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SIMPLIFIED METHOD FOR ESTIMATING THE ELASTOPLASTIC DEFORMATIONS OF RC STRUCTURES

ZHANG Lingxin [1] JIANG Jinren [2]

SUMMARY

In this paper, a simplified method for estimating the elastoplastic deformations of RC structures subjected to major earthquake is developed, which reduces the multi-degree of freedom (MDOF) structure to an equivalent single degree of freedom (SDOF) system. As an example, the top displacement and the interstory displacement of a RC frame-shear wall structure is estimated by this method.

INTRODUCTION

Considering the balance between safety and economy, the seismic design of buildings usually permits the structures to enter the elastoplastic working state. The current seismic design codes adopt elastic design in form and strength design that is not applicable to performance-based seismic design. In response to the need of performance-based seismic design, simplified nonlinear static procedure (pushover analysis) has been put forward to determine the inelastic seismic displacement of buildings. Besides, the current Chinese Code for Seismic Design of Buildings (GB50011-2001) requires that the high buildings need checking the seismic deformation under major earthquake and provides the method only for the RC frame structures with lower than 12-story and the well-distributed stiffness. As above-mentioned, the computation of the inelastic seismic deformations may adopt nonlinear static procedure. In this paper, based on the nonlinear static procedure, a simplified inelastic seismic response analysis method using an equivalent SDOF system to estimate the top displacement and the interstory displacement of multi-story and high-rise buildings is developed. And as an example, a RC frame-shear wall structure is analyzed by the suggested method using the design response spectrum of the code (GB50011-2001).

EQUIVALENT SDOF MODEL

In order to establish the equivalent SDOF model of MDOF system, many researches have been done. For example, Saiidi and Sozen[2] put forward to Q-model method in 1979. Later, Qi, Mochle[3] and Collins[4] improved this method. The method of this paper is mainly based on the research works of Qi, Mochle and Collins.

[1] Professor, Institute of Engineering Mechanics, CEA, Harbin 150080, China
[2] Professor, Institute of Engineering Mechanics, CEA, Harbin 150080, China

For inelastic MDOF system, if the top (roof) displacement response is chosen as the value of solution, we can imitate the mode decomposition method of the elastic MDOF to solve the equivalent SDOF system.
Multi-story structure is usually simplified to a MDOF system with dump masses on the floors. Its motion equation can be expressed as:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+\{R\}=-[M]\{I\}\ddot{u}_g \tag{1}$$

Where, [M] is the mass matrix, which is hypothesized to be diagonal; [C] is the damping matrix; {R} is the restoring force vector; {u} is the vector of relative lateral displacement of floor; {I} is the influence vector of the ground motion; u_g is the displacement of ground motion; the dot indicates time derivative.
It is hypothesized that {u(t)} can be shown as $\{\phi_1\}D(t)$, in which $\{\phi_1\}$ is a hypothetic fixed- displacement profile and is normalized according to the component vector of top displacement being 1; D(t) is the top displacement. The displacement profile $\{\phi_1\}$ is solved using the inelastic static analysis of structure which is applied a stipulated lateral load distribution {f}. The vector {f} is normalized so that the base shear force becomes 1, i.e. $\{I\}^T\{f\}=1$. Then by increasing the lateral load step by step, and repeating the inelastic static analysis, the applied load distribution can be expressed as V{f}={Vf}, where V is a scalar quantity factor and can indicate the base shear force in physics. Through this kind of increment analysis, the variation of V with D can be obtained. The curve of V and D is drawn, as shown in Fig. 3, which can be expressed using the mathematic equation V = KG(D), where K is the slope of the initial part of the curve; G(D) is the scalar quantity function indicating the shape of curve. It is hypothesized that the restoring force vector {R} can be also shown by a group of force in static analysis, i.e. KG(D){f}. Substituting KG(D){f} and $\{\phi_1\}$D with {R} and {u}, then the motion equation can be written as:

$$[M]\{\phi_1\}\ddot{D}+[C]\{\phi_1\}\dot{D}+KG(D)\{f\}=-[M]\{I\}\ddot{u}_g \tag{2}$$

The two sides of the above vector equation are multiplied by the second vector $\{\phi_2\}^T$:

$$\{\phi_2\}^T[M]\{\phi_1\}\ddot{D}+\{\phi_2\}^T[C]\{\phi_1\}\dot{D}+KG(D)\{\phi_2\}^T\{f\}=-\{\phi_2\}^T[M]\{I\}\ddot{u}_g \tag{3}$$

Defining, $M^*=\{\phi_2\}^T[M]\{\phi_1\}$;

$C^*=\{\phi_2\}^T[C]\{\phi_1\}$;

$K^*=K\{\phi_2\}^T\{f\}$;

$L^*=\{\phi_2\}^T[M]\{I\}$;

$P^*=L^*/M^*$.

Then Eq.3 can be written as follow:

$$M^*\ddot{D}+C^*\dot{D}+K^*G(D)=-L^*\,\ddot{u}_g \tag{4}$$

The two sides of Eq.4 are divided by M*:

$$\ddot{D}+2\xi\omega^*\dot{D}+\omega^{*2}\,G(D)=-P^*\ddot{u}_g \tag{5}$$

Where $\omega^{*2}=K^*/M^*$; $\xi=C^*/(2M^*\omega^*)$.

Eq.5 can be explained to be the motion equation of SDOF with initial (pure elastic) frequency ω* and damping ratio ξ.
In the above deduction, the vector $\{\phi_2\}$ is not clear definition. If $\{\phi_2\}$={I} is chosen, K*=K. therefore K*G(D) corresponds to the base shear force V. If $\{\phi_2\}=\{\phi_1\}$ is chosen, the equation of SDOF obtained coincides with equivalent SDOF equation deduced using the virtual work theory[5]. The item K*G(D) in

Eq.4 is not expressed as the base shear force any more, but it includes lateral load distribution and displacement profile, and can reflect much better the dynamic response characteristics of structures.
Eq.5 provides a simplified method estimating the top displacement of MDOF structures. In the seismic design and evaluation of structures, the interstory displacement ratio is usually regarded as an index evaluating performance of structures. If the maximum top displacement is determined by the equivalent SDOF model, the interstory displacement ratio can be given by the method as follows.
The vector of the maximum lateral displacement of floors is shown as $\{u_{max}\}=\{\phi_1\}D_{max}$, and then the interstory displacement ratio θ_i of any floor i can be expressed as:

$$\theta_i = D_{max}\left(\phi_{1,i} - \phi_{1,i-1}\right) / h_i \tag{6}$$

Where, $\phi_{1,i}$ and $\phi_{1,i-1}$ is the components of displacement vector $\{\phi_1\}$at up and down floor of any floor i; h_i is the story height of any floor i. The entire displacement ratio is defined as $\theta_Z = D_{max}/H$, where H is the entire height of structure. Then the interstory displacement ratio of the ith floor can be written as:

$$\theta_i = \frac{\theta_i}{\theta_Z}\theta_Z = H\left[\frac{\phi_{1,i} - \phi_{1,i-1}}{h_i}\right]\theta_Z = \beta\theta_Z \tag{7}$$

And the maximum interstory displacement ratio θ_{max} is:

$$\theta_{max} = H\left[\frac{\phi_{1,i} - \phi_{1,i-1}}{h_i}\right]_{max}\theta_Z = \beta_{max}\theta_Z \tag{8}$$

Where, β and β_{max} are the factors of the interstory displacement ratio and the maximum interstory displacement ratio transformed from the entire displacement ratio, respectively. It can be seen that β and β_{max} give the relationship between the equivalent SDOF analysis results and the MDOF responses.
In the analysis, the lateral load distribution can adopt the reverse triangle distribution used often in the base shear force method, which is determined by response spectrum mode decomposition method.
Eq.4 or 5 can be solved by direct integration method or inelastic response spectrum method. In this paper, in order to use conveniently the standard elastic response spectrum stipulated by code, the equivalent linerization method and elastic response spectrum method are adopted to solve two equations.

EQUIVALENT LINER SYSTEM AND ELASTIC RESPONSE SPECTRUM METHOD

The equivalent SDOF system of Eq.4 can be substituted by equivalent linear system with the equivalent damping C_e and the equivalent stiffness K_e:

$$M^*\ddot{D} + C_e\dot{D} + K_e D = -L^*\ddot{u}_g \tag{9}$$

This paper adopts the equivalent stiffness given by the average stiffness method and the equivalent damping based on results of experiments[5]. For the two-linear or the trilinear system, the equivalent linear stiffness can be expressed as[5]:

$$K_e(\mu) = K^*\left[(1-\alpha)(1+\ln\mu) + \alpha\mu\right]/\mu \tag{10}$$

Where, μ is the ductility ratio, which is defined as the ratio of the maximum displacement to the yield displacement; α is the stiffness ratio of post-yield to pre-yield.
The equivalent damping ratio can be shown as follows[5]:

$$\xi_e(\mu) = 0.05 + 0.15\times\left(1 - \sqrt{\frac{1+\alpha(\mu-1)}{\mu}}\right) \tag{11}$$

Owing to these equivalent linear parameters depending on the displacement responses, the iteration method on the basis of the elastic response spectrum is used to solve responses.

The displacement response spectrum corresponding to the design spectrum of the code (GB50011-2001) can be expressed as:

$$\Delta(T,\xi) = \frac{1}{\omega^2} g\alpha(T) = 24.85T^2\alpha(T) \quad \text{(cm)} \tag{12}$$

Where, ω and T is the inherent frequency and period of structures, respectively; α (T) is the seismic influence coefficient, which includes a damping revised coefficient for damping ratio other than 0.05.

The last displacement required is the one obtained using the above equation multiplied by P*.

EXAMPLE

The application example of this method is illustrated by a seven-story RC frame-shear wall structure located in Tianjin. The site classification is III, and its plan is shown in Fig.1.

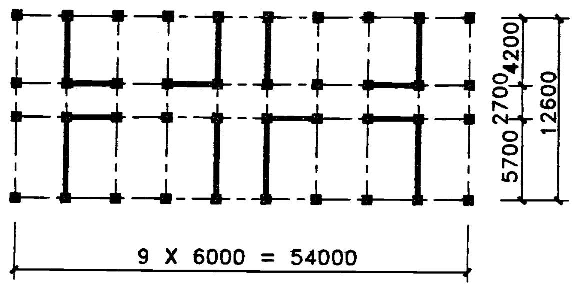

Fig.1 Plan

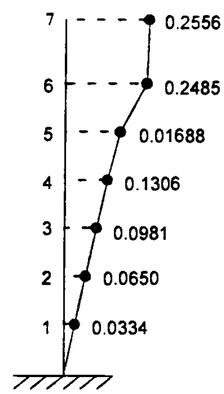

Fig.2 Lateral load distribution

The lateral load distribution {f} determined by the response spectrum mode decomposition method is shown in Fig.2. From this figure, we can see that its shape is similar to the reverse triangle distribution except that the load in the 6th floor is larger. Applying the lateral load distribution on the structure and increasing the value of the distribution, we can complete the inelastic static analysis step by step, and gain the relationship between the base shear force V and the top displacement D, as shown in Fig.3. Fig.4 shows the lateral displacement profile of structure at different loading stage. In order to obtain the function G(D) of the equivalent SDOF model using the curve in Fig.3, a two-linear force-displacement relationship (dashed line) is adopted to approximate the curve in Fig.3, in which the post-yield stiffness is taken the same one as that of the real force-displacement curve and the initial elastic stiffness is obtained by the energy equalization principle.

Then the equation of the force-displacement relationship G(D) of the equivalent SDOF model can be expressed as:

$$V = \begin{cases} K_0 D & \text{when} D \le 0.03945\text{m} \\ \alpha K_0 D & \text{when} D \succ 0.03945\text{m} \end{cases} \tag{13}$$

Where, K_0 = 307990.515 kN/m, is elastic stiffness; α = 0.038, is the ratio of plastic stiffness to elastic stiffness. The initial viscous damping ratio is 0.05.

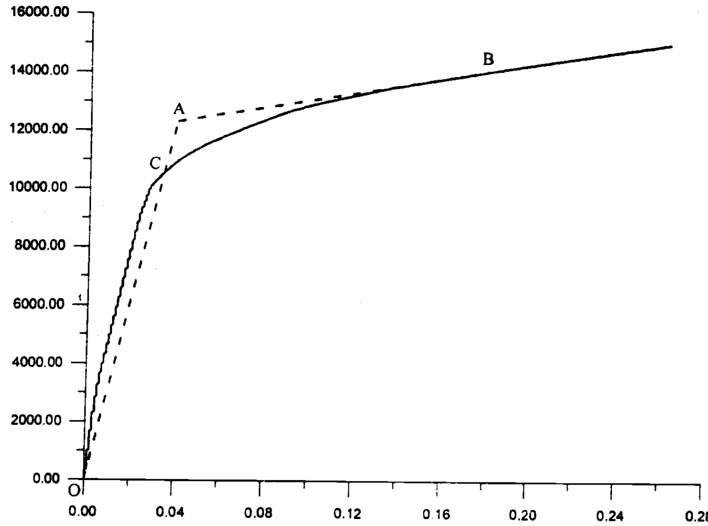

Fig.3 Top displacement (D) - base shear force (V) relationship curve

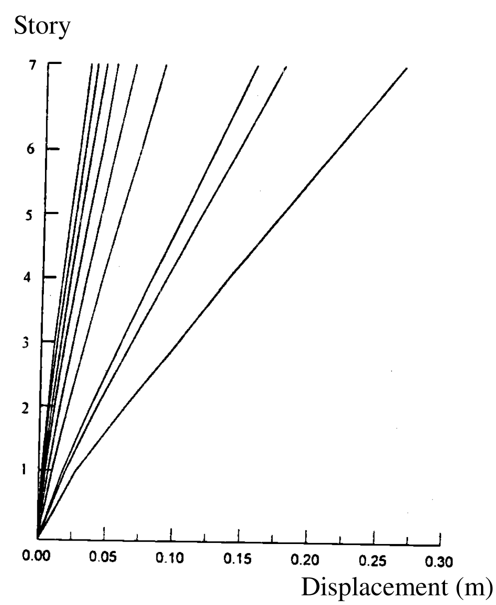

Fig.4 Displacement profile at different loading stage

In this example, we are interested in the expected inelastic displacement of structures subjected to major earthquake. So we need select a certain deflection profile corresponding to the expected inelastic displacement to demarcate the parameters of the above equivalent SDOF equation. To the frame-shear wall structures, referring to the frame structures, without a doubt the displacement profile should take the one with smaller top displacement ratio. By trial and error method, this paper adopts the displacement profile with the top displacement ratio being 1/150 to calculate the approximate inelastic top displacement of frame-shear wall structure subjected to major earthquake. The parameters of the equivalent SDOF equation thus obtained are list in Tab.1.

TABLE 1 THE PARAMETERS OF THE EQUIVALENT SDOF EQUATION

M*	K*	P*	$T_e(\mu)$	$\xi_e(\mu)$	$\eta(\xi)$
1843.442	213654.5	1.47	0.617	0.085	0.817

The peak ground acceleration corresponding to major earthquake of intensity VIII is taken as 400gal to analyze the response of structure. In order to check the accuracy of this method, three men-made waves signed by I, II and III respectively are adopted to conduct accurate inelastic dynamic response analysis. The analysis software used here is the revised inelastic dynamic response analysis software DRAIN-2D. All the calculated results are list in Tab.2. The compared results with results of simplified method are also list in Tab.2. It can be seen from the table that the results of the simplified method are agreeable to the average results of the time history analysis for three men-made waves. The difference of the top displacement is only 2.9%; the difference of the maximum interstory displacement ratio is a bit large, but it's less than 8%. All the maximum interstory displacement ratios are appeared in the 6th story. The checking results of this example indicate that this method has enough engineering accuracy to estimate the inelastic deformation of multi-story frame-shear wall structure subjected to major earthquake. In the mean time, it is shown that the target displacement selected to the top displacement ratio being 1/150 is suitable.

TABLE 2 COMPARISON OF NONLINEAR RESPONSE RESULTS BETWEEN ACCURATE AND SIMPLIFIED METHODS OF THE EXAMPLE

Analysis method	History time analysis method				Simplified method	Difference
	Men-made wave I	Men-made wave II	Men-made wave III	Average		
Top displacement Δ(m)	0.068	0.072	0.074	0.0713	0.06925	2.9 %
Top displacement ratio Δ/H	1/392.6	1/370.8	1/360.8	1/374.5	1/385.5	2.9 %
The maximum interstory displacement ratio θ_{max}	0.00300 (1/333)	0.00308 (1/324)	0.00305 (1/327)	0.00305 (1/328)	0.00281 (1/356)	7.87 %
The floor where θ_{max} appeared	6	6	6	6	6	

CONCLUSIONS

A simplified method for estimating the top displacement and the interstory displacement by reducing the multi-degree of freedom earthquake resistant structure to an equivalent single degree of freedom system is developed. The seismic response of the equivalent SDOF system is solved by equivalent linerization and design spectrum of seismic code. The method is applied to a seven-story RC frame-shear wall structure, and the target displacement corresponding to the displacement profile is taken as the top displacement ratio being 1/150. The good agreement between the results of simplified method with those of time history analysis for the application example shows that the simplified method has enough engineering accuracy and the target displacement selected to the top displacement ratio being 1/150 under major earthquake is suitable. So it could be used for the displacement computation of RC buildings under major earthquake in Chinese code and also for the displacement evaluation for performance-based seismic design.

REFERENCES

1. National Standard of the People's Republic of China: Code for seismic design of buildings (GB50011-2001), Architectural Industry Press of China. (in Chinese).
2. M. Saiidi and M. A. Sozen: Simple and Complex Models for Nonlinear Seismic Response of Reinforced Concrete Structure, Structural Research Series №. 465, University of Illinois Urbana, IL, August 1979.
3. X. Qi and J. P. Mochle: Displacement Design Approach for Reinforced Concrete Structures Subjected to Earthquake, UCB/EERC-91/02, Jan. 1991.
4. K. R. Collins, A Reliability-Based Dual Level Seismic Design Procedure for Building Structures, Earthquake Spectra, V1, N3, 1995.
5. Jiang Jinren and Lu Qinnian, Stochastic Seismic Response Analysis of Hysteretic MDF Structures Using Men Response Spectra, Earthquake Engineering and Engineering Vibration, Vol.4, No.4, Dec., 1984, pp1-13. (in Chinese).