



## **NEW CONCEPTS IN ASSESSING THE EFFECTS OF THE SEISMIC SPATIAL VARIABILITY ON EMBEDDED STRUCTURE**

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### **SUMMARY**

The effects of the spatial variability of seismic motion on extended structures have been extensively studied using several models of coherency established either by theoretical or empirical methods. Although these effects have shed light on the need to take into account the spatial variability of earthquake motion in the seismic design, they remain to be refined for extended structures below the ground surface. This paper aims to present new concepts in estimating the effects of the seismic spatial variability on embedded structures using the Complete Stochastic Deamplification Approach which is basically a theoretical approach dedicated to identify the spatial variability of seismic motion at any point within the soil mass. The results obtained show that the effects of the spatial variability of earthquake motion are higher for structures resting on stratified soil rather than on homogeneous soil. Also, we found that when we use a coherency model developed or established at the free surface to perform seismic analysis of embedded structure we underestimate spectral acceleration by 30%. This result seems to be in accordance with recent observations made on the collapse of structures during seismic events such as the 1994 Northridge earthquake.

### **INTRODUCTION**

The spatial variability of seismic motion is regarded as one of the seismic features that could lead to dramatic effects on extended and/or embedded structures. Several models have been developed during past decades to describe this spatial variability based on either statistical approach Harichandran [1] or analytical approach Der Kiureghian [2]. Hence, it is now possible to understand the behavior of extended structure subjected to spatial variability of ground motion using one of the above approaches. However, most of these approaches have been developed at the free surface and thus could not be used and applied to determine the seismic response of embedded structure. To overcome this restriction a recent approach named Complete Stochastic Deamplification Approach (CSDA) has been recently developed Zendagui [3]. This approach aims to describe the spatial variability at the free surface but also at depth. The main question addressed in the present work is: what if a coherency model developed at the free surface is used to determine the seismic response of an embedded structure? This question is very important since most of the seismic design of embedded use coherency model developed at the free surface.

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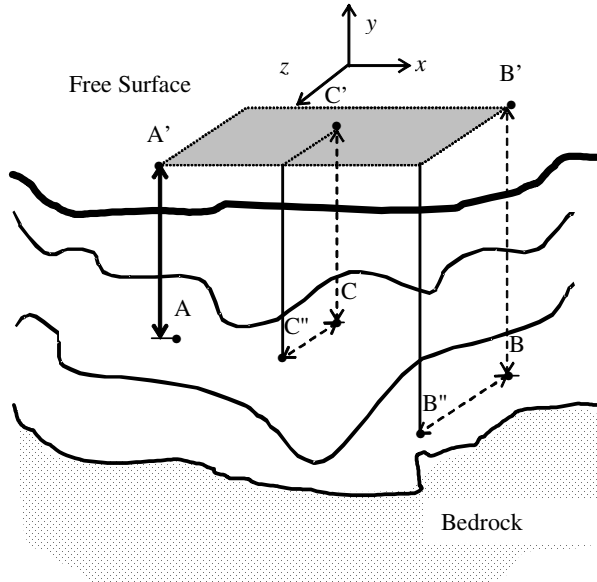
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## MODEL OF COHERENCY BASED ON CSDA

### Theoretical background

One of the major concerns of the engineer is to perform seismic analysis on embedded and extended structures. Most of the current models can describe the spatial variability at the free surface; hence they can for instance establish model of coherency between points A' and C' which are situated at the free surface (Figure 1). At depth, for example between points A and C no model of coherency are proposed. What if embedded buried lifeline or underground structure has A and C as an attached points? We must hence have a coherency function between those points which could be derived through CSDA. Let assume that the seismic motion, at a point located through it coordinates  $(x, y, z)$ , is  $(u, v, w)$ . Although the CSDA is able to compute the coherency function between any points, we will focus on points having the same  $(y, z)$  coordinates.



**Figure 1 Soil configuration system**

The matrix of cross spectral density functions of the motion at two points having the same  $(y, z)$  coordinates but separated by  $x$  horizontal distance equal to  $\lambda$  could be assessed using the CSDA Zendagui [3]

$$\mathbf{S}(\lambda, y, \omega) = \begin{bmatrix} S_{uu}(\lambda, y, \omega) & 0 & 0 \\ 0 & S_{vv}(\lambda, y, \omega) & 0 \\ 0 & 0 & S_{ww}(\lambda, y, \omega) \end{bmatrix} \quad (1)$$

where

$$S_{ii}(\lambda, y, \omega) = \int_{-\infty}^{\infty} (T_{ii}(\lambda, y, \omega))^2 S_{ii}(k, 0, \omega) dk \quad i = (u, v, w) \quad (2)$$

$S_{ii}(k, 0, \omega)$  is the Fourier transform of the cross spectral density function  $S_{ii}(\lambda, 0, \omega)$  recorded or computed at the free surface and  $T_{ii}(\lambda, y, \omega)$  is the transfer function of the motion between points located at the free surface and at depth. Hence based on these functions, it is now possible to compute  $S_{ii}(\lambda, y, \omega)$  which enable the evaluation of the matrix of coherency functions. Hence:

$$\rho(\lambda, y, \omega) = \begin{bmatrix} \rho_{uu}(\lambda, y, \omega) & 0 & 0 \\ 0 & \rho_{vv}(\lambda, y, \omega) & 0 \\ 0 & 0 & \rho_{ww}(\lambda, y, \omega) \end{bmatrix} \quad (3)$$

where

$\rho_{uu}(\lambda, y, \omega)$  is the coherency function with respect to  $x$  coordinate,

$\rho_{vv}(\lambda, y, \omega)$  is the coherency function with respect to  $y$  coordinate,

$\rho_{ww}(\lambda, y, \omega)$  is the coherency function with respect to  $z$  coordinate.

It is worth noting that current models make possible the computation of  $\rho_{jj}(\lambda, 0, \omega)$ ,  $j = (u, w)$  only.

### Coherency functions for the case of stratified soil

#### General consideration

Once the general approach CSDA established one could derive easily the statistical properties of the motion for the special case P-SV-SH waves propagating through a halfspace as well as stratified soil resting on a halfspace. The SH waves generate motion with respect to  $z$  coordinate, thus if we consider that the seismic motion is a set of SH waves then  $\rho_{uu}(\lambda, y, \omega) = \rho_{vv}(\lambda, y, \omega) = 0$ . In turn, the P-SV waves generate motion with respect to  $(x, y)$  coordinates, thus if we consider that the seismic motion is a set of P-SV waves then  $\rho_{ww}(\lambda, y, \omega) = 0$ . So  $\rho_{uu}(\lambda, y, \omega)$  and  $\rho_{vv}(\lambda, y, \omega)$  are evaluated using P-SV waves whereas  $\rho_{ww}(\lambda, y, \omega)$  is computed using SH waves. Below are the coherency functions obtained at depth using the CSDA for the separated case of halfspace and a stratified soil Zendagui[3,5], Berrah [4].

#### Case of a halfspace

Assume that the halfspace is defined by its P-wave velocity  $C_p$  and S-wave velocity  $C_s$ . One supposes that a set of SH waves, with direction defined by the angle of incidence  $\theta \equiv \theta_{SH}$  with respect to the vertical axis, propagate through the halfspace then the coherency function is obtained by:

$$\rho_{ww}(\lambda, y, \omega) = \frac{S_{ww}(\lambda, y, \omega)}{S_{ww}(0, y, \omega)} \quad (4)$$

where

$$S_{ww}(\lambda, y, \omega) = 4S(\omega) \int_{-\pi/2}^{\pi/2} g^2(\theta) e^{-i\omega s_{y\theta} \lambda} \{ \cos(\omega s_{y\theta} y) \}^2 d\theta \quad (5)$$

$s_{x\theta} = \sin \theta / C_s$ ,  $s_{y\theta} = \cos \theta / C_s$ ,  $S(\omega)$  is the PSD of each wave and  $g(\theta)$  is its amplitude.

On assume that a set of P waves, with direction defined by the angle of incidence  $\theta \equiv \theta_p$  with respect to the vertical axis, propagate then the coherency functions is obtained by:

$$\rho_{uu}(\lambda, y, \omega) = \frac{S_{uu}(\lambda, y, \omega)}{S_{uu}(0, y, \omega)} \quad (6)$$

$$\rho_{vv}(\lambda, y, \omega) = \frac{S_{vv}(\lambda, y, \omega)}{S_{vv}(0, y, \omega)} \quad (7)$$

Where

$$S_{uu}(\lambda, y, \omega) \equiv S_{uu}^P(\lambda, y, \omega) = C_P^2 S(\omega) \int_{-\pi/2}^{\pi/2} g^2(\theta) e^{-i\omega p_{x\theta} \lambda} \{ p_{x\theta}^2 [1 + A_\theta^2 + 2A_\theta \cos(2\omega p_{y\theta} y)] + 2B_\theta \frac{p_{x\theta} s_{y\theta}}{\kappa} [\cos(\omega(s_{y\theta} + p_{y\theta})y) + A_\theta \cos(\omega(s_{y\theta} - p_{y\theta})y)] + B_\theta^2 s_{y\theta}^2 / \kappa^2 \} d\theta \quad (8)$$

$$S_{vv}(\lambda, y, \omega) \equiv S_{vv}^P(\lambda, y, \omega) = C_P^2 S(\omega) \int_{-\pi/2}^{\pi/2} g^2(\theta) e^{-i\omega p_{x\theta} \lambda} \{ p_{y\theta}^2 [1 + A_\theta^2 - 2A_\theta \cos(2\omega p_{y\theta} y)] + 2B_\theta \frac{p_{x\theta} p_{y\theta}}{\kappa} [\cos(\omega(s_{y\theta} + p_{y\theta})y) - A_\theta \cos(\omega(s_{y\theta} - p_{y\theta})y)] + B_\theta^2 p_{x\theta}^2 / \kappa^2 \} d\theta \quad (9)$$

The superscript <sup>P</sup> is introduced to mention that the  $S_{uu}^P(\lambda, y, \omega)$  and  $S_{vv}^P(\lambda, y, \omega)$  is introduced to mention that only P-waves are incident.  $s_{x\theta} = \sin \theta / C_S$ ,  $s_{y\theta} = \cos \theta / C_S$ ,  $p_{x\theta} = \sin \theta / C_P$ ,  $p_{y\theta} = \cos \theta / C_P$ ,  $\kappa = C_P / C_S$ ,  $S(\omega)$  is the PSD of each wave and  $g(\theta)$  is its amplitude.  $A_\theta$  and  $B_\theta$  are the amplitude of the reflected P and SV waves respectively.

On assume that a set of SV waves, with direction defined by the angle of incidence  $\theta \equiv \theta_{SV}$  with respect to the vertical axis, propagate then the coherency functions is obtained by:

$$\rho_{uu}(\lambda, y, \omega) = \frac{S_{uu}(\lambda, y, \omega)}{S_{uu}(0, y, \omega)} \quad (10)$$

$$\rho_{vv}(\lambda, y, \omega) = \frac{S_{vv}(\lambda, y, \omega)}{S_{vv}(0, y, \omega)} \quad (11)$$

where

$$S_{uu}(\lambda, y, \omega) \equiv S_{uu}^{SV}(\lambda, y, \omega) = C_S^2 S(\omega) \int_{-\pi/2}^{\pi/2} g^2(\theta) e^{-i\omega s_{x\theta} \lambda} \{ s_{y\theta}^2 [1 + A_\theta^2 - 2A_\theta \cos(2\omega s_{y\theta} y)] - 2B_\theta s_{x\theta} s_{y\theta} \kappa [\cos(\omega(s_{y\theta} + p_{y\theta})y) - A_\theta \cos(\omega(p_{y\theta} - s_{y\theta})y)] + B_\theta^2 p_{x\theta}^2 \kappa^2 \} d\theta \quad (12)$$

$$S_{vv}(\lambda, y, \omega) \equiv S_{vv}^{SV}(\lambda, y, \omega) = C_s^2 S(\omega) \int_{-\pi/2}^{\pi/2} g^2(\theta_s) e^{-i\omega s_{x\theta} \lambda} \left\{ s_{x\theta}^2 [1 + A_\theta^2 + 2A_\theta \cos(2\omega s_{y\theta} y)] - \right. \\ \left. - 2B_\theta s_{x\theta} p_{y\theta} \kappa [\cos(\omega(s_{y\theta} + p_{y\theta})y) + A_\theta \cos(\omega(p_{y\theta} - s_{y\theta})y)] + B_\theta^2 p_{y\theta}^2 \kappa^2 \right\} d\theta \quad (13)$$

The superscript  $P$  is introduced to mention that the  $S_{uu}^{SV}(\lambda, y, \omega)$  and  $S_{vv}^{SV}(\lambda, y, \omega)$  is introduced to mention that only SV-waves are incident  $s_{x\theta} = \sin \theta / C_s$ ,  $s_{y\theta} = \cos \theta / C_s$ ,  $p_{x\theta} = \sin \theta / C_p$ ,  $p_{y\theta} = \cos \theta / C_p$ .  $S(\omega)$  is the PSD of each wave and  $g(\theta)$  is its amplitude.  $A_\theta$  and  $B_\theta$  are the amplitude of the reflected SV and P waves respectively. Hence knowing the wave content, one could derive easily the coherency functions at any depth and particularly at the free surface by setting  $y = 0$ .

#### Case of a stratified soil resting on a halfspace

The main assumption in this section is about the geometrical properties of the stratified which is considered as infinite laterally. This stratified soil is supposed to have  $(N-1)$  layers resting on a substratum or a halfspace. Each layer  $(j)$  is completely defined by its P-waves velocity  $C_p^{(j)}$ , S-waves velocity  $C_s^{(j)}$  and thickness  $h^{(j)}$ . Let assume that a set of SH waves arrive at the top of the substratum which support the stratified soil. These waves will propagate through the stratified soil and arrive at the free surface. The coherency function is then simply redefined using the equation (4) but by considering

$$S_{ww}(\lambda, y, \omega) = (2A)^2 S(\omega) \int_{-\pi/2}^{\pi/2} g^2(\theta) e^{-i\omega s_{x\theta} \lambda} (m_{11})^2 d\theta \quad (14)$$

where  $m_{11}$  is the transfer function of the motion between the free surface and depth Zendagui [3], Berrah [4],  $\theta = \theta^{(1)}$  is the angle of incidence of the SH waves at the free surface and  $A$  is the amplitude of the waves at the free surface. When considering that a set of P waves arrive at the top of the substratum, many reflection and refractions occur at the edge of the layers and lead to the incidence of P-SV waves at the free surface. In order to use the CSDA we have to establish  $S_{ii}(k, 0, \omega)$  and  $T_{ii}(\lambda, y, \omega)$   $i = (u, v)$ . The latter could be derived straightforwardly using for example the Haskell-Thomson matrix whereas the former could be derived as follow Zendagui [5]:

$$S_{uu}(k, 0, \omega) = S(\omega) \frac{C_p^{(1)}}{\omega} g^2 \left( \text{Arc sin} \left( \frac{kC_p^{(1)}}{\omega} \right) \right) \left\{ E \left[ \text{Arc sin} \left( \frac{kC_p^{(1)}}{\omega} \right), \omega, 0 \right] \right\}^2 / \sqrt{1 - \left( \frac{kC_p^{(1)}}{\omega} \right)^2} \quad (15)$$

$$S_{vv}(k, 0, \omega) = S(\omega) \frac{C_p^{(1)}}{\omega} g^2 \left( \text{Arc sin} \left( \frac{kC_p^{(1)}}{\omega} \right) \right) \left\{ G \left[ \text{Arc sin} \left( \frac{kC_p^{(1)}}{\omega} \right), \omega, 0 \right] \right\}^2 / \sqrt{1 - \left( \frac{kC_p^{(1)}}{\omega} \right)^2} \quad (16)$$

where

$$E \left[ \text{Arc sin} \left( \frac{kC_p^{(1)}}{\omega} \right), \omega, 0 \right] = A_\theta [(1 + B_\theta) \sin \theta_{p\theta} + D_\theta \cos \theta_{s\theta}] + C_\theta [(-1 + D_\theta) \cos \theta_{s\theta} + B_\theta \sin \theta_{p\theta}] \quad (17)$$

$$G\left[\text{Arc sin}\left(\frac{kC_P^{(l)}}{\omega}\right), \omega, 0\right] = A_\theta[(1 - B_\theta)\cos\theta_{p\theta} + D_\theta \sin\theta_{s\theta}] + C_\theta[(1 + D_\theta)\sin\theta_{s\theta} - B_\theta \cos\theta_{p\theta}] \quad (18)$$

$A_\theta$ ,  $B_\theta$ ,  $C_\theta$  and  $D_\theta$  are the amplitude of the incident and reflected P SV at the free surface.

Hence using Eqs 15-18 and Eq 2 one could determine the coherency functions which are depicted in Eq. 10-11.

If we consider that a set of SV waves travel a substratum and reach a stratified soil, we can see, that at the free surface, this system of incidence lead also to P SV incident waves and thus  $S_{uu}(k, 0, \omega)$  and  $S_{vv}(k, 0, \omega)$  are in their shape identical to the one obtained for the above case. The main difference is in the establishment of the transfer functions which can be found elsewhere Achenbach [6].

#### *Multiple waves at the substratum*

The seismic motion is the combination of many waves, and thus an "accurate" representation of the earthquake motion is the combination of P-SV-SH waves. The methodology presented above could be extended for the case of the propagation of P-SV waves by simply superposing the cross spectral density function  $S_{ii}(\lambda, y, \omega)$   $i = (u, v)$  obtained for each case. Thus it is possible to obtain a new coherency functions:

$$\rho_{uu}(\lambda, y, \omega) = \frac{S_{uu}^P(\lambda, y, \omega) + S_{uu}^{SV}(\lambda, y, \omega)}{S_{uu}^P(\lambda, 0, \omega) + S_{uu}^{SV}(\lambda, 0, \omega)} \quad (19)$$

$$\rho_{vv}(\lambda, y, \omega) = \frac{S_{vv}^P(\lambda, y, \omega) + S_{vv}^{SV}(\lambda, y, \omega)}{S_{vv}^P(\lambda, 0, \omega) + S_{vv}^{SV}(\lambda, 0, \omega)} \quad (20)$$

#### *Conclusion*

This section was dedicated to the establishment of the coherency functions for both horizontal and vertical component using the CSDA and based on the assumptions that the seismic motion is a set of P-SH-SV waves propagating through a halfspace or a stratified soil. The next section is devoted to the derivation of response spectra.

### **DERIVATION OF RESPONSE SEPCTRA USING THE CSDA**

#### **Background**

Engineers use generally response spectra as a general tool to understand the behavior of structure under seismic motion which is characterized by the time history. However, coherency function remains the main tools to describe the spatial variability of seismic motion. Thus if we intend to use response spectra, we have to simulate time history from a coherency function. Basically this is the idea that motivates the development of Simqke-II Vanmarcke [7]. To use this software we have to introduce the Power spectral density function PSD of the motion at the desired point and then use one of the coherency model that are proposed This tool has been modified to take into account the coherency functions developed through CSDA. Let first describe the soil properties used in the application.

#### **Soil configuration**

##### *Halfspace*

Let assume that the soil considered as halfspace is defined by its shear wave velocity  $C_s = 100m/s$  (Figure 2a). The control point are defined as A, C, A' and C' (Figure 2b).



$T$  is the natural period of vibration of the SDOF

It is worth noting that currently engineer use  $RS_A(T)$  because there is no model of coherency at depth. So if we examine carefully those response spectra it is straightforward to see that there is an error in estimating the response of a structure. To view this error we introduce the concept of characterized inaccuracy  $CI(T)\%$  which is a function of  $T$

$$CI(T)\% = \frac{RS_E(T) - RS_A(T)}{RS_E(T)} \cdot 100 \quad (21)$$

These functions are computed in the next section.

## EFFECTS OF THE SPATIAL VARIABILITY ON EMBEDDED STRUCTURES

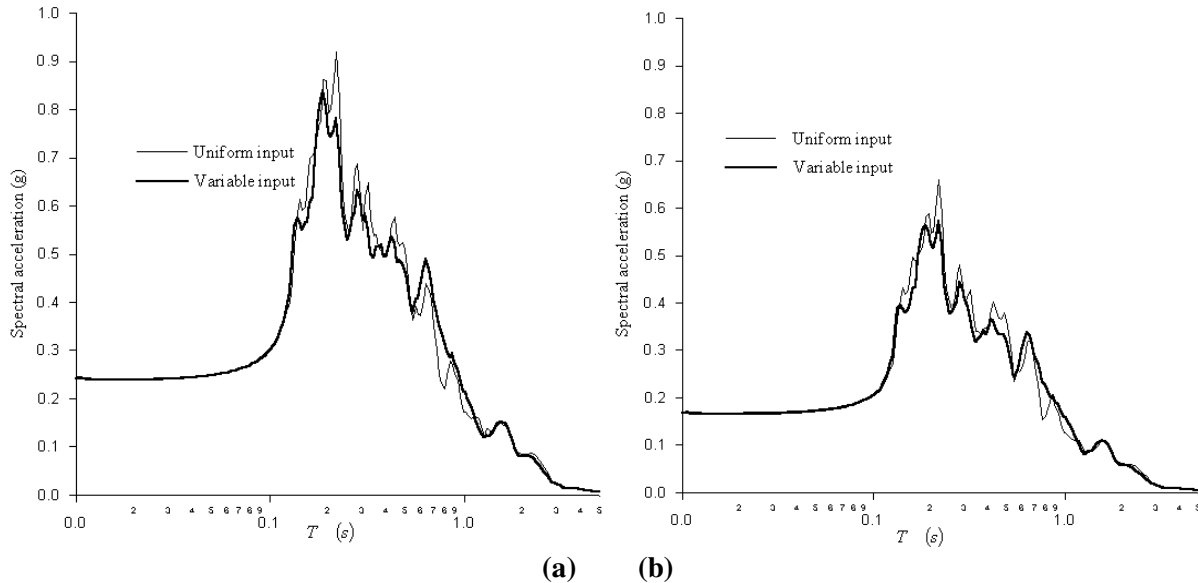
### General statement

The response spectra related to the horizontal and vertical components for the cases of P-SV-SH waves propagating through a halfspace and stratified soil are computed. We suppose that the wave content is identical in the halfspace or in the substratum supporting the stratified soil. These response spectra are computed at the control points located both at surface and at depth. In order to assess the effects of variability, a comparative study between the case of uniform input and variable input has been also conducted. The results are presented for the control points A and C (Figure 2,3b) at the free surface and A' and C' (Figure 2,3b) at depth.

### Case of a halfspace

#### Horizontal component

The response spectra obtained at the free surface (Figure 4a) is as expected higher than the one obtained at depth (Figure 4b). On the other hand, it is obvious to see that the use of uniform input over-estimates the responses at some locations and under-estimates the responses at others (Figure 4a,b).



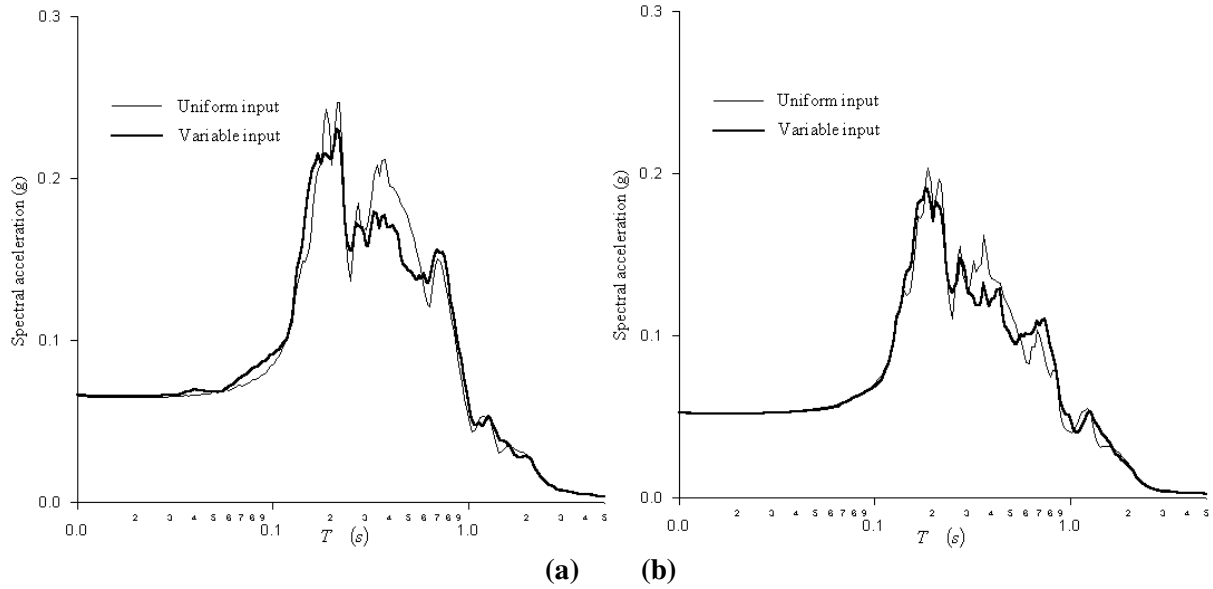
**Figure 4 Response Spectra –Horizontal component – Halfspace (a) at the free surface (point A and C) (b) at depth (point A' and C')**

#### Vertical component

One of the major difference between results obtained for horizontal component (Figure 4a,b) and vertical component (Figure 5a,b) is that the latter leads to values of response spectra lesser than the former. For



instance, the maximum of the spectral acceleration related to the vertical component (Figure 5a) is three times lesser than the one obtained for the horizontal component (Figure 4a). Also it is found that the use of uniform input or variable input underestimates values of the response spectra on some range of the period and overestimates on others.

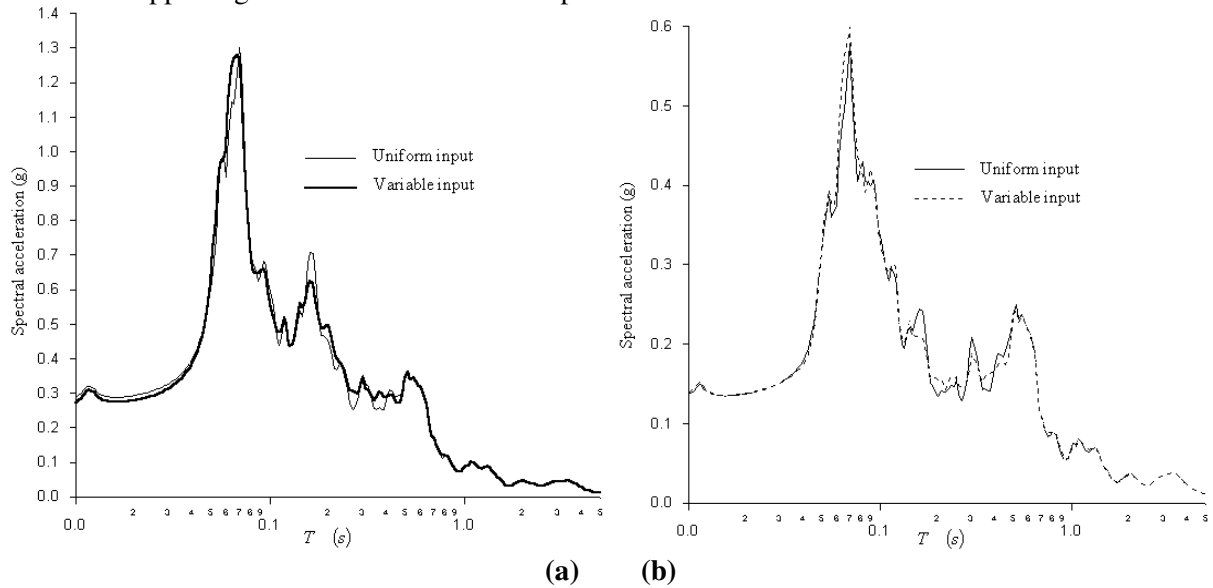


**Figure 5 Response Spectra –Vertical component – Halfspace (a) at the free surface (point A and C) (b) at depth (point A' and C')**

### Case of a stratified soil

#### Horizontal component

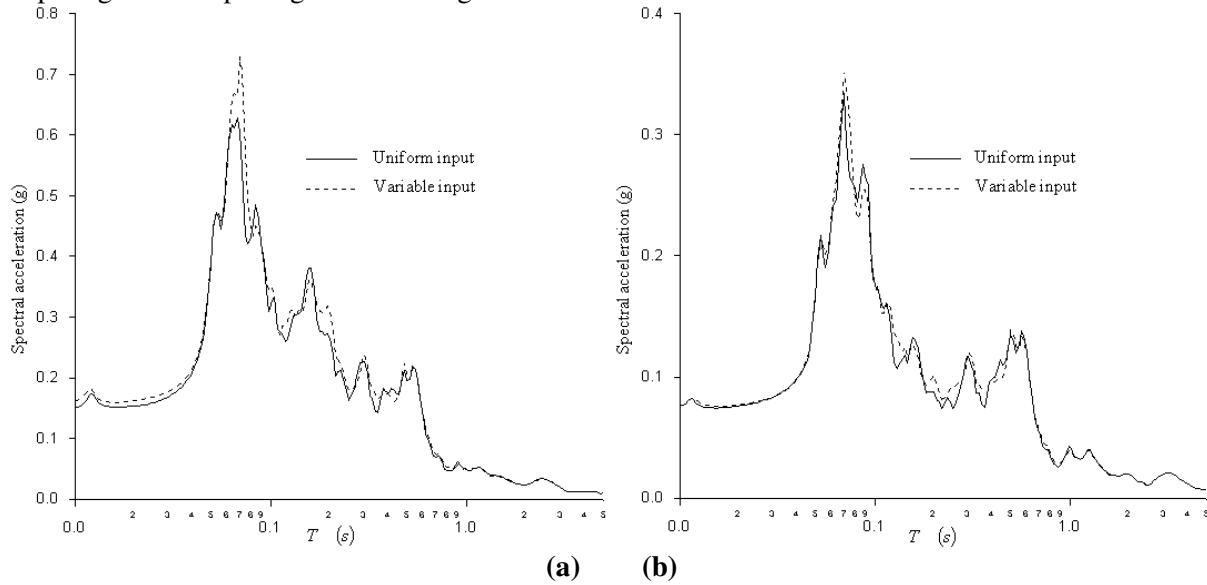
It is obvious to see that the amplification of the motion is important for the case of a stratified soil (Figure 6a) than for the case of a halfspace (Figure 4a) although the same wave content has been assumed at the substratum supporting the stratified and the halfspace.



**Figure 6 Response Spectra –Horizontal component – Stratified soil (a) at the free surface (point A and C) (b) at depth (point A' and C')**

### Vertical component

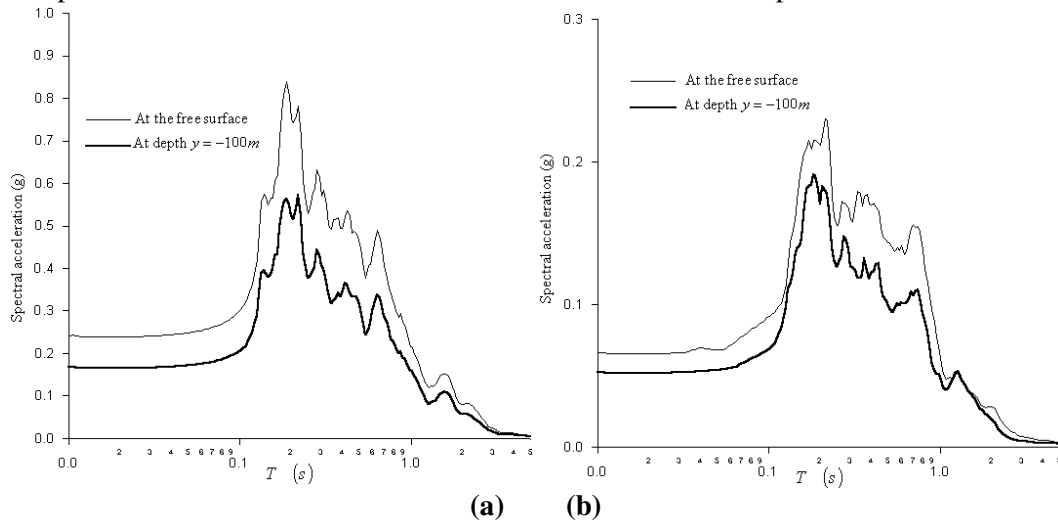
As for the case of a halfspace, we found that spectral acceleration associated with vertical component presents values higher than the one obtained for horizontal component. This could be easily noticed by comparing for example Figure 6a and Figure 7b.



**Figure 7 Response Spectra –Vertical component – Stratified soil (a) at the free surface (point A and C) (b) at depth (point A' and C')**

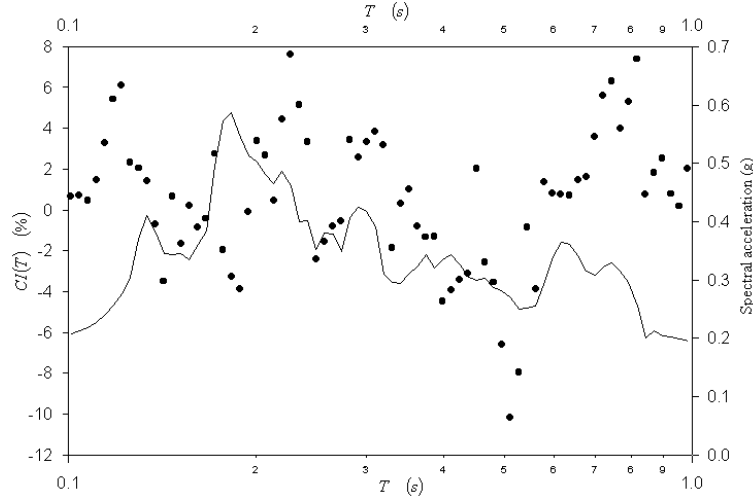
### NEW IDEAS ON HOW TO TAKE INTO ACCOUNT THE SPATIAL VARIABILITY AT DEPTH

It is obvious to note that the use of variable input depict comparable outcome in the variation of response spectra for both horizontal and vertical components. It has been also found that spatial variability of the ground motion could lead to value of response spectra higher than for the case of uniform input for some values of  $T$ . At depth, the response spectra have been computed using a coherency function and a PSD evaluated at depth (Figure 8a,b). It is interesting to compute the characterized inaccuracy  $CI(T)\%$  for the case of halfspace and stratified related to both horizontal and vertical component.

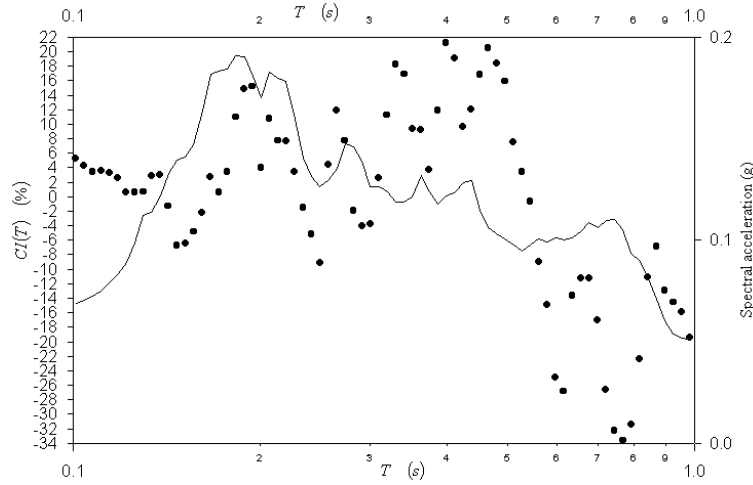


**Figure 8 Response Spectra - Halfspace (a) Horizontal component (b) Vertical component.**

A careful examination of Figures 9 and 10 shows that positive values of  $CI(T)$  reaches 8% for the horizontal component (Figure 9) and 22% for the vertical component (Figure 10). Therefore, for the case of a halfspace current design of the extended structure at depth under estimates the forces by a rate of 8-22%.



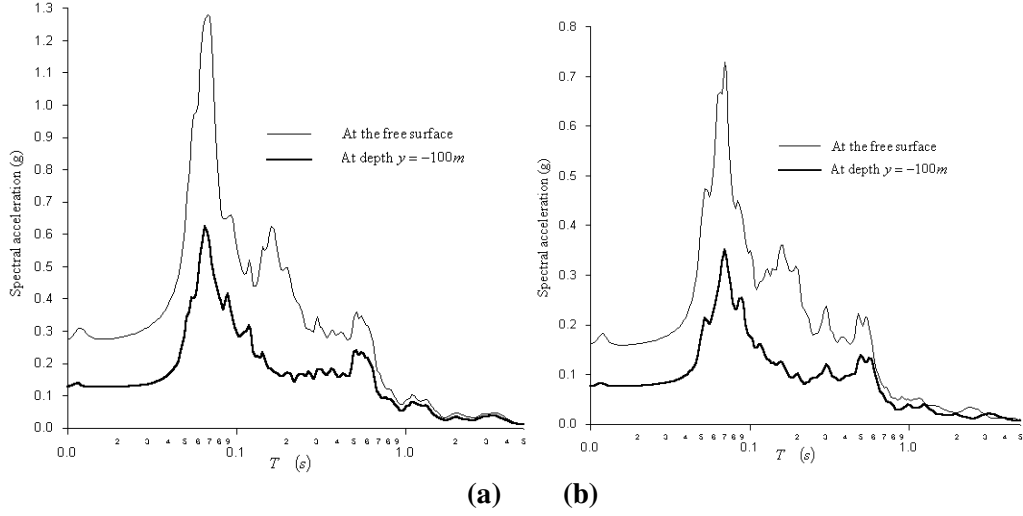
**Figure 9 Characterized inaccuracy ( $CI(T)$ ) over the range  $T = [0.1-1]s$ , Horizontal component, Halfspace. Continuous line:  $SP_E(T)$ , Dark point:  $CI(T)$**



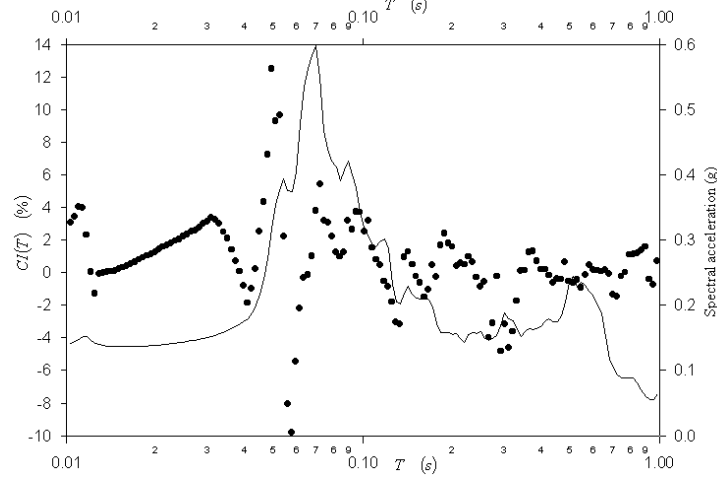
**Figure 10 Characterized inaccuracy ( $CI(T)$ ) over the range  $T = [0.1-1]s$ , Vertical component, Halfspace. Continuous line:  $SP_E(T)$ , Dark point:  $CI(T)$**

What about the case of stratified soil? The positive values of  $CI(T)$  are, for some case, greater than 12% for horizontal component (Figure 12) and 20% for vertical component. These values could reach 30%, which means that the forces acting on embedded structures are under estimates by 30%. This results seems to be in accordance with recent post seismic investigation which explain the damage of some structure to an underestimation of forces by 15% Fenves [8].

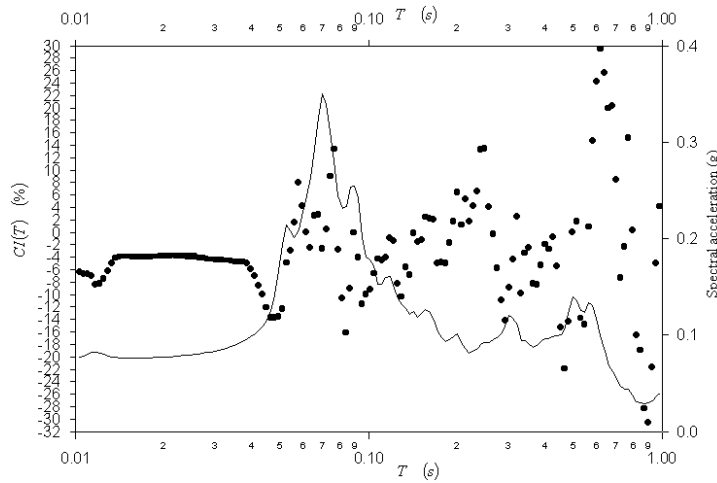
Based on these results, we strongly recommend using a proper coherency function at depth to perform seismic analysis of embedded structure



**Figure 11 Response Spectra – Stratified soil (a) Horizontal component (b) Vertical component**



**Figure 12 Characterized inaccuracy ( $CI(T)$ ) over the range  $T=[0.1-1]s$ , Horizontal component, Stratified soil. Continuous line:  $SP_E(T)$ , Dark point:  $CI(T)$**



**Figure 13 Characterized inaccuracy ( $CI(T)$ ) over the range  $T=[0.1-1]s$ , Vertical component, Stratified soil. Continuous line:  $SP_E(T)$ , Dark point:  $CI(T)$**

## CONCLUSIONS

The effects of the spatial variability of seismic motion on extended and /or structures could be regarded as one of the major cause of structural damage. This paper aims to present new concepts in estimating the effects of the seismic spatial variability on embedded structures using the Complete Stochastic Deamplification Approach. The results obtained show that the effects of the spatial variability of earthquake motion are higher for structures resting on stratified soil rather than on homogenous soil. Also, we found that when we use a coherency model developed or established at the free surface to perform seismic analysis of embedded structure we underestimate spectral acceleration by 30%. This result seems to be in accordance with recent observations made on the collapse of structures during seismic events such as the 1994 Northridge earthquake.

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