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SIMPLIFIED SEISMIC ANALYSIS OF SOIL-FOUNDATION-STRUCTURE SYSTEMS INCLUDING SOIL-STRUCTURE INTERACTION EFFECTS

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SUMMARY

This work presents simplified, yet accurate, soil foundation models suitable for dynamic and seismic analysis of structures accounting for Soil Structure interaction effects. The model parameters are extracted from impulse response functions (B-IRF) of the soil foundation system to B-Spline impulse excitations. The B-IRF are obtained using a direct time domain 3-D B Spline BEM methodology for elastodynamics. These models are suitable for direct time domain analysis and they accommodate nonlinear structural behavior. The ease of use, accuracy and versatility of the proposed models is demonstrated. A series of studies address the effectiveness of seismic isolation devices when SSI effects are accounted for.

INTRODUCTION

Soil-Structure Interaction (SSI) is a collection of phenomena in the response of structures caused by the flexibility of the foundation soils, as well as in the response of soils caused by the presence of structures. Analytic and numerical models for dynamic analysis typically ignore SSI effects of the coupled in nature structure-foundation-soil system. It has been recognized that SSI effects may have a significant impact especially in cases involving heavier structures and soft soil conditions. Mathematical models suitable for SSI analysis are based on simplifying assumptions and solutions require advanced methods and techniques, such as the well established Finite Element Methods (FEM) and the Boundary Element Methods (BEM). General literature reviews on BEM and FEM methods for problems in SSI analysis have been reported [1-4]. Such models and methods yield highly accurate results but tend to be computationally expensive. Further simplifications of the soil-foundation system lead to lumped parameters models which represent the soil-foundation system as an equivalent system consisting of discrete oscillators. The properties of the lumped parameter models are commonly assumed to be independent of the excitation frequency [5-8], or, in more advanced models, frequency dependent [9, 10]. While the latter exhibit a better representation of the system behavior, their implementation by the

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practitioner engineer is rather involved and not easily comprehended. The frequency independent models, although less accurate, are more widely used due to their ease of implementation and integration into global analysis procedures [11-13]. Probably one of the better known simple models is presented by Wolf [14] wherein calibration of SDOF oscillators is based on a double asymptotic approximation of the foundation impedance functions. Lumped parameter models, while easier to use and implement than FEM and BEM methods, tend to be problem dependent and less accurate. Therefore, there is a need to further improve simplified procedures for SSI analysis that yield similar levels of accuracy as the more refined techniques.

The present work discusses simplified, yet accurate models of rigid foundation-soil systems for the dynamic analysis of structures including SSI effects [15, 16]. The proposed models are based on a system identification approach that determines the natural frequency, ω_n, and damping ratio, ξ, of equivalent SDOF systems. To this end, the B-Spline BEM methodology [17,18] is used to compute the characteristic B-Spline Impulse Response Function (B-IRF) of 3-D continuous soil-foundation systems. Subsequently, the SDOF parameters are defined based on a nonlinear regression that fits the response of the equivalent SDOF to the free vibration phase of the B-IRF of the system. The proposed lumped parameter models are incorporated directly in Finite Element formulations for seismic analysis of structures [15, 19]. Investigations are conducted to quantify the SSI effects on the seismic response of bridges with seismic isolation devices. In the present paper, the first section introduces the B-IRF of rigid surface foundations used in this work. The subsequent section presents a number of aspects of the regression analysis and the parameters of the proposed equivalent lumped parameter SDOF systems. The last section presents validation studies and the findings of the parametric investigations.

DEFINITION OF SOIL-FOUNDATION SYSTEM AND THE B-IRF FUNCTIONS

The foundation B-IRF functions are calculated for a massless, rigid, square foundation, of side w = 120 in., resting on a homogeneous elastic half-space. This system is considered as a reference system and is shown in the inset of Figure 1. The properties of the half space are shown in Table 1. Due to the rigid conditions, the foundation response can be expressed by the three translations and the three rotations of

Table 1 Reference System Properties

Reference System Properties		
Property	***Symbol***	***Value***
Foundation width (in)	w	120
Lame Constant	λ	6.63×10^6
Shear Modulus (lb/in^2)	G	3.315×10^6
Density (slug/in^3)	ρ	2.82×10^{-4}
B-Spline Support (sec)	Δt	1.0×10^{-4}
Analysis Time Step (sec)	δt	2.5×10^{-5}
Pressure Wave Velocity (in/sec)	v_p	2.17×10^5
Shear Wave Velocity (in/sec)	v_s	1.08×10^5

the foundation reference point. However, due to symmetry the two horizontal translation modes are identical and similarly the two rocking modes. Furthermore, under the assumption of non-relaxed boundary conditions the coupling modes are considered, and should be equal. Consequently, the response of the square rigid surface foundation is described by five distinct vibration modes, i.e., horizontal (*H*) and vertical (*V*) translation, rocking (*R*), twist (*T*), and coupling modes (*Q*). In order to compute the impulse response of the foundation for each of the five modes, an impulse force (or moment) of cubic B-Spline modulation is applied in the direction of each of the four degrees of freedom (d.o.f.), i.e., H,V,R, or T. The duration of the B Spline impulse (*Spline Support*) is Δt=0.0001 sec. The corresponding B-IRF are computed for each d.o.f. using the direct time domain BEM reported by Rizos [18]. To this end, the boundary of the half space is discretized into 8 node Boundary Elements. The foundation is assumed to remain always in contact with the soil and the rigid surface boundary element introduced in [17] is adopted in this work. The computed B-IRF in each case is normalized as

$$\overline{U}_i = \begin{cases} Gwu_i & \textit{due to force} \\ Gw^2 u_i & \textit{due to moment} \end{cases} \qquad \overline{\Theta}_i = \begin{cases} Gw^2 \vartheta_i & \textit{due to force} \\ Gw^3 \vartheta_i & \textit{due to moment} \end{cases} \qquad i=x,\, y,\, z \tag{1}$$

where $\overline{U}_i$ *and* $\overline{\Theta}_i$ are the non-dimensional amplitudes of the translation and rotation, respectively, u_i and ϑ_i are the corresponding dimensional amplitudes, and G is the soil shear modulus. The time parameter, t, is expressed in a nondimensional form, τ, as,

$$\tau = \frac{tv_s}{w} \tag{2}$$

Figure 1 shows the normalized vertical and horizontal B-IRF functions as obtained from the BEM solution in a discrete form in time. Similar B-IRF functions have been computed for all vibration modes of the system. It is observed that the resulting B-Spline impulse responses have characteristic shapes for each mode. In particular, two phases of the response are identified: (i) the forced vibration phase of duration Δt and (ii) a free vibration phase that resembles the free vibration response of a single degree of freedom (SDOF) system with relatively high damping. It is observed that, the B-IRF function for each translation mode reaches equilibrium monotonically indicating that the equivalent SDOF system is over- critically damped. The B-IRF function for each rotational mode, however, shows slight oscillation about the initial

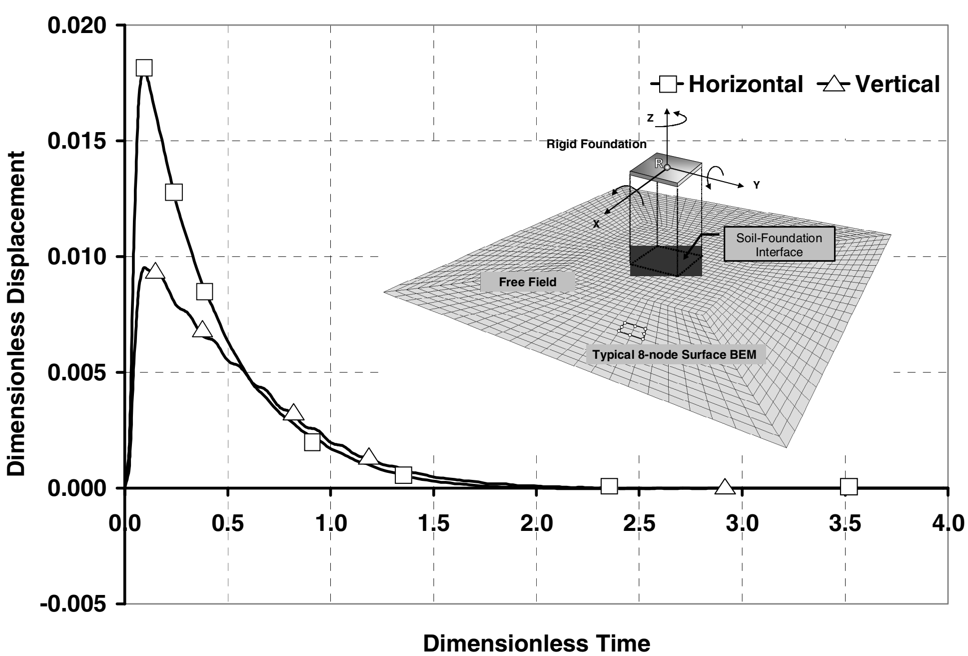


Figure 1. B-Spline Impulse Response for Horizontal and Vertical Modes

equilibrium indicating that the equivalent SDOF system is under-critically damped. It has been observed that the free vibration phase of the B-IRF of the coupling mode does not have the characteristic shape of the dynamic response of a SDOF subjected to an impulse excitation and is not considered further in this study.

SYSTEM IDENTIFICATION AND LUMPED PARAMETER MODEL

The free vibration phase of the B-IRF functions, as calculated by the BEM and presented in the preceding section, are used next to extract the dynamic properties of the equivalent SDOF that represents the soil foundation system. To this end, least square nonlinear regression techniques are adopted within a system identification approach, as reported by the authors in [15, 16].

System Identification

The normalized response of the primary modes of the foundation exhibit similar characteristics to the free vibration response of an SDOF oscillator subjected to initial conditions. Therefore, the corresponding analytic solutions are used in a non-linear least square regression analysis for a best fit approximation of the discrete B-IRF values. The solution for the free vibration response of an under-critically damped SDOF is adopted in this work to represent the normalized response, $\overline{U}(\tau)$, of a rotational vibration mode (*R* or *T*) and is expressed as,

$$\overline{U}(\tau) = [A\cos(\omega_D(\tau - \Delta\tau)) + \frac{B}{\omega_D}\sin(\omega_D(\tau - \Delta\tau))]e^{-\xi\omega_n(\tau-\Delta\tau)}$$
$$\text{where}:\ A = \overline{U}(\Delta\tau);\ \omega_D = \omega_n\sqrt{1-\xi^2};\ \text{and}\ B = \frac{d\overline{U}(\Delta\tau)}{d\tau} + \overline{U}(\Delta\tau)\xi\omega_n \tag{3}$$

where $\Delta\tau$ is the normalized support of the B-Spline impulse. The initial displacement/rotation is denoted as $\overline{U}(\Delta\tau)$, ω_n is the natural frequency of the system, ω_D is the damped natural frequency of the system, ξ is the damping ratio, and $\frac{d\overline{U}(\Delta\tau)}{d\tau}$ is the initial velocity. Using the same notation, the solution for the free vibration of an over critically damped SDOF is adopted to represent the translational modes and can be defined in a similar form as,

$$\overline{U}(\tau) = \left[\left(\frac{B + (A\,\xi\,\omega_n)}{\omega_D}\right)\sinh(\omega_D(\tau - \Delta\tau)) + A\cosh(\omega_D(\tau - \Delta\tau))\right]e^{-\xi\,\omega_n(\tau-\Delta\tau)}$$
$$\text{where}:\ A = \overline{U}(\Delta\tau);\ \omega_D = \omega_n\sqrt{\xi^2 - 1};\ \text{and}\ B = \frac{d\overline{U}(\Delta\tau)}{d\tau} \tag{4}$$

It should be noted that the evaluation of Equations (3) and (4) are on a shifted normalized time scale due to the fact that they start at the end of the forced vibration stage at time $\tau = \Delta\tau$ that provides the initial displacement of the free vibration phase. The initial velocity is defined based on the descending trend of the first few data points in the corresponding B-IRF function. In view of the normalized B-IRF data, the curve fitting procedures pertain to the calibration of the natural frequency, ω_n, and damping ratio, ξ, appearing in Equations (3) or (4), for each vibration mode using a nonlinear least squares regression routine. Table 2 summarizes the values of the initial conditions and the identified parameters for the four vibration modes as evaluated from the nonlinear regression [15, 16].

Table 2 Equivalent SDOF Properties

Property \ Mode	Horizontal (H)	Vertical (V)	Rocking (R)	Twist (T)
$\overline{U}(\Delta\tau)$	0.01816832	0.00949191	0.09773068	0.09175623
$d\overline{U}(\Delta\tau)/d\tau$	-0.025	-0.002	-0.15	-0.325
$\overline{\omega}_n$	7.74	3.5	4.1138	3.8
$\overline{\xi}$	1.58	1.1	0.6343	0.76

Equivalent Lumped Parameter Model

The proposed lumped parameter model represents a soil-foundation system by discrete spring, mass, and damper elements for each vibration mode of the foundation. The stiffness of the spring element is assumed to be the static stiffness of each vibration mode of the soil-foundation system. The equivalent static stiffness is reported in the literature, e.g. [14]. In the present work the coefficient of static stiffness is computed using the procedure reported in [18]. To this end, a unit amplitude constant force is suddenly applied on the reference system in the direction of each dof and the system is allowed to reach its new state of equilibrium. The inverse of the amplitude of the steady state displacement represents the static stiffness for each mode, and is expressed in a dimensionless form in view of Equation (1). The computed dimensionless static stiffness coefficients for each primary mode are,

$$\begin{aligned} \overline{k}_H &= 2.87 \quad \overline{k}_V = 3.68 \\ \overline{k}_R &= 0.889 \quad \overline{k}_T = 1.189 \end{aligned} \tag{5}$$

Using the identified dimensionless natural frequencies and damping ratios of Table 2, along with the static stiffness, for each of the primary modes, an equivalent dimensionless mass, $\overline{m}_i$, and damping coefficient, $\overline{c}_i$, can be calculated as,

$$\overline{m}_i = \frac{\overline{k}_i}{\left(\overline{\omega}_n\right)_i^2} \tag{6}$$

$$\overline{c}_i = 2\left(\overline{\xi}_n\right)_i \sqrt{\overline{k}_i \overline{m}_i} \tag{7}$$

where the subscript i=H, V, R or T indicates the vibration mode of the response. Subsequently, the equivalent system properties for a particular physical problem (new soil and foundation size) can be calculated as,

$$\begin{aligned} k_i &= \overline{k}_i\, G\, w \\ m_i &= \overline{m}_i\, \rho\, w^3 \qquad i = H, V \\ c_i &= \overline{c}_i \left(G\, w^2\right)/v_s \end{aligned} \tag{8}$$

$$k_i = \bar{k}_i \, G \, w^3$$
$$m_i = \bar{m}_i \, \rho \, w^5 \qquad i = R, T \tag{9}$$
$$c_i = \bar{c}_i \left(G \, w^4 \right) / v_s$$

These equivalent SDOF systems model the soil-foundation components of a typical structural system. These models can be easily implemented into commercially available software for the global transient analysis of structural systems to account for dynamic SSI effects. In the following studies the soil foundation continuous system is represented by the lumped parameter system shown in Figure 2 in plane view. This model is implemented in standard FEM procedures for time history analysis using Newmark's-β method.

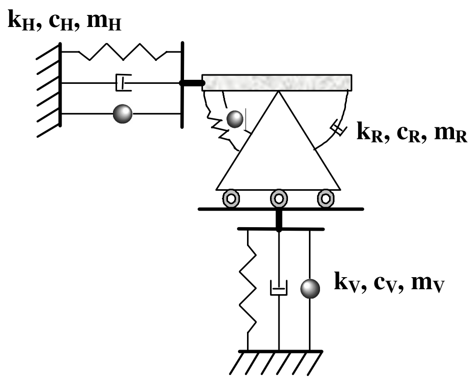


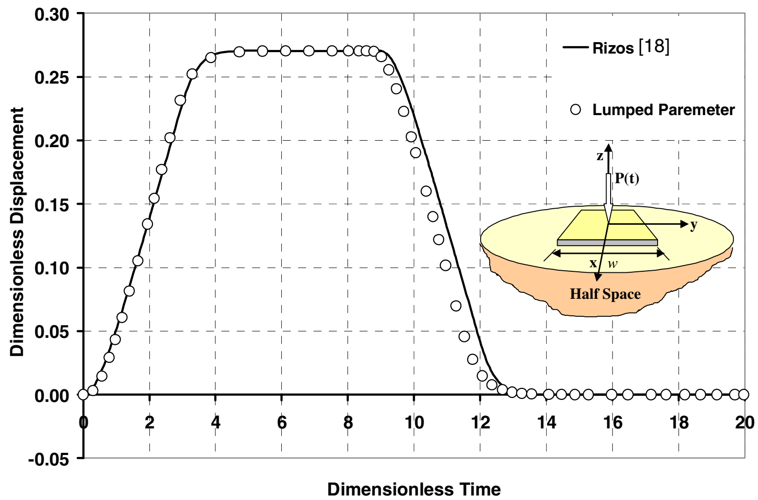
Figure 2. Lumped Parameter

VALIDATION STUDY

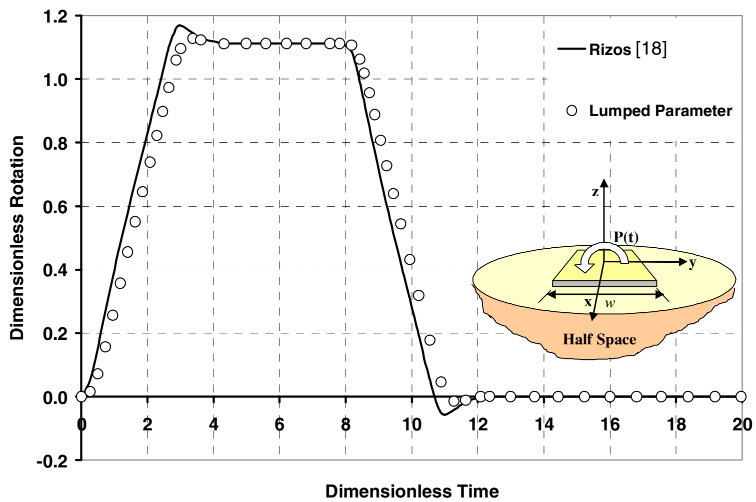
The proposed model is validated in both the time and frequency domains. To this end, analysis results using the proposed model are compared to rigorous BEM solutions and other lumped parameter models reported in the literature in the form of response time histories [18] or compliance curves [14,18,20].

Solution to Arbitrary Loading – Time Domain

The reference soil-foundation system is subjected to a trapezoidal load in the direction of each primary mode of vibration. The total duration is selected to be approximately 7 times the natural period of vibration of each mode, $(T_n)_i$, as computed from the non-dimensional natural frequencies. For each load time history the governing equations are expressed in a FEM sense and the system response is computed directly in time domain using the Newmark-β time marching scheme. Figure 3 shows a comparison of the time history of the response for the vertical, Figure 3a, and rocking, Figure 3b, modes, as obtained by the proposed model and the BEM method reported in [18]. It is evident that proposed lumped parameter model yields accurate predictions of the foundation response as compared to the rigorous BEM time domain solutions. A number of arbitrary load studies have been performed that consider a variety of excitation time histories and all have demonstrated similar accuracy for the proposed simplified approaches [15, 16].



(a) Vertical Response



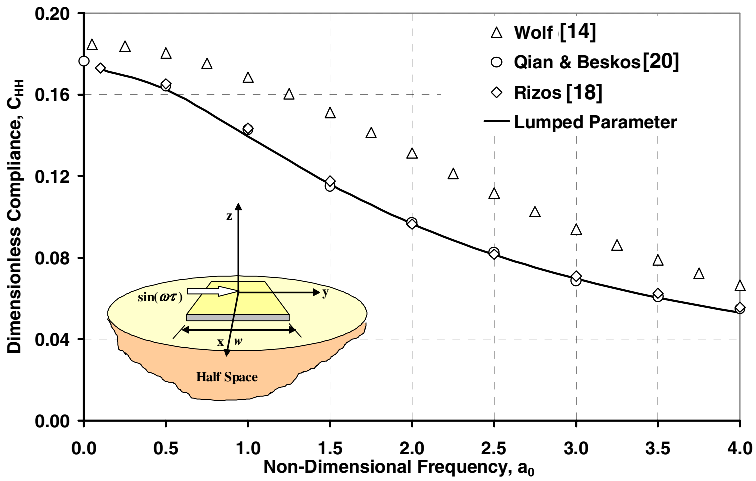
(b) Rocking Response

Figure 3. Validation Study: Time History of Foundation Response to (a) Vertical and (b) Rocking excitations

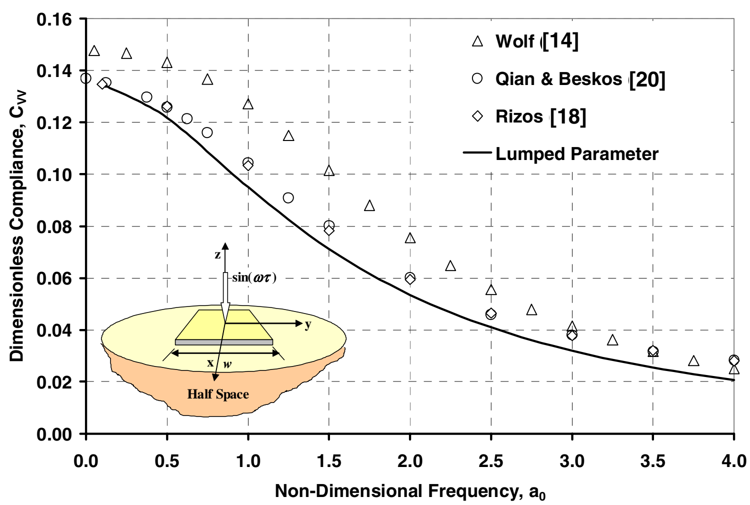
Solution to Harmonic Loading – Frequency Domain

The proposed lumped parameter model has also been validated in the frequency domain [15, 16]. In particular, comparisons are performed based on compliance functions of the soil foundation system. The soil-foundation system is subjected to a unit amplitude harmonic load with dimensionless frequencies, $a_0 = \omega w / 2 v_s$, in the range of zero to four. For each excitation frequency, the time history of the system response is calculated and the amplitude of the steady state is monitored. The dimensionless compliance values, C*ij*, are calculated using the normalization factors of Equation (1) for each of the primary modes. Subscript *i* indicates the vibration response mode and subscript *j* indicates the excitation mode. The vertical and horizontal compliances as computed based on the lumped parameter systems are shown in Figures 4a and 4b respectively, along with the compliances reported in the literature, where available [14,18,20]. It is evident that the proposed models produce accurate results for the translational modes.

Compliance comparison for the rotational modes, however, shows higher discrepancies at higher frequencies, despite the good agreement in the time domain [18].



(a) Horizontal Compliance



(b) Vertical Compliance

Figure 4. Validation Study: (a) Horizontal Compliance and (b) Vertical Compliance

EFFECTS OF SSI ON RESPONSE OF SEISMICALLY ISOLATED STRUCTURES

This section studies the dynamic and seismic response of coupled soil-foundation-structure systems in reference to a typical two-span highway bridge model with non-integral abutments [15, 19]. In this study seismic isolation devices are also considered. A stick model is used to represent the vibration of bridge structure in the transverse direction. It is assumed that the mass of the bridge deck is lumped at the top of the pier. A simplification of the bridge foundation assumes that the pier rests on surface rigid footings that are modeled using the proposed lumped parameter models. The bridge model is shown in Figure 5. Standard FEM analysis procedures are followed, as discussed in the preceding validation study. The bridge model is subjected to seismic loading records from Imperial Valley, El Centro 1940 E-W, component. The impact of SSI on the structural response for isolated and non-isolated conditions is then quantified using rigid base analysis of the structure as a baseline. The equivalent force time history vector is computed as

$$P(t) = M_{deck} a_g(t) \tag{10}$$

where $a_g(t)$ is the ground acceleration time history, and M_{deck} is the superstructure lumped mass. The equivalent force is applied to the lumped mass at the superstructure level for all comparative studies as shown in Figure 5. During these studies, the maximum displacement of excitation, and damped period of free vibration phase, T_D, during the free vibration phase of the system response are monitored. The composite damping ratio, ξ of the system is calculated based on the assumption of logarithmic decrement.

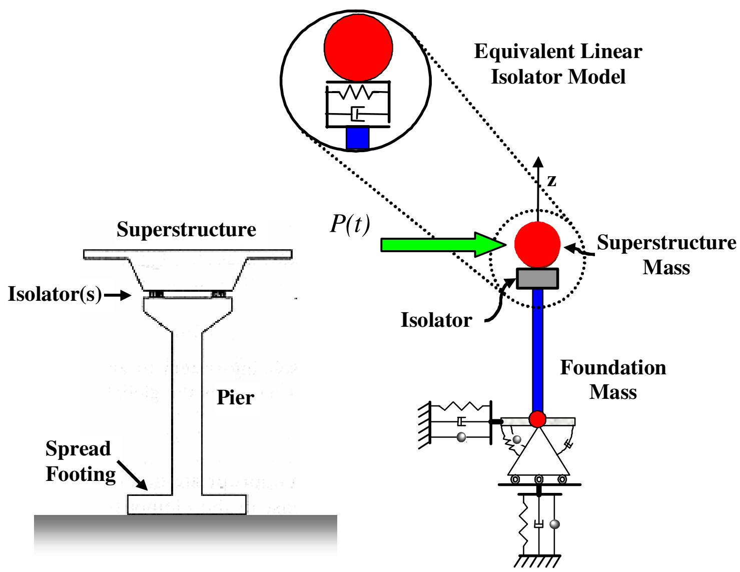


Figure 5. Bridge structure and FEM model

Identifying the Effects of SSI on Structural Response

Figure 6 shows the relative horizontal deck displacement when subjected to an equivalent force time history from El Centro 1940, for the rigid base assumption as compared to the case where SSI effects are accounted for. Different soil conditions ranging from soft (Soil 1) to stiff (Soil 4) are considered in this study. It is observed that the SSI effects are more pronounced for softer soils and less significant, although still present, for stiffer soils. In particular, the effects of SSI on the structural response are: (i) increased maximum relative deck displacement, (ii) increased number of significant cycles of large amplitude displacement, and (iii) damped period of vibration of the structure significantly lengthened for softer soils. In addition, the composite damping ratio when SSI is accounted for is higher for softer soils and approaches the rigid base damping ratio for stiffer soils. This higher value of damping ratio is attributed to the added radiation damping that is characteristic of softer soils. In addition, the measured damped period ratio and composite damping ratio are plotted in Figure 7 in terms of the soil shear modulus for rigid base and SSI analysis. The larger period elongation and added damping for softer soils is evident.

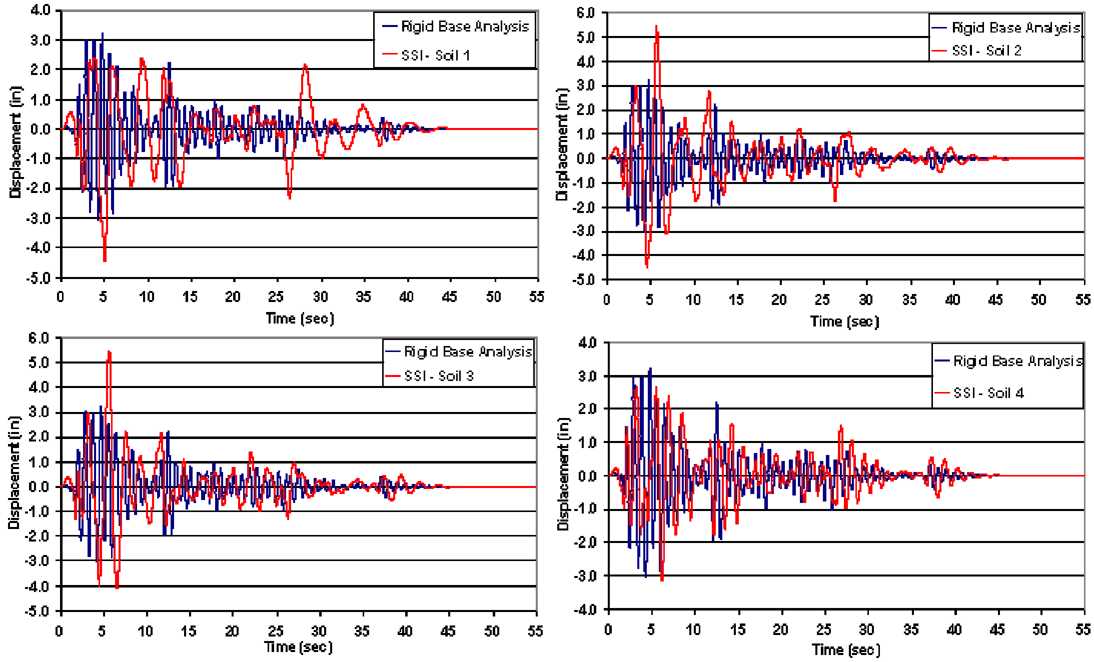


Figure 6. Bridge Seismic Response for Different Soil Conditions

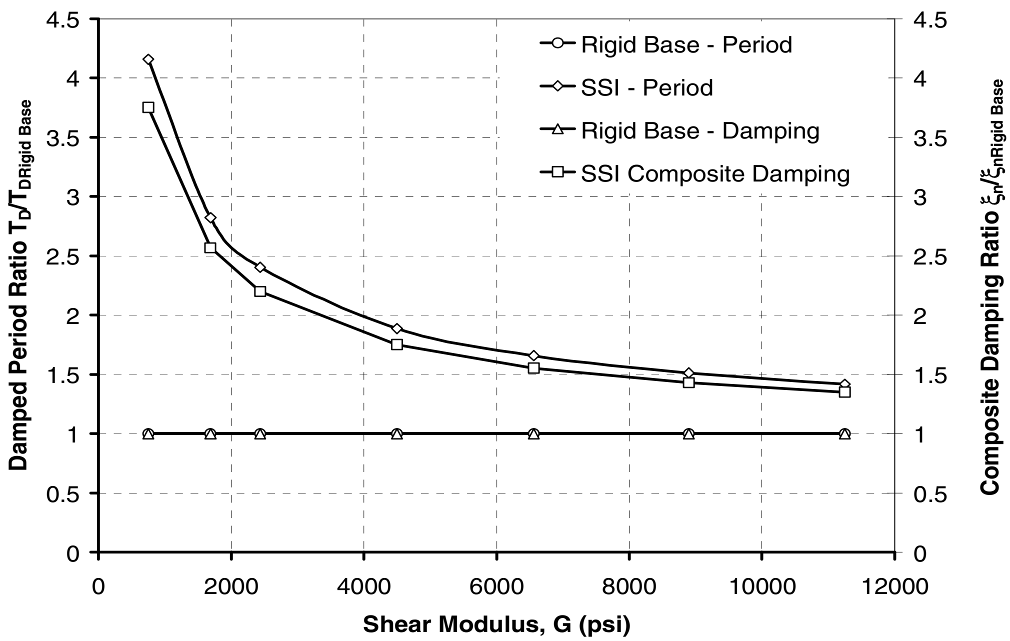


Figure 7. Period Elongation and Increased Composite Damping

SSI Effects on Efficiency of Seismic Isolation Devices

The isolated bridge shown Figure 5 is also subjected to an equivalent force time history from El Centro 1940 for two different seismic isolation devices in the structure [19]. The un-retrofitted structure has a natural period of 0.9 seconds. The isolation devices are designed using the rigid base assumption for a target structural natural period of vibration of 2.0 and 3.0 seconds for Isolation Device (Iso. Dev.) #1 and #2 respectively. These devices are modeled as equivalent linear springs and dampers that are indicative of the average horizontal stiffness variability and dissipation characteristics of a bearing. In particular, the properties of equivalent effective stiffness, k_{eff}, and effective damping ratio, ξ_{eff}, are associated with the springs and dampers. Using the methods presented in [21], an effective horizontal stiffness for each isolation device is calculated as 16.17 kip/in and 6.3 kip/in for Iso. Dev. #1 and #2 respectively. These values for horizontal stiffness are within the range of typical values for elastomeric bearings for bridge structures [21]. Additionally, the isolation devices are assumed to be able to provide 15% of critical viscous damping. The combined effects of considering SSI along with seismic isolation of the two bridge models are investigated next. The models are subjected to the same excitation from El Centro 1940, and the relative horizontal displacement of the top of the pier is monitored. Figure 8 shows the results of the relative deck (pier) displacements using Iso. Dev. #2 in rigid base analysis as compared to considering SSI effects for the four different soil conditions.

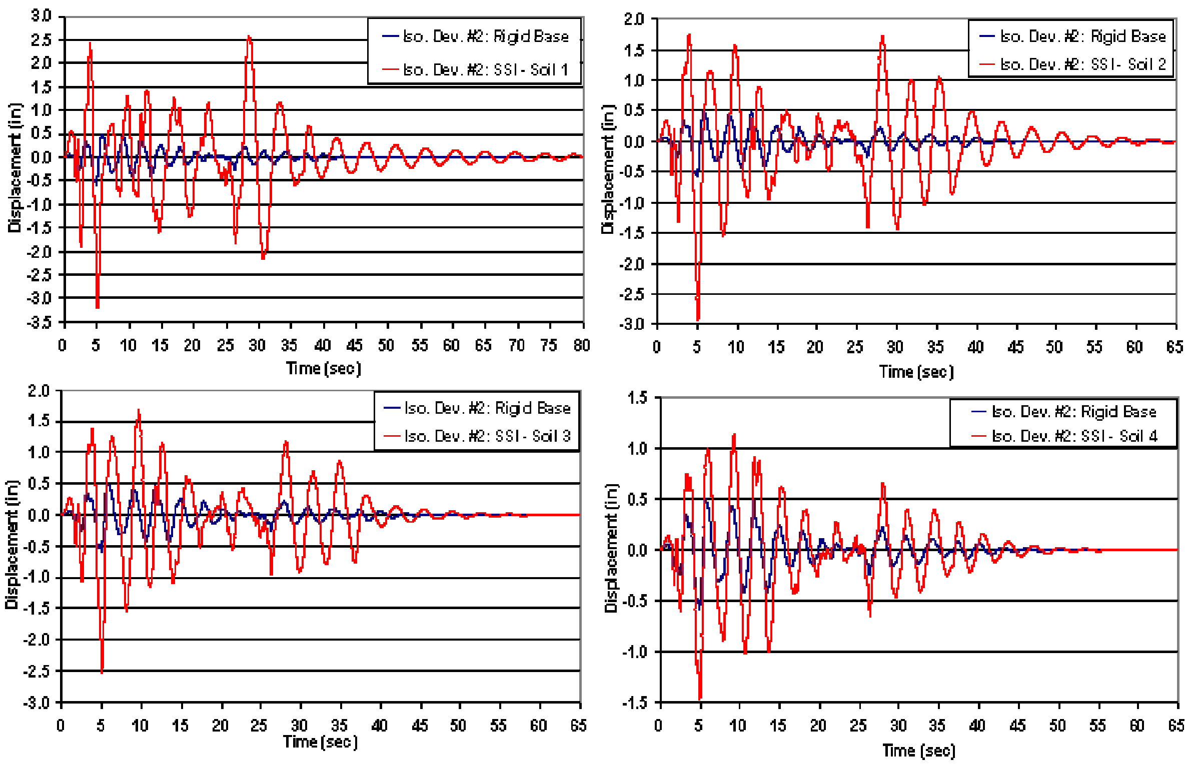


Figure 8. Isolated Bridge Seismic Response for Different Soil Conditions

SSI effects can be seen in the response amplification for the pier for all of the soil conditions. As was identified earlier for the un-retrofitted structure when SSI is considered, a period shift is noticeable for each of the soils with the softest soil (Soil 1) having the most pronounced elongation. Also, the composite damping ratio of the structure appears to be very small, suggesting that SSI effects counteract the damping characteristics of the isolation system. Similar observations are present when Iso. Dev. #1 is utilized. It is interesting to note that as the soil gets stiffer, the effects of SSI are less significant in the response of the pier, but still tend to amplify the response of the pier by approximately 40% for the stiffest soil condition considered as compared to the pier demand in the rigid base analysis. Figure 9 illustrates the pier amplitude amplification for different soil conditions considering the un-retrofitted structure, and the isolated structures along with the consideration of SSI and rigid base conditions. It is apparent from these results that the inclusion of SSI for this system shows the ineffectiveness of the isolators to reduce the demand on the structure.

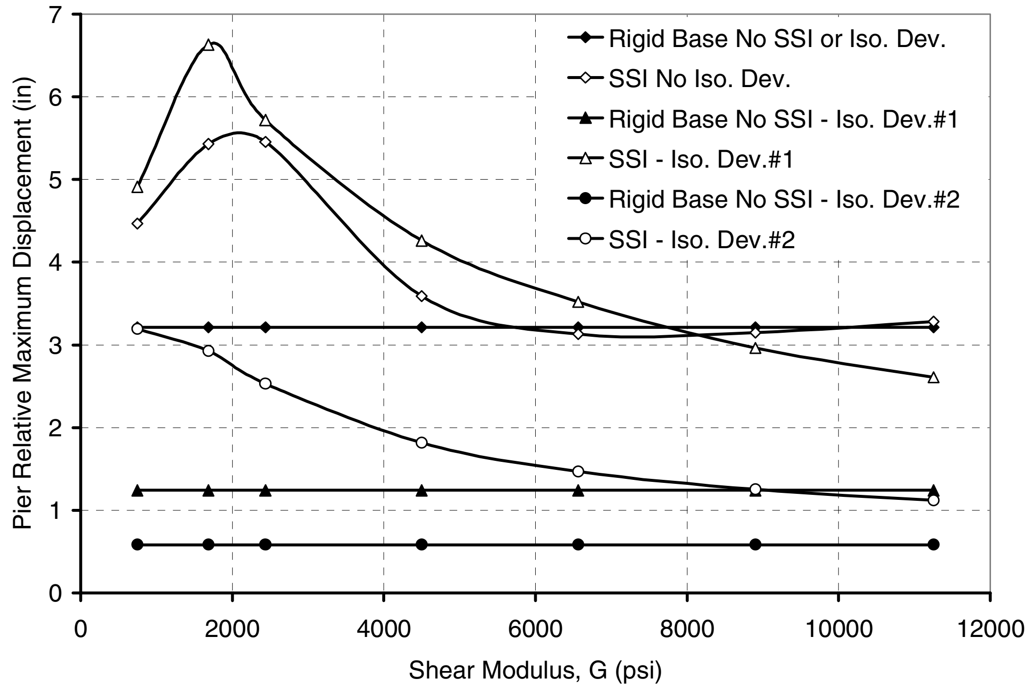


Figure 9. Pier Relative Maximum Displacement using SSI and Iso. Dev.

Considering SSI effects in the analysis of bridge structures can have a significant impact on the structural response during a given event. Previously, SSI effects have been identified as important, in particular, for heavy structures resting on soft soils. It is evident from the results presented here that the effects of SSI are important for this relatively light structure over a range of soil conditions. Perhaps then, it is not a function of particular mass or stiffness of the structure, but rather a ratio of the relative rigidity or frequency ratio between the structure and the soil-foundation system.

CONCLUSIONS

This work discussed simplified, yet accurate, models of soil-foundation systems. These models pertain to rigid square surface foundations and the development is based on a system identification approach. To this end, the dynamic parameters of SDOF systems are calibrated through nonlinear regression to fit the B-Spline impulse response functions of the vibration modes of the soil-foundation system. The proposed models are validated in both the frequency and time domain. They are used in standard FEM analysis procedures for assessing the effects of Soil-Structure Interaction (SSI) on the seismic response of bridge structures, as well as assessing the SSI effects on the effectiveness of seismic isolation devices. Preliminary investigations have shown that the significance of the SSI effects depends on the relative rigidity between the superstructure and the soil-foundation systems. It is identified further, that the natural frequency ratio between the bridge structure and the soil-foundation system may be the governing criterion in assessing the impact of SSI effects.

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