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SEISMIC HAZARD CHARACTERIZATION AND RISK EVALUATION USING GUMBEL'S METHOD OF EXTREMES (G1 AND G3) AND G-R FORMULA FOR IRAQ

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SUMMARY

The earthquake catalog for Iraq covering an area between latitude 39 ° – 50° E and longitude 29° – 50° N and containing more than thousand events for the period 1905 - 2000 has been compiled and statistical analysis was carried out for its homogeneity and completeness in various magnitude ranges estimating the MMC (Minimum Magnitude of Complete reporting). Based on the newly compiled catalog, earthquake activity parameters (b-value and maximum regional magnitude, Mmax) and recurrence rates of moderate to large magnitudes were estimated using methods developed by Gumbel and by Gutenberg and Richter (G-R).

The statistical parameters for Gumbel's first distribution (G1) and G-R have been estimated using both the least squares and maximum likelihood techniques while in case of Gumbel's third distribution (G3) only the maximum likelihood technique is applied to estimate the parameters. The goodness of fit is evaluated employing Kolmogorov-Smirnov test with respect to G1 and G3 distributions. The results reveal that the procedures of the theory of extremes gives large probable errors in determinations of recurrence rates for moderate to large magnitudes. However, it is observed that G3 is more suited than G1 for modeling the magnitude uncertainties and results in better statistical estimation of maximum magnitude expected in the region with medium to low seismicity of Iraq corresponding to various structure design periods. Such estimation of seismic hazard and risk provide very important information for design of various structures and other critical facilities in Iraq.

INTRODUCTION

The ultimate goal of seismic hazard assessment and risk evaluation for a particular site or area is to condense seismotectonic knowledge and experience into parameters used for predicting ground motion which in turn can be applied by engineers in design and subsequent earthquake resistant construction.

The primary advantage of probabilistic seismic hazard analysis (PSHA) is that it integrates over all seismicity: temporal and spatial alongwith ground motions to calculate a combined probability of exceedence, which incorporates the relative frequencies of occurrence of different earthquakes and ground-motion characteristics. Practically in any earthquake catalogue the quality of different parts (periods and areas) varies significantly with respect to completeness, magnitude reliability, homogeneity and location accuracy. Ameer[1,2,3]

Statistical theory of extreme values has been used to analyze the observed extremes of any phenomena and to forecast the further extremes based on the appropriate distribution Gumbel [4]. In earthquake engineering, this theory has been applied successfully by many researchers in the past few decades (Nordquist[5]; Epstein [6]; Yegulalp[7]; Al-Abbasi[8] and Jaiswal[9]. This theory does not require analysis of the complete record of earthquake occurrence, but uses the sequence of earthquakes constructed from the largest values of the magnitude over a set of predetermined intervals.

The primary aim of the present study is to estimate new model of FMD (Frequency Magnitude Distribution considering the potentially damaging earthquakes throughout Iraq and using the newly compiled data set for the Iraq which was replaced from 1964-2000 by pruned and final determinations of ISC (International Seismological Centre) data. The other objective is to examine the applicability of extreme value distribution for the evaluation of seismic hazard and risk using the updated earthquake catalog for the Iraq region.

SEISMIC ACTIVITY AND EARTHQUAKE DATA FILE

Iraq experiences an appreciable level of earthquake activity since it is tectonically positioned in one of the seismically active zones of the world. Owing to this unique geographical location, the eastern and northeastern parts of the country are directly influenced by the seismicity of the Alpine-Himalayan orogenic system (Zagros-Tauros range), Al-Abbasi and Fahmi, [10]. This system, identified with high mountain ranges and shallow, somewhat diffuse seismicity constitute one of the most seismically active continental regions of the world with a long and well-documented history of earthquakes. Studies on historical, Alsinawi & Ghalib [11] as well as on recent activity, Alsinawi & Ghalib [12] have shown the general association and conformity of earthquake epicenters with the geological formation and structural setting of the country.

The seismicity catalogue prepared for the present study includes published data from various sources by Fahmi and Al-Abbasi [13] over the period 1905-1963. The earthquake catalog for Iraq has been updated and replaced by the pruned and final determinations of ISC over the period 1964-2000. The database consists of more than 1761 seismic events for the last 95 years from 1905 to 2000. The magnitudes of events have been re-evaluated in terms of surface wave magnitude Ms (out of a total earthquake catalog entries for the Iraq, 478 entries originate from actual Ms readings observed by ISC and 1283 entries converted from observed ISC's mb readings using Marshall's [14] formula; Ms=2.08 mb – 5.65).

FREQUENCY MAGNITUDE DISTRIBUTION FOR IRAQ

Assuming that the times of occurrence of earthquakes are random and the number decreases exponentially with size and that there is no upper or lower limit on the sizes of the earthquakes, Gutenberg and Richter [15], have shown that the average number per year, N of earthquakes of any given magnitude, M, or larger is related to M by:

$$\text{Log } N(M) = a - b\,M \tag{1}$$

where a and b are constants. The Frequency Magnitude Distribution (FMD) for the whole of Iraq has been re-evaluated after correcting for incomplete reporting in the data sample, i.e. by creating an artificially complete sample of data below the magnitude, which was completely reported over the whole 95 years of data. The complete mean rate of recurrence for each incomplete magnitude classes was used to "fill in" the lower end of FMD. For example Fig. 1 illustrates this treatment for the whole of Iraq, where the open squares represent the complete data below the complete magnitude reporting over 95 years and solid squares show the incomplete data. The regression analysis (least squares, LS) was then applied to the incomplete and complete data sets to select the minimum magnitude of complete reporting, MMC, as the value where the FMD graph clearly departs from the straight line plot, while the largest earthquake in the sample was usually taken to be the upper magnitude.

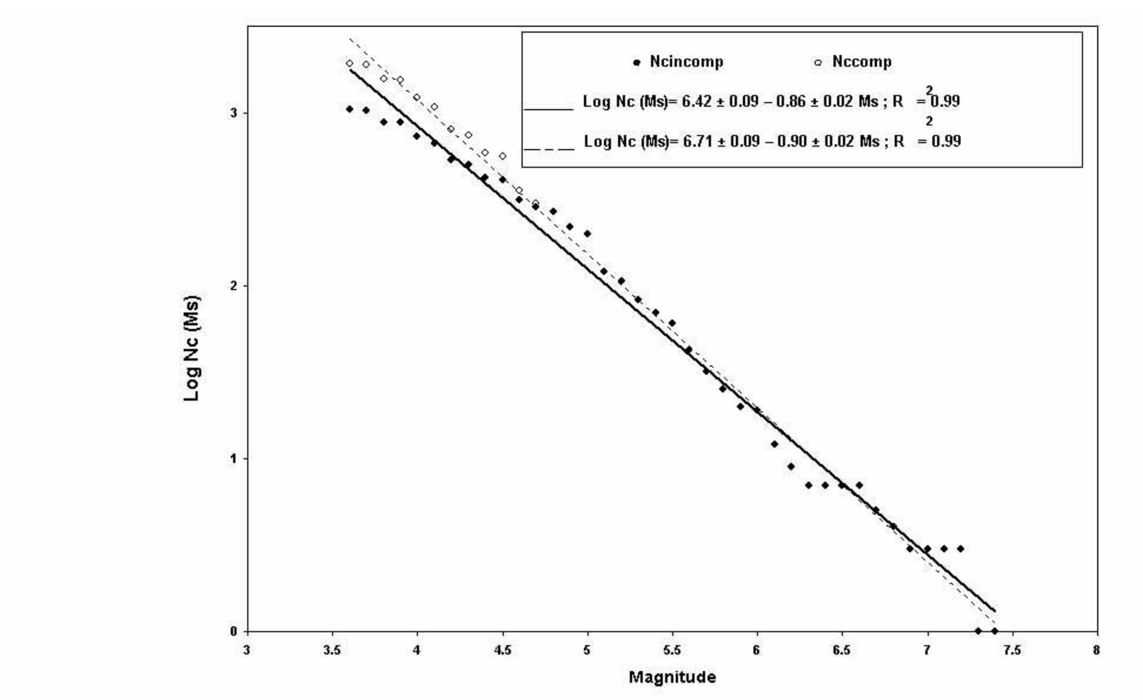

Fig. 1 Frequency Magnitude Distribution for Iraq

The recurrence relationship of the incomplete and complete catalog data with MMC=4.0 respectively is:

$$\text{LogNc }(Ms) = (6.4150 \pm 0.0938) - (0.8562 \pm 0.0162)\,Ms \;(R^2 = 0.99) \tag{2a}$$

$$\text{LogNc }(Ms) = (6.7143 \pm 0.0865) - (0.9030 \pm 0.0150)\,Ms \;(R^2 = 0.99) \tag{2b}$$

where Nc(Ms) is the cumulative number of events greater than Ms and R^2 is the coefficient of determination from equations (2a-2b). We may determine the expected largest magnitude earthquake for whole of Iraq since 1905 to be 7.5 and 7.4, respectively. The value from equation

3 agrees exceptionally well with the observed maximum earthquake magnitude of 7.4 Ms and equation (2b) also shows the better estimation of b-slop with lesser uncertainty. Fig 1 reveals decreasing the MMC for the whole of Iraq to Ms=4.0 while it was been estimated by Al-Abbasi and Fahmi [8] as Ms=4.4. This change is due to improvement in the method of analysis and increase of the number of seismographs in the conterminous regions. To illustrate the effect of MMC on the parameters of G-R recurrence relation, comparative analysis can be carried out using the plot of the logarithm of the number of earthquakes per year N (Ms) occurring in Iraq (fig. 2). Taking the magnitude ranges $3.2 \leq Ms \leq 7.4$(that includes the entire range of seismic activity) $4.0 \leq Ms \leq 7.4$ (to illustrate the effect of MMC and this range based on engineering importance of seismic hazard) and $4.0 \leq Ms \leq 6.0$ (that includes the effect of sets of small to moderate earthquakes on seismicity, seismic hazard and limitation of seismic potentials within Iraq territory), then the recurrence relationships are:

$$\text{LogN (Ms)}=(4.9681\pm0.3889)-(0.6577\pm0.0727)\text{ Ms}; (R^2 =0.90) \quad 3.2 \leq Ms \leq 7.4 \qquad (3a)$$

$$\text{LogN (Ms)}=(5.7244\pm0.4768)-(0.7825\pm0.0837)\text{Ms}; (R^2 =0.93) \quad 4.0 \leq Ms \leq 7.4 \qquad (3b)$$

$$\text{LogN (Ms)}=(5.3919\pm0.4126)-(0.7075\pm0.0818)\text{ Ms}; (R^2 =0.95) \quad 4.0 \leq Ms \leq 6.0 \qquad (3c)$$

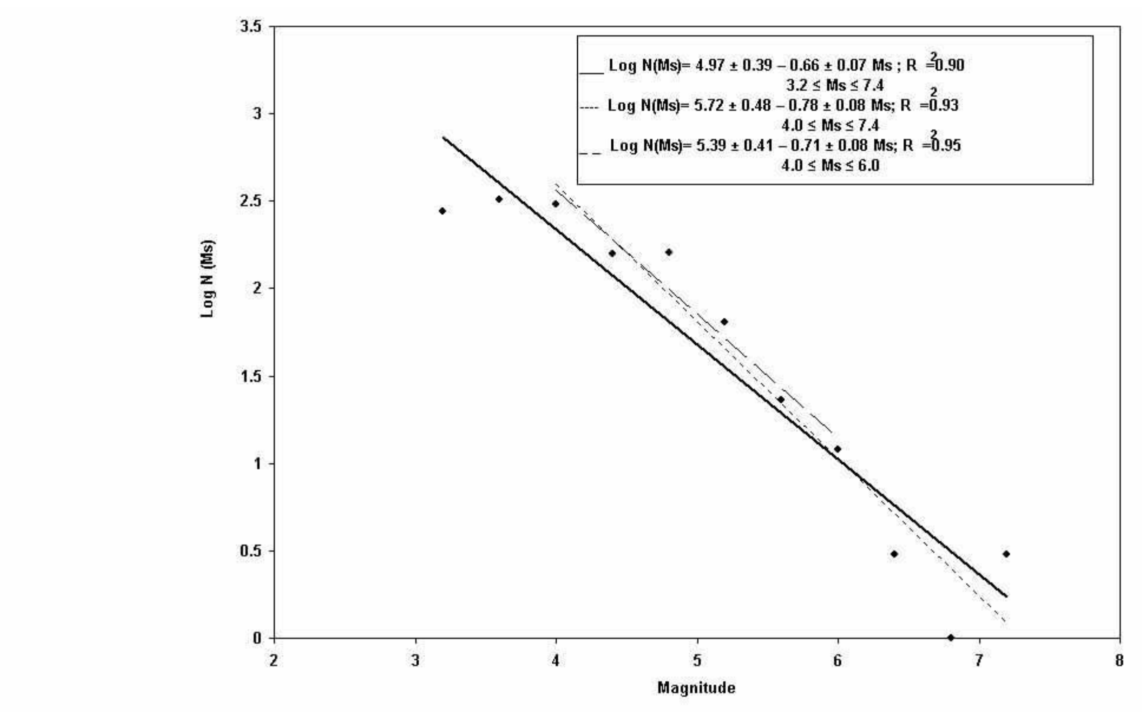

Fig. 2. Log-Linear Magnitude Frequency Relationship for Iraq with different MMC

EXTREME VALUE STATISTICS

The theory of extremes developed by Gumbel [4] provides a convenient method to obtain estimates of the frequencies of occurrence of earthquake magnitudes on the extreme of a statistical distribution and to estimate recurrence times after rearranging the magnitudes in ascending order such that:

$M_1 < M_2 < \ldots\ldots\ldots < M_n$

Then the ith largest value of magnitude with a probability P_i of being an extreme in any one year is given by Gumbel [4]:

$$P_i\,(Ms)=(i+r)/(n+r+1) \qquad (4a)$$

where P_i is the effective plotting position of the ith magnitude observation on the special Gumbel probability paper; n is the total time sample and r (added to Gumbel's formula) is the missing data(years of no reporting of less than MMC). According to Knopoff and Kagan, [16] there is significant bias in using plotting rule (4a). Another competitive plotting rule is (Jenkinson, [17] which is also for the largest of the annual earthquakes, Kimball [18]:

$$P_i = (i-0.5+r)(n+r) \qquad (4b)$$

A third plotting rule is given by Gringorten, [19]

$$P_i = (i-0.44+r)/(n+r+0.12) \qquad (4c)$$

which, minimizes the bias in the long return period end in the distribution. The basic formulation of Type-I and Type-III asymptotic distributions can be briefed as following.

TYPE-I ASYMPTOTIC DISTRIBUTION

The Cumulative Distribution Function (CDF) for the asymptotic Type-I distribution of largest value of annual extreme magnitude Ms is given by

$$G_I\,(Ms)=\exp\,\{-\exp\,[-A\,(Ms-B]\} \qquad (5a)$$

$$Ms=B-\,A^{-1}\,(\ln\,(-\ln P_i) \qquad (5b)$$

where A is the characteristic largest value of the initial variate Ms and B is the inverse measure of dispersion.

Type-III Asymptotic Distribution

The CDF of the asymptotic Type-III distribution of the largest value of annual extreme magnitude Ms is given by

$$G_{III}(Ms)=\exp[-\{(W-Ms)/(W-U)\}^K] \qquad (6a)$$

which is equivalent to

$$\ln(W-Ms)=\ln(W-U)+K^{-1}\ln(-\ln P_i) \qquad (6b)$$

where W is upper limit to the initial variate Ms, U is its characteristic largest value and K is the shape parameter which is also termed as inverse of the measure of dispersion. The Type-III distribution has a finite upper bound.

PARAMETER ESTIMATION

Least Squares Method

Many researchers have described the application of least squares technique for the evaluation of parameters of the asymptotic distribution (see for example Yegulalp and Kuo, [7] and Burton, [20]. In the present study the least square method was applied for estimation of G-R parameters for incomplete and complete data sets. Furthermore we proceed to estimate the linear regression fits for the three different magnitude ranges (figs. 2). The parameters for the Gumbel's Type-I asymptotic distribution were estimated applying the least squares method using different plotting position rules to simplify the inspection of the Type-I asymptotic distribution and to illustrate the effectiveness of this plotting formula. Moreover the parameters A and B have been estimated by graphical and moment methods. Type-III asymptotic parameters have been estimated by least squares method by assuming the value of (W) same as found for the world leaving only (K) and (U) in (6b) as unknowns. Furthermore, the Type-III parameters have been estimated by least square method using different plotting position rules.

Maximum Likelihood Method

The maximum likelihood estimator of a distribution parameter is that value of the parameter which causes the likelihood function to be a maximum. Al-Abbasi and Fahmi, [10] as well as other investigators (e.g.; Kijko and Sellevoll, [21]; Kijko, [22] have demonstrated the significance, reliability, and efficiency of using the ML instead of the LS in parameter estimation of the Gumbel distributions for more details refer Al-Abbasi and Fahmi, [10].

GOODNESS OF FIT TEST AND STATISTICAL HAZARD PREDICTION

The problem of fit is to test the hypothesis that the sample of observations (in our example, earthquake magnitudes) is from some specified distribution against the alternative that it is from some other distribution. The Kolmogorov-Smirnov (K-S) test was used to check for goodness of fit between the observations and the Gumbel's models employing least square and maximum likelihood methods.

Having evaluated the best estimate of extreme value distributions and corresponding critical fits, it is possible to express the earthquake hazard R(Ms) associated with particular a magnitude level Ms at return period T years for design period D, provided that T > D. Thus for the Type-I distribution we have

$$R_I(Ms|\, D)=1-\exp\{D/T\ \ln[1-R_I(Ms|\, T)]\} \qquad (7a)$$

and similarly for the Type-III distribution, we have

$R_{III}(Ms| D)=1- \exp\{D/T \ln[1 – R_{III}(Ms| T)]\}$ (7b)

The calculation of excedence probabilities and return period associated with particular level of design period has been carried out using computer program developed by Al-Abbasi and Fahmi, [8].

RESULTS AND DISCUSSION

The results obtained in the present study raise several interesting observations. The first concerns the significance of the Minimum Magnitude of Complete reporting MMC in the light of the temporal heterogeneity of the seismic catalogue. The regression analysis (least squares) was applied both to the incomplete and complete data sets to select MMC (fig 1 and Eqs. 2a-2b). The new MMC for whole of Iraq was estimated to be Ms=4.0 (see fig1 and Eq. 2a). Equation 2b shows the better estimation of b-slop with lesser uncertainty and exact maximum observed magnitude for the Iraq. Fig 2 illustrates a comparative analysis to check the effect of MMC and different magnitude ranges on G-R recurrence relations (see Eqs. 3a-3c also).

The second observation concerns the significant seismic rate changes. The FMD for the whole of Iraq has been re-evaluated after correcting for incomplete reporting in the data sample using the newly compiled data set for the Iraq which was replaced from 1964-2000 by the pruned and final determinations of ISC (International Seismological Centre) data. The occurrence of maximum magnitude earthquakes has been modeled using Gumbel's Type-I and Type-III asymptotic distributions and exceedence probabilities and return period for various magnitude levels are estimated. The parameters are estimated from LS for different plotting position rules and ML methods. The maximum likelihood estimators of Gumbel's distributions are obtained using Newton Raphson methods (Table 1 and figures 3 & 4). Tables 2 &3 show the comparison of return periods and seismic risks respectively associated with different level of earthquake magnitudes for two asymptotic distributions. It is obvious that the Gumbel's Type-III asymptotic distribution with maximum likelihood estimator gives better approximation for return period and risk calculations for the earthquakes of Iraq.

Table 1: Estimated parameters for Gumbel's Type (I & III) asymptotic distributions using LS and ML and their goodness of fit for the Iraq

Gumbel's Method	Gumbel's Type - I Distribution			Gumbel's Type – III Distribution		
	Gumbel*	Jenkinson**	Gringorten***	Gumbel*	Jenkinson**	Gringorten***
LS Method	A=1.73 B=4.96	A=1.84 B=4.98	A=1.82 B=4.97	W=8.70 U=4.91 K=4.97	W=8.70 U=4.93 K=5.04	W=8.70 U=4.93 K=5.00
K-S test	0.1213	0.1311	0.1280	0.0906	0.0849	0.0844
ML method with N-R procedure	A=1.1848 B=4.7488			W=8.0247 U=4.8355 K=3.3821		
K-S test	0.1563			0.1177		

***Pi=(i+r)/(n+r+1) {Gumbel's, 1958}; ** Pi=(i-0.5+r)/(n+r) {Jenkinson, 1955}**
***** Pi=(i-0.44+r)/(n+r+0.12) {Gringorten, 1963a}**

Table 2: Predicted return periods for selected maximum surface wave magnitudes expected in Iraq using Gumbel's Type (I & III) asymptotic distributions based on LS and ML methods.

*[* - Gumbel ** - Jenkinson *** - Gringorten]*

Mag. (Ms)	Return Period (year)-Gumbel's Type - I Distribution				Return Period (year)-Gumbel's Type – III Distribution			
	LS *	LS **	LS ***	ML	LS *	LS **	LS ***	ML
5.0	1.65	1.62	1.63	1.91	1.70	1.67	1.67	1.77
5.5	3.08	3.14	3.16	2.97	2.79	2.82	2.81	2.74
6.0	6.56	7.05	7.03	4.92	5.59	5.89	5.82	5.17
6.5	14.86	16.90	16.70	8.47	14.04	15.60	15.28	12.63
7.0	34.60	41.63	40.73	14.90	47.04	55.87	54.14	46.80
7.5	81.48	103.71	100.44	26.54	247.33	320.90	306.56	440.15

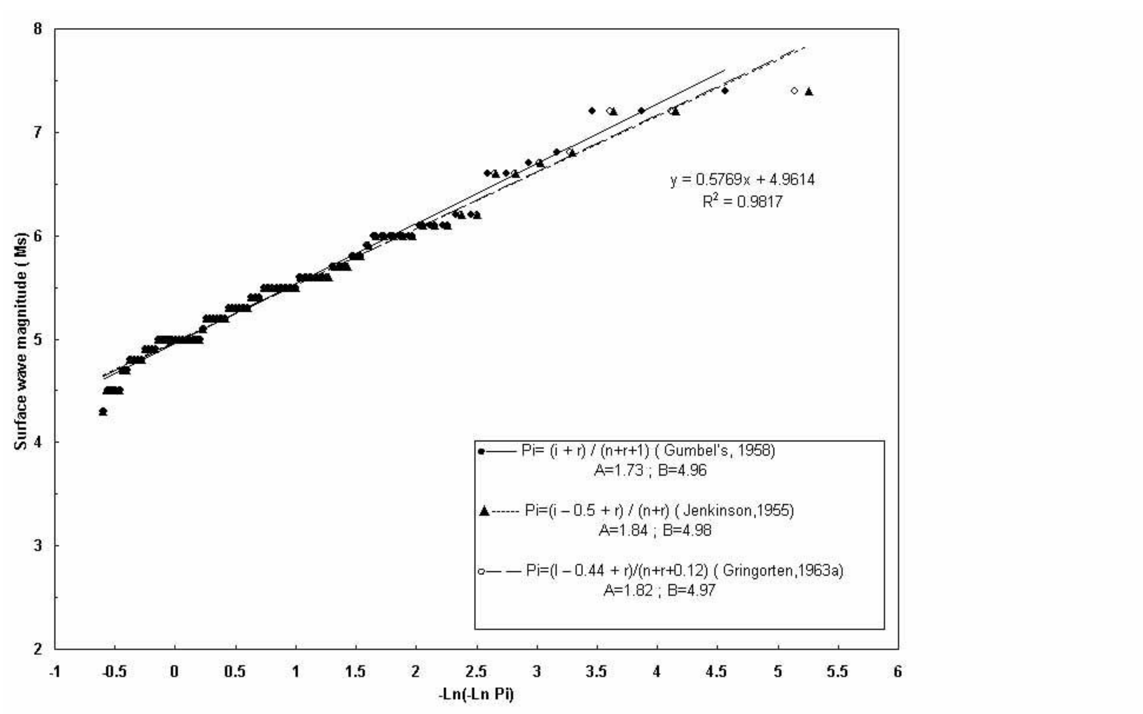

Fig. 3 Gumble Type I Asymptotic distribution for earthquakes in Iraq based on plotting positions over the period 1905-2000 estimated through LS

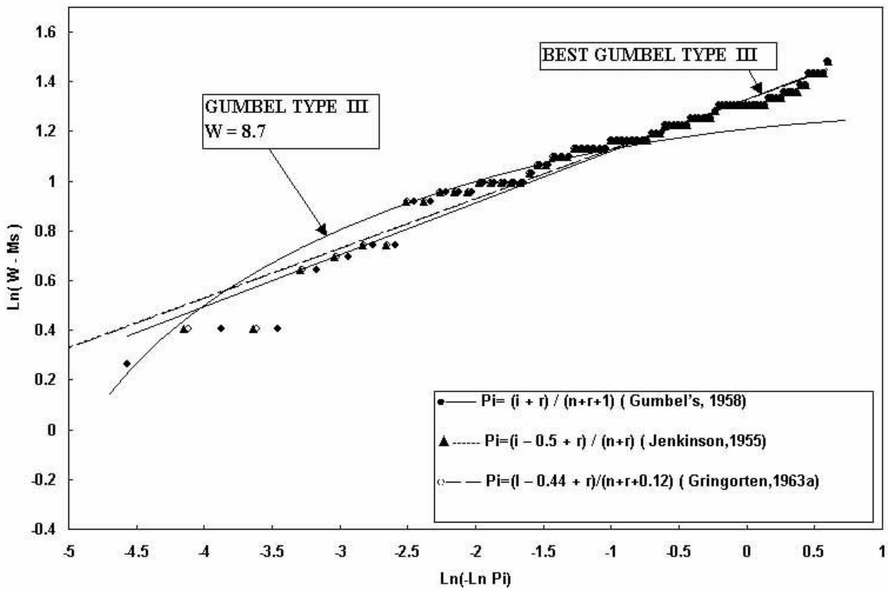

Fig. 3 Gumble Type III asymptotic distribution for earthquakes in Iraq based on plotting positions over the period 1905-2000 estimated through LS

CONCLUSION

In this investigation, the alternative statistical technique Maximum Likelihood method (ML) alongside the more frequently used Least Square method (LS) are applied for Gutenberg and Richter (G-R) and Gumbel's Type-I and Type-III asymptotic distributions parameters estimation using a newly compiled earthquakes catalogue for Iraq on the basis of the ISC data sets. The M_L displayed better results for the analysed 96 year magnitude sample, particularly for the overall fit of Gumbel's Type-III distribution. It has been found that conventional approach based on Gutenberg and Richter relation is not appropriate for modeling of rare large earthquake sequence. The suitability of extreme value theory in predicting large earthquakes in Iraq has been re-examined. Gumbel's Type-III asymptotic distribution using M_L approach with Gringoten's plotting positions rule is found to be more appropriate as it fits better to the data sets based on the Kolmogorov-Smirnov goodness of fit test. Extreme value theory can be effectively used for the seismic hazard assessment and risk evaluation in the regions with lesser microseismic records especially in less instrumental areas. The conventional approach of hazard estimation based on magnitude frequency relationship is useful only when the data set is complete for the entire magnitude range. Small change in the G-R's recurrence relationship parameters is observed to significantly change the hazard and risk estimates associated with a region. With good and complete data sets the G-R method is more appropriate and accurate for seismic hazard and risk estimation than the Gumbel's method.

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TABLE 3:ESTIMATED PROBABILITIES OF SELECTED MAXIMUM SURFACE WAVE MAGNITUDES EXPECTED IN IRAQ USING GUMBEL'S TYPE (I & III) ASYMPTOTIC DISTRIBUTION BASED ON LS AND ML METHODS.

		Gumbel's Type- I Asymptotic Distribution							Gumbel's Type-III Asymptotic Distribution						
	MAG	Probability of Occurrence in Design Life of D Years													
	(Ms)	50	75	100	125	150	175	200	50	75	100	125	150	175	200
LS - Gumbel	5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	6.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	6.5	0.969	0.995	0.999	1.000	1.000	1.000	1.000	0.975	0.996	0.999	1.000	1.000	1.000	1.000
	7.0	0.769	0.889	0.947	0.974	0.988	0.994	0.997	0.658	0.800	0.883	0.932	0.960	0.977	0.986
	7.5	0.461	0.604	0.709	0.786	0.843	0.885	0.915	0.183	0.262	0.333	0.397	0.455	0.508	0.555
LS-Jenkinson	5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	6.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	6.5	0.953	0.990	0.998	1.000	1.000	1.000	1.000	0.964	0.993	0.999	1.000	1.000	1.000	1.000
	7.0	0.703	0.839	0.912	0.952	0.974	0.986	0.992	0.595	0.742	0.836	0.895	0.933	0.958	0.973
	7.5	0.384	0.516	0.620	0.702	0.766	0.816	0.856	0.144	0.209	0.268	0.323	0.374	0.421	0.464
LS-Gringorten	5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	6.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	6.5	0.954	0.990	0.998	1.000	1.000	1.000	1.000	0.966	0.994	0.999	1.000	1.000	1.000	1.000
	7.0	0.711	0.845	0.917	0.955	0.976	0.987	0.993	0.606	0.753	0.845	0.903	0.939	0.962	0.962
	7.5	0.394	0.528	0.632	0.714	0.777	0.826	0.865	0.151	0.217	0.279	0.335	0.387	0.435	0.435
ML	5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	6.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	6.5	0.998	1.000	1.000	1.000	1.000	1.000	1.000	0.984	0.998	1.000	1.000	1.000	1.000	1.000
	7.0	0.969	0.995	0.999	1.000	1.000	1.000	1.000	0.660	0.802	0.885	0.933	0.961	0.977	0.977
	7.5	0.853	0.944	0.979	992	0.997	0.999	1.000	0.107	0.157	0.203	0.247	0.289	0.328	0.328