



## **SIMPLE TUNED MASS DAMPER TO CONTROL SEISMIC RESPONSE OF ELEVATED TANKS**

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### **SUMMARY**

A simple tuned mass damper is proposed to control seismic response of RCC elevated tanks. The simplicity of proposed TMD lies in the fact that it has been derived from the existing components of tank. This TMD consists of roof slab of tank and container columns, which support the roof slab. Usually tanks are analysed as 2-DOF system, in which sloshing and impulsive modes of vibration are included. With the deployment of such a TMD, tank becomes a 3-DOF system, in which sloshing mass and TMD are not attached to each other. To retain the simplicity of proposed TMD, its damping is kept as structural damping of its material. In this sense, proposed TMD is a non-optimum TMD. Effectiveness of proposed TMD is demonstrated by considering an example tank. Response spectrum analysis using design acceleration spectra of IS 1893 [12] has shown that such a non-optimum TMD reduces the tank response by 20%. Further it is noted that for a TMD with mass equal to 5% mass of tank, the required sizes of container column and roof slab thickness are practically feasible and stresses in TMD columns are within permissible limits. Using time history analysis, performance of such a TMD is also shown to be effective under past earthquakes. Some observations are noted on further enhancement in TMD's performance by increasing its damping and by including frictional damping.

### **INTRODUCTION**

In India, elevated tanks are commonly used in the public water distribution systems. These elevated tanks are generally of reinforced cement concrete (RCC). A typical elevated tank consists of RCC container supported on RCC tower also known as staging. Containers are usually circular in shapes, though rectangular, truncated conical or intze type containers are also used. Supporting tower (or staging) could be of frame type or pedestal type. In India, capacity of these elevated water tanks generally varies from 50 to 2000 kiloliters and height of staging usually ranges from 10 to 25 m. Since these elevated tanks are integral parts of lifeline systems, their seismic safety is of considerable importance. Due to its large height and heavy mass at the top, seismic analysis of elevated tank needs special considerations. Seismic design of elevated tank uses two degree-of-freedom model, in which sloshing (or convective) and impulsive modes of vibration are considered (Housner [1]).

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Study reported in this paper is concerned with the control of seismic response of elevated tanks using tuned mass damper (TMD) with main emphasis on using a very simple type of TMD, which is derived from the existing components of the tank. In this connection it may be mentioned that many investigations have been reported on use of tuned mass dampers to control seismic response of buildings (Miyama [2], Villaverde and Koyama [3], Sadek et. al. [4], Brito and Ruiz [5] and Lukkunaprasit et. al. [6] ). From these studies it has been established that TMDs are effective in controlling seismic response of buildings. Some of the recent studies on application of TMD for seismic response have been directed towards use of simple type of TMD. For example, Villaverde and Mosqueda [7] and Villaverde [8] have provided laminated rubber bearings between building roof and column. In such an arrangement, the roof slab and rubber bearing constitute the TMD. Similarly, Johnson et. al. [9] have used a TMD in the form of roof top frame. This timber roof top moment frame is quite flexible and is tuned suitably to act as TMD. They have shown the effectiveness of such a TMD by analyzing six structures under five different earthquakes. Jaiswal and Bakre [10] have proposed tuned mass damper in the form of a soft storey at building top. This soft storey which covers the entire plan area of building, consists of columns of smaller sizes. It does not have walls and is only a bare three dimensional frame. It is shown that by properly tuning the frequency of such a soft storey, seismic response of the building can be controlled. They have also shown that such a soft storey is effective in controlling seismic response of unsymmetrical buildings.

As far as elevated tanks are concerned, there are very few studies on control of their seismic response. Shenton III and Hampton [11] have studied performance of a base isolation system in controlling the seismic response of elevated tanks. Present work describes use of a tuned mass damper to control seismic response of elevated tanks. Here, TMD is derived from the existing components of tank and no additional constructional efforts are required. Effectiveness of such a TMD is demonstrated by comparing the seismic response of a tank with and without TMD. Response spectrum and time history methods are used for analysis. Effect of sloshing mode of vibration is included and various issues associated with the deployment of this kind of TMD are discussed.

## **DESCRIPTION OF PROPOSED TMD AND MATHEMATICAL MODEL**

A typical elevated tank consists of staging, base slab, container wall, container columns and roof slab (Fig. 1a). Container columns are the columns inside the container and they support the roof slab which is also supported and monolithically attached to the container wall. The proposed TMD is derived by disconnecting the roof slab from the wall (Fig. 2). By doing so, container columns and roof slab together becomes an independent structural system, which can act as TMD. The meaning of disconnection of roof slab with container wall is disconnection in lateral direction, i.e. roof slab may be resting on the wall but should be free to slide over it. Thus the proposed TMD is constituted from the existing components of the tank. The secondary system, consisting of container columns and roof slab can be suitably tuned i.e. its natural period will be tuned with that of the tank (i.e. primary system). Here primary system means tank without container columns and roof slab. It may be noted that the natural period of tank used for tuning the TMD, corresponds to tank full condition. The tuning of TMD can be achieved by suitably changing the sizes of container columns and roof slab thickness. While doing this, due consideration needs to be given to issues like minimum size of container columns and their strength. Similarly, roof slab should be of reasonable thickness. Another point to be noted is that for a TMD to be most effective (i.e. for optimum TMD), its damping should be higher than the damping of the primary structure. In the present case, TMD and the primary structure are of RCC and will have same damping properties. In this sense, this kind of TMD will not be an optimum one. However, it will be interesting to evaluate effectiveness of such a non-optimum TMD.

For further work, an elevated tank in which roof slab is monolithically attached to the wall is termed as tank without TMD and one in which roof slab is detached from the wall is called tank with TMD. Further, sloshing mode of vibration is also considered in the analysis. Accordingly, tank without TMD is modeled as two degree-of-freedom (2-DOF) system as shown in Fig. 1b. In this 2-DOF model,  $M_c$  and  $K_c$  represent mass and stiffness of sloshing part of water,  $K_t$  represents stiffness of staging and mass of tank  $M_t$  is taken as  $M_t = M_i + M_{cont} + M_s/3$ . Here,  $M_i$  = impulsive water mass,  $M_{cont}$  = mass of empty container and  $M_s$  = mass of staging. The container mass includes mass of roof slab and container columns. The convective and impulsive masses are obtained as per Housner [1]. For tank with TMD, the roof slab is considered to be detached from the wall (Fig. 3a) and it is modeled as three degree-of-freedom (3-DOF) system as shown in Fig. 3b. In this 3-DOF model,  $M_{tmd}$  and  $K_{tmd}$  represents respectively the mass and stiffness of TMD (i.e. roof slab and container column system). It may be noted that in this case, mass of container ( $M_{cont}^*$ ) does not include mass of roof slab and container columns. Further, it may be noted that in the 3-DOF model TMD mass is not connected to sloshing mass.

### EXAMPLE TANK

The effectiveness of proposed TMD is demonstrated by analyzing an example tank of 350 kiloliter capacity and staging height 12m. Various details of this tank are given in Fig. 4. The tank is of RCC with M20 grade of concrete. For this tank values of various masses and stiffness are:  $M_c = 146.7t$  ;  $M_i = 197.4t$ ;  $M_{cont}^* = 146t$ ;  $M_s = 77t$ ;  $K_c = 535.1$  kN/m and  $K_t = 5111$  kN/m. Free vibration characteristics of this tank are given in Table 1. The first mode corresponds to sloshing mode of vibration and second corresponds to tank mode. It may be noted here that parameters of TMD i.e. sizes of container column and roof slab will be decided on the basis of requirement of mass and stiffness of TMD, which will be tuned to the tank mode of vibration, i.e. second mode of vibration.

#### TMD parameters

The TMD consists of four container columns of height 5.3 m, which originate from the base slab at the locations of internal columns of staging. These container columns support the roof slab and roof beams. Container columns are braced at mid height. This TMD will be tuned to the tank mode of vibration or the second vibration mode. The frequency of TMD is obtained following the approach of Sadek et. al. [4], wherein, ratio of frequency of TMD and tank,  $f$  is given by

$$f = \frac{1}{1 + \phi_2 M_{tmd} / M_2} \left( 1 - \xi_t \sqrt{\frac{\phi_2 M_{tmd} / M_2}{1 + \phi_2 M_{tmd} / M_2}} \right)$$

where,  $\phi_2$  is the normalized mode shape coefficient of tank mass in second mode,  $M_2$  is the modal mass in second mode and  $\xi_t$  is damping of tank mode, which is taken as 5%. TMDs with five different masses varying from 3 to 7% of tank mass are considered and their stiffness are evaluated. Details of TMD parameters thus obtained are given in Table 2. It is seen that for TMD with 3% mass, the required sizes of container column and roof slab are quite small. However, for TMD with 5% mass these sizes are reasonable. It may be noted that for TMD to be optimum, its damping can be evaluated based on the criteria similar to one given by Sadek et. al. [4], however, since the proposed TMD is considered to be of RCC, its damping is considered as 5%. In this sense, this TMD is not an optimum one. However, it would be interesting to see if such a non-optimum damping will lead to reduction in tank response. Free vibration characteristics of tank without TMD (2-DOF model) and tank with TMD (3-DOF model) are given in Table 3. Qualitative description of mode shapes of 3-DOF model is given in Fig. 5. From Table 3 it is seen that due to presence of TMD, time period of tank mode (i.e. 2<sup>nd</sup> mode) increases, however TMD does not affect the time period of sloshing mode (i.e. 1<sup>st</sup> mode).

## RESPONSE SPECTRUM ANALYSIS

Seismic response of tank is obtained using response spectrum of IS 1893 (Part 1):2002 [12]. Tank is considered to be in zone II and on hard soil. Importance factor is taken as 1.5 and response reduction factor is 5.0. Damping for sloshing mode (1<sup>st</sup> mode) is taken as 0.5% and for tank and TMD mode (2<sup>nd</sup> and 3<sup>rd</sup> mode) 5% damping is considered. Response spectrum analysis is performed and modal responses are combined using Square Root of Sum of Square (SRSS) rule. Results of response spectrum analysis of tank without and with TMD are shown in Table 4. It is seen that presence of TMD reduces the base shear by about 20%. It is also seen that TMD mass has no significant effect on base shear and presence of TMD does not affect seismic force corresponding to sloshing mass. A reduction of 20% in base shear of tank is quite satisfactory knowing that the TMD used here does not have optimum damping. As described earlier, in this analysis damping of TMD which is of RCC is retained as 5% i.e. damping of RCC. Another point to be noted is that since TMD mass does not have any sizable effect on tank base shear, one can choose TMD of any mass up to 7% mass of tank, depending on other factors such as stress levels in TMD columns. Forces in the container columns due to seismic load and gravity loads are given in Table 5. It is seen that bending moment and axial force in container column is not very excessive. This is evident from the value of design interaction coefficient  $R^*$  given in Table 5. For stresses to be within permissible limits, value of  $R^*$  should be less than 1.33. Thus, one finds that for all the values of TMD mass considered here, the stresses in columns are safe. However, from Table 2, one finds that roof slab thickness is reasonable for TMD with 5% mass. Thus, one can say that TMD with 5% mass can be practically deployed in this tank.

### Effect of mistuning of TMD

An important step in the design of TMD is that its natural period has to be properly tuned with that of tank. However, actual time period of tank can be slightly different than the one obtained by considering it as 2-DOF system. Apart from this, there could be other factors, which may influence the time period of tank and thus exact time period of tank may not be known. In such a situation, it will be logical to assess the effect of mistuning of time period of TMD and tank. Results on base shear of tank with mistuned TMD of 5% mass are shown in Table 6. Here frequency ratio indicates the ratio of natural frequency of TMD and tank, which is varied from 0.7 to 1.1 and frequency ratio of 0.92 represents the tuned case. It is seen that within the range of frequency ratio from 0.7 to 1.1, the TMD does reduce the base shear as compared to the case of tank without TMD (Table 4), though amount of reduction decreases with mistuning. Nevertheless TMD does not give adverse effect i.e. it does not give higher seismic force than the case of tank without TMD. It is interesting to note the effect of mistuning on modal masses in 2<sup>nd</sup> and 3<sup>rd</sup> mode.

Another point to be noted is that tuning of TMD is being done for tank full condition. For partially filled or empty tank conditions, the time period of tank will change and for this case also, mistuning will occur. However, in partially filled or empty tank conditions mass of the tank itself gets reduced, and these conditions are usually not critical. Hence, mistuning corresponding to these conditions will not be of much concern.

## TIME HISTORY ANALYSIS

In the previous section effectiveness of TMD was demonstrated using response spectrum method of analysis, wherein, design spectra from IS 1893 [12] was used. It would be natural to assess the effectiveness of TMD under real earthquake loadings. For this purpose recorded time histories of five past earthquakes are chosen and time history analysis is performed. Details of these five earthquakes are given in Table 7. Time history compatible with the design spectra of IS 1893 [12] is also considered, and is denoted as SCTH in Table 7. Modal superposition method is used and damping in first mode is taken as 0.5% and in second and third mode 5% damping is considered. It may be noted that first mode

corresponds to sloshing and second and third one correspond to tank-TMD modes. A comparison of peak responses obtained from analysis of 2-DOF model (i.e. tank without TMD) and 3-DOF model (i.e. tank with TMD) is shown in Table 8. Response quantities compared in Table 8 are maximum displacement of tank ( $X_t$ ) and sloshing mass ( $X_c$ ) along with the base shear of tank. It is seen that with the deployment of TMD, tank displacement and base shear reduces. This reduction varies from 20 to 50% for different earthquakes. The variation in effectiveness of TMD under different earthquake excitations is quite well known (Johnson et. al. [9]). Thus, time history analysis also reveals that the proposed TMD could be quite effective in reducing the seismic response of tank. Comparison of time history of tank displacement with and without TMD is shown in Fig. 6.

The results so far presented correspond to 5% damping of 2<sup>nd</sup> and 3<sup>rd</sup> mode of vibration. To study the effect of damping of TMD on response, time history analysis is also performed by considering 10% damping in 3<sup>rd</sup> mode. These results are given in Table 9. It is seen that with the increase in damping of 3<sup>rd</sup> mode, tank response further reduces for almost all the earthquakes. However, one should note that 3<sup>rd</sup> mode of vibration is not a purely TMD mode. This is evident from the % mass excited in 3<sup>rd</sup> mode (Table 3). As far as damping in TMD is concerned, it can be enhanced by providing viscous dampers as shown in Fig. 7. However, analysis of such a system is not covered in this study. In this context one may also be noted that Sadek et. al. [4] have given expressions for obtaining optimum damping ratio of TMD. However, one will have to ascertain if these will be applicable to the present case also in which the tank-TMD system is different than the usual MDOF system with TMD.

## DISCUSSION

Past research has clearly established tuned mass dampers can be used to achieve reduction of the order of 30% in the seismic response. Recent studies (Villaverde [8], Johnson et. al. [9]) have put emphasis on use of simple and cost effective type of TMDs. In this paper also a very simple type of TMD has been proposed for RCC elevated tanks. The simplicity of the proposed TMD lies in the fact that it has been derived from the existing components of tank (Fig. 2). This TMD consists of container columns and roof slab of the tank. With the deployment of such a TMD the tank needs to be analysed as a 3-DOF model, which without TMD is a 2-DOF model. It is interesting to note that in this 3-DOF model, TMD mass is not attached to the sloshing mass. Thus, tank with TMD is a slightly different system than classical MDOF system with TMD, wherein, TMD is attached to the top most mass of the structure. Another, point which makes this 3-DOF model different than other systems is that damping in first modes is 0.5% whereas damping in 2<sup>nd</sup> mode is 5%. Further, to retain the simplicity of TMD, it is decided to keep its damping as 5%, which is structural damping ratio of its material i.e. RCC. From this point of view, this TMD is not an optimum one since its damping is not optimum one. Response spectrum analysis has revealed that even such a non-optimum leads to about 20% reduction in seismic response of tank (Table 4) under design acceleration spectrum. By varying the TMD mass from 3 to 7% of mass of tank it is found that TMD mass has no significant effect on tank response. Thus, one can choose a TMD of suitable mass depending on other factors like strength of TMD columns (or container columns) and thickness of roof slab. It is noted that stresses in TMD columns (or container columns) are within permissible limits (Table 5). Sizes of container column and roof slab thickness for TMD with mass equal to 5% of tank mass are practically feasible.

The crucial step in the effective functioning of TMD is tuning of its frequency with that of the main structure. One may argue that the estimated natural frequency of tank to which TMD has been tuned itself may be different than actual frequency of tank. This difference could be due to so many unforeseen factors like: evaluation of stiffness of staging and sloshing mass. Under such a situation, one needs to ensure that TMD does not give adverse effect. To ascertain the effectiveness of TMD against such a case, results are also obtained for those cases in which frequency of TMD is not tuned with that of tank. It is

noted that even if frequency of TMD is mistuned by 20%, it does not give adverse effect. In fact TMD with 20% mistuned frequency also reduces the tank response though by lesser extent (Table 6).

Effectiveness of the proposed TMD is also assessed for five past earthquakes. Time history analysis revealed that a reduction of 20 to 50% is achieved under these earthquakes (Table 8). The variation in effectiveness of TMD under different earthquakes is quite logical and this has been discussed in detail by Johnson et. al. [9]. A very limited attempt is also made to study the effect of damping of TMD on its performance. It is noted that if damping of third mode is 10%, then one gets further reduction in the tank response. However, there are some unresolved issues regarding the optimum damping of the proposed TMD. One will have to ascertain if optimum damping proposed by Sadek et. al. [4] can be applied to present tank-TMD system. In this regard it may also be noted that in the proposed TMD, friction damping can also be included. This frictional damping can be evoked by making the roof slab slide over the tank wall i.e. roof slab will be resting on wall but will be free to slide over it. In such a frictional damping mechanism the friction force will depend on mass of the TMD itself. Further investigations are needed on optimum damping of proposed TMD and for including frictional damping in this TMD.

## CONCLUSIONS

Following conclusions are drawn from the present study:

- 1) Existing components of elevated tank are suitably adjusted to act like TMD. Such a TMD is simple and cost effective.
- 2) The proposed TMD has shown a reduction of 20% in seismic response of tank under design acceleration spectra.
- 3) The proposed TMD is also found to be quite effective under five past recorded earthquakes.
- 4) TMD with mass equal to 5% mass of tank is found to have stresses within permissible limits and has practically feasible sizes of TMD columns and roof slab thickness.
- 5) More detailed investigations are needed to ascertain the optimum damping of proposed TMD and to include frictional damping in it.

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**Table1 Free vibration Characteristics of tank used for obtaining TMD parameters**

Mode no.	T (Sec)	Mode shape coeff.		Modal mass (t)
		$\phi_{1j}$	$\phi_{2j}$	
1	3.51	0.973	8.110	239.95
2	1.58	5.113	-1.543	275.90

Note: j = mode number

**Table 2 Parameters of TMD**

% mass of TMD	Freq. Ratio ( f )	Time period (Sec)	$M_{tmd}$ (t)	$K_{tmd}$ (kN/m)	Column & braces (mm)	Roof beam (mm)	Roof slab thickness (mm)
3	0.96	1.65	15.6	227.5	125x125	95x95	70.3
4	0.94	1.70	20.9	286.9	135x135	95x95	96.5
5	0.92	1.72	26.1	347.7	144x144	95x95	122.2
6	0.91	1.74	31.3	408.9	152x152	95x95	148.6
7	0.90	1.75	36.5	470.2	159x159	95x95	174.7

**Table 3 Free vibration characteristics of tank without and with TMD**

% Mass of TMD	Mode no.	Tank without TMD (2-DOF model)				Tank with TMD (3-DOF model)				
		Time period (Sec)	Mode shape coeff.		% Mass exited	Time period (Sec)	Mode shape coeff.			% Mass exited
			$\phi_{1j}$	$\phi_{2j}$			$\phi_{1j}$	$\phi_{2j}$	$\phi_{3j}$	
3	1	3.51	0.98	8.10	46.2	3.51	0.99	8.10	1.26	46.4
	2	1.62	5.0	-1.60	53.8	1.79	3.02	-1.26	20.2	29.2
	3	-	-	-	-	1.46	4.13	-1.01	-15.1	24.4
4	1	3.51	0.99	8.10	46.1	3.51	0.99	8.10	1.28	46.5
	2	1.63	4.97	-1.61	53.9	1.83	2.84	-1.28	18.0	28.5
	3	-	-	-	-	1.45	4.25	-1.03	-12.4	25.0
5	1	3.51	0.99	8.09	46.0	3.51	0.99	8.09	1.31	46.5
	2	1.64	4.93	-1.62	54.0	1.88	2.69	-1.30	16.5	28.0
	3	-	-	-	-	1.45	4.34	-1.04	-10.5	25.5
6	1	3.51	0.99	8.09	46.0	3.51	1.00	8.08	1.32	46.5
	2	1.65	4.90	-1.64	54.0	1.91	2.63	-1.35	15.1	28.5
	3	-	-	-	-	1.44	4.38	-1.03	-9.41	25.0
7	1	3.51	1.00	8.09	45.8	3.52	1.00	8.07	1.34	46.5
	2	1.66	4.86	-1.66	54.2	1.95	2.58	-1.39	14.1	28.9
	3	-	-	-	-	1.43	4.41	-1.02	-8.56	25.6

Notes: j = Mode number.

$\phi_{ij}$ , i = 1,2,3, represent coefficient corresponding to tank, sloshing and TMD mass respectively.

**Table 4 Effect of TMD on seismic forces and base shear**

% Mass of TMD	Tank Without TMD			Tank With TMD				% Reduction in base shear
	$Q_t$ (kN)	$Q_c$ (kN)	Base Shear (kN)	$Q_t$ (kN)	$Q_c$ (kN)	$Q_{TMD}$ (kN)	Base Shear (kN)	
3	17.5	14.1	31.6	7.0	13.9	4.2	25.1	20.6
4	17.7	14.2	31.9	6.4	14.0	4.8	25.2	21.0
5	17.8	14.2	32.0	6.1	14.0	5.3	25.4	20.6
6	18.0	14.3	32.3	5.7	14.1	5.8	25.6	20.7
7	18.0	14.3	32.3	5.4	14.1	6.2	25.7	20.5

Note:  $Q_t$ ,  $Q_c$ ,  $Q_{TMD}$  respectively represent seismic force on tank, sloshing and TMD mass.

**Table 5 Forces in container column**

% Mass of TMD	Size Container columns	Due to Seismic loads		Due to gravity loads		R*
		Axial force (kN)	Bending Moment (kN-m)	Axial force (kN)	Bending Moment (kN-m)	
3	125 x 125	2.053	1.860	15.74	-	1.04
4	135 x 135	2.372	2.144	20.39	-	0.99
5	144 x 144	2.627	2.368	24.92	-	0.94
6	152 x 152	2.887	2.598	29.56	-	0.91
7	159x 159	3.140	2.820	34.11	-	0.89

Note: R\* is interaction design check coefficient. For safe design R\* should be less than 1.33

**Table 6 Effect of mistuning of TMD on response (5% mass TMD)**

Freq. Ratio	T <sub>1</sub> (Sec)	T <sub>2</sub> (Sec)	T <sub>3</sub> (Sec)	% modal mass			Base shear (kN)
				M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	
0.7	3.51	2.32	1.54	47	10	43	27.9
0.8	3.51	2.07	1.51	47	16	37	26.6
0.92*	3.51	1.88	1.45	47	28	25	25.4
1.0	3.51	1.80	1.39	47	36	17	26.1
1.1	3.51	1.74	1.30	47	43	10	27.9

Note: \* indicates Optimum frequency ratio

**Table 7 Details of earthquake time histories used in the analysis**

Sr. No.	Earthquake	Station	PGA (g)	Duration (sec)
1	El Centro	Imperial valley	0.32	31.18
2	Loma-Prieta	Oakland Harbour	0.27	40.0
3	Northridge	Santa Monica	0.88	60.0
4	San Fernando	Pacoima dam	1.17	41.82
5	Uttarkashi (India)	Aalmora	0.02	21.32
6	SCTH	-	1.14	30.0

Note: SCTH denotes spectrum compatible time history corresponding to spectra of IS 1893 [12]

**Table 8 Effect of TMD (with 5% damping) on peak response**

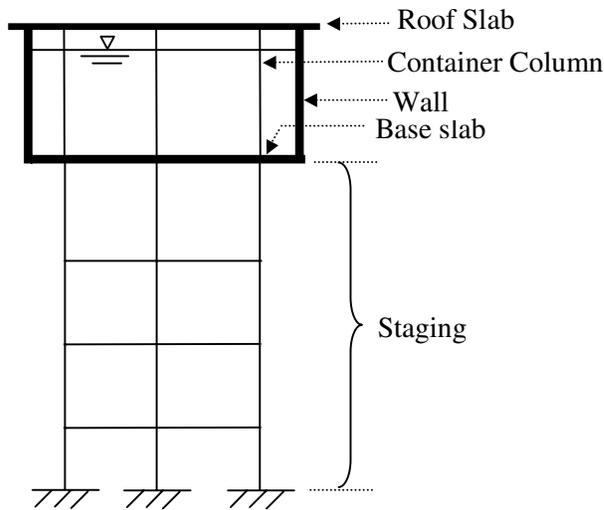
Earthquake	Tank without TMD (2-DOF model)			Tank with TMD (3-DOF model)				% Reduction		
	Displacement		Base shear (kN)	Displacement			Base shear (kN)	Displacement		Base shear (kN)
	X <sub>t</sub> (mm)	X <sub>c</sub> (mm)		X <sub>t</sub> (mm)	X <sub>c</sub> (mm)	X <sub>tmd</sub> (mm)		X <sub>t</sub> (mm)	X <sub>c</sub> (mm)	
Elcentro	1.8	6.8	9.2	1.63	6.9	4.8	8.3	9.4	-1.5	9.8
Loma-Prieta	2.5	2.6	12.7	2.0	2.9	6.3	10.2	20.0	-11.5	19.7
Northridge	2.9	6.8	15.0	1.6	6.8	8.5	8.2	44.8	0.0	45.3
San Fernando	6.9	9.3	35.1	5.1	9.3	19.3	26.1	25.0	0.0	25.6
Uttarkashi	0.1	0.4	0.5	0.08	0.4	0.2	0.4	20.0	0.0	20.0
SCTH	8.4	21.7	42.8	6.8	20.5	19.3	34.7	19.0	5.5	18.9

Note: X<sub>t</sub> and X<sub>c</sub> denote displacement of tank and sloshing mass respectively.

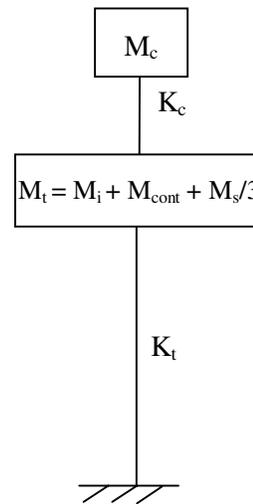
**Table 9 Effect of TMD (with 10% damping) on peak response**

Earthquake	Tank without TMD (2-DOF model)			Tank with TMD (3-DOF model)				% Reduction		
	Displacement		Base shear (kN)	Displacement			Base shear (kN)	Displacement		Base shear (kN)
	$X_t$ (mm)	$X_c$ (mm)		$X_t$ (mm)	$X_c$ (mm)	$X_{tmd}$ (mm)		$X_t$ (mm)	$X_c$ (mm)	
Elcentro	1.8	6.8	9.2	1.47	6.85	4.3	7.5	18.3	-0.7	18.5
Loma-Prieta	2.5	2.7	12.7	1.6	2.9	5.4	8.2	35.4	-9.8	35.4
Northridge	2.9	6.8	15.0	1.4	6.9	7.5	7.4	50.6	-0.5	50.6
San Fernando	6.9	9.3	35.1	4.6	9.3	17.1	23.7	32.3	0.3	32.5
Uttarkashi	0.1	0.4	0.5	0.08	0.4	0.2	0.4	20.0	0.0	20.0
SCTH	8.4	21.7	42.9	6.7	20.7	17.0	34.1	20.5	4.8	20.5

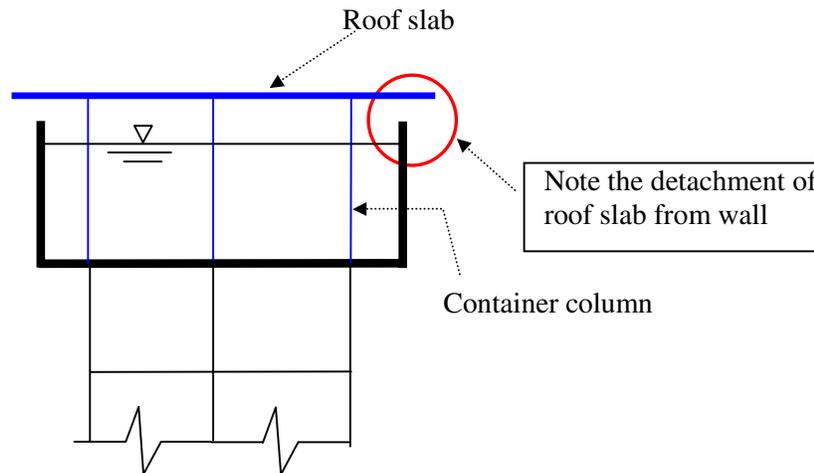
Note:  $X_t$ ,  $X_c$  and  $X_{tmd}$  denote displacement of tank, sloshing mass and TMD respectively.



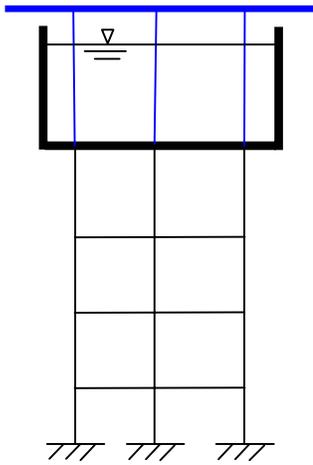
**Fig 1a Typical Elevated tank**



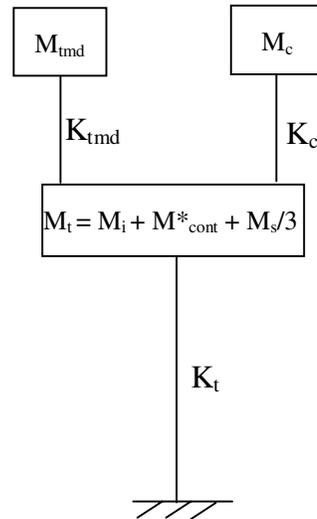
**Fig. 1b Mathematical model of elevated tank**



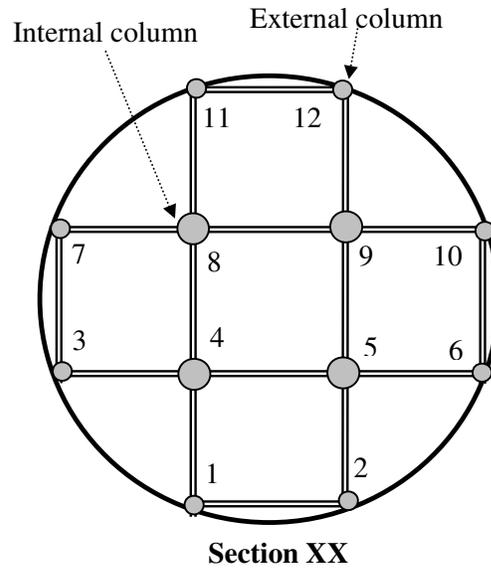
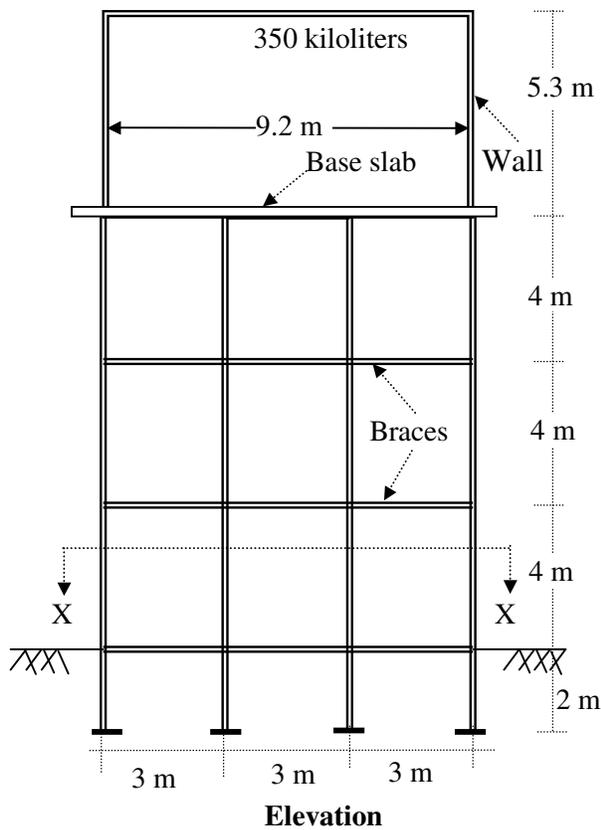
**Fig. 2 Description of TMD; Roof slab supported on container column constitute the TMD.**



**Fig. 3a** Elevated tank with TMD



**Fig. 3b** Mathematical model of tank with TMD



Internal columns - 4Nos. 450 mm dia  
 External columns 8 Nos. 350 mm dia  
 Braces - 250x200mm  
 Base beams - 250x600 mm;  
 Base slab - 220 mm  
 Wall - 200mm thick; Gallery 1m wide

**Fig 4** Details of example tank

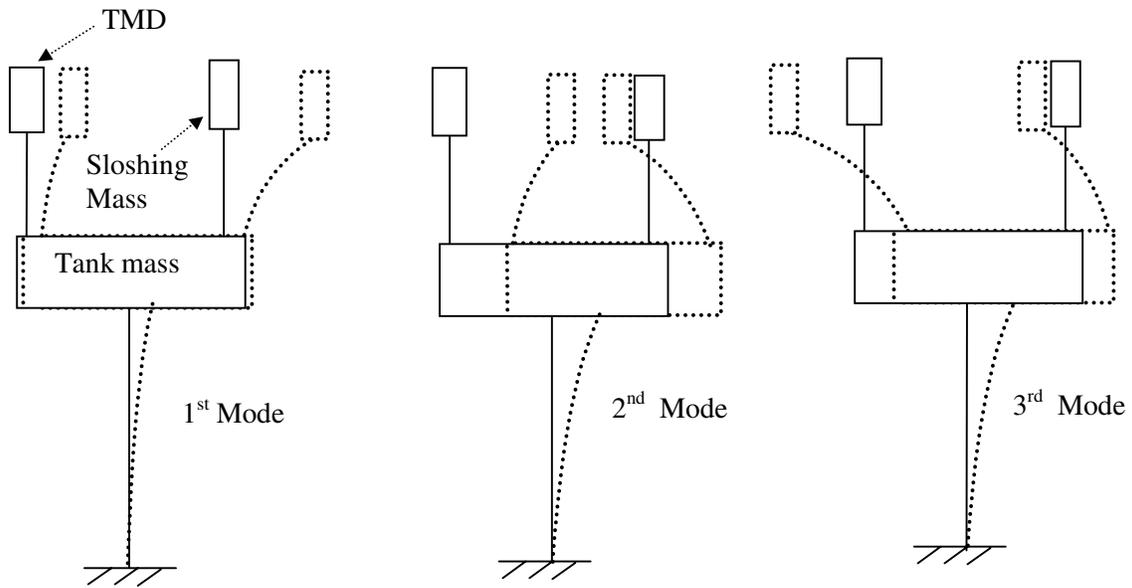


Fig. 5 Qualitative description of mode shapes of 3-DOF model

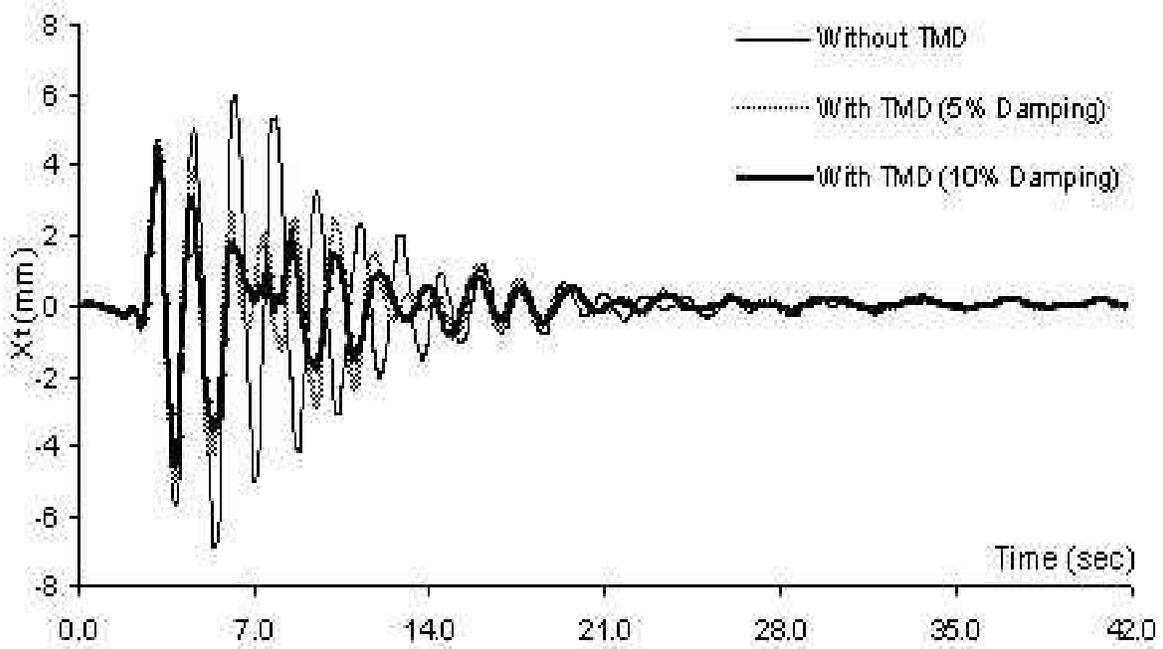
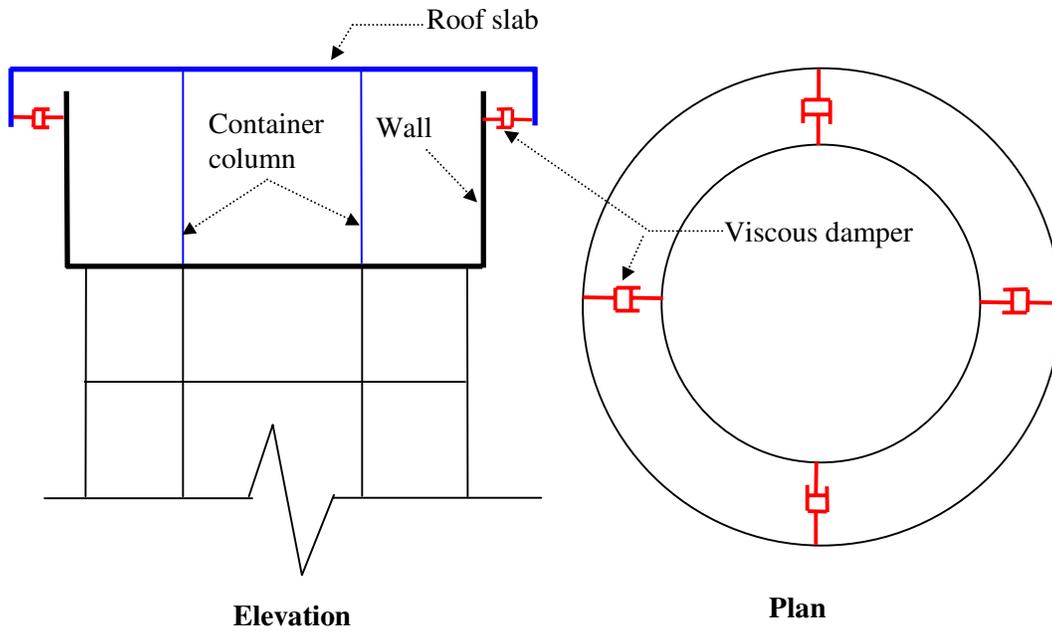


Fig. 6 Time history of tank displacement without and with TMD (San Fernando Earthquake)



**Fig. 7 Use of viscous dampers to increase damping of TMD**