13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 2949

IDENTIFICATION METHOD FOR FLEXURE AND SHEAR BEHAVIOR OF SHEAR WALL BUILDINGS

F. GARCÉS[1], C. GENATIOS[2], M. LAFUENTE[2], A. MÉBARKI[3]

SUMMARY

This article presents an original identification method for the assessment of flexure and shear stiffness of shear wall buildings. Required data includes an initial (theoretical) model, the estimation of lumped mass values (by floor) and the experimental evaluation of two eigenvalues (modal frequency and its modal shapes). The method estimates stiffness whenever flexural (EI) or shear (GA) values are relevant or are irrelevant. An initial formula includes both shear and flexural components. Furthermore, particular developments are carried out for particular cases of irrelevant shear or flexural deformations. The method is applied for two numerical examples of shear wall buildings showing its simple and efficient implementation. Numeric errors are very small, in the order of exp(10-18) up to exp(10-12) in the case of structures with 5 up to 12 stories. This method allows the correction of structural models based on reduced experimental data (no more than two frequencies and experimental modes).

Keywords: experimental testing, buildings, structural systems, evaluation and retrofit, structural response, modal analysis.

INTRODUCTION

There is always a difference between the theoretical model of a building and the real one. On the other hand, structural stiffness can vary due to degradation through time, due to building modifications, damage, overloads or seismic effects. The goal is the determination of the actual structural stiffness in a given moment Genatios [1], Ventura [2].

Several stiffness identification methods have been developed taking as data modal experimental results and structural typology, leading to stiffness changes and damage evaluation, Baruch[3], Kabe[4], Papadoupulus[5], Sawyer[6], Zhang[7]. Some of this methods come from aeronautics, and have been extended for shear wall building identification, Woodgate [8], Yua [9], modeled as flexural structures with lumped masses on each stage, Ling [10], Garcés[11].

[1] Doctorate Student, Laboratoire de Mécanique, Université de Marne-la-Vallée, France
[2] Titular Professor, Instituto de Materiales y Modelos Estructurales, Universidad Central de Venezuela
[3] Professor, Laboratoire de Mécanique, Université de Marne-la-Vallée, France

The present article proposes an identification procedure for flexural stiffness (EI) and shear stiffness evaluation (GA/γ) for shear wall buildings with predominant behavior on flexure, shear or both.

Given a particular typology, stiffness values can be evaluated for each level of the building. This requires two eigenvalues evaluation: two modal frequencies and the corresponding modal shapes. For shear or flexural predominant behavior, only one eigenvalue is needed.

SHEAR AND FLEXURAL STIFFNESS EVALUATION FOR SHEAR WALL BUILDINGS

Structural idealization

The structure is idealized with N dynamic degrees of freedom (dof) with lumped masses on each story, with an unknown flexibility matrix F (or its corresponding stiffness matrix K) and a known mass matrix M. It is accepted that the dynamic analysis allows to know m modal frequencies and its corresponding modal shapes with m<N.

General Methodology for flexural and shear stiffness evaluation.

Fig. 1 shows the structural model that considers flexural and shear behavior, rigid floors, non vertical deformations and lumped masses.

Dynamic parameters can be obtained from:

$$(\lambda_i^{-1} - F.M)\phi_i = 0 \qquad (1)$$

with F=flexibility matrix, M= Mass matrix, $\lambda_i = \omega_i^2$, ω_i= i^{th} modal frequency, i= 1 to N, ϕ_i = i^{th} eigenvector.

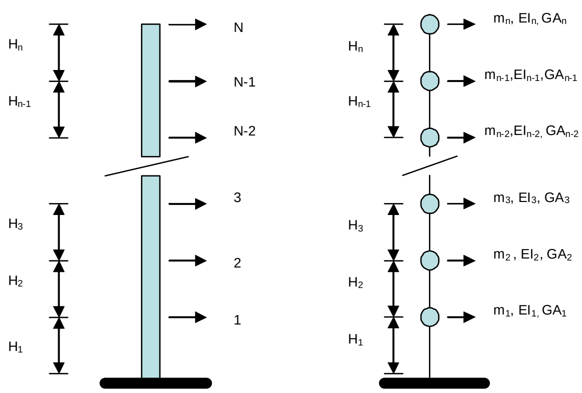

Figure 1. N dof Shear wall structure

Flexibility and Mass matrix have the following expressions:

$$F = \begin{bmatrix} f_{11} & & f_{1j} & \cdots & f_{1N} \\ \cdots & \cdots & f_{ij} & \cdots & \vdots \\ \vdots & \cdots & \ddots & \cdots & \\ f_{N1} & \cdots & f_{Nj} & & f_{NN} \end{bmatrix} \qquad (2)$$

$$M = \begin{bmatrix} m_1 & & \cdots & & 0 \\ & m_2 & & & \\ \vdots & & \ddots & & \vdots \\ & & & m_{N-1} & \\ 0 & & \cdots & & m_N \end{bmatrix} \qquad (3)$$

Equation (1) corresponds to:

$$\left(\begin{bmatrix} 1/\lambda_a & \cdots & 0 \\ \vdots & 1/\lambda_a & \vdots \\ 0 & \cdots & 1/\lambda_a \end{bmatrix} - \begin{bmatrix} f_{11} & \cdots & \\ \vdots & f_{ij} & \\ 0 & \cdots & f_{NN} \end{bmatrix} \cdot \begin{bmatrix} m_1 & 0 & \cdots & & 0 \\ 0 & & & & \\ \vdots & & m_i & & \vdots \\ & & & & 0 \\ 0 & \cdots & & 0 & m_N \end{bmatrix} \right) \begin{Bmatrix} \phi_a^1 \\ \\ \phi_a^i \\ \\ \phi_a^N \end{Bmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} \qquad (4)$$

with : m_i = lumped mass in the i[th] level f_{ij} = flexibility coefficient of the flexibility matrix F, ϕ_a^i = modal coordinate of the i[th] level of modal shape “a”, $1/\lambda_a = 1/\omega_a^2$ and ω_a modal frequency of mode “a”.

Considering two eigenvalues corresponding to the modal frequencies ω_a and ω_b, and the modal shapes ϕ_a and ϕ_b, equation (4) becomes:

$$\begin{gathered} f_{11}m_1\phi_a^1 + ... + f_{1j}m_j\phi_a^j + \cdots + f_{1N}m_N\phi_a^N = \frac{1}{\omega_a^2}\phi_a^1 \\ f_{11}m_1\phi_b^1 + ... + f_{1j}m_j\phi_b^j + \cdots + f_{1N}m_N\phi_b^N = \frac{1}{\omega_b^2}\phi_b^1 \\ \vdots \\ f_{i1}m_1\phi_a^1 + f_{i2}m_2\phi_a^2 + \cdots + f_{iN}m_N\phi_a^N = \frac{1}{\omega_a^2}\phi_a^i \\ f_{i1}m_1\phi_b^1 + f_{i2}m_2\phi_b^2 + \cdots + f_{iN}m_N\phi_b^N = \frac{1}{\omega_b^2}\phi_b^N \end{gathered} \qquad (5)$$

Each flexibility value can be evaluated as follows:

$$f_{ij} = \sum_{k=1}^{i} \frac{1}{(EI)_k}\alpha_{ijk} + \sum_{k=1}^{i} \frac{1}{(GA/\gamma)_k}\beta_k \qquad \text{si} \quad i \le j \qquad (6)$$

with:

$$\alpha_{ijk} = H_k \cdot \left[\left(\sum_{l=k+1}^{i} H_l \right) \cdot \left(\sum_{l=k+1}^{j} H_l \right) + \frac{H_k}{2} \left(\sum_{l=k+1}^{i} H_l + \sum_{l=k+1}^{j} H_l \right) + \frac{H_k^2}{3} \right] \tag{7}$$

$$\beta_k = H_k$$

where: H_k = story height of level "k", E_k = elasticity modulus of the material of level "k", I_k = Inertia modulus of level "k", G_k = Transversal elasticity modulus of level "k", A_k = transversal surface of level "k", (A_k/γ)=shear transversal surface of level "k".

Equation (6) can be rewritten as follows, with "i" and "j", values from 1 to N:

$$f_{ij} = \sum_{k=1}^{Min(i,j)} \frac{1}{(EI)_k} \alpha_{ijk} + \sum_{k=1}^{Min(i,j} \frac{1}{(GA/\gamma)_k} \beta_k \qquad \forall \;\; i, j \tag{8}$$

It can be also expressed as:

$$f_{ij} = \sum_{k=1}^{N} \frac{1}{(EI)_k} \alpha_{ijk} . 1_{k \le Min(i,j)} + \sum_{k=1}^{N} \frac{1}{(GA/\gamma)_k} \beta_k . 1_{k \le Min(i,j)} \qquad \forall \;\; i, j$$

$$1_{k \le Min(i,j)} = \begin{cases} 1: \text{ if } k \le Min(i,j) \\ 0: \text{ otherwise} \end{cases} \tag{9}$$

The goal of this procedure is the evaluation of the stiffness coefficients $(EI)_k$ and $(GA/\gamma)_k$, for each level "k", with k = 1 to N. Considering equations (5) and (6) we come to:

$$\begin{bmatrix} a_{11} & b_{11} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ & & & \cdots & \cdots & \cdots & \cdots & 0 \\ & & & & & \cdots & \cdots & 0 \\ a_{i1} & b_{i1} & \ldots & a_{ij} & b_{ij} & \cdots & \cdots & 0 \\ \vdots & & & & \ddots & & & \vdots \\ & & \cdots & \cdots & \cdots & \cdots & & \\ & & \cdots & \cdots & \cdots & \cdots & \cdots & \\ a_{N1} & b_{N1} & & a_{Nj} & b_{Nj} & & a_{NN} & b_{NN} \end{bmatrix} . \begin{bmatrix} \frac{1}{(EI)_1} \\ \frac{1}{(GA/\gamma)_1} \\ .. \\ \frac{1}{(EI)_i} \\ \frac{1}{(GA/\gamma)_i} \\ \frac{1}{(EI)_N} \\ \frac{1}{(GA/\gamma)_N} \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega^2} \phi^1 \\ \ldots \\ \ldots \\ \frac{1}{\omega^2} \phi^i \\ \ldots \\ \ldots \\ \ldots. \\ \frac{1}{\omega^2} \phi^N \end{bmatrix} \tag{10}$$

i and k = 1 to N:

The values are:

$$\begin{aligned} a_{ik} &= \sum_{j=1}^{N} m_j.\phi^j.\alpha_{ijk}.1_{k\le Min(i,j)} \\ b_{ik} &= \beta_k.\sum_{j=1}^{N} m_j.\phi^j.1_{k\le Min(i,j)} \end{aligned} \tag{11}$$

Equation (10) defines a system of N equations with 2N unknowns, as two unknown coefficients (EI) and (GA/γ) are considered for each level. This fact imposes the requirement of two modal shapes and their corresponding frequencies in order to produce two sets of N equations, so 2N equations. The corresponding two modal shapes are:

$$\phi_a = \begin{Bmatrix} \phi_a^1 \\ \cdots \\ \phi_a^i \\ \cdots \\ \phi_a^N \end{Bmatrix} \quad \text{and} \quad \phi_b = \begin{Bmatrix} \phi_b^1 \\ \cdots \\ \phi_b^i \\ \cdots \\ \phi_b^N \end{Bmatrix} \tag{12}$$

The two modal shapes define the following coefficients:

$$\begin{aligned} a_{ik}^a &= \sum_{j=1}^{N} m_j.\phi_a^j.\alpha_{ijk}.1_{k\le Min(i,j)} \quad \text{and} \quad a_{ik}^b = \sum_{j=1}^{N} m_j.\phi_b^j.\alpha_{ijk}.1_{k\le Min(i,j)} \\ b_{ik}^a &= \beta_k.\sum_{j=1}^{N} m_j.\phi_a^j.1_{k\le Min(i,j)} \quad \text{and} \quad b_{ik}^b = \beta_k.\sum_{j=1}^{N} m_j.\phi_b^j.1_{k\le Min(i,j)} \end{aligned} \tag{13}$$

Leading to the 2N equations system with 2N unknowns that include shear and flexure coefficients:

$$\begin{bmatrix} a_{11}^a & b_{11}^a & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ a_{11}^b & b_{11}^b & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ & & & & 0 & \cdots & \cdots & 0 \\ a_{i1}^a & b_{i1}^a & ... & a_{ij}^a & b_{ij}^a & .. & \cdots & 0 \\ a_{i1}^b & b_{i1}^b & ... & a_{ij}^b & b_{ij}^b & .. & & \vdots \\ .. & & \cdots & \cdots & \cdots & \cdots & & 0 \\ a_{N1}^a & b_{N1}^a & \cdots & a_{Nj}^a & b_{Nj}^a & \cdots & a_{NN}^a & b_{NN}^a \\ a_{N1}^b & b_{N1}^b & & a_{Nj}^b & b_{Nj}^b & & a_{NN}^b & b_{NN}^b \end{bmatrix} . \begin{bmatrix} \frac{1}{(EI)_1} \\ \frac{1}{(GA/\gamma)_1} \\ .... \\ \frac{1}{(EI)_j} \\ \frac{1}{(GA/\gamma)_j} \\ ...... \\ \frac{1}{(EI)_N} \\ \frac{1}{(GA/\gamma)_N} \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega_a^2}\phi_a^1 \\ \frac{1}{\omega_b^2}\phi_b^1 \\ .... \\ \frac{1}{\omega_a^2}\phi_a^i \\ \frac{1}{\omega_b^2}\phi_b^i \\ .... \\ \frac{1}{\omega_a^2}\phi_a^N \\ \frac{1}{\omega_b^2}\phi_b^N \end{bmatrix} \tag{14}$$

The system can be rewritten as :

$$[A]\{x\} = \{c\} \tag{15}$$

{x} is the unknowns vector that includes the stiffness coefficients.

1 story structure and some singularities

A one store system leads to a singularity. Given matrix [A], equations (14)-(15):

$$\begin{bmatrix} a^a_{11} & b^a_{11} \\ a^b_{11} & b^b_{11} \end{bmatrix} . \begin{Bmatrix} \dfrac{1}{(EI)_1} \\ \dfrac{1}{(GA/\gamma)_1} \end{Bmatrix} = \begin{Bmatrix} \dfrac{1}{\omega_a^2}\phi_a^1 \\ \dfrac{1}{\omega_b^2}\phi_b^1 \end{Bmatrix} \qquad (16)$$

As there is only 1 dof $\omega_a=\omega_b=\omega$. The determinant is equal to:

$$\begin{vmatrix} a^a_{11} & b^a_{11} \\ a^b_{11} & b^b_{11} \end{vmatrix} = m_1^2 . \frac{H_1^4}{3} . \left(\phi_a^1 . \phi_b^1 - \phi_b^1 . \phi_a^1\right) = 0 \qquad (17)$$

leading to:

$$a^a_{11} . \frac{1}{(EI)_1} + b^a_{11} . \frac{1}{(GA/\gamma)_1} = \frac{1}{\omega_a^2}\phi_a^1 \quad \text{sea} \quad m_1 . \left(\frac{H_1^3}{3.(EI)_1} + \frac{H_1}{(\frac{GA}{\gamma})_1} \right) = \frac{1}{\omega^2} \qquad (18)$$

General case and singularity in the highest level « N »

A singularity is found at the highest level “N”, so the last two equations of eq. (14) shall have a particular treatment. The solution for the first N-1 levels is (i = 1, N-1):

$$\begin{bmatrix} a^a_{11} & b^a_{11} & 0 & \cdots & \cdots & .. & \cdots & 0 \\ a^b_{11} & b^b_{11} & 0 & \cdots & \cdots & .. & \cdots & 0 \\ & & & & 0 & .. & \cdots & 0 \\ a^a_{i1} & b^a_{i1} & .. & a^a_{ij} & b^a_{ij} & .. & \cdots & 0 \\ a^b_{i1} & b^b_{i1} & .. & a^b_{ij} & b^b_{ij} & .. & & \vdots \\ .. & & .. & \cdots & \cdots & .. & & 0 \\ a^a_{N-1,1} & b^a_{N-1,1} & .. & a^a_{N-1,j} & b^a_{N-1,j} & .. & a^a_{N-1,N-1} & b^a_{N-1,N-1} \\ a^b_{N-1,1} & b^b_{N-1,1} & & a^b_{N-1,j} & b^b_{N-1,j} & & a^b_{N-1,N-1} & b^b_{N-1,N-1} \end{bmatrix} . \begin{bmatrix} \dfrac{1}{(EI)_1} \\ \dfrac{1}{(\frac{GA}{\gamma})_1} \\ \cdots \\ \dfrac{1}{(EI)_i} \\ \dfrac{1}{(\frac{GA}{\gamma})_i} \\ \cdots\cdot \\ \dfrac{1}{(EI)_{N-1}} \\ \dfrac{1}{(\frac{GA}{\gamma})_{N-1}} \end{bmatrix} = \begin{bmatrix} \dfrac{\phi_a^1}{\omega_a^2} \\ \dfrac{\phi_b^1}{\omega_b^2} \\ \cdots \\ \dfrac{\phi_a^i}{\omega_a^2} \\ \dfrac{\phi_b^i}{\omega_b^2} \\ \cdots \\ \dfrac{\phi_a^{N-1}}{\omega_a^2} \\ \dfrac{\phi_b^{N-1}}{\omega_b^2} \end{bmatrix} \qquad (19)$$

Once the solution is obtained for the 2(N-1) equations, the highest level can be evaluated by the following shear-flexural relationship:

$$a^a_{NN} . \frac{1}{(EI)_N} + b^a_{NN} . \frac{1}{(GA/\gamma)_N} = \frac{\phi_a^N}{\omega_a^2} - \sum_{k=1}^{N-1} \left(a^a_{Nk} . \frac{1}{(EI)_k} + b^a_{Nk} . \frac{1}{(GA/\gamma)_k} \right) \qquad (20)$$

$$a^{b}_{NN}\cdot\frac{1}{(EI)_N}+b^{b}_{NN}\cdot\frac{1}{(GA/\gamma)_N}=\frac{\phi_b^N}{\omega_b^2}-\sum_{k=1}^{N-1}\left(a^{b}_{Nk}\cdot\frac{1}{(EI)_k}+b^{b}_{Nk}\cdot\frac{1}{(GA/\gamma)_k}\right)\qquad(21)$$

Damage and residual properties

This methodology can be applied for a structural system with known properties before an earthquake affects its properties, producing new or damaged condition due to the change of shear and flexural properties.

Initial properties, before the seismic event are: (for "N" levels):

$$\left\{(EI)_1^0 \quad (GA/\gamma)_1^0 \quad .... \quad (EI)_i^0 \quad (GA/\gamma)_i^0 \quad ...... \quad (EI)_N^0 \quad (GA/\gamma)_N^0\right\}\qquad(22)$$

Properties can be evaluated following equations (19)-(21) once the damage is introduced, and can be related to the initial ones by:

$$\begin{aligned}(EI)_i &= (1-D_i^f).(EI)_i^0\\ (GA/\gamma)_i &= (1-D_i^v).(GA/\gamma)_i^0\end{aligned}\qquad(23)$$

D^f= indicates flexure damage and D^v= indicates shear damage. Both indexes have positive values from 0 (undamaged condition) to 1 (totally damaged condition).

For the last level only one relationship is found for shear and flexure coefficients so we can accept an equal damage value for both coefficients:

$$D_N^f = D_N^v = 1-\frac{a^{b}_{NN}\cdot\frac{1}{(EI)_N^0}+b^{b}_{NN}\cdot\frac{1}{(GA/\gamma)_N^0}}{\frac{\phi_b^N}{\omega_b^2}-\sum_{k=1}^{N-1}\left(a^{b}_{Nk}\cdot\frac{1}{(EI)_k}+b^{b}_{Nk}\cdot\frac{1}{(GA/\gamma)_k}\right)}$$

(24)

$$D_N^f = D_N^v = 1-\frac{a^{a}_{NN}\cdot\frac{1}{(EI)_N^0}+b^{a}_{NN}\cdot\frac{1}{(GA/\gamma)_N^0}}{\frac{\phi_a^N}{\omega_a^2}-\sum_{k=1}^{N-1}\left(a^{a}_{Nk}\cdot\frac{1}{(EI)_k}+b^{a}_{Nk}\cdot\frac{1}{(GA/\gamma)_k}\right)}$$

FLEXURE STIFFNESS EVALUATION

Structures with flexure predominant behavior (so shear terms can be neglected), only (EI) terms are significant, and (GA/γ) terms can be eliminated. In this particular case only one eigenpair is required so the flexure stiffness values can be obtained from the simplified system of equations obtained from eq. (14):

$$\begin{bmatrix} a^a_{11} & 0 & \dots & \dots & \dots \\ \dots & \dots & 0 & \dots & \dots \\ a^a_{i1} & \dots & a^a_{ij} & .. & \dots \\ \dots & \dots & \dots & \dots & 0 \\ a^a_{N1} & \dots & a^a_{Nj} & \dots & a^a_{NN} \end{bmatrix} . \begin{bmatrix} \frac{1}{(EI)_1} \\ \dots \\ \frac{1}{(EI)_i} \\ \dots \\ \frac{1}{(EI)_N} \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega_a^2}\phi_a^1 \\ \dots \\ \frac{1}{\omega_a^2}\phi_a^i \\ \dots \\ \frac{1}{\omega_a^2}\phi_a^N \end{bmatrix} \qquad (25)$$

The solution leads to the stiffness evaluation. Damage can be described by the following indicator, defined for each "i" level, with i=1,N:

$$(EI)_i = (1 - \mathrm{D}_i^f).(EI)_i^0 \qquad (26)$$

SHEAR STIFFNESS EVALUATION

In the case of significant values of shear stiffness compared to the flexure stiffness so this last can be neglected, only (GA/γ) terms are significant. Only one eigenpair is required and flexure stiffness coefficients are given by:

$$\begin{bmatrix} b^a_{11} & 0 & \dots & \dots & \dots \\ \dots & \dots & 0 & \dots & \dots \\ b^a_{i1} & \dots & b^a_{ij} & .. & \dots \\ \dots & \dots & \dots & \dots & 0 \\ b^a_{N1} & \dots & b^a_{Nj} & \dots & b^a_{NN} \end{bmatrix} . \begin{bmatrix} \frac{1}{(GA/\gamma)_1} \\ \dots \\ \frac{1}{(GA/\gamma)_i} \\ \dots \\ \frac{1}{(GA/\gamma)_N} \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega_a^2}\phi_a^1 \\ \dots \\ \frac{1}{\omega_a^2}\phi_a^i \\ \dots \\ \frac{1}{\omega_a^2}\phi_a^N \end{bmatrix} \qquad (27)$$

Solution leads to the stiffness evaluation. Damage can be described by the following indicator, defined for each "i" level, with i=1,N:

$$(GA/\gamma)_i = (1 - \mathrm{D}_i^v).(GA/\gamma)_i^0 \qquad (28)$$

EXAMPLE AND NUMERICAL ROBUSTNESS

As an example, a composed structure is considered (Fig 2):

Shear wall height is 3m , length 3m and thickness 0,5 m.

N levels

Floors 3 m x 6m, with 0.5m thickness.

Initial geometric and mechanical characteristics are given in Table 1.

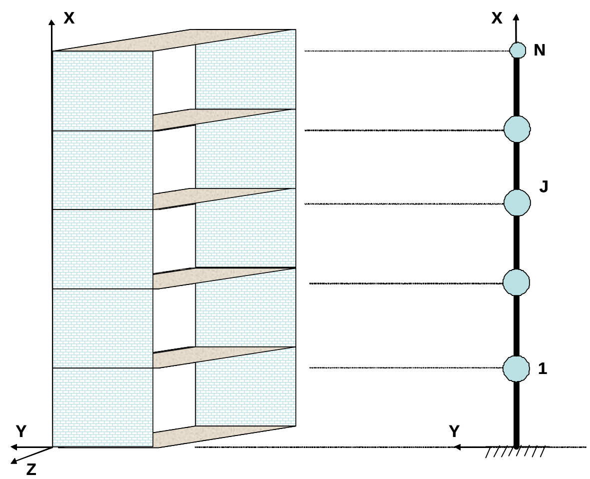

Fig. 2 – N dof structure

Initial calculations for this example require theoretical values of frequencies and modal shapes of the model so two of those eigenpairs can be employed in the identification procedure following eq. (19) and (21). Mode 1 and mode N where chosen in this particular case. Relative precision of stiffness coefficients (EI) and (GA/γ), allows robustness evaluation by comparing the initial stiffness values (table 1) with the numerically obtained values (tables 2 and 3) following eq. (29) and (30). Obtained precision is very high ranging 10^{-14} to 10^{-12} for stiffness coefficients of all floors but the last (level N). The method proposes for the last floor, the quotient between shear and flexure values, and in this particular case precision range was 10^{-17}.

Table 1 – Mechanical and Geometric Characterístics

	Units	Values	Level
E: Elasticity modulus	MPa	21400	All
G: Transversal Elasticity modulus	MPa	8917	All
H: wall height	m	3	All
L: walls width	m	3	All
e: walls thickness	m	0, 5	All
D: distance between walls	m	5	All
h: floor thickness	m	0,25	All
μ: Volumetric mass for materials	Kg/m^3	2500	All
I: Z-Z axis quadratic inertia value	m^4	1,125	All
(A/γ): reduced section for Y-Y shear	M^2	1,25	All
M: lumped mass for each floor	kg	15938	1 a (N-1)
	kg	10313	Higher level (N)
$(EI)_0$: Initial stiffness condition for flexure behavior Z-Z axis	$MN.m^2$	24075	All
$(GA/\gamma)_0$: Initial stiffness condition for	MN	11146	All
walls Y-Y axis			

For error evaluation:

$$\Delta(EI) = \frac{\left|(EI) - (EI)_0\right|}{(EI)_0} \quad \text{and} \quad \Delta(GA/\gamma) = \frac{\left|(GA/\gamma) - (GA/\gamma)_0\right|}{(GA/\gamma)_0} \tag{29}$$

$$\Delta\left[(EI)_N, \left(\frac{GA}{\gamma}\right)_N\right] = \frac{\left|\left[\frac{\phi^N}{\omega^2} - \sum_{k=1}^{N-1}\left(\frac{a_{Nk}}{(EI)_k} + \frac{b_{Nk}}{(GA/\gamma)_k}\right)\right] - \left[\frac{a_{NN}}{(EI)_N^0} + \frac{b_{NN}}{(GA/\gamma)_N^0}\right]\right|}{\left[\frac{a_{NN}}{(EI)_N^0} + \frac{b_{NN}}{(GA/\gamma)_N^0}\right]} \tag{30}$$

Table 2- Identification results of a 5 level building

Item	Units	Precision	Levels
$\Delta(EI)/(EI)_0$: Precision of (EI)	--	10^{-14}	All but last
$\Delta(GA/\gamma)/(GA/\gamma)_0$: Precision of (GA/γ)	--	10^{-14}	All but last
$\Delta\left[(EI)_N, \left(GA/\gamma\right)_N\right]$, *Eq.(30)* Relative Precision (EI) and (GA/γ) modes “**a**” and “**b**”	--	10^{-19}	Last floor
Frequency last mode: ω_a	rd/s	746,5	
Period last mode	*s*	*0,0084*	
Modal Coordinates	--	0,8032	1
		-1	2
		0,7456	3
		-0,3575	4
Frequency first mode: ω_b	rd/s	45,9	
Period first mode	*s*	*0,1369*	
Modal Coordinates	--	0,1061	1
		0,3481	2
		0,66287	3
		1,0000	4

Table 3- Identification results of a 10 level building

Item	Units	Precision	level
$\Delta(EI)/(EI)_0$: Precision of (EI)	--	10^{-12}	All but last
$\Delta(GA/\gamma)/(GA/\gamma)_0$: Precision of (GA/γ)	--	10^{-12}	All but last
$\Delta\left[(EI)_N, \left(GA/\gamma\right)_N\right]$, *Eq.(30)* Relative Precision (EI) and (GA/γ) modes "**a**" and "**b**"	--	10^{-17}	Last level
Modal frequency last mode: $\omega_{\mathbf{a}}$	rd/s	816	
Period last mode	*s*	*0,0077*	
Modal Coordinates	--	0,3813	1
		-0,6192	2
		0,8254	3
		-0,9542	4
		1	5
		-0,9578	6
		0,8304	7
		-0,6342	8
		0,3619	9
		-0,1557	10
Frequency first mode: $\omega_{\mathbf{b}}$	rd/s	8	
Period first mode	*s*	*0,785*	
Modal Coordinates	--	0,0176	1
		0,0650	2
		0,1376	3
		0,2307	4
		0,3401	5
		0,4614	6
		0,5909	7
		0,7255	8
		0,86241	9
		1	10

CONCLUSIONS

This article presents a method for the evaluation of the flexure (EI) and shear (GA/γ) stiffness coefficients for each level of a building considering 1 dof per level. Required data are level masses, considered as lumped for each level, two natural frequencies and their corresponding modal shapes. Whenever either one of shear or flexure coefficients predominates so the other can be neglected, only one modal shape and its corresponding frequency is needed.

In the general case, once two eigenpairs are known, it is easy to evaluate the shear and flexure stiffness coefficients for each level, excepting the highest one, for the last one, only the quotient between coefficients is obtained.

Once the stiffness coefficients are known, it is easy to obtain the damage values affecting mechanical properties of each level.

For the analyzed examples corresponding to 4 to 10 floors buildings with mechanical and geometric properties with few numerical changes, the developed methodology gives shear and flexure stiffness coefficients with a 10^{-19} to 10^{-12} precision.

REFERENCES

1. Genatios C., Cascante G., Lafuente M., Bellan P., Lorrain M., "Evaluation Expérimentale des Caractéristiques Modales et ajustement du Modéle Dynamique d´un Bâtiment de Portiques", Revue Materials and Structures RILEM, vol. 32, pp 117-124, Paris, Mars 1999.
2. Ventura & Horyna , "Measured and Calculated Modal Characteristics of the Heritage Court Tower in Vancouver, B,C,", International Modal Analysis Conference-XVIII, pp.1070-1074.
3. Baruch; "Optimization Procedure to Correct Stiffness and Flexibility Matrices Using Vibrations Test", *AIAA Journal*, Vol. **16**, Nº 11, pp.1208-11210. 1978
4. Kabe; "Stiffness Matrix Adjustment Using Mode Data", *AIAA Journal*, Vol. **23**, Nº 9, pp.1431-1436. 1985
5. Papadopoulos & Garcia; "Structural Damage Identification: A Probabilistic Approach", *AIAA Journal*, Vol.**36**, Nº11, pp.2137-2145, 1985.
6. Sawyer & Rao; "Structural Damage Detection and Identification Using Fuzzy Logic", *AIAA Journal*, Vol, **38**, Nº 12, pp.1431-1436, 2000.
7. Zhang & Zerva; "Stiffness Matrix Adjustment Using Incomplete Measured Modes", *AIAA Journal*, Vol. **35**, Nº 5, pp.917-919, 1997.
8. Woodgate; "Efficient Stiffness Matrix Estimation for Elastic Structures", *Computers and Structures*, Vol.**69**, pp.79-84, 1998.
9. Yuan, Wu & Ma; "Estimated Mass and Stiffness Matrices of Shear Building from Modal Test", *Earthquake Engineering and Structural Dynamics*, Vol.**27**, pp.415-421, 1998.
10. Li, Hao, Lu & Chen; "A Flexibility Approach for Damage Identification of Cantilever-Type Structures with Bending and Shear Deformation", *Computers and Structures*, Vol.**73**, pp., 565-572, 1999.
11. Garcés F., Genatios C., Mébarki A. & Lafuente M.; "Reajuste de Matrices de Rigidez y Flexibilidad para Sistemas Aporticados", *Boletín Técnico del IMME,* Vol.**40**, N°3, pp.19-26, ISSN 0376-723X, 2000.