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RESPONSE MEAN UPCROSSING RATE FOR LINEAR MDOF SYSTEMS SUBJECTED TO FULLY NONSTATIONARY EARTHQUAKE GROUND MOTION MODEL

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SUMMARY

New explicit closed-form solutions are derived for the mean upcrossing rate of nonstationary response quantities of linear elastic, both classically and non-classically damped, multi-degree-of-freedom (MDOF) systems subjected to a fully nonstationary earthquake ground motion process. The stochastic earthquake ground motion model used in this study captures the temporal variation of both the amplitude and frequency content typical of real earthquake ground motions. The analytical results obtained are applied to single-degree-of-freedom (SDOF) systems and a three-dimensional unsymmetrical building equipped with viscous bracings. Each of the stochastic earthquake processes used in the application examples was calibrated against an actual earthquake record.

Using the derived closed-form solutions for the mean upcrossing rates of various structural response quantities, approximate analytical solutions are developed for the time-variant structural reliability problem (i.e., evaluation of failure probability cumulative over a time interval such as the duration of an earthquake process) using the Poisson assumption. An analytical upper bound for the time-variant probability of failure is also obtained. These approximate analytical solutions and analytical upper bound for the time-variant probability of failure are verified via Monte Carlo simulation.

The analytical solutions presented for the mean upcrossing rate of structural response quantities are extremely useful in gaining better physical insight into the nonstationary seismic response behavior of linear dynamic systems. They can also be used in benchmark studies to evaluate the accuracy of numerical procedures devoted to the computation of the time-variant probability of failure of linear and nonlinear structural systems subjected to realistic stochastic earthquake loading.

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INTRODUCTION

In many engineering fields, the importance of using stochastic processes to model dynamic loads such as earthquake ground motions, wind effects on civil and aerospace structures, and ocean wave induced forces on offshore structures, has been widely recognized. Extensive research has been devoted to the development of analytical methods and numerical simulation techniques related to modeling of stochastic loads and analysis of their effects on structures [1,2,3]. In particular, in earthquake engineering, the non-stationarity in both amplitude and frequency content of earthquake ground motions has been recognized as an essential ingredient to capture realistically the seismic response of structures [4,5,6,7]. Therefore, significant attention has been given to nonstationary earthquake ground motion models, with particular emphasis on their accurate but compact representation [8,9,10,11,12].

The probability of failure over a given interval of time (i.e., probability of a response vector process outcrossing a general limit-state surface during an exposure time) is the fundamental result required in a time-variant reliability analysis. For a large class of structural applications, the failure condition can be identified as the exceedance of a deterministic threshold by a linear combination of scalar response quantities. To date, no exact closed-form solution of this problem (also called the first-passage problem in the literature) is available, even for the simplest case of structural model (deterministic linear elastic SDOF system) subjected to the simplest stochastic load model (stationary Gaussian white noise). The Monte Carlo simulation technique is the only general method accommodating for non-stationarity and non-Gaussianess of the excitation as well as nonlinearity in the structural behavior and uncertainty/randomness in the structural parameters. However, it is computationally extremely expensive. Nevertheless, an analytical upper bound of the time-variant probability of failure can be obtained readily when response mean outcrossing rates are available [1] and several direct approximations of this failure probability have been developed making use of different statistics of the response quantities of interest [13,14]. In particular, Poisson's and Vanmarcke's approximations have been shown to offer a good compromise between accuracy and computational effort [15,16,17].

In this paper, explicit closed-form solutions are derived for the mean upcrossing rate of response quantities of linear elastic multi-degree-of-freedom (MDOF) systems subjected to a fully nonstationary earthquake ground motion process previously developed by the authors. The stochastic earthquake ground motion model used herein accounts for the temporal variation of both the amplitude and frequency content typical of real earthquake ground motions and it has been calibrated against well-known historic earthquake records. These closed-form solutions are used for the numerical evaluation of the analytical upper bound (obtained in integral form) and of the Poisson's approximation (also obtained in integral form) of the time-variant probability of failure of the subject MDOF system. All the analytical and semi-analytical results obtained in this study are compared and validated with Monte Carlo simulation results.

STOCHASTIC EARTHQUAKE GROUND MOTION MODEL

The stochastic earthquake ground motion model used herein is a sigma-oscillatory process, nonstationary in both amplitude and frequency content [12]. This earthquake ground acceleration model, $\ddot{U}_g(t)$, is defined as the sum of a finite number of pairwise independent, uniformly modulated Gaussian processes, i.e.,

$$\ddot{U}_g(t) = \sum_{k=1}^{p} X_k(t) = \sum_{k=1}^{p} A_k(t) S_k(t) \tag{1}$$

where p represents the number of component processes or sub-process, $A_k(t)$ is the time modulating function of the k-th sub-process $X_k(t)$, and $S_k(t)$ is the k-th Gaussian stationary process. The time modulating function $A_k(t)$ is defined as

$$A_k(t) = \alpha_k (t - \theta_k)^{\beta_k} e^{-\gamma_k (t - \theta_k)} H(t - \theta_k) \tag{2}$$

where α_k and γ_k are positive constants, β_k is a positive integer, and θ_k represents the "arrival time" of the k-th sub-process, $X_k(t)$; H(t) denotes the Heaviside unit step function. The k-th zero-mean, stationary Gaussian process, $S_k(t)$, is characterized by its autocorrelation function

$$R_{S_k S_k}(\tau) = e^{-\nu_k|\tau|}\cos(\eta_k\tau) \tag{3}$$

and the corresponding power spectral density (PSD) function

$$\Phi_{S_k S_k}(\omega) = \frac{\nu_k}{2\pi}\left[\frac{1}{\nu_k^2 + (\omega+\eta_k)^2} + \frac{1}{\nu_k^2 + (\omega-\eta_k)^2}\right] \tag{4}$$

in which ν_k and η_k are the two free parameters representing the frequency bandwidth and the predominant or central frequency of the process $S_k(t)$, respectively. It can be shown [12] that the mean square function of the above ground acceleration model can be expressed as

$$E[|\ddot{U}_g(t)|^2] = \int_{-\infty}^{\infty}\sum_{k=1}^{p}|A_k(t)|^2\Phi_{S_k S_k}(\omega)d\omega = \sum_{k=1}^{p}|A_k(t)|^2 \tag{5}$$

where E[…] denotes the expectation operator, and the corresponding evolutionary (time-varying) power spectral density function is given by

$$\Phi_{\ddot{U}_g\ddot{U}_g}(t,\omega) = \sum_{k=1}^{p}|A_k(t)|^2\Phi_{S_k S_k}(\omega) \tag{6}$$

The above evolutionary PSD function gives the time-frequency distribution of the earthquake ground acceleration process.

A very important property of the Gaussian process $S_k(t)$, described by the autocorrelation function in Equation (3), is that it can be realized as a linear combination of the displacement and velocity responses of the same SDOF system (with natural period and damping ratio obtained from the two parameters ν_k and η_k) subjected separately to two uncorrelated white noise processes. This property enables a very efficient simulation procedure of $S_k(t)$ and thus of the entire nonstationary process $\ddot{U}_g(t)$.

STATE-SPACE FORMULATION OF EQUATIONS OF MOTION AND COMPLEX MODAL ANALYSIS OF LINEAR MDOF SYSTEMS

The general equations of motion of an n-degree-of-freedom linear system can be expressed in matrix form as

$$\mathbf{M}\ddot{\mathbf{U}}(t) + \mathbf{C}\dot{\mathbf{U}}(t) + \mathbf{K}\mathbf{U}(t) = \mathbf{P}F(t) \tag{7}$$

where **M**, **C**, and **K** are the $n \times n$ time-invariant mass, damping and stiffness matrices, respectively; $\mathbf{U}(t)$, $\dot{\mathbf{U}}(t)$, and $\ddot{\mathbf{U}}(t)$ are the length-n vectors of nodal displacements, velocities and accelerations, respectively; **P** is the length-n load distribution vector, and F(t) is an external, scalar loading function which, in the case of random excitations, is modeled as a random process. Defining the following length-2n 'state vector'

$$\mathbf{Z}(t) = \begin{bmatrix}\mathbf{U}(t)\\ \dot{\mathbf{U}}(t)\end{bmatrix}_{(2n\times 1)}, \tag{8}$$

the matrix equation of motion (7) can be recast into the following first-order matrix equation:

$$\dot{\mathbf{Z}}(t) = \mathbf{G}\mathbf{Z}(t) + \tilde{\mathbf{P}}F(t) \tag{9}$$

where

$$\mathbf{G} = \begin{bmatrix} \mathbf{0}_{(n \times n)} & \mathbf{I}_{(n \times n)} \\ (-\mathbf{M}^{-1}\mathbf{K}) & (-\mathbf{M}^{-1}\mathbf{C}) \end{bmatrix}_{(2n \times 2n)} \quad \text{and} \quad \tilde{\mathbf{P}} = \begin{bmatrix} \mathbf{0}_{n \times 1} \\ \mathbf{M}^{-1}\mathbf{P} \end{bmatrix}_{(2n \times 1)} \tag{10}$$

The complex modal matrix, $\mathbf{T}$, formed by the complex eigenmodes can be used as an appropriate transformation matrix to decouple the first-order matrix equation (9). Introducing the transformed state vector $\mathbf{V}(t)$ of complex modal coordinates defined by

$$\mathbf{Z}(t) = \mathbf{T}\mathbf{V}(t), \tag{11}$$

substituting Equation (11) into Equation (9), performing some algebraic manipulations considering that $\mathbf{T}^{-1}\mathbf{G}\mathbf{T} = \mathbf{D}$ [18] (where $\mathbf{D}$ is the diagonal matrix containing the 2n complex eigenvalues, λ_1, λ_2, ..., λ_{2n}, of the system matrix $\mathbf{G}$) and that $\mathbf{T}^{-1}\tilde{\mathbf{P}} = [\Gamma_1, ..., \Gamma_{2n}]^T$ (where Γ_i is the i-th complex-valued modal participation factor), one obtains the normalized first-order complex modal equations

$$\dot{S}_i(t) = \lambda_i S_i(t) + F(t), \quad i = 1, 2, ..., 2n \tag{12}$$

where the normalized complex modal responses $S_i(t)$ have been defined by

$$V_i(t) = \Gamma_i S_i(t), \quad i = 1, 2, ..., 2n \tag{13}$$

The unit impulse response function for the i-th mode, $h_i(t)$, defined as the solution of Equation (12) when $F(t) = \delta(t)$ where $\delta(t)$ denotes the Dirac delta function and for at rest initial conditions at time $t = 0^-$, is simply given by $h_i(t) = e^{\lambda_i t}$, $t > 0$. Assuming for simplicity that the system is initially at rest, the solution of Equation (12) can be expressed by the Duhamel integral

$$S_i(t) = \int_0^t e^{\lambda_i(t-\tau)} F(\tau)d\tau, \quad i = 1, 2, ..., 2n \tag{14}$$

It is worth mentioning that the normalized complex modal responses $S_i(t)$, $i = 1, 2, ..., 2n$, are complex conjugate by pairs. Combining Equations (11) and (13) yields

$$\mathbf{Z}(t) = \mathbf{T}\mathbf{V}(t) = \mathbf{T}\mathbf{\Gamma}\mathbf{S}(t) = \tilde{\mathbf{T}}\mathbf{S}(t) \tag{15}$$

in which $\mathbf{\Gamma}$ is the diagonal matrix containing the 2n modal participation factors Γ_i, $\tilde{\mathbf{T}} = \mathbf{T}\mathbf{\Gamma}$ is the effective modal participation matrix and $\mathbf{S} = [S_1(t), S_2(t), \ldots, S_{2n}(t)]^T$ is the normalized complex modal response vector.

EXPLICIT CLOSED-FORM SOLUTIONS FOR THE STOCHASTIC RESPONSE OF LINEAR MDOF SYSTEMS SUBJECTED TO NONSTATIONARY GROUND MOTION MODEL

The second-order statistics of the response of a linear MDOF system subjected to the fully nonstationary earthquake ground motion model presented above is derived using state-space and complex modal analysis. Due to the assumption in the ground motion model that the component processes of the sigma-oscillatory process are pairwise statistically independent, the second-order response statistics can be obtained by simply adding the contributions of the individual component processes.

In the time domain, the cross-correlation function of normalized complex modal responses $S_i(t)$ and

$S_j(t)$ due to the k-th earthquake component process $X_k(t)$ can be derived as

$$R^{(k)}_{S_iS_j}(t,\tau) = E[S_i^{(k)*}(t)S_j^{(k)}(t+\tau)] \;=\; \int_0^t e^{\lambda_i^*(t-u)}\int_0^{t+\tau} e^{\lambda_j(t+\tau-s)}E[X_k^*(u)X_k(s)]dsdu \tag{16}$$

in which $\tau \geq 0$ and the superscript * denotes the complex conjugate operator. For $\tau < 0$, the following relationships can be used

$$\begin{aligned} R^{(k)}_{S_iS_j}(t,\tau) &= R^{(k)*}_{S_jS_i}(t+\tau,-\tau) \qquad \text{if } t+\tau \geq 0 \\ &= 0 \qquad\qquad\qquad\qquad\quad \text{if } t+\tau < 0 \end{aligned} \tag{17}$$

The explicit closed-form solution has been obtained for the above complex cross-modal cross-correlation function $R^{(k)}_{S_iS_j}(t,\tau)$ and is presented elsewhere [19].

The second-order statistics of the nodal relative displacement and velocity responses of a linear MDOF system can be obtained from the second-order statistics of the normalized complex modal responses, simply by summing over all modes and over all sub-processes of the ground motion model accounting for their different arrival times θ_k, as

$$\mathbf{R_{ZZ}}(t,\tau) \;=\; \tilde{\mathbf{T}}^*\mathbf{R_{SS}}(t,\tau)\tilde{\mathbf{T}}^T \tag{18}$$

where $\mathbf{R_{SS}}(t,\tau)$ is the complex cross-modal cross-correlation matrix obtained summing $R^{(k)}_{S_iS_j}(t,\tau)$ over all sub-processes of the ground motion model.

The second-order statistics of the nodal absolute acceleration responses can also be derived through a simple linear transformation as explained below. Consider the following alternative form of the governing equation of motion of the MDOF system

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{U}}(t) + \mathbf{KU} \;=\; \mathbf{0}_{(n\ \times\ 1)} \tag{19}$$

in which $\ddot{\mathbf{X}}(t)$ denotes the length-n absolute acceleration response vector. Thus,

$$\ddot{\mathbf{X}}(t) \;=\; \begin{bmatrix} (-\mathbf{M}^{-1}\mathbf{K}) & (-\mathbf{M}^{-1}\mathbf{C}) \end{bmatrix}\begin{bmatrix} \mathbf{U}(t) \\ \dot{\mathbf{U}}(t) \end{bmatrix} \;=\; \mathbf{AZ}(t) \tag{20}$$

where matrix $\mathbf{A}$ is an $(n \times 2n)$ transformation matrix defined as

$$\mathbf{A} \;=\; \begin{bmatrix} (-\mathbf{M}^{-1}\mathbf{K}) & (-\mathbf{M}^{-1}\mathbf{C}) \end{bmatrix} \tag{21}$$

Therefore, the correlation matrix and evolutionary power spectral density matrix of the absolute acceleration response vector $\ddot{\mathbf{X}}(t)$ are given by, respectively,

$$\mathbf{R}_{\ddot{\mathbf{X}}\ddot{\mathbf{X}}}(t,\tau) \;=\; \mathbf{A}\mathbf{R_{ZZ}}(t,\tau)\mathbf{A}^T \quad \text{and} \quad \mathbf{\Phi}_{\ddot{\mathbf{X}}\ddot{\mathbf{X}}}(\omega,t) \;=\; \mathbf{A}\mathbf{\Phi_{ZZ}}(\omega,t)\mathbf{A}^T \tag{22}$$

MEAN UPCROSSING RATE OF RESPONSE QUANTITIES FOR LINEAR MDOF SYSTEMS

The mean upcrossing rates of the response of a linear MDOF system subjected to the fully nonstationary earthquake ground motion model presented earlier can be derived readily from the second-order statistics of the normalized complex modal responses. In fact, given that the input processes are Gaussian and the system (filter) is linear, the output processes are also Gaussian. Using the well-known Rice formula [3,15], the mean upcrossing rate of level $U_i \;=\; \xi_i$, $\nu_i(\xi_i^+, t)$, of the i-th nodal displacement response $U_i(t)$ can be obtained as

$$\nu_i(\xi_i^+, t) = \int_0^\infty \dot{u}_i f_{U_i\dot{U}_i}(\xi_i, \dot{u}_i, t)d\dot{u}_i = \frac{1}{2\pi}\frac{\sigma_{\dot{U}_i}}{\sigma_{U_i}}\sqrt{1-\rho^2_{U_i\dot{U}_i}} \cdot e^{-\frac{1}{2\left(1-\rho^2_{U_i\dot{U}_i}\right)}\left(\frac{\xi_i}{\sigma_{U_i}}\right)^2} \cdot$$

$$\left\{1 + \frac{\sqrt{\pi}\rho_{U_i\dot{U}_i}}{\sqrt{2(1-\rho^2_{U_i\dot{U}_i})}}\left(\frac{\xi_i}{\sigma_{U_i}}\right)e^{\frac{\rho^2_{U_i\dot{U}_i}}{2\left(1-\rho^2_{U_i\dot{U}_i}\right)}\left(\frac{\xi_i}{\sigma_{U_i}}\right)^2} \operatorname{erfc}\left(-\frac{\rho_{U_i\dot{U}_i}}{\sqrt{2(1-\rho^2_{U_i\dot{U}_i})}}\frac{\xi_i}{\sigma_{U_i}}\right)\right\} \tag{23}$$

where

$$\sigma^2_{U_i} = R_{Z_iZ_i}(t, 0), \quad \sigma^2_{\dot{U}_i} = R_{Z_{n+i}Z_{n+i}}(t, 0), \quad \rho_{U_i\dot{U}_i} = \frac{R_{U_i\dot{U}_i}(t, 0)}{\sigma_{U_i}\sigma_{\dot{U}_i}} = \frac{R_{Z_iZ_{n+i}}(t, 0)}{\sqrt{R_{Z_iZ_i}(t, 0)R_{Z_{n+i}Z_{n+i}}(t, 0)}} \tag{24}$$

The mean outcrossing rate of the level $|U_i| = \xi_i$, $\nu_i(\xi_i, t)$, (symmetric double barrier problem [3]) is given by

$$\nu_i(\xi_i, t) = \nu_i(\xi_i^+, t) + \nu_i(\xi_i^-, t) = 2\nu_i(\xi_i^+, t) \tag{25}$$

The mean outcrossing rate of any response quantity linearly related to the nodal relative displacement and/or relative velocity responses, $\mathbf{Y}(t) = \mathbf{BZ}(t)$ (such as inter-story drifts, internal forces, absolute floor accelerations), can be obtained by substituting in Equation (23) the quantities $\sigma^2_{U_i}$, $\sigma^2_{\dot{U}_i}$ and $\rho_{U_i\dot{U}_i}$, with $\sigma^2_{Y_i}$, $\sigma^2_{\dot{Y}_i}$ and $\rho_{Y_i\dot{Y}_i}$, respectively. The latter three response statistics are given by

$$\sigma^2_{Y_i} = R_{Y_iY_i}(t, 0), \quad \sigma^2_{\dot{Y}_i} = R_{\dot{Y}_i\dot{Y}_i}(t, 0), \quad \rho_{Y_i\dot{Y}_i} = \frac{R_{Y_i\dot{Y}_i}(t, 0)}{\sigma_{Y_i}\sigma_{\dot{Y}_i}} \tag{26}$$

where $\mathbf{R_{YY}}(t, 0) = \mathbf{BR_{ZZ}}(t, 0)\mathbf{B}^T$, $\mathbf{R_{Y\dot{Y}}}(t, 0) = \mathbf{BR_{Z\dot{Z}}}(t, 0)\mathbf{B}^T$, $\mathbf{R_{\dot{Y}\dot{Y}}}(t, 0) = \mathbf{BR_{\dot{Z}\dot{Z}}}(t, 0)\mathbf{B}^T$, $\mathbf{R}_{\mathbf{Z}^{(p)}\mathbf{Z}^{(q)}}(t, \tau) = \partial^p/\partial t^p(\partial^q/\partial\tau^q\mathbf{R_{ZZ}}(t, \tau))$, $\mathbf{Z}^{(p)}(t) = \partial\mathbf{Z}^p/\partial t^p$, and $Y_i(t)$ denotes the i-th component of the generic structural response vector $\mathbf{Y}(t)$. Closed-form frequency-domain solutions for $\mathbf{R}_{\mathbf{Z}^{(p)}\mathbf{Z}^{(q)}}(t, \tau)$ are also given elsewhere [19].

EVALUATION OF FAILURE PROBABILITY CUMULATIVE OVER A TIME INTERVAL

The most important quantity that has to be evaluated in a reliability analysis of a structure is the probability of failure over a given interval of time. In the present study, the probability of failure is identified as the probability of exceeding a given (deterministic and time-invariant) threshold by a scalar response quantity $Y(t)$ (single barrier problem).

It is known [1] that an upper bound of the probability of failure over the time interval [0, t], $P_f(t)$, is obtained by integrating in time the mean out-crossing rate $\nu(\xi, t)$ of level ξ by the subject response quantity $Y(t)$, i.e.,

$$P_f(t) = P[\mathrm{Max}(Y(\tau)) \geq \xi] = \sum_{n=1}^{\infty} P[N(t) = n] \leq \sum_{n=1}^{\infty} nP[N(t) = n] = E[N(t)] = \int_0^t \nu(\xi, \tau)d\tau \tag{27}$$

where $N(t)$ denotes the number of out-crossing events in the time interval [0, t]. Moreover, it is common to express the probability of failure $P_f(t)$ as [3,17]

$$P_f(t) = 1 - P[Y(t=0) < \xi] \cdot \exp\left\{-\int_0^t h(\tau)d\tau\right\} \tag{28}$$

where $P[Y(t=0) < \xi]$ denotes the probability that, at time t = 0, the subject response quantity $Y(t)$ is below the failure threshold ξ, and $h(t)$ is the so-called hazard function, i.e., it is the mean out-crossing rate conditioned on zero out-crossing prior to time t. In this paper, at rest initial conditions are assumed (i.e., $Y(t=0) = \dot{Y}(t=0) = 0$) resulting in $P[Y(t=0) < \xi] = 1$). To date, no exact closed-form solution is available for the hazard function even for the simplest case of structural model (linear elastic SDOF oscillator). Nevertheless, many approximations have been developed and are described in the literature [13,16,17].

The most well-known and simplest approximation is the Poisson hazard function [15], obtained by assuming that the out-crossing events follow the memoryless Poisson random occurrence model (i.e., out-crossing events are statistically independent). This simplifying assumption leads to

$$h(t) \approx \nu(\xi, t) \tag{29}$$

For low thresholds and/or narrow-band processes, the Poisson hazard function tends to give a very conservative estimate of the probability of failure, while for high barrier/threshold levels and broad-band processes, it is asymptotically correct.

APPLICATION EXAMPLES

Earthquake models

In this study, the stochastic earthquake ground motion model presented earlier has been calibrated to three actual ground motion records: the S00E (N-S) component of the Imperial Valley earthquake of May 18, 1940, recorded at the El Centro station; the N00W (N-S) component of the San Fernando earthquake of February 9, 1971, recorded at the Orion Blvd. station; and the N90W (W-E) component of the Loma Prieta earthquake of October 17, 1989, recorded at the Capitola site. The three calibrated stochastic earthquake models will be referred hereafter as the El Centro, the Orion Blvd., and the Capitola earthquake, respectively. The parameters for each of these stochastic ground motion models have been estimated by adaptively least-square fitting the analytical evolutionary power spectral density (EPSD) function of the model to the EPSD estimated from the target actual earthquake record using the short-time Thomson's multiple-window spectrum estimation method [12]. The model parameter values for the El Centro and the Orion Blvd. earthquakes are given in the same reference, while the model parameters for the Capitola earthquake are provided in Table 1. In each case, the very good agreement between the calibrated stochastic ground motion model and the target deterministic record (in terms of time-frequency distribution of the ground motion energy, mean-square envelope function, global power spectral density function, and various commonly used ground motion parameters) has been described elsewhere [12,20]. An illustration of this agreement is given in Fig. 1 which compares the estimated and the model EPSD for the El Centro earthquake in parts (a) and (b), respectively, and shows the actual and a simulated earthquake ground motion in part (c). For each of the three target earthquake records, a set of 10,000 ground motion realizations was generated using the corresponding stochastic ground motion model and a very efficient simulation technique based on the physical interpretation of the stationary Gaussian subprocesses $S_k(t)$ mentioned earlier. The simulated ground motions are baseline-corrected in the frequency domain by using a simple rectangular high-pass filter with a cut-off frequency of 0.10 Hz and by applying a least-square straight line fitting to both the integrated ground velocity and ground displacement motions. As shown in Fig. 1(d), excellent agreement is obtained between the analytical mean square ground acceleration function given in Equation (5) and its counterpart estimated from the simulations. Finally, these artificial earthquake ground motions are used as input in the Monte Carlo simulation of the structural response, in order to validate the analytical closed-form solution developed for the mean out-crossing rate of various response quantities as well as the approximate solution and upper bound for the time-variant probability of failure.

Table 1. Estimated parameters of ground acceleration model for Capitola earthquake

k # 	α_k $[cm/s^{2+\beta_k}]$	β_k [-]	γ_k [1/s]	θ_k [s]	ν_k [rad/s]	η_k [rad/s]
1	0.00051984	10	1.7348	2.7148	0.2429	9.3424
2	44.469	3	0.7484	10.677	0.8074	7.7278
3	3.4284	3	0.6430	18.132	1.3313	5.6751
4	0.00025921	8	1.1304	-0.2135	2.5585	15.461
5	0.10602	4	0.6009	6.0153	2.0788	18.367
6	3.0913	3	0.5300	0.255	2.1052	29.731
7	21.748	3	1.0678	11.672	0.2289	21.77
8	4.8589	3	0.5471	-0.4698	2.2722	39.641
9	4.6704	4	0.8466	-0.1082	1.5603	46.114
10	15.485	4	1.0249	0.7955	1.9604	51.334
11	0.4639	4	0.6769	-0.3986	8.2081	59.656

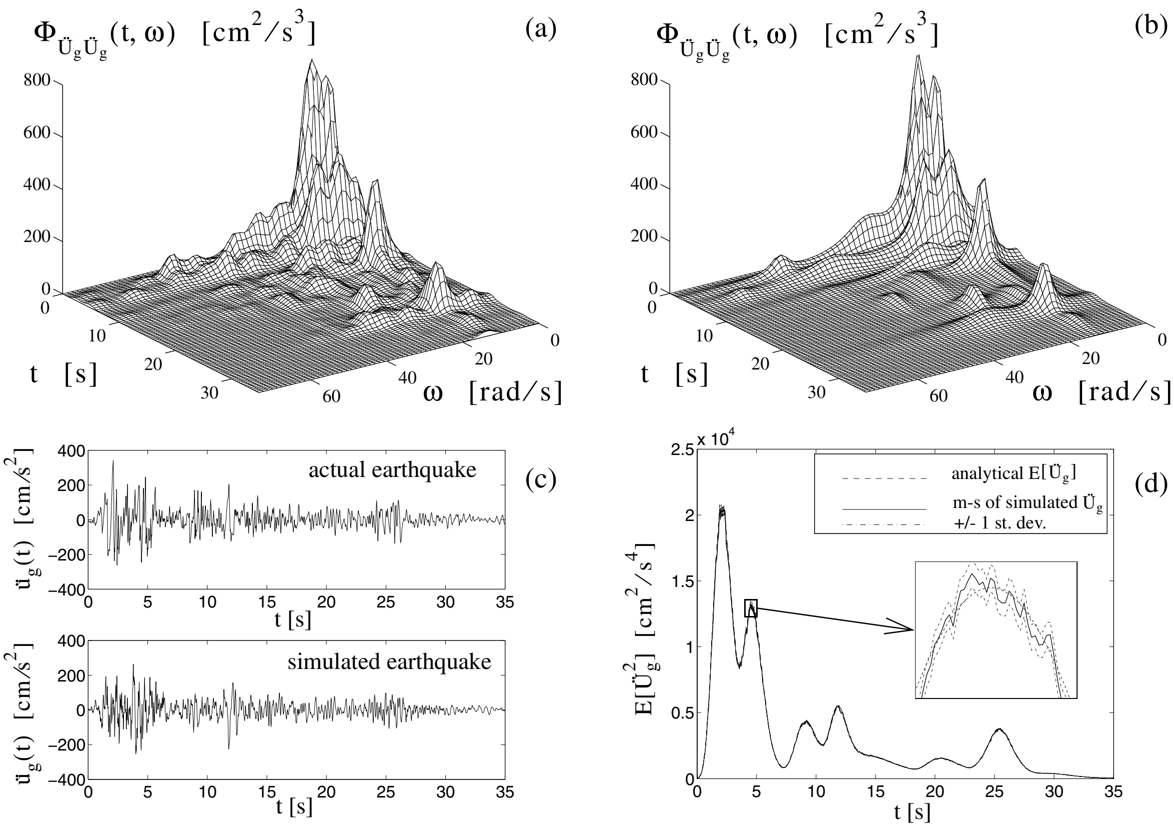


Fig. 1. El Centro earthquake: (a) estimated EPSD, (b) model EPSD, (c) comparison between actual and simulated ground motion acceleration, (d) analytical and simulated mean square ground acceleration functions

SDOF linear oscillator

The first application example consists of a linear elastic SDOF system subjected to base excitation defined as the stochastic earthquake ground motion models defined above. A SDOF system is a particular case of classically damped MDOF systems; its equation of motion can be readily formulated and solved in state-space format. The undamped natural period T_n (or the undamped natural circular frequency $\omega_n = 2\pi/T_n$) and the damping ratio ζ completely define the structure in terms of its kinematic response quantities. Several natural periods, damping ratios and normalized thresholds ξ for displacement response have been considered. In the sequel, the symbol $\hat{r}$ indicates the Monte Carlo Simulation (MCS) estimate (ensemble average) of a response statistics r, while the symbol $\bar{r}$ denotes the average over an interval of time (temporal average) of a response statistics r.

Figs. 2 and 3 display the results obtained for a SDOF system with parameters $T_n = 2$ s and $\zeta = 0.10$ subjected to the El Centro earthquake process. Fig. 2 (a) compares the analytical mean square (relative) displacement response, $E[U^2(t)]$, and its MCS estimate given with ± 2 standard deviations interval. A close-up of the second peak of the mean square displacement response is given in the inset. Fig. 2(b) compares the analytical displacement mean upcrossing rate for three different normalized threshold levels ($\xi = 2\sigma_{max}, 2.5\sigma_{max}, 3\sigma_{max}$ where $\sigma_{max} = \max(\sigma_U(t))$) and their MCS estimates obtained through ensemble averaging of upcrossing over a single time step $\Delta t = 0.02$ s followed by temporal averaging of this ensemble average over ten time steps. The results shown in Fig. 3 relate to the normalized displacement threshold level $\xi = 2\sigma_{max}$. Fig. 3(a) plots the MCS estimate of the sums $\sum_{n=1}^{n_{max}} nP[N = n]$ ($n_{max} = 1, 2, 3, 4$ as $n_{max} = 4$ is the maximum number of upcrossing events in a single realization over the ensemble of 10,000 response realizations), see Equation (27). These estimated sum quantities are important since their limit for $n_{max} \to \infty$ is the estimate of the mean number of upcrossing events in the time interval [0, t], which corresponds to the MCS estimate of the analytical upper bound of the time-variant probability of failure. Furthermore, Fig. 3(a) shows the relative contribution of the terms $nP[N = n]$ to the upper bound of $P_f(t)$ in Equation (27). Fig. 3(b) compares the MCS estimate of the mean number of upcrossings, $\hat{E}[N]$, (given with the ± 1 standard deviation interval) and its analytical counterpart E[N]. Fig. 3(b) compares also the analytical Poisson approximation of the time-variant probability of failure, $P_{f,\,Poisson}$, and the MCS estimate of the time-variant probability of failure (given with the ± 1 standard deviation interval). In this case, the Poisson approximation given in Equations (28) and (29) provides a significantly better estimate of the time-variant probability of failure than the analytical upper bound given in Equation (27). The comparisons made in Fig. 3 are repeated in Figs. 4 and 5 for the normalized threshold $\xi = 3\sigma_{max}$ and a damping ratio $\zeta = 0.10$. SDOF systems with natural periods $T_n = 1$ s and $T_n = 2$ s, subjected to the El Centro earthquake process are considered in Fig. 4, while Fig. 5 is concerned with the SDOF system with parameters ($T_n = 2$ s and $\zeta = 0.10$) subjected to the Orion Blvd. earthquake process in part (a) and the Capitola earthquake process in part (b).

All the analytical solutions for the expected number of upcrossings, E[N], are in good agreement with the corresponding MCS estimates, $\hat{E}[N]$, thus validating the closed-form solution of the mean upcrossing rate developed for SDOF systems. It is worth noting that for the examples considered with the normalized threshold $\xi = 3\sigma_{max}$ (Figs. 4 and 5), the Poisson approximation does not provide a significantly improved probability of failure compared to the analytical upper bound E[N]. These two approximations converge asymptotically to the exact value of the probability of failure P_f, but the convergence of $P_{f,Poisson}$ to E[N] appears faster than the convergence to P_f.

In the range of natural periods and damping ratios considered in this study, it appears that the classical Poisson approximation is quite conservative in its evaluation of the time-variant probability of failure for normalized thresholds ξ ranging between $2\sigma_{max}$ and $3\sigma_{max}$. More extensive studies are needed to assess the behavior of SDOF systems with a wider range of natural periods and further research is in progress to develop better analytical and numerical estimates of the time-variant probability of failure.

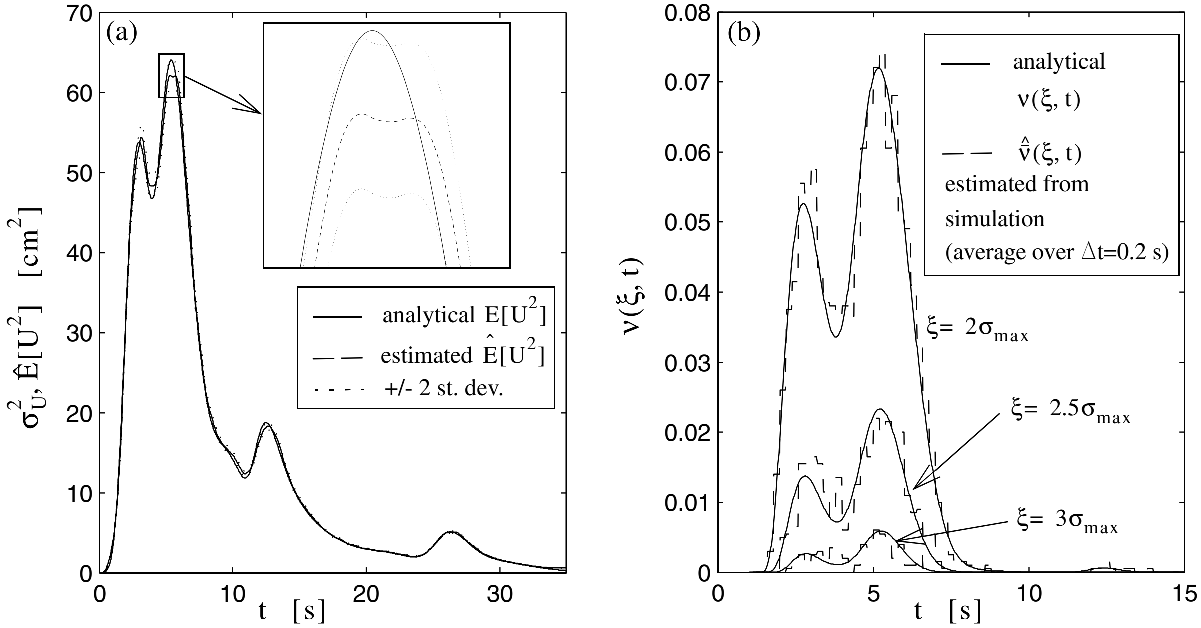


Fig. 2. Comparison of analytical response statistics and their MCS estimates for a SDOF system with T_n=2 s and ζ=0.10 subjected to the El Centro earthquake process: (a) mean square relative displacement response, (b) mean upcrossing rates

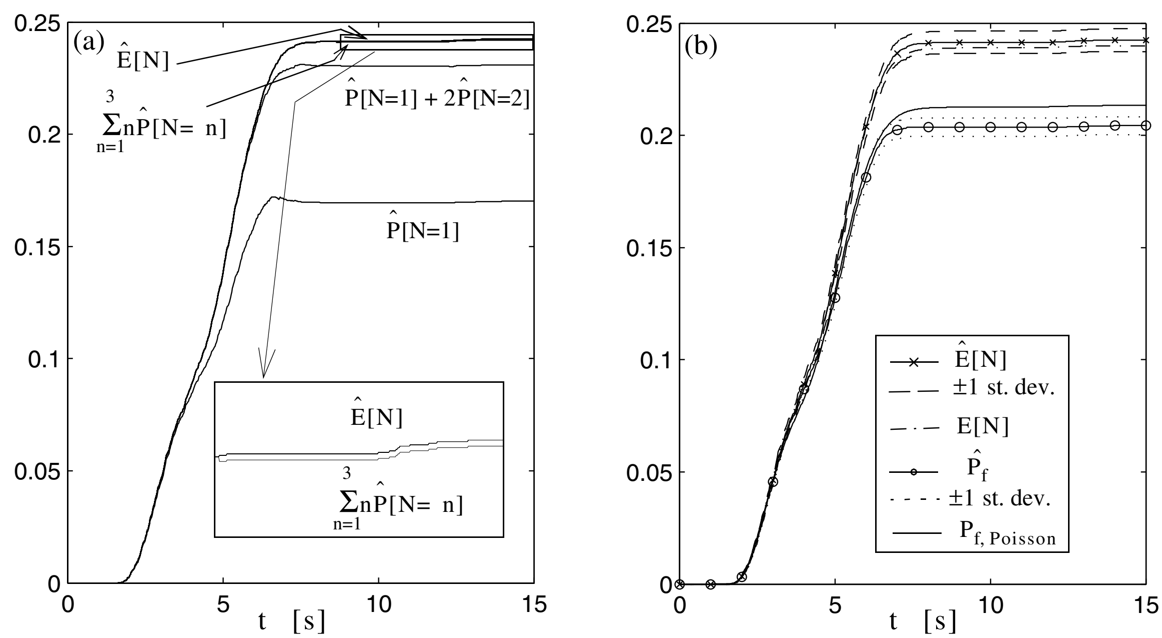


Fig. 3. SDOF system with T_n=2 s and ζ=0.10 subjected to the El Centro earthquake process: (a) MCS estimate of mean number of upcrossings, (b) comparison between MCS estimates of and analytical evaluations of E[N] and P_f, ξ=2σ_{max}

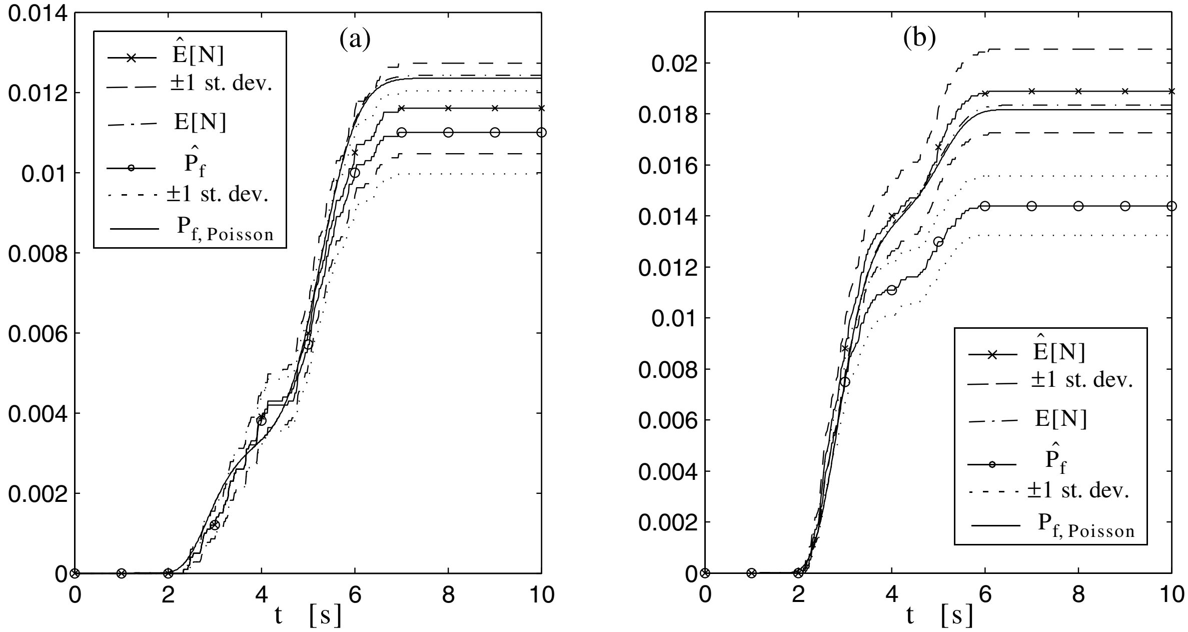


Fig. 4. Comparison between MCS estimates of and analytical evaluations of E[N] and P_f, corresponding to ξ=3σ_{max} for SDOF system subjected to the El Centro earthquake process: (a) T_n=2 s and ζ=0.10, (b) T_n=1 s and ζ=0.10

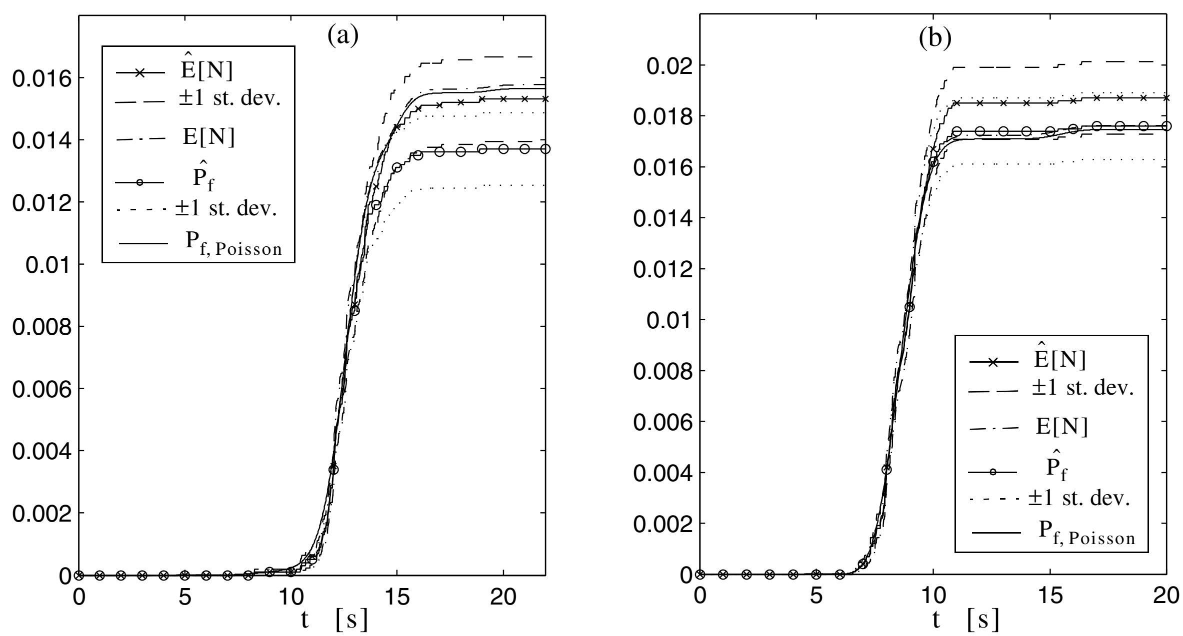
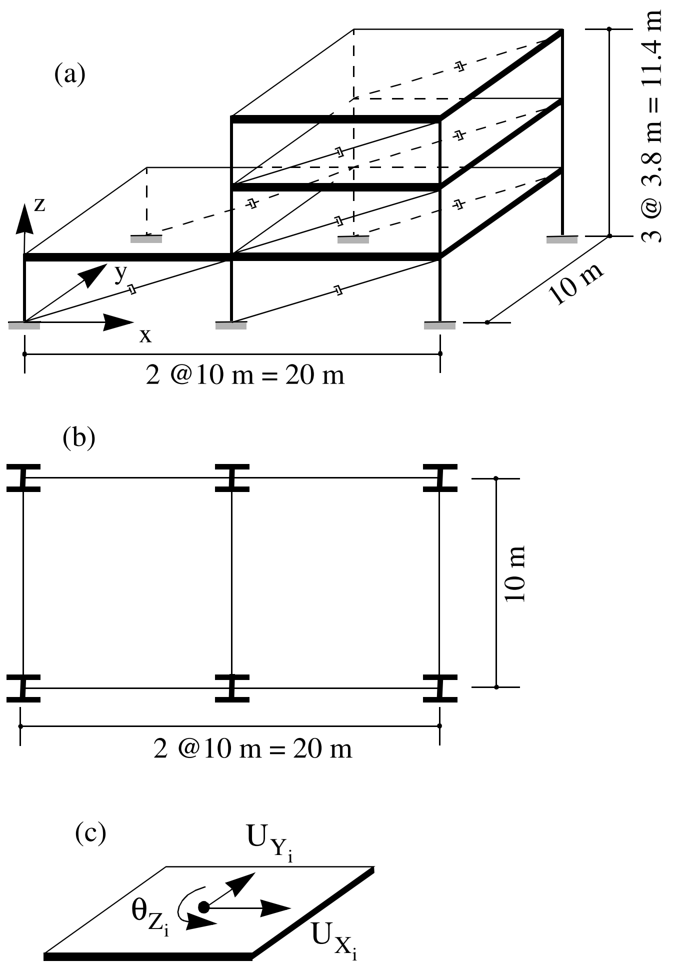


Fig. 5. Comparison between MCS estimates of and analytical evaluations of E[N] and P_f, corresponding to ξ=3σ_{max} for SDOF with T_n=2 s and ζ=0.10 subjected to: (a) the Orion Blvd. earthquake process, (b) the Capitola earthquake process

Three-dimensional unsymmetrical building (linear MDOF)

The idealized three-dimensional unsymmetrical building shown in Fig. 6 is used to illustrate the application of complex modal analysis and the derived closed-form solutions for the threshold-crossing and time-variant reliability problems applied to linear MDOF systems subjected to the fully nonstationary earthquake ground motion model defined above. This application example is identical to the one used in previous work by the authors [19]. This building structure consists of three floor diaphragms, assumed infinitely rigid in their own plane, supported by wide flange steel columns of size $\mathrm{W}\ 14 \times 145$. Each floor diaphragm is assumed to be made of reinforced concrete with a weight density of 3.6 kN/m^3 and a depth of 18 cm. The columns are assumed inextensible. The modulus of elasticity of steel is taken as 200 GPa. The motion of each floor diaphragm is completely defined by three DOF's defined at the floor center of mass (CM), namely the relative displacements with respect to the ground in the x-direction, $U_{X_i}(t)$, in the y-direction, $U_{Y_i}(t)$, and the rotation about the vertical z-axis, $\theta_{Z_i}(t)$. The earthquake ground motion excitation is assumed to act at 45 degrees with respect to the x-axis. Both classically and non-classically damped structural models are considered. For the case of classical damping, each modal damping ratio is taken as 2 percent. To physically realize the non-classical damping case, diagonal viscous damping elements (fluid viscous braces) are added as shown in Fig. 6(a). The damping coefficient of each viscous damping element is taken as 0.1 $\mathrm{kN \cdot s/mm}$. The undamped natural circular frequencies of this building are given in Fig. 6(d).



(d)

Mode #	ω_i [rad/s]	Description
1	15.97	x-translation
2	24.12	y-translation + torsion
3	36.56	x-translation
4	41.21	y-translation + torsion
5	56.74	y-translation + torsion
6	56.98	x-translation
7	73.88	y-translation + torsion
8	95.15	y-translation + torsion
9	127.69	y-translation + torsion

Fig. 6. MDOF unsymmetrical building: (a) three-dimensional view, (b) plan view, (c) i-th floor with DOF's noted, (d) undamped modes of vibration

Figs. 7 and 8 show some of the results obtained from the analytical closed-form solutions presented above and through Monte Carlo simulation for the building subjected to the El Centro earthquake excitation process. Fig. 7 corresponds to the classically damped case (i.e., building without viscous dampers) and presents the results for the roof translational degree of freedom in the x direction, $U_{X_3}(t)$. The normalized displacement threshold level is taken as $\xi = 3\sigma_{max}$ where $\sigma_{max} = \max(\sigma_{U_{X_3}}(t))$. Fig. 7(a) shows the MCS results for the expected number of upcrossings and the probabilities of N = 1, 2, ..., 8 upcrossing events during the earthquake. Fig. 7(b) compares (1) the analytical prediction E[N] of the expected number of upcrossings during the earthquake and the corresponding estimate $\hat{E}[N]$ (given with the ± 1 standard deviation interval) obtained via Monte Carlo simulation, and (2) the analytical Poisson approximation, $P_{f,Poisson}$, and the Monte Carlo simulation estimate $\hat{P}_f$ (given with the ± 1 standard deviation) of the probability of failure during the earthquake, P_f. Fig. 8 shows the same results as in Fig. 7, but for the non-classically damped building (i.e., building with viscous dampers) and for the rotational degree of freedom of the roof diaphragm $\theta_{Z_3}(t)$.

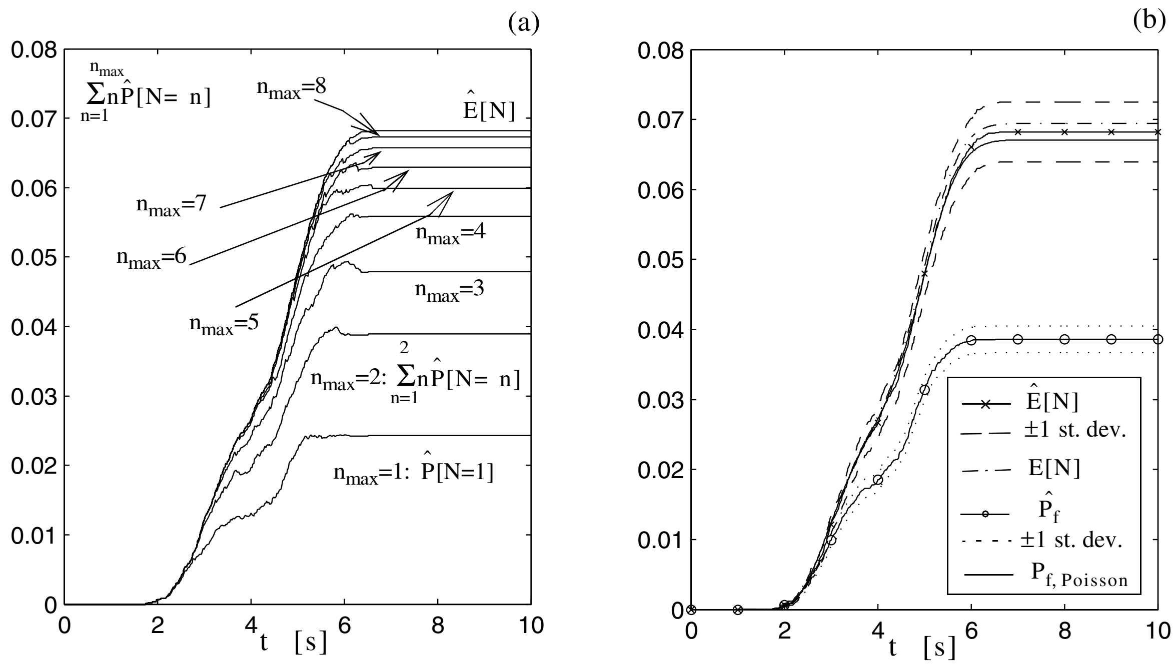


Fig. 7. Unsymmetric building with classical damping subjected to the El Centro earthquake process, (a) MCS estimates of upcrossing probabilities, (b) comparison between MCS estimates and analytical evaluation of E[N] and P_f, corresponding to $\xi=3\sigma_{max}$ for DOF $U_{X_3}(t)$

The results for this MDOF application example exhibit features similar to the ones observed for the SDOF linear oscillator in the previous section. In particular, the analytical mean upcrossing rate (not shown here) and the analytical expected value of upcrossings during the earthquake are in good agreement with their respective MCS estimates, thus validating the derived closed-form solutions for linear MDOF systems. The Poisson approximation of the probability of failure is very close to the analytical upper bound and overestimates significantly the probability of failure estimated via Monte Carlo simulation, even for the relatively large normalized displacement threshold $\xi = 3\sigma_{max}$ considered here. It is likely that significant improvement in the evaluation of the time-variant probability of failure can be achieved by taking into account the “memory properties” of the response process, by using for example the Vanmarcke’s approximation [16,17] of the probability of failure.

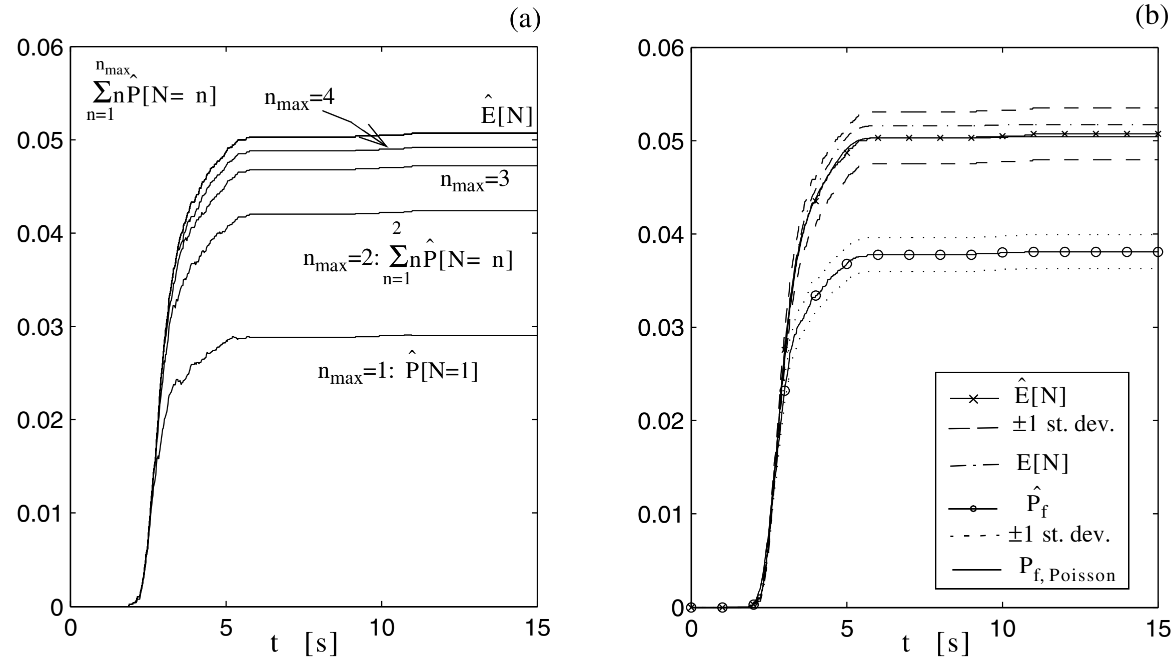


Fig. 8. Unsymmetric building with non-classical damping subjected to the El Centro earthquake process, (a) MCS estimates of upcrossing probabilities and (b) comparison between MCS estimates and analytical evaluation of E[N] and P_f, corresponding to ξ=3σ_{max} for DOF θ_{Z_3}

CONCLUSIONS

New explicit closed-form solutions are developed for the mean upcrossing rates of nonstationary response quantities of linear elastic, both classically and non-classically damped, multi-degree-of-freedom (MDOF) systems subjected to a fully nonstationary earthquake ground motion process. These closed-form solutions are based on state-space complex modal analysis. Application examples are presented in which single-degree-of-freedom (SDOF) systems and a three-dimensional idealized unsymmetrical building structure with and without viscous bracing elements are subjected to the nonstationary earthquake ground motion model. The latter is calibrated against three historic, well-known earthquake ground motion records. All the analytical results are validated via Monte Carlo simulation.

The closed-form solutions obtained for the analytical mean upcrossing rates are then used to evaluate an analytical upper bound (the expected number of upcrossings, E[N]) and the Poisson approximation, $P_{f,Poisson}$, of the time-variant probability of failure, P_f, for a given scalar response threshold, ξ. Again, the analytical results are compared with their counterparts obtained via Monte Carlo simulation, showing a very good agreement for the expected number of upcrossings. It is found that (1) for low normalized response thresholds (i.e., $\xi \le 2\sigma_{max}$ where σ_{max} denotes the maximum value of the response standard deviation during the earthquake), the Poisson approximation provides a significant improvement over the upper bound estimate of the time-variant probability of failure, but remains short of being accurate; and (2) for normalized response thresholds as high as $\xi = 3\sigma_{max}$, the Poisson approximation and the upper bound estimate are very close, but may still overestimate significantly the time-variant probability of failure depending on the relative frequency properties of the system and earthquake excitation process. Therefore, further studies are underway to find better analytical/numerical approximations of the time-variant probability of failure.

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