13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 2897

SEQUENTIAL OPTIMAL CONTROL FOR CALCULATING IDEAL CONTROL FORCE IN SMART SEISMIC ISOLATED BUILDINGS

Yongfeng DU [1] Hui Li [1] Guofan Zhao [2]

SUMMARY

This paper derives a new model for optimal control of smart seismic isolated buildings. When classical optimal control is applied to seismic control, a non-homogeneous term will appear in the traditional Riccati equation, which causes the model unsolvable. Two current optimal control algorithms were derived based on a lot of approximation. In this paper, the control objective function is expressed using impulse response of both seismic excitation and the control force, which leads to a new style model of optimal control. By defining a dual system according to the resemblance between the companion equation and the state equation, the optimal control force is directly calculated by state transition algorithm. An improved optimal control algorithm is thus developed, and is named as Sequential Optimal Control (SOC). A 7-story isolated RC frame structure is employed as a numerical example, and simulation result shows that the proposed SOC not only provide a more reasonable concept for deriving the model, a more accurate way of solving the equation, but also numerically improves the control effect of the current optimal control algorithms.

1. INTRODUCTION

Seismic isolation has become one of the most widely accepted means for seismic protection of building structures in the past decade. To date, more than one thousand of practical engineering projects worldwide have been implemented with isolation system. However, base isolation is a kind of passive, narrow band control method. Passive control systems may not always be efficient in reducing the random vibration caused by earthquakes. A lot of research work has been carried out to improve the adaptiveness of isolation method by combining active or semi-active devices with the isolation system. Typical examples of such devices may be found as the active tendon Soong [1], Housner [2], active viscous dampers Ribakov [3], controllable friction devices Feng [4], semi-active ER dampers Makris [5], and MR dampers Spencer [6].

To achieve desired control effect, proper control algorithms must be worked out to generate control forces according to sensed structural responses and earthquake excitation. Optimal control theory is one of the most useful tools, and has found wide application in structural control Soong [7]. Because the earthquake

[1] Professor, Institute of Earthquake Protection and Disaster Mitigation, Lanzhou University of Technology, Lanzhou 730050, PR China Email: dooyf@sohu.com
[2] Professor, Academician, Civil and Hydraulic School, Dalian University of Technology, Dalian 116023, PR China

excitation is not known *a priori*, the control force in current optimal control algorithm is solved using either of the following approximate methods:

(1) Omit the earthquake excitation and use traditional Riccati Equation to get a control gain matrix based on approximate classical optimal control (COC) Abdel-Rohman [8], which is usually regarded not to be a true optimal control;

(2) Replace objective function with a different one, i.e., the instantaneous optimal control (IOC) Yang [9], Yang [10]. After examining the model of IOC, the senior author found out that the expression of performance index of IOC is not reasonable, and the formula for calculating the control force in IOC is mathematically contradicting with each other.

This paper presents a new optimal control algorithm by expressing the performance index using impulse response of both the earthquake excitation and the control force at each time step, so as to simulate the way in which the earthquake excitation is input to a structural system. A new model of optimal control is derived by directly performing functional variation on the performance index. The control force is calculated by introducing a concept of dual system according to the resemblance between the state equation and the companion equation. The obtained expression of the control force includes both the effect of the structural response and the external seismic excitation. Since the control gain matrix is solved in a step by step manner, the new optimal control algorithm is named as "Sequential Optimal Control (SOC)" by analogy with the sequential linear programming.

A 7-story isolated RC frame structure is employed as a numerical example, and the simulation result shows that the proposed SOC not only provide a more reasonable concept for deriving the model, a more accurate way of solving the equation, but also numerically improved the control effect of the current two optimal control algorithms in that it can better suppress the peak response and causes less increase in the response of the superstructure. The result also shows that the proposed algorithm has almost the same stability range with COC, and has a better settling speed.

2. EQUATION OF MOTION

The equation of motion in state space can be expressed as

$$\{\dot{U}\} = [A]\{U\} + [B]\{f_c\} + \{E\}\ddot{u}_g \tag{1a}$$

$$\{y\} = [C_y]\{U\} + [B]\{f_c\} + \{W\}\ddot{u}_g \tag{1b}$$

$$\{U\} = \begin{Bmatrix} \{u\} \\ \{\dot{u}\} \end{Bmatrix},\ [A] = \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix},\ [B] = \begin{bmatrix} [0] \\ [M]^{-1}[L] \end{bmatrix},\ \{E\} = \begin{Bmatrix} \{0\} \\ -\{\delta\} \end{Bmatrix} \tag{1c}$$

$$[C_y] = \begin{bmatrix} [\Delta] & [0] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix},\ \{W\} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \tag{1d}$$

where $\{u\}$ and $\{\dot{u}\}$ are the displacement and velocity, respectively, relative to the ground. $[M]$, $[C]$ and $[K]$ are the $n \times n$ mass, stiffness and damping matrices, respectively, $\{\delta\}$ is the dynamic load effect vector with all n elements being 1, $\ddot{u}_g$ is the seismic ground acceleration. $\{f_c\}$ is control force vector, $[L]$ is the vector for describing the position of the control force, $[\Delta]$ is a matrix that can generate relative displacement output, and n is the number of degree of freedom of the isolated structure.

3. MATHEMATICAL CONDITION FOR EXTREME POINT

3.1 Main improvement on current optimal control algorithms

For linear structure implemented with ideal controller, optimal control is a powerful tool for determining the control force, and has found wide application in civil engineering. However, when classical optimal control is applied to seismic control, a non-homogeneous term will appear in the traditional Riccati equation, which causes the model unsolvable. Two current optimal control algorithms, i.e., COC and IOC, were derived based on a lot of additional assumptions. The authors examined the model of traditional form of optimal control, and pointed out two difficult things that must be solved before deriving the ideal control force:

1) The earthquake signal is not known *a priori*, which is usually considered to be a widely recognized fact.

2) Another fact worthwhile noting is that even if the earthquake signal is known, the control force still can not be solved directly by using the Riccati equation in the traditional form of expression, because when the model of conventional optimal control algorithms is applied to seismic control, a non-homogeneous term will appear Du [11].

To counter for the first difficulty, this paper proposed a model by expressing the control objective function at each time step. This procedure keeps the advantage of the IOC that better reflects the order in which the excitation (e.g., earthquake or control force) is input to the structural system. In this way, the excitation information is equally available for both cases of determinate and random excitation. The latter difficulty is overcome by defining a dual system according to the resemblance between the companion equation and the state equation, the optimal control force is directly calculated by state transition algorithm. While the first difficulty is solved in this subsection, the second difficulty will be handled in the next subsection.

3.2 Stepwise expression of performance index

While working with the dynamic test of smart isolated model, the senior author noticed a very important fact: no matter whether the external excitation is known (determined) or unknown (random), the excitation is input to the structural system in a step by step manner, and the response of the structure at the present moment can only be related to the state vector and the earthquake excitation of the past and present moment, while can not be related to the earthquake excitation of the future moment (because the excitation of the future moment is not input to the system yet). Therefore, for the purpose of control force calculation at each time step, the excitation information is equally available for both determinate and random excitation. In other words, even if the excitation at all moment in the control time period is known, the control force is still calculated only using the excitation information up to the present moment. The only difference between determinate and random excitation is that in the former case, the control force can be calculated offline in advance, while in the latter case, the control force can only be calculated in a real time manner.

Because once the earthquake excitation and control force are exerted on the structural system, their effect is determined. Therefore, calculating the control force by minimizing the performance index at each moment is reasonable. The advantage of the IOC is that it can better reflect the order in which the earthquake and control force are input to the structural system. However, after examining the model of IOC, the senior author found out that the expression of performance index of IOC is not reasonable, and the formula of IOC for calculating the control force is mathematically contradicting with each other. Based on this understanding,

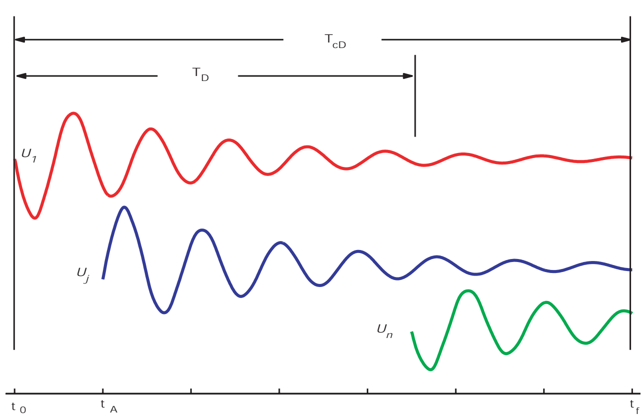


Fig. 1 Performance index expressed by impulse response

this paper makes an improvement on the expression of the performance index of IOC, i.e., expresses the control objective function using impulse response of both seismic excitation and the control force, as shown in Fig. 1. The main difference between the proposed method and the IOC is that the former expresses the performance index using the responses over the duration of $t_\mathrm{f} - t_A$, while the latter using the response at only one time point, i.e., the response at moment t_A. Assuming that the number of time steps between t_0 and t_A is j, the response of the system at arbitrary time t by the proposed method will be the superposition of the first j impulse responses

$$\{U(t)\} = \{U_1(t)\} + \{U_2(t)\} + \cdots + \{U_j(t)\} = \sum_{i=1}^{j} \{U_i(t)\} \tag{2a}$$

Separating the response induced by the present pulse of the excitation from the effect of the past in Equation (2a), leads to

$$\{U(t)\} = \{U_{\Sigma(j-1)}(t)\} + \{U_j(t)\} \tag{2b}$$

where the subscript $\sum(j-1)$ denote the response induced by the pulse in the past. Express the performance index over the duration from t_A to t_f

$$J = \int_{t_A}^{t_f} \left[\frac{1}{2}\{U(t)\}^T [Q] \{U(t)\} + \frac{1}{2}\{f_c(t)\}^T [R] \{f_c(t)\} \right] dt \tag{3}$$

Combining Equations (1a), (2) and (3), one obtains the Lagrange function corresponding to stepwise performance index at time t_A

$$\begin{aligned}
&L(\{U_{\Sigma(j-1)}\},\{U_j\},\{f_c(t_{\Sigma(j-1)})\},\{f_c(t_A)\},\{\lambda\},t) = -\{\lambda(t_f)\}^T\{U_{\Sigma(j-1)}(t_f)\} \\
&\quad +\{\lambda(t_0)\}^T\{U_{\Sigma(j-1)}(t_0)\} - \{\lambda(t_f)\}^T\{U_j(t_f)\} + \{\lambda(t_A)\}^T\{U_j(t_A)\} \\
&\quad +\frac{1}{2}\int_{t_0}^{t_f}\{U_{\Sigma(j-1)}\}^T[Q]\{U_{\Sigma(j-1)}\}dt + \frac{1}{2}\int_{t_A}^{t_f}\{U_j\}^T[Q]\{U_j\}dt + \int_{t_A}^{t_f}\{U_j\}^T[Q]\{U_{\Sigma(j-1)}\}dt \\
&\quad +\frac{\Delta t}{2}\{f_c(t_{\Sigma(j-1)})\}^T[R]\{f_c(t_{\Sigma(j-1)})\} + \frac{\Delta t}{2}\{f_c(t_A)\}^T[R]\{f_c(t_A)\} \\
&\quad +\int_{t_0}^{t_f}\{\lambda\}^\mathrm{T}[A]\left\{U_{\Sigma(j-1)}(t)\right\}dt + \int_{t_A}^{t_f}\{\lambda\}^\mathrm{T}[A]\left\{U_j(t)\right\}dt + \{\lambda\}^\mathrm{T}[B]\{f_c(t_{\Sigma(j-1)})\}\Delta t \\
&\quad +\{\lambda\}^\mathrm{T}[B]\{f_c(t_A)\}\Delta t + \{\lambda\}^\mathrm{T}\{E\}\{\ddot{u}_g(t_{\Sigma(j-1)})\}\Delta t + \{\lambda\}^\mathrm{T}\{E\}\{\ddot{u}_g(t_A)\}\Delta t \\
&\quad +\int_{t_0}^{t_f}\{\dot{\lambda}\}^\mathrm{T}\left\{U_{\Sigma(j-1)}(t)\right\}dt + \int_{t_A}^{t_f}\{\dot{\lambda}\}^\mathrm{T}\left\{U_j(t)\right\}dt
\end{aligned} \tag{4}$$

3.3 Condition of extreme point for SOC

This subsection derives the condition of extreme point for SOC by performing functional analysis on the Lagrange function. Separating the part of Lagrange function which contains pure effect of the past excitation in Equation (4), leads to

$$L(\bullet) = L_{j-1}(\{U_{\Sigma(j-1)}\},\{f_c(t_{\Sigma(j-1)})\},\{\lambda\},t) + L_j(\{U_{\Sigma(j-1)}\},\{U_j\},\{f_c(t_A)\},\{\lambda\},t) \tag{5}$$

where L_{j-1} is the Lagrange function which contains pure effect of the past excitation, and L_j is given by

$$\begin{aligned}
&L_j(\{U_{\Sigma(j-1)}\},\{U_j\},\{f_c(t_A)\},\{\lambda\},t) = -\{\lambda(t_f)\}^T\{U_j(t_f)\} + \{\lambda(t_A)\}^T\{U_j(t_A)\} \\
&\quad +\frac{1}{2}\int_{t_A}^{t_f}\{U_j\}^T[Q]\{U_j\}dt + \int_{t_A}^{t_f}\{U_j\}^T[Q]\{U_{\Sigma(j-1)}\}dt
\end{aligned}$$

$$+\frac{\Delta t}{2}\{f_c(t_A)\}^T[R]\{f_c(t_A)\}+\int_{t_A}^{t_f}\{\lambda\}^{\mathrm{T}}[A]\{U_j(t)\}dt$$

$$+\{\lambda\}^{\mathrm{T}}[B]\{f_c(t_A)\}\Delta t+\{\lambda\}^{\mathrm{T}}\{E\}\{\ddot{u}_g(t_A)\}\Delta t+\int_{t_A}^{t_f}\{\dot{\lambda}\}^{\mathrm{T}}\{U_j(t)\}dt \tag{6}$$

Considering that the pure effect of the past excitation has become a constant at the present time, and have no contribution on variation, one obtains

$$\delta L=\delta L_j \tag{7}$$

Substituting Equation (6) into Equation (7), yields the variation of the Lagrange function

$$\delta L=-\{\lambda(t_f)\}^T\{\delta U_j(t_f)\}+\{\lambda(t_A)\}^T\{\delta U_j(t_A)\}$$

$$+\frac{\partial L_j}{\partial\{U_{\Sigma(j-1)}\}}\{\delta U_{\Sigma(j-1)}\}+\frac{\partial L_j}{\partial\{U_j\}}\{\delta U_j\}+\frac{\partial L_j}{\partial\{f_c(t_A)\}}\{\delta f_c(t_A)\} \tag{8}$$

The condition of extreme point for SOC can be expressed as follows Du [11]

$$\{f_c(t_A)\}=-[R]^{-1}[B]^{\mathrm{T}}\{\lambda(t_A)\} \tag{9a}$$

$$\{\dot{\lambda}\}=-[A]^{\mathrm{T}}\{\lambda\}-[Q]\{U\},\quad \{\lambda(t_f)\}=0 \tag{9b}$$

$$\{\dot{U}\}=[A]\{U\}+[B]\{f_c\}+\{E\}\ddot{u}_g \tag{9c}$$

One can easily observe that Equations (9) have almost the same form with the expression of ordinary optimal control model except for the definition region of Equation (9a) which is only meaningful at the moment t_A in Equations (9). Equation (9b) is usually referred to as “companion equation” in optimal control theory.

4. EXPRESSION OF CONTROL FORCES

4.1 Expressions of control force for current optimal control algorithms and their deficiency

In current optimal structural control algorithms, the following relationship is presumed

$$\{\lambda(t)\}=[P(t)]\{U(t)\}+\{q(t)\} \tag{10}$$

where $[P(t)]$ and $\{q(t)\}$ are feedback matrix and feed-forward vector, respectively. Substituting Equation (10) into (9b), and then applying Equations (9a) and (9c), one obtains

$$\left(\left[\dot{P}(t)\right]+[P(t)]-[P(t)][B][R]^{-1}[B]^T[P(t)]+[Q]\right)\{U(t)\}$$

$$+\{\dot{q}(t)\}-\left([P(t)][B][R]^{-1}[B]^T-[A]^T\right)\{q(t)\}+[P(t)]\{E\}\ddot{u}_g(t)=\{0\} \tag{11}$$

Since the earthquake excitation is not known *a priori*, direct solving of Equation (11) is usually not possible. One of the commonly used approximate methods is to set both $\{q(t)\}$ and $\ddot{u}_g(t)$ to 0, and Equation (11) is simplified to traditional Riccati equation

$$\left[\dot{P}(t)\right]+[P(t)][A]+[A]^T[P(t)]-[P(t)][B][R]^{-1}[B]^T[P(t)]+[Q]=[0],\ [P(t_f)]=0 \tag{12}$$

With $[P(t)]$ solved from Equation (12), optimal control force can be obtained

$$\{f_c\}=-[R]^{-1}[B]^{\mathrm{T}}[P(t)]\{U(t)\} \tag{13}$$

Equation (12) and (13) is the approximate classical optimal control (COC) algorithm. Because the expression of optimal control force of COC is obtained by omitting earthquake excitation, it is usually regarded not to be a true optimal control.

Another approximate method is to transform Equation (9c) into the following finite difference equation

$$\left\{U(t_A)\right\}=\left\{D(t_{j-1})\right\}+\frac{\Delta t}{2}\left([B]\{f_c(t_A)\}+\left\{E\right\}\ddot{u}_g(t_A)\right) \tag{14}$$

where $\left\{D(t_{j-1})\right\}$ is a function related to the response of the past moment. Express the performance index using only the response at the current moment t_A

$$J(t_A)=\frac{1}{2}\{U(t_A)\}^T[Q]\{U(t_A)\}+\frac{1}{2}\{f_c(t_A)\}^T[R]\{f_c(t_A)\} \tag{15}$$

Calculating the extreme value, the control force can be expressed as

$$\left\{\lambda(t_A)\right\}=-\left[Q\right]\left\{U(t_A)\right\} \tag{16}$$

$$\{f_c(t_A)\}=\frac{\Delta t}{2}[R]^{-1}[B]^{\mathrm{T}}\{\lambda(t_A)\} \tag{17}$$

Equations (16) and (17) is the model of the so-called instantaneous optimal control (IOC) algorithm. Bearing in mind that $\left\{D(t-\Delta t)\right\}$ and $\ddot{u}_g(t)$ are all known at the present moment, Equation (14) contains the following relationship between control force, response and excitation:

$$\{f_c(t_A)\}=\frac{2}{\Delta t}[B]^{-1}\left(\left\{U(t_A)\right\}-\left\{D(t_{j-1})\right\}\right)-\left\{E\right\}\ddot{u}_g(t_A) \tag{18}$$

One can easily observe that Equation (18) is mathematically contradicting with Equation (17).

4.2 Expressions of control forces for SOC with state feedback

Unlike the idea of the above two approximate algorithms, this paper proposed a method that directly solves the Lagrange multiplier $\{\lambda\}$ from Equation (9b). Noting the resemblance between the format of companion equation (9b) and that of the state Equation (9c), the former may be regarded as a state equation of a special system

$$\{\dot{\lambda}\}=[A_\lambda]\{\lambda\}+[Q_\lambda]\{U\},\quad \{\lambda(t_f)\}=0 \tag{19}$$

Equation (19) is defined as the dual system with the state matrices being

$$[A_\lambda]=-[A]^T,\quad [Q_\lambda]=-[Q] \tag{20}$$

With this understanding, $\{\lambda\}$ can be calculated directly using state transition algorithm, and optimal control force can be obtained using Equation (9a). The main procedure of derivation is shown below

1) The response vector is first calculated from the state equation (9c)

$$\left\{U(t_j)\right\}=e^{[A]\Delta t}\left\{U(t_{j-1})\right\}+\Delta t\left\{E\right\}\ddot{u}_g(t_{j-1}),\quad t_j=t_A+j\Delta t,\quad j=1,2,\cdots,m \tag{21}$$

where t_A is the present time, and m is the number of time increment (steps) from the present time to the end of control period.

2) Since Equation (9b) is regarded as a state equation of a dual system, the vector $\{\lambda\}$ can be numerically solved in a similar way as the response is calculated from the state equation in Equation (21). Considering the terminal condition, $\{\lambda\}$ should be calculated inversely in terms of time order

$$\left\{\lambda(t_{k-1})\right\}=e^{-[A_\lambda]\Delta t}\left\{\lambda(t_k)\right\}-\Delta t e^{-[A_\lambda]\Delta t}[Q_\lambda]\{U(t_k)\},\ t_k=t_A+k\Delta t,\ k=m,\cdots,2,1 \tag{22}$$

3) The control force at the current moment t_A can be obtained

$$\{f_c(t_A)\}=-[R]^{-1}[B]^{\mathrm{T}}\{\lambda(t_A)\} \tag{23}$$

Simulating the order in which earthquake is input to a structural system, the response of the structure can be regarded as the free vibration induced by the pulse of the excitation at the current moment, and the expressions in Equations (21)-(23) can be further simplified. Let $j=1$ in Equation (21), and noting that no excitation is exerted on the system for $j>0$, the response can be calculated as

$$\left\{U(t_A+\Delta t)\right\}=e^{[A]\Delta t}\left\{U(t_A)\right\}+\Delta t e^{[A]\Delta t}\left\{E\right\}\ddot{u}_g(t_A)=e^{[A]\Delta t}\left(\left\{U(t_A)\right\}+\Delta t\left\{E\right\}\ddot{u}_g(t_A)\right) \tag{24}$$

the response for other values of j can be derived similarly

$$\left\{U(t_j)\right\}=e^{[A]j\Delta t}\left(\left\{U(t_A)\right\}+\Delta t\left\{E\right\}\ddot{u}_g(t_A)\right),\quad t_j=t_A+j\Delta t,\quad j=2,3,\cdots,m \tag{25}$$

Let $k=m$ in Equation (22)

$$\left\{\lambda[t_A+(m-1)\Delta t]\right\}=e^{-[A_\lambda]\Delta t}\left\{\lambda(t_A+m\Delta t)\right\}-\Delta t e^{-[A_\lambda]\Delta t}[Q_\lambda]\{U(t_A+m\Delta t)\} \tag{26}$$

Applying the terminal condition, i.e., $\left\{\lambda(t_A+m\Delta t)\right\}=0$, in Equation (26), and applying Equation (25), yields

$$\left\{\lambda[t_A+(m-1)\Delta t]\right\}=-\Delta t e^{-[A_\lambda]\Delta t}[Q_\lambda]e^{[A]m\Delta t}\left(\left\{U(t_A)\right\}+\Delta t\left\{E\right\}\ddot{u}_g(t_A)\right) \tag{27}$$

Repeating Equation (27) till $m = 1$, yields

$$\left\{\lambda(t_A)\right\}=-\Delta t[Q_A(m)]\left(\left\{U(t_A)\right\}+\Delta t\left\{E\right\}\ddot{u}_g(t_A)\right) \tag{28}$$

$$[Q_A(m)]=e^{-[A_\lambda]m\Delta t}[Q_\lambda]e^{[A]m\Delta t}+e^{-[A_\lambda](m-1)\Delta t}[Q_\lambda]e^{[A](m-1)\Delta t}+\cdots+e^{-[A_\lambda]\Delta t}[Q_\lambda]e^{[A]\Delta t} \tag{29}$$

For the case of state feedback, the ideal control force at the current moment is

$$\{f_c(t_A)\}=\Delta t[I_{RU}(m)]\left\{U(t_A)\right\}+(\Delta t)^2\left\{E_{RU}(m)\right\}\ddot{u}_g(t_A) \tag{30}$$

$$[I_{RU}(m)]=[I_Q(m)]^{-1}[R_Q(m)],\qquad \left\{E_{RU}(m)\right\}=[I_{RU}(m)]\left\{E\right\} \tag{31a}$$

$$[I_Q(m)]=[I]-(\Delta t)^2[R_Q(m)][B],\qquad [R_Q(m)]=[R]^{-1}[B]^{\mathrm{T}}[Q_A(m)] \tag{31b}$$

4.3 Expressions of control forces with output feedback

For output feedback, the performance index can be expressed as

$$J=\int_{t_A}^{t_f}\left[\frac{1}{2}\{y(t)\}^T[Q]\{y(t)\}+\frac{1}{2}\{u_c(t)\}^T[R]\{u_c(t)\}\right]dt \tag{32}$$

The control force can be derived in a similar way as in Equations (24)-(29) except that the state matrices of the dual system is replaced by

$$[A_\lambda]=-\left([A]^{\mathrm{T}}-[Q_d][r_1]^{-1}[B]^{\mathrm{T}}\right),\quad [Q_\lambda]=-\left([Q_c]-[Q_d][r_1]^{-1}[Q_d]^{\mathrm{T}}\right) \tag{33a}$$

$$[Q_c]=[C_y]^T[Q][C_y],\quad [Q_d]=[C_y]^T[Q][D_y],\quad [r_1]=[R]+[D_y]^T[Q][D_y] \tag{33b}$$

The ideal control force is given by

$$\{f_c(t_A)\}=[I_{Ry}(m)]\left\{U(t_A)\right\}+(\Delta t)^2\left\{E_{Ry}(m)\right\}\ddot{u}_g(t_A) \tag{34}$$

$$[I_{Ry}(m)]=[I_{Qy}(m)]^{-1}[r_Q(m)],\qquad \left\{E_y(m)\right\}=[I_{Qy}(m)]^{-1}\left\{E_r(m)\right\} \tag{35a}$$

$$[r_Q(m)]=[r_1]^{-1}\left(\Delta t[B]^{\mathrm{T}}[Q_A(m)]-[Q_d]^{\mathrm{T}}\right),\quad \left\{E_r(m)\right\}=[r_1]^{-1}[B]^{\mathrm{T}}[Q_A(m)]\left\{E\right\} \tag{35b}$$

$$[I_{Qy}(m)]=[I]-(\Delta t)^2[r_1]^{-1}[B]^{\mathrm{T}}[Q_A(m)][B] \tag{35c}$$

5. NUMERICAL EXAMPLE

A 7-story isolated RC frame structure is employed as numerical example to illustrate the control effect of the proposed method. The structural parameters of the building is listed in Table 1, and earthquake signal of Northridge (1994), El Centro (1940, S00E) and a sinoidal wave are adopted as the input. The PGA of the two natural earthquakes is adjusted to $0.706ms^{-2}$, and the expression of the sine wave is

$$\ddot{u}_g(t) = 0.706\sin(2.2\omega_1 t)$$

Table 1 Structural parameters of numerical example

Story No.	1	2	3	4	5	6	7	isolator
m_i /kg	1125402	1239436	1239436	1239436	1239662	1454693	512669	1103057
k_i /kNm^{-1}	1614600	1613825	1613825	1613825	1613825	1371778	434464	138200

To facilitate comparison between different control algorithms, a control energy index E_c is defined by the absolute integral of the ideal control force along the control duration:

$$E_c = \int_{t_0}^{t_f} | f_c(t) | dt$$

Setting the values of E_c to be approximately identical for different control algorithms, one can observe the following:

1) No mater whether the excitation is natural earthquake records or sine wave, the proposed method (SOC) is effective in suppressing peak response and has good control speed. Especially, the effect of SOC for suppressing peak response is the best among all the control algorithms under comparison, and causes less increase in the response of the superstructure than IOC and COC, as shown in Figs 2 and 3. The upper half of each figure shows the controlled response of the isolator, while the lower half shows the superstructure.

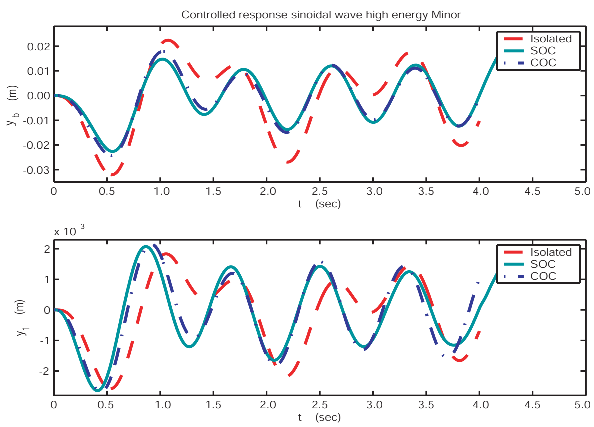
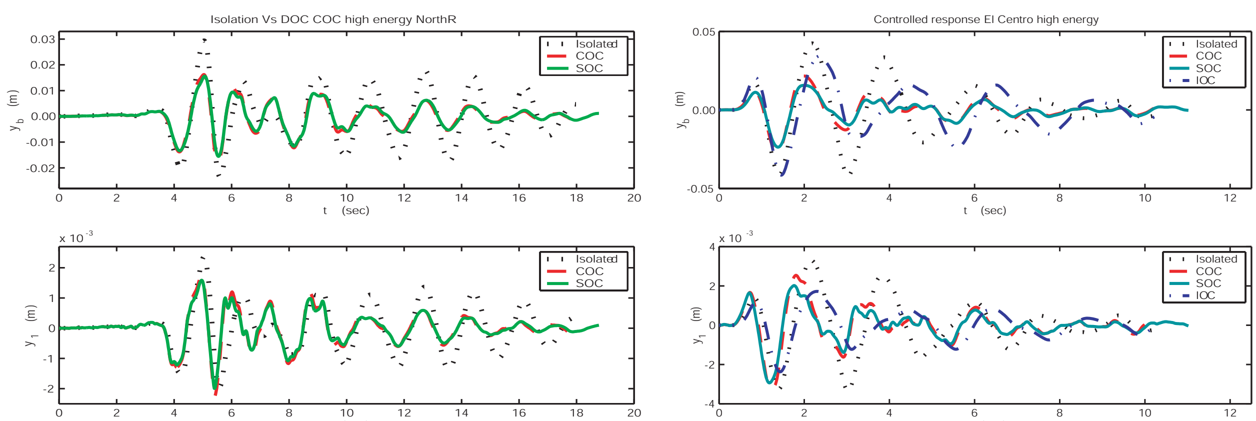


Fig. 2   Controlled response with sine wave input

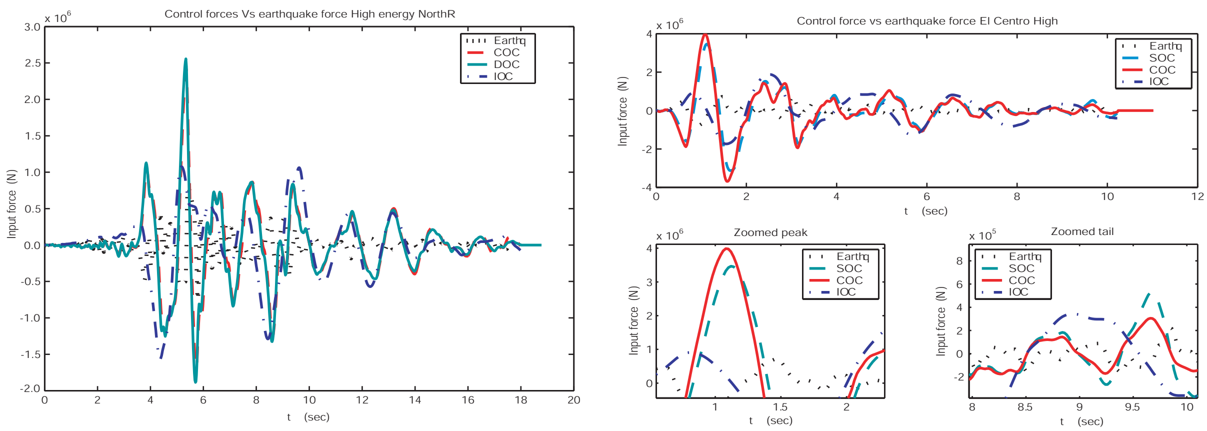


(a) Northridge   (b) El Centro

Fig. 3   Comparison of controlled response with natural earthquake signal input

2) The controlled response of COC in the same group of figures is very close to that of SOC, but the controlled response of IOC, on the other hand, is very different. This is because though COC is derived based on the assumption of omitting the earthquake excitation, only the excitation at the present moment is omitted, the cumulative response by the excitation at the previous moment has already been included in the present state vector.

3) Fig 4 compares the control forces generated by different control algorithms. It can be seen that the control force of SOC follows the characteristics of the excitation very well, in that the value of the control forces near the peak of the excitation is large, and the value decreases rapidly when the excitation becomes smaller. The control force of COC is very close to that of SOC, while the control force generated by IOC is different in terms of both magnitude and phase.



(a) Northridge (b) El Centro

Fig. 4 Control forces generated by different control algorithms

4) For the special case in which the input excitation is a sinoidal wave, the magnitude of control force of COC is almost constant, because the control gain matrix of COC is constant for linear time invariant system (LTI). In contrast, the magnitude of the control force generated by SOC decreases slightly as the time approaches to the end of control period, as shown in Fig 5. In the upper half of Fig 5, the control force calculated from Equations (21)-(23) is shown to be completely identical to the result of Equations (30)-(31), while in the lower half of the same figure the control force of IOC is again shown to be apparently different from that of both SOC and COC.

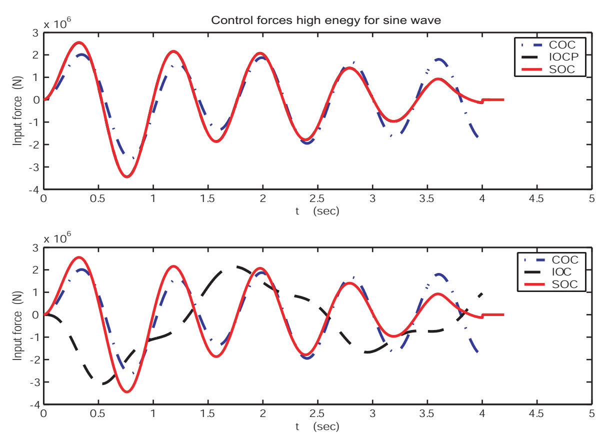
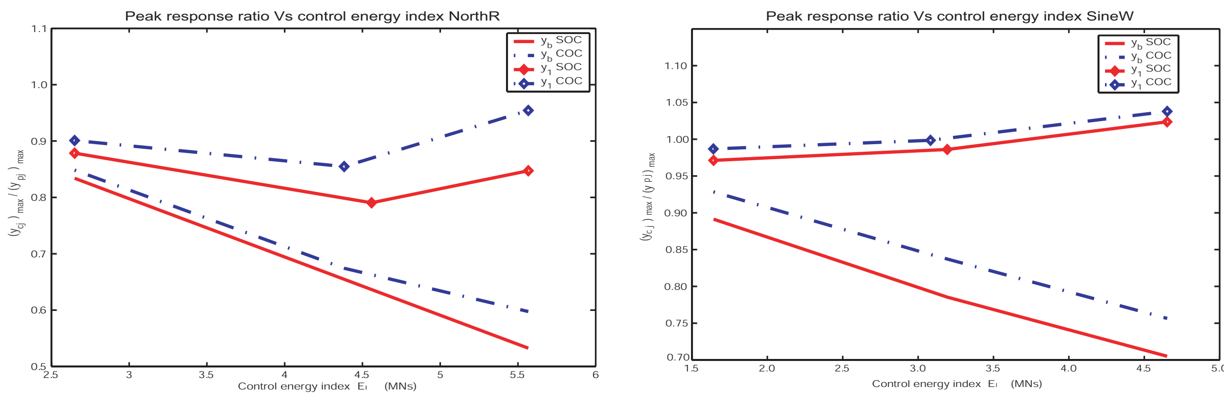


Fig. 5 Control force with Sine wave input

5) Fig 6 compares the response reduction ratio of smart to passive isolated system for different control algorithms. The response reduction ratio is defined by dividing the peak response of smart isolated system with the peak response of passive isolated system. One can observe that at the same level of control energy input, SOC reduces the response of base drift more and causes increase in the story drift of the superstructure less. (The weighting matrix in this paper is selected based on the principle of reducing the base drift as much as possible for given level of control energy input). For ordinary isolated building, this feature is an very important index reflecting a better vibration mitigation effect of the SOC. This

advantage may partly be aroused from the rapid decrease of the control force with the decrease of the excitation, which saves control energy when the control force is not necessarily large. In other words, the proposed algorithm SOC has a higher control efficiency Du [12].



(a) Northridge (b) El Centro

Fig. 6 Response reduction ratios for different control algorithms

6) Substituting Equation (30) or (34) into Equation (9c), an equivalent dynamic property matrix for closed-loop controlled system can be obtained for SOC. Deducing the elements of the equivalent dynamic property matrix of SOC with the elements of the corresponding matrix for SOC, an SRSS error can be obtained. A comparison of the SRSS error at each time step for different level of control energy is shown in Fig. 7. At the initial stage in the control duration, the error is very large because the information for the input excitation is limited. As the input information accumulates, the error becomes less, i.e., the difference of the equivalent dynamic property matrices of the two control algorithms becomes less, reflecting that SOC and COC are originated form the same theoretical source. However, this difference will not vanish because the assumptions for the two algorithms are different.

7) When examined using Lyapunov direct method, the proposed SOC method can be proved to be of good stability. The stability range of SOC is almost the same with that of the COC, and has a better settling speed Du [13], while the stability range of IOC with output feedback is apparently smaller when compared with that of the SOC and COC. Fig 8 shows an example of phase plane plot for the initial response of base drift with the same initial value.

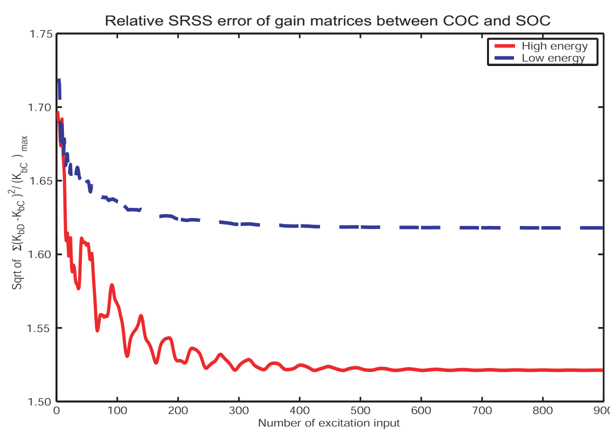


Fig. 7 SRSS error of dynamic property matrices

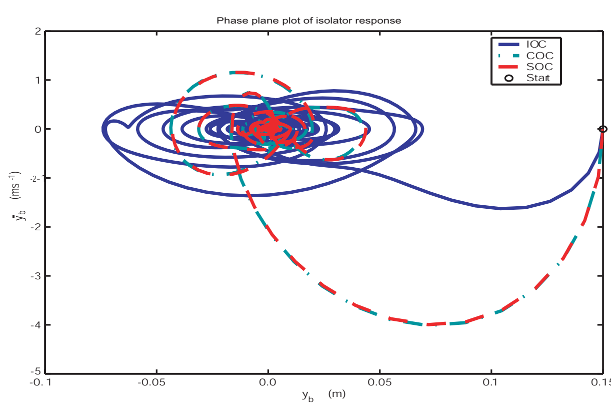


Fig. 8 Phase plane plot of initial response

6. CONCLUSION

This paper derived a new model for optimal control of smart seismic isolated buildings. The control objective function was expressed using impulse response of both seismic excitation and the control force. The optimal control force was directly calculated using state transition algorithm by defining a dual system according to the resemblance between the companion equation and the state equation. An improved optimal control algorithm has thus been developed, and was named as Sequential Optimal Control (SOC). Simulation result has shown that the proposed SOC not only provided a more reasonable concept for deriving the model, a more accurate way of solving the equation, but also numerically improved the control effect of the current optimal control algorithms for seismic response control.

ACKNOWLEDGEMENT

This research was supported by Natural Science Foundation, Gansu Province, PR China under Project No. ZS021-A25-015-G. This support is gratefully acknowledged. The writers are grateful to Prof Billie F. Spencer at the University of Illinois at Urbana-Champaign, USA, for valuable discussions, and Prof Yi-Kwei Wen at the same university for providing the first author with references concerning the Bouc-Wen model.

REFERENCES

1. Soong, TT. Active Structural Control: Theory and Practice. New York: Wiley, 1990.
2. Housner, GW, Bergman, LA, Caughey, TK, Chassiakos, AG, Claus, RO, Masri, SF, Skelton, RF, Soong, TT, Spencer, BF, Yao, TP. Structural control: past, present, and future. Journal of Engineering Mechanics, ASCE. 1997, 123(9): 897-971.
3. Ribakov, Y, Gluck, J, and Reinhorn. Active viscous damping system for control of MDOF structures. Earthquake Engineering and Structural Dynamics, 2001, 30(1): 195-212.
4. Feng, MQ, Shinozuka, M., and Fuji, S. Friction controllable sliding isolation system [J]. J. Engineering Mechanics. ASCE, 1993,119(9): 1845-1864.
5. Makris, N. Rigidity-plasticity-viscosity: Can electrorheological dampers protect base-isolated structures from near-source Ground Motions? Earthquake Engineering and Structural Dynamics [J], 1997, 26: 571-591.
6. Spencer, Jr., B.F., Johnson, E.A. and Ramallo, J.C. (2000). "'Smart' Isolation for Seismic Control," JSME International Journal: Special Issue on Frontiers of Motion and Vibration Control, Series C, Vol. 43, No. 3, pp. 704-711.
7. Soong, TT. State-of-the-art review: Active structural control in civil engineering. Engineering Structures. 1988, 10:74-84
8. Abdel-Rohman, M., and Leipholz, H.H.E. Structural control by pole assignment method. Journal of Engineering Mechanics Div, ASCE, 1978, 104: 1157-1175.
9. Yang, JN, Akbarpour, A and Ghaemmaghami, P. New optimal control algorithms for structural control. Journal of Engineering Mechanics, ASCE. 1987, 113(9): 1369-1386.
10. Yang, JN, Li, Z and Liu, SC. Stable controllers for instantaneous optimal control. Journal of Engineering Mechanics, ASCE. 1992, 118(8): 1612-1630.
11. Du, YF, Li, H. Comparison of models of optimal control and improvement. World Seismic Engineering. To appear in 2004 (in Chinese).
12. Du, YF, Li, H, Dang, Y., et al. Numerical simulation of smart isolated structures in high earthquake intensity region. Proc. 4^{th} National Conference on Seismic Vibration and Control, Guangzhou, China, November, 2003 (in Chinese).
13. Du, YF, Zhang, LJ, Li, H., et al. Expression of performance index and stability issue for optimal control of seismic response of structures. To appear in Journal of Lanzhou University of Technology, 2004 (in Chinese).