_i is the moment arm. After obtaining the equivalent strain of the i^{th} state from the moment-curvature relationship of the original member section, the stress increment $\Delta\sigma_i$ in Equation (9) can be solved to give the equivalent stress as follows

$$\sigma_{(eq)i} = \sigma_{(eq)i-1} + \Delta\sigma_i \tag{9}$$

where $\sigma_{(eq)i}$ is the equivalent stress at the i^{th} state. A linear interpolation is assumed between two consecutive states in the stress-strain relationship of the equivalent material. By repeating the derivation procedures as given in Equations (3) to (8) for each selected states of the original member behaviour, the complete stress-strain relationship of the equivalent member material can be obtained.

ENERGY BASED FORMULATION

Numerical results from preliminary studies (Deng et al. [3]) using the direct formulations presented in previous section have shown that the derived stress-strain relationship after the elastic limit may become unstable in the inelastic range when applied to the modeling of some reinforced concrete member sections. An alternative approach to derive the equivalent stress-strain relationship by the energy method is formulated. In the energy based formulation, similar procedures are applied to selected states in the behavior of the reinforced concrete member by consideration of strain energy instead of direct equilibrium in the derivation of the nonlinear equivalent member material. For structural member under flexure, the strain energy is determined by

$$U = \int_L \frac{M^2}{EI} dx = \int_V \sigma \varepsilon dV \tag{10}$$

where E is the elastic modulus and I is the moment of inertia of the member. Linear relationship is assumed for the stress-strain behaviour between two consecutive modeling states. The strain energy of an equivalent member of solid cross-sectional area can be expressed by strain energy density in terms of equivalent stress and strain as follows

$$\Delta u_{(eq)j} = \frac{\sigma_{(eq)j} + \sigma_{(eq)j-1}}{2} \{\varepsilon_{(eq)j} - \varepsilon_{(eq)j-1}\} \tag{11}$$

where $\Delta u_{(eq)j}$ is the change of strain energy density from the (j-1)th to the jth state, $\sigma_{(eq)j}$ is the equivalent stress at the jth state, $\varepsilon_{(eq)j}$ is the strain at the jth state, $\sigma_{(eq)j-1}$ and $\varepsilon_{(eq)j-1}$ are the stress and strain at the (j-1)th state, respectively. After the strain energy density $\Delta u_{(eq)j}$ is obtained from two consecutive states of the reinforced concrete member, the equivalent stress $\sigma_{(eq)j}$ in Equation (11) can be solved.

COMPUTER IMPLIMENTATION

In the present study, computer modules based on the computer-aided toolbox Mathcad have been developed for the implementation of the simplified equivalent analysis model and procedures to generate the required stress-strain relationship of the nonlinear equivalent member material. The developed computer modules can be used in conjunction with other computer structural analysis programs with user specified nonlinear material modeling capacities for detailed analysis nonlinear reinforced concrete structural members of arbitrary shape.

Open System for Earthquake Engineering Simulation framework (OpenSees) is a software framework in development at the University of California, Berkeley (Mazzoni et al. [8]) for simulating the seismic response of structural and geotechnical systems subjected to earthquakes. It has a library of different material and structural element modeling modules, and computational platforms which can be selected and assembled into specific analysis tools for advanced analysis of complex structural systems. In the present study, modules of the simplified method for monotonic loading are implemented in the OpenSees. The hysteretic material model in the OpenSees is adopted and modified for analyzing the behavior of structural members subjected to cyclic loading using the simplified method. The derived equivalent material model adopted for cyclic response analysis of reinforced concrete member has the hysteretic relationship shown in Figure 5. The unloading stiffness is evaluated based on the initial modulus of elasticity E_0 and ductility μ_i, which is calculated by Equation (12) as follows

$$\mu_i = \frac{\varepsilon_i}{\varepsilon_{(eq)y}} \tag{12}$$

where ε_i is the strain at the i$^{\text{th}}$ time-step, $\varepsilon_{(eq)y}$ is the equivalent yield strain calculated by the simplified method. The parameter β shown in Figure 5 is an optional parameter used to determine the unloading stiffness and is set to zero in the present investigation.

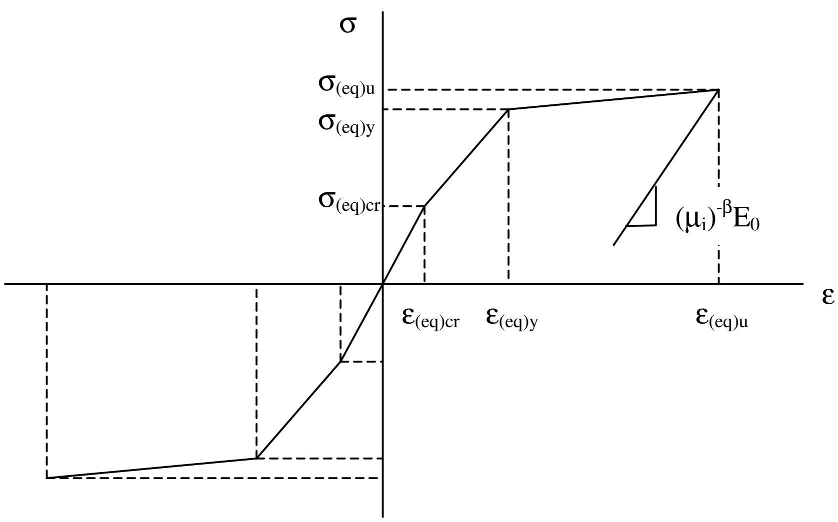


Figure 5 Equivalent material model for cyclic loading

NUMERICAL EXAMPLES

To demonstrate the validity of the equivalent model and analysis procedure, two examples of reinforced concrete structural members are analyzed using the simplified method. The two examples are a reinforced concrete cantilever beam and a double skinned concrete filled tubular (DSCFT) member, which is an innovative system recently developed in Taiwan for seismic construction. Results from the simplified method are compared with fibre layer model using OpenSees, and finite element model using the computer program ABAQUS [9]. ABAQUS is a finite element numerical analysis program with extensive linear and nonlinear material modeling and structural analysis capabilities. The simplified method results on the DSCFT columns are also compared to test measurements obtained in experimental research by Tsai et al. [10].

Cantilever Beam Example

The cantilever beam analyzed in this study is a doubly reinforced concrete structural member with only longitudinal reinforcement subjected to an end applied moment. The rectangular section is reinforced by compression steel (As' = 6300 mm^2) at an effective depth of 76.2mm, and tension steel (As = 6300 mm^2) at an effective depth of 732.8mm. The compression strength of the concrete is 30MPa with an ultimate strain of 0.0035. For the example here, the concrete material is assumed to be unconfined due to the absence of transverse reinforcement, and the steel is modeled by a bilinear model with perfectly plastic behaviour after yielding. Figure 6 and Table 1 show the dimensions and material properties, respectively, of the example cantilever beam.

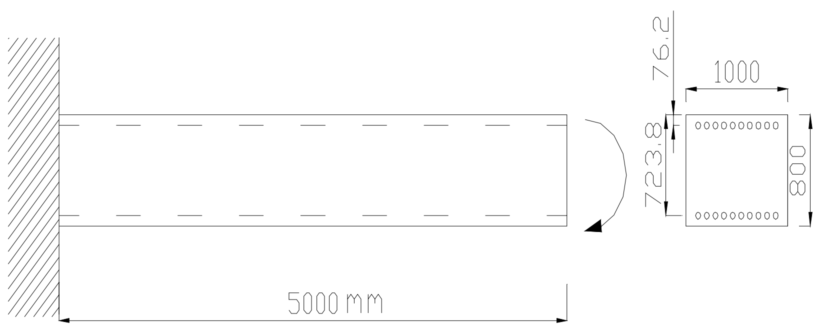


Figure 6 Cantilever beam dimension

Table 1 Material properties of cantilever beam

Concrete		Steel	
Parameter	Value	Parameter	Value
Ec	24648 MPa	Es	200 GPa
$f_c^{'}$	30 MPa	f_y	400 MPa
$f_{cr}^{'}$	3.286 MPa	f_u	400 MPa
ε_o	0.002	ε_u	0.159

For the cantilever beam, the derived equivalent stress-strain relationship for the reinforced cantilever beam is presented in Figure 7. The direct formulation gives a fluctuating stress-strain relationship, whereas as the strain energy approach gives a smooth equivalent stress-strain relationship. The moment-curvature behavior at the tip of the cantilever beam obtained by the simplified method using ABAQUS are shown in Figure 8. From comparison of the numerical results show in Figure 8, it is observed that the equivalent stress-strain relationship obtained by the energy approach results in a lower yielding capacity of the equivalent model. The strain energy of the equivalent member by the energy approach closely follows the behaviour of the original reinforced concrete member. This is particularly noticeable for response after the elastic limit. In comparison, the results by the direct formulation give fluctuating behavior in the moment-curvature relationship in the inelastic ranges. Thus, the results indicate that the simplified method by the energy approach can accurately predict the behaviour of the cantilever beam up to the ultimate capacity. It is noted that since the concrete material in the cantilever beam is unconfined, therefore large portion of the concrete material does not contribute to the resistance of the load when the strain of the concrete fibre exceeds 0.002. Consequently, this leads to a softening behaviour in its moment capacity and the behaviour of the cantilever beam is governed by the reinforcing steel.

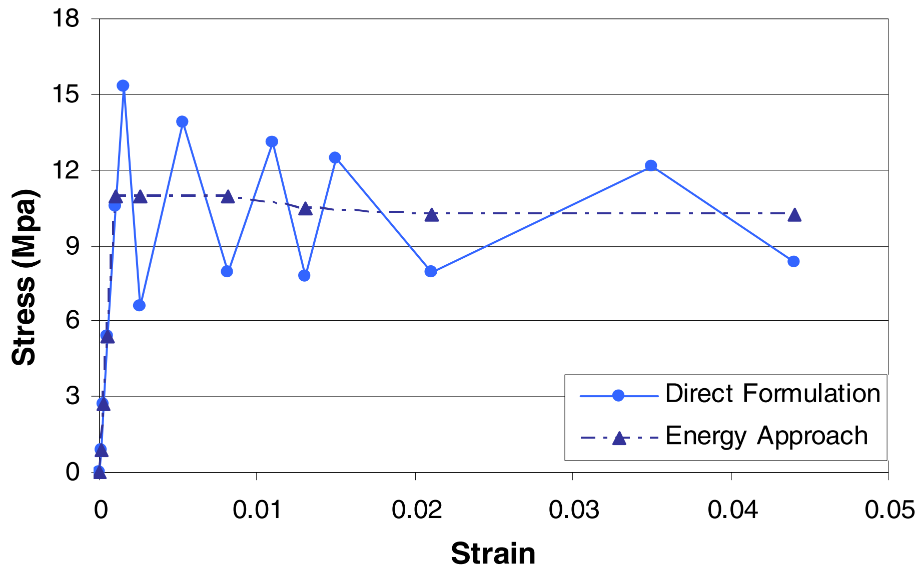


Figure 7 Equivalent stress-strain relationship of cantilever beam

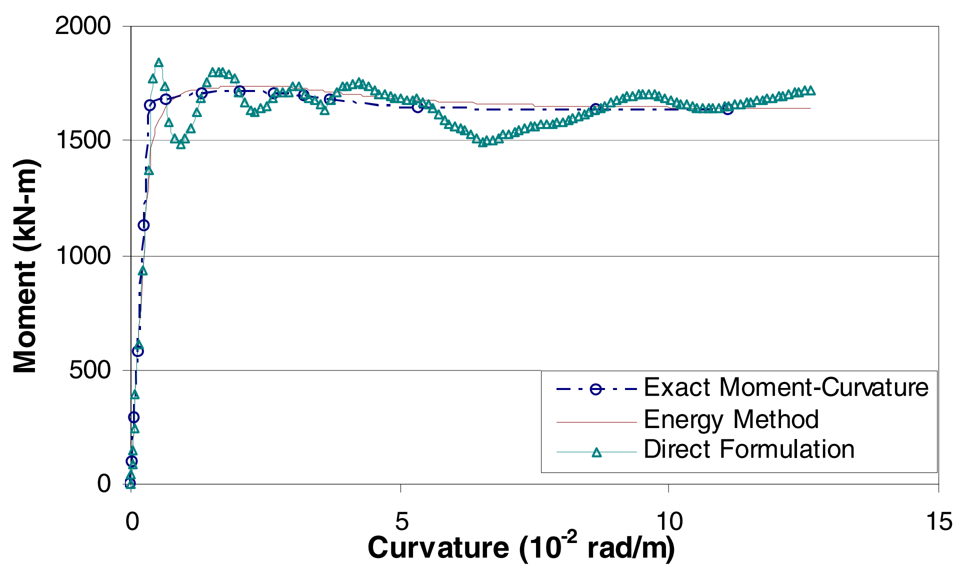


Figure 8 Moment-curvature relationship of cantilever beam

Double Skinned Concrete Filled Tube Example

Figure 9 shows the DSCFT member consisting of two concentric circular thin steel tubes with concrete filled between them. Research carried out in Taiwan (Tsai et al. [9]; Lin [11]; Tsai et al. [12]; Tsai et al. [13]) has demonstrated that the DSCFT members possess light-weight, moderated axial capacity and high flexural strength characteristics. An important advantage of the DSCFT column system is that the reduced weight of the column can lead to a significant reduction of seismic force on the member. Therefore, DSCFT is an innovative structural system particularly suitable for construction of tall bridge priers in seismic regions. Since the DSCFT is a complex reinforced concrete structural system, it is difficult to analyze using general analysis methods. The simplified method is applied here to analyze the behaviour of the DSCFT.

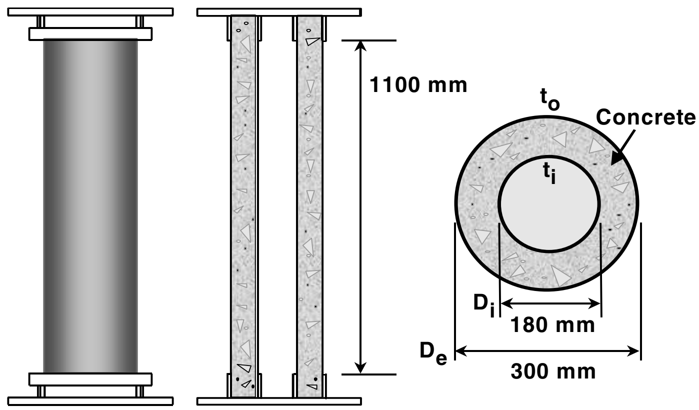


Figure 9 DSCFT section (Tsai [10])

The experimental specimen analyzed using the simplified approach has an effective length of 1100mm. The external and internal tube diameters are 300mm and 180mm, respectively. The thickness is 2mm for both the external and internal steel tubes. Table 2 lists the material properties used in the analysis. For the case study here, the concrete filler is initially modeled as confined concrete with confinement coefficient k of 1.231, as determined by the modified Kent and Park confined concrete model for the amount of steel in the double skinned steel tube. A bilinear model is adopted for representing the behaviour of the steel tube material.

Table 2 Material properties of DSCFT

Concrete		Steel	
Parameter	Value	Parameter	Value
Ec	25045 MPa	Es	201 GPa
$f_c^{'}$	28 MPa	f_y	305.2 MPa
$f_{cr}^{'}$	3.175 MPa	f_u	416.1 MPa
ε_o	0.002	ε_u	0.159

Figure 10 shows the comparison of the derived stress-strain relationships by the direct formulation and the strain energy formulation of the simplified method. Figure 11 shows the comparison of the numerical results of the moment-curvature behavior obtained by ABAQUS using the derived stress-strain relationships, shown in Figure 10, and test results from the experiments conducted by Tsai [10]. When comparing with the experimental results, the simplified method gives a conservative lower moment capacity. One possible reason for the lower moment capacity is due to the inadequate modeling of strength and stiffness of the confined concrete core. To more accurately account for the total confinement of the concrete core by the steel tubes, a parametric study on the confinement coefficient k is carried out by increasing k to 2.5 and 3.7, which are twice and three times of that determined by the modified Kent and Park model developed for open spiral transverse reinforcement. This means that the confinement of the double skinned steel tube is three times more efficient in providing confinement to the core concrete than open transverse reinforcement assumed in the modified Kent and Park confined concrete model. The results from Figures 10 and 11 show that the confinement behaviour of the concrete core due to the double skinned steel tubes is more accurately represented by the confinement coefficient k = 3.7.

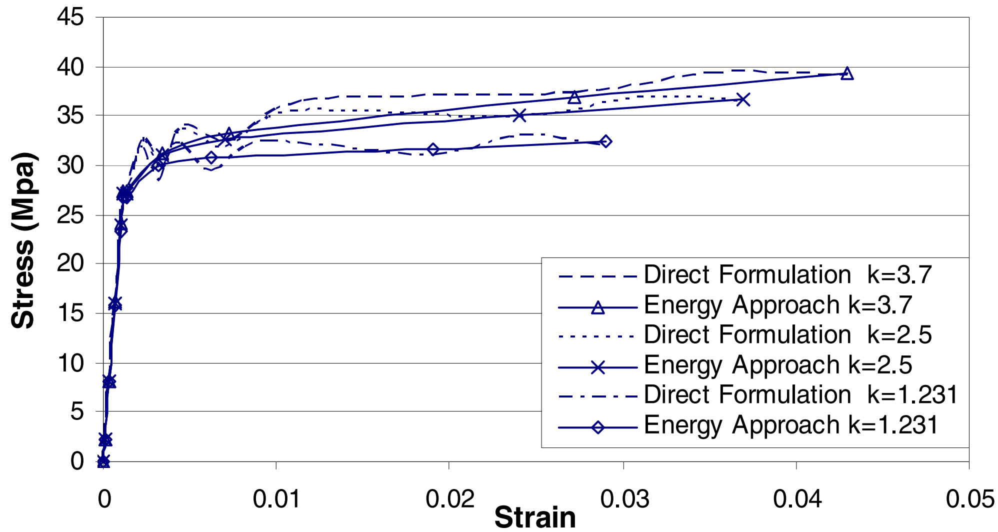


Figure 10 Equivalent stress-strain relationship of DSCFT member

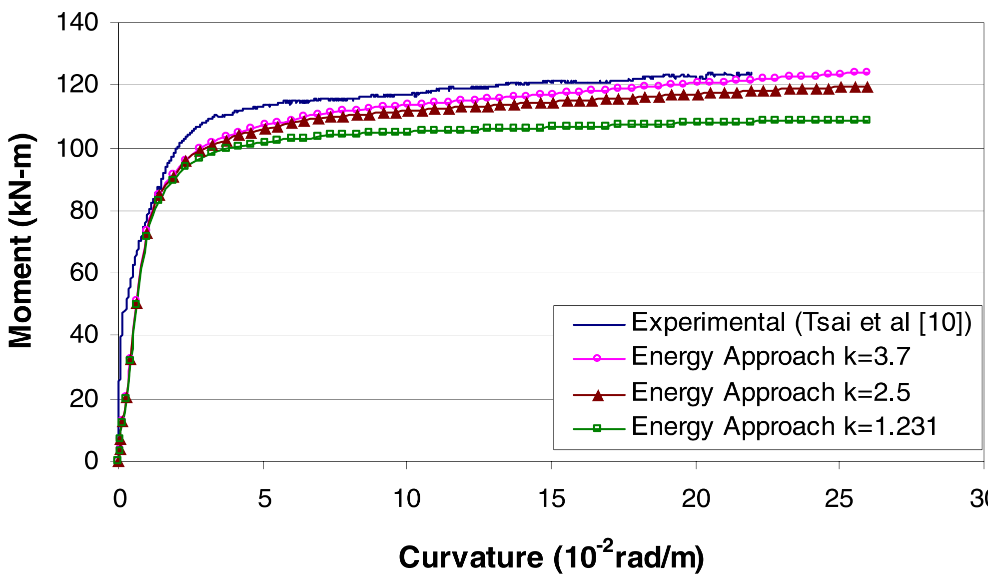


Figure 11 Moment-curvature relationships of DSCFT member

Figure 12 shows the comparison of the analytical results obtained by OpenSees based on the derived stress-strain relationship of the equivalent member material with the confinement coefficient k of 2.5 and test results from the experiments conducted by Tsai [10]. The effects of strength deterioration and stiffness degradation are not modeled in this example. The simplified model gives reasonable correlation with the experimental results. The ultimate moment capacity predicted by the computer simulation agrees well with the test data.

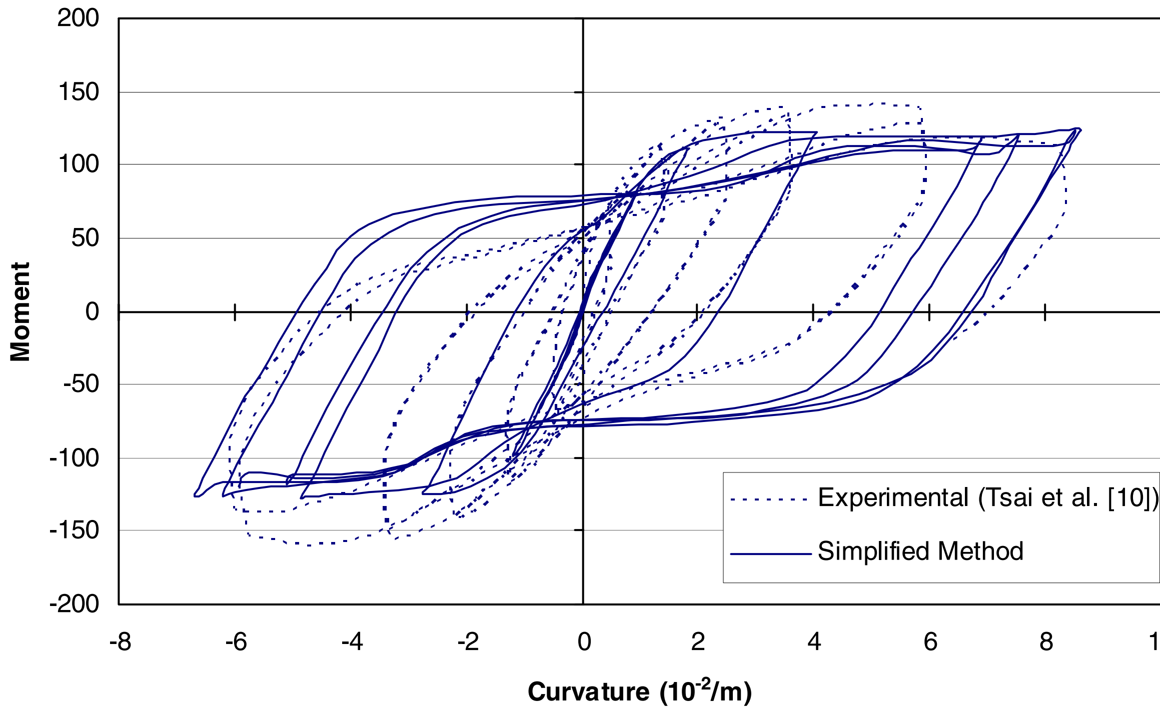


Figure 12 Analysis result of cyclic loadings for DSCFT member

CONCLUSIONS

This paper presents the derivation of a simplified modeling and analysis approach for the nonlinear analysis of reinforced structural members. In the simplified method, the original structural member section is modeled by an equivalent member section of homogeneous nonlinear material with a derived stress-strain relationship such that the equivalent section has the same moment-curvature behaviour as the original reinforced concrete section. Two numerical examples are presented to illustrate the procedures of the simplified analysis method. A rectangular reinforced concrete cantilever beam subjected to an end moment and a double skinned concrete filled tubular (DSCFT) member subject to a lateral load are analyzed. Results obtained from the simplified method are in good agreement with the analysis results obtained from the structural analysis software framework OpenSees and finite element program ABAQUS. The results also correlate well with test data. The research shows that the nonlinear response of reinforced concrete members can be accurately predicted by the simplified method. The simplified method is an efficient tool for evaluating the nonlinear behaviour of reinforced concrete structural members up to ultimate failure. The advantage of the simplified method is that it can be easily implemented with most structural analysis programs with the capabilities of user specified nonlinear material models. The developed model can accurately predict the nonlinear hysteretic responses of reinforced concrete and other reinforced concrete structures with frame members of arbitrary shapes and reinforcing details under severe earthquake excitations.

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