13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 310

EXPERIMENTAL DETERMINATION OF MODAL DAMPING FROM FULL SCALE TESTING

John BUTTERWORTH[1], Jin Hee LEE[2], Barry DAVIDSON[3]

SUMMARY

Damping properties are of significant importance in determining the dynamic response of structures, and accurate prediction of them at the design stage, especially in the case of light-weight, wind-sensitive buildings, is very desirable. Unfortunately, damping parameters can not be deduced deterministically from other structural properties and recourse is generally made to data from experiments conducted on completed structures of similar characteristics. Such data is scarce but valuable, both for direct use in design and for furthering research into the phenomenon and modelling of damping.

This paper presents the results of an experimental investigation of the damping properties of an 11-storey reinforced concrete shear-core office building based on forced vibration tests. Excitation was by means of a rotating eccentric mass exciter driven by a precisely controlled electric motor with electromagnetic and resistive braking capabilities, allowing sinusoidal force amplitudes of up 40kN. Accelerometers were used to measure structural response. It proved possible to excite the structure in four distinct modes, two translational and two torsional. Amplitudes were constrained to avoid structural damage, but were large enough to induce motion sickness in some occupants, resulting in the testing having to be conducted when the building was empty. The acquired data was processed using both frequency and time-domain methods to arrive at modal damping ratios. The methods used included traditional ones such as free vibration decay and half-power bandwidth, with some adaptations, such as a direct application of the half- power method to the un-normalised spectrum, and a hybrid method based on experimental resonant response combined with modal data from an associated free vibration analysis. Damping for the lowest modes was around 1.5%, rising to approximately 6% for the 2nd translational mode. It was also observed that damping coefficients and resonant frequencies were amplitude-dependent.

INTRODUCTION

Advances in analytical techniques, combined with the availability of newer and more efficient materials, have facilitated the construction of lighter and more flexible buildings. These modern structures are known to be much more responsive to dynamic loadings than their more solid predecessors. As a

[1] Dept of Civil & Env. Engineering, The University of Auckland, NZ, jw.butterworth@auckland.ac.nz
[2] Graduate Student, Dept of Civil and Environmental Engineering, The Universityof Auckland, NZ
[3] Dept of Civil & Env. Engineering, The University of Auckland, NZ, bj.davidson@auckland.ac.nz

consequence, it has become more important to accurately assess dynamic properties, such as damping and natural frequencies. In particular, damping, a measure of energy dissipation in a vibrating system, has been recognised as playing a major role in the assessment of serviceability limit states. Unfortunately, damping remains the most difficult dynamic property to predict at the design stage, as it cannot be deduced from the physical properties of a structure, unlike the mass and stiffness properties. As a result the assessment of damping relies largely on measured data from similar structures. Over many years, considerable attention has been given to assessing damping in full-scale structures throughout the world. These measured damping values have contributed to a valuable database. However, the number of careful systematic studies, particularly in New Zealand (Early, [1]), is relatively few and more data is required to increase the accuracy of predicted damping values for different types of structure. Kareem and Gurley [2] stated that accurate information about damping values might alleviate a major source of uncertainty at the design stage, particularly of wind sensitive structures.

Full scale testing involves measuring the dynamic response of a target building to excitation which is typically applied in one of three ways. The simplest approach employs ambient excitation, typically from wind, but occasionally from traffic, earthquake or other influences. The drawback is the almost complete lack of control over the timing, duration, amplitude and direction of the applied effects. A second approach is the snap-back method in which the structure is suddenly released from an imposed initial displacement and the ensuing free vibration recorded. This approach can be difficult to apply to large structures and also struggles to excite higher modes of vibration. The most comprehensive and accurate method remains forced vibration testing using a mechanical exciter to apply a harmonically varying force of known frequency and amplitude to the structure (Littler, [3]). The excitation device commonly consists of contra-rotating eccentric masses, usually rotating in a horizontal plane, generating a sinusoidal force in the horizontal direction. The amplitude and frequency of the applied force can be controlled by varying the rotational speed and the magnitude and eccentricity of the attached masses. Servo-controlled hydraulic actuators attached between the building and a sliding inertial mass have also been used to apply direct mechanical excitation. The results reported herein are based on tests conducted using mechanical excitation generated by an eccentric mass type vibrator.

TEST BUILDING AND INSTRUMENTATION

Test Building

The building chosen for testing was an 11-storey reinforced concrete office block forming part of the Engineering School at Auckland University (Figure 1). Constructed in 1968, the building consisted of a central shear core and 12 perimeter columns supporting 11 floors, square in plan apart from the lowest two floors. The shear core accommodates elevators, stairs and services and extends above the 11th floor to support a 12th floor of reduced area (area is the same as the shear core) housing lift machinery and other services. The floor levels within the building complex are numbered from 2 (ground floor) to 13 (roof slab) and 14 for the elevator machinery room. For convenience these numbers were used during testing and are retained for this paper. Level 3 houses student facilities, level 4 is an atrium and levels 5 to 12 comprise staff offices. Seismic isolation joints separate the test building from adjoining buildings at levels 3 and 4. A typical floor plan is shown in Figure 1.

The core consisted of a 6m square prestressed concrete box with 305mm thick walls. Floors were 200mm thick, reinforced concrete flat slabs apart from the roof which was 120mm, and were supported at the shear core and the 460mm square perimeter columns. Apart from the lowest two floors there were no supporting beams, allowing storey heights of 2.76m for most storeys, and 3.96m for the lower three. The building was founded on circular reinforced concrete end-bearing piles together with a system of deep foundation beams. Due to the slope of the underlying ground there were foundation supports positioned at various points over levels 2, 3 and 4.

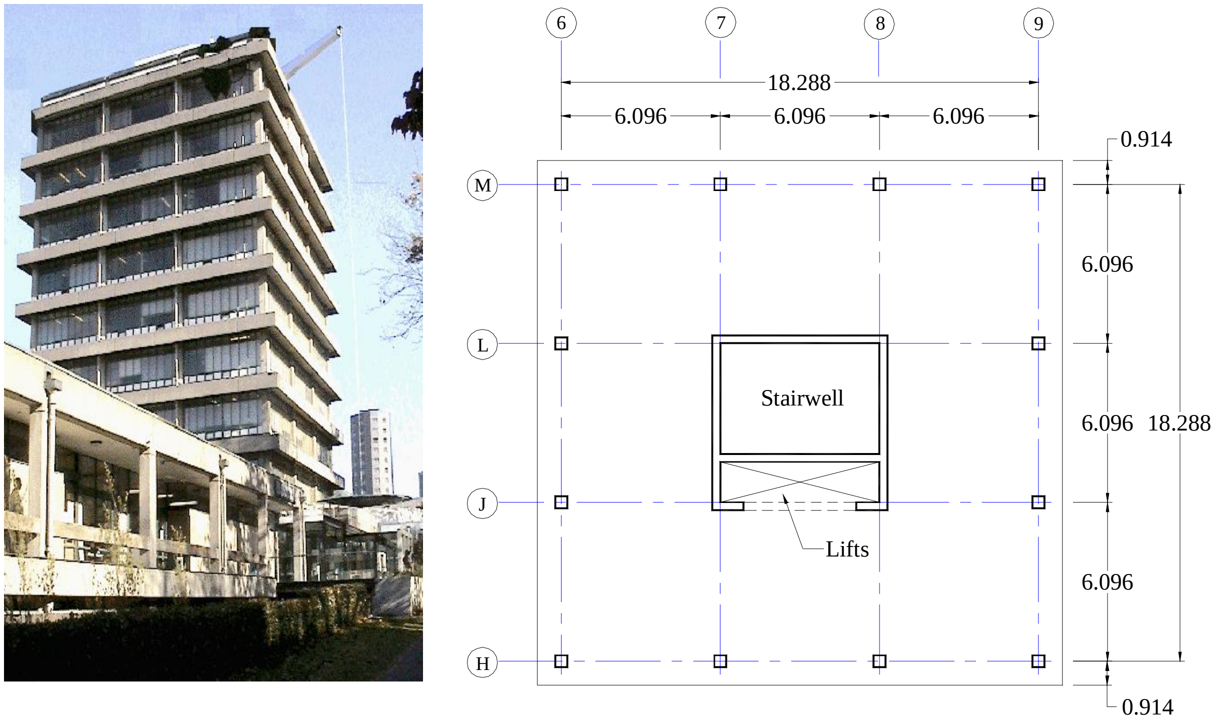

Figure 1 Test building on left with typical floor plan

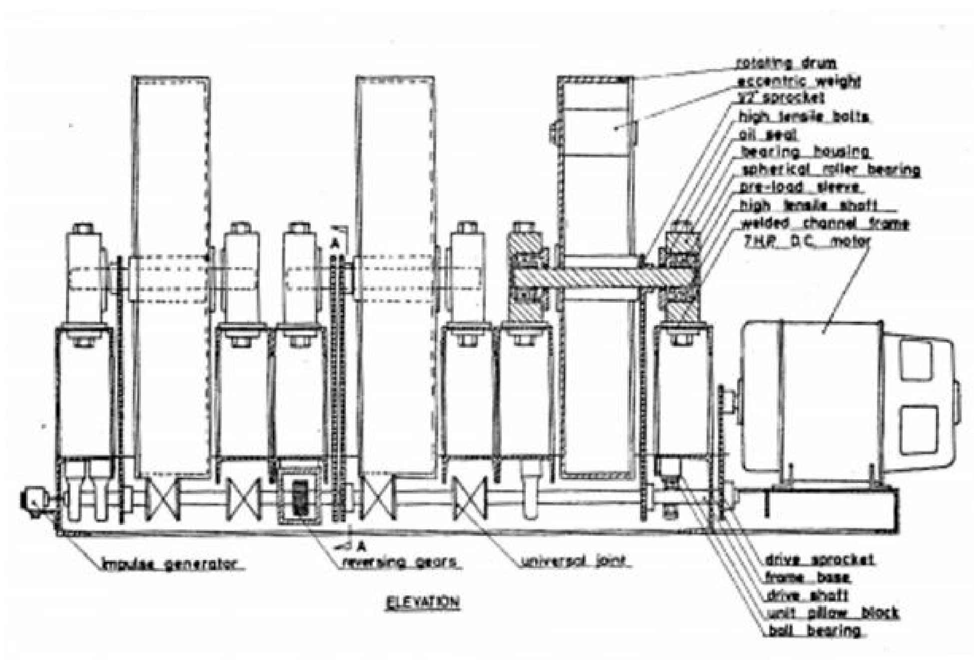

Figure 2 Eccentric mass exciter - original drawing (Reay [4])

Eccentric Mass Exciter

The eccentric mass exciter was originally built in the late 1960s for research at Canterbury University by Reay [4] and subsequently installed in the lift machinery room (highest level) of the test building in the early 1970s. As shown in Figure 2, the exciter consisted of three identical circular drums mounted on three separate high tensile shafts having a collinear horizontal axis. The drums were driven by a 7.5 H.P. DC motor via a drive shaft which was connected to the drums by a chain and sprocket system. Each of the drums contained an eccentric weight, with the outer two drums holding individual weights equal to half that of the inner drum. The drums were 914 mm in diameter, with eccentric masses mounted at a radius of approximately 325 mm. The exciter was designed to produce a uni-directional sinusoidal force by using the principle of contra-rotating masses: the outer two drums rotated in the opposite direction to that of the inner drum, each carrying a mass equal to half that of the inner drum.

The exciter was extensively overhauled and upgraded by installing a new 3kW AC motor and gearbox, new bearings, chains, toothed belts, etc, and a high precision variable speed controller incorporating electromagnetic and resistive braking facilities. The frame and its attachment to the floor slab of the lift room strengthened. Unfortunately space constraints limited the positioning of the exciter such that it could exert force only in the North-South direction (left-right direction in Figure 1). Figure 3 shows the upgraded exciter installed on level 14, the highest floor in the building. It is worth noting that the control of horizontal axis exciters of this type is complicated by the alternating gravity torque and considerable care is needed to achieve precise control. Vertical axis exciters do not have this problem.

The controller permitted the exciter to operate in the range 0.15Hz to 10Hz with increments as small as 0.004Hz. Total eccentric mass could be selected as 10, 20 or 40kg with corresponding force amplitudes up to 40kN. However, for safety reasons the force amplitude was limited to 13kN. Operating frequency was determined from the rotational speed of the motor and independently from a Hall-effect sensor counting pulses from a toothed wheel in the drive train.

Figure 3 Exciter installed at level 14

Response measurement
Building response was measured by an array of six horizontal and two vertical axis ±0.25g servo-accelerometers which were repositioned as necessary for the various tests. The accelerometer signals were passed through a variable gain amplifier and low-pass filter before being digitised and recorded at 1000 samples per second (each channel). The accelerometers, which could also be used as inclinometers, were tilt calibrated.

FREE AND FORCED VIBRATION TESTING

Preliminary tests together with theoretical modelling pointed to four measurable natural frequencies, two predominantly translational and two predominantly torsional (due to small asymmetries in the structure the modes were not purely translational or torsional). The three main types of test carried out were as follows:

Free vibration decay
The structure was vibrated at a resonant frequency until it settled into a steady state at which point the mechanical exciter was brought to a sudden stop leaving the structure to vibrate freely until brought to rest by the effects of damping. The resulting damped free vibration record could then be used with methods such as the well-known log-decrement to determine the equivalent viscous damping coefficient. Whilst this approach is ideal for the lowest mode, it becomes progressively more difficult with the higher modes due to their tendency to revert to the lowest mode as decay proceeds and it was successfully applied to only the two lowest modes (one translational and one torsional). Tests were conducted with three different amplitudes of excitation (corresponding to the 10, 20 and 40kg eccentric mass sets).

Frequency sweep
Frequency sweep testing was used to determine acceleration and displacement response spectra by operating the exciter at a range of gradually incrementing frequencies and recording the steady state response data at each increment. Peaks in the resulting spectra indicated natural frequencies, so in order to generate data from which damping results could be deduced with maximum accuracy, the frequency sweep was repeated in the vicinity of the peaks with smaller increments (approx. 0.005Hz). Because eccentric mass type exciters generate forces which are proportional to the square of the excitation frequency, $\overline{\omega}$, the recorded data was generally normalised by dividing by $\overline{\omega}^2$. .

Mode shapes
Mode shape tests were conducted by distributing accelerometers at different levels down the building, exciting the structure at each resonant frequency, and recording accelerations at the same instant. Double integrating of the acceleration records permitted displacement records to be obtained, revealing the mode shapes. Pairs of parallel oriented accelerometers, spaced apart, enabled separation of torsional and translational displacements.

Testing was generally carried out at night or weekends when the building was largely unoccupied and when passing road traffic also tended to be lighter. Likewise, periods of strong winds were avoided in order to minimise sources of vibration other than that which was deliberately applied.

RESULTS

Frequency sweep
The section of the normalised response spectrum in the vicinity of the first N-S translational mode is shown in Figure 4 (left) for three amplitudes of excitation. The respective peaks occur at frequencies of

1.911, 1.900 and 1.880Hz revealing a slight decrease in natural frequency with increasing amplitude. Similar variations of natural frequency with amplitude have been observed before, for example by Littler [3] and Ellis [5]. Similarly, Figure 4 (right) shows the spectra around the frequency of the 1st torsional mode. Again the resonant peaks show a decrease in natural frequency with increasing amplitude, being 2.480, 2.463 and 2.447Hz respectively for 10, 20 and 40kg exciter masses.

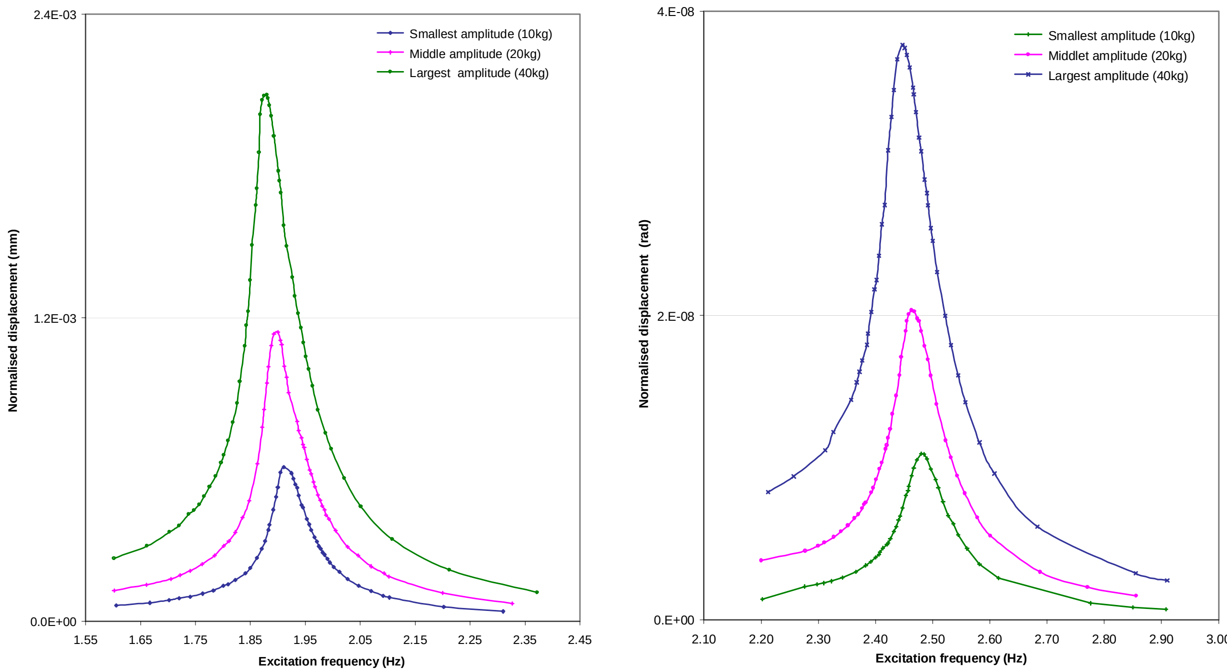

Figure 4 **Normalised displacement response spectra -**
Left: 1st translational mode (N-S) Right: 1st torsional mode

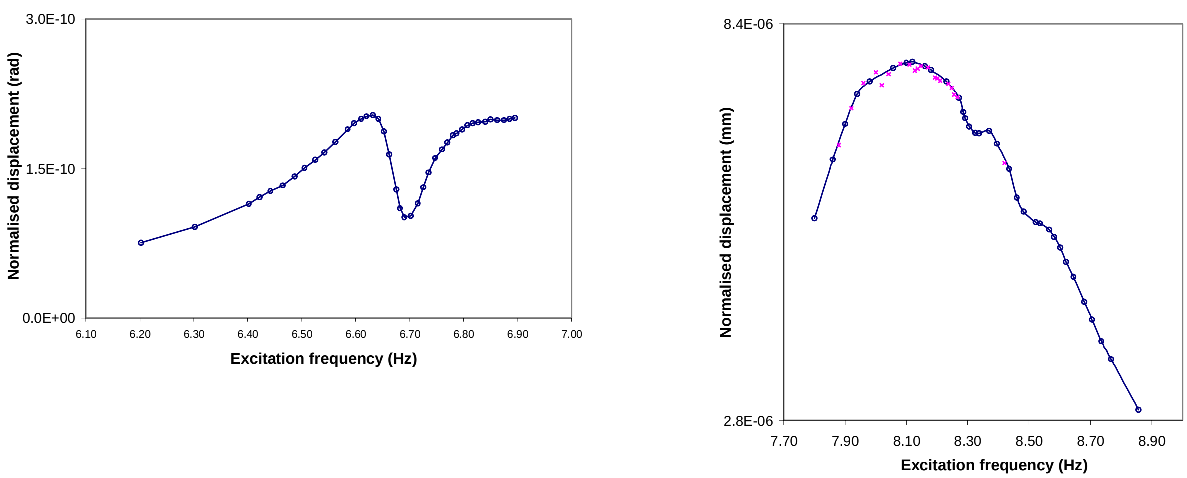

Figure 5 **Normalised displacement response spectra -**
Left: 2nd torsional mode Right: 2nd translational mode (N-S)

The displacement spectrum in the vicinity of the 2nd torsional and translational modes are shown in Figure 5 for the minimum excitation amplitude. The peaks reveal natural frequencies for these modes as 6.632 and 8.120Hz respectively. It is noticeable that the spectral curve is distorted to the right of the peak in each case by the proximity of higher modes – also the smoothness evident in the preceding cases is less apparent, even after rejecting a number of biased data points. It is not unusual to fit smooth curves to the data points in such cases (Ellis [5]), however that approach has not been used here.

Damping from normalised spectra

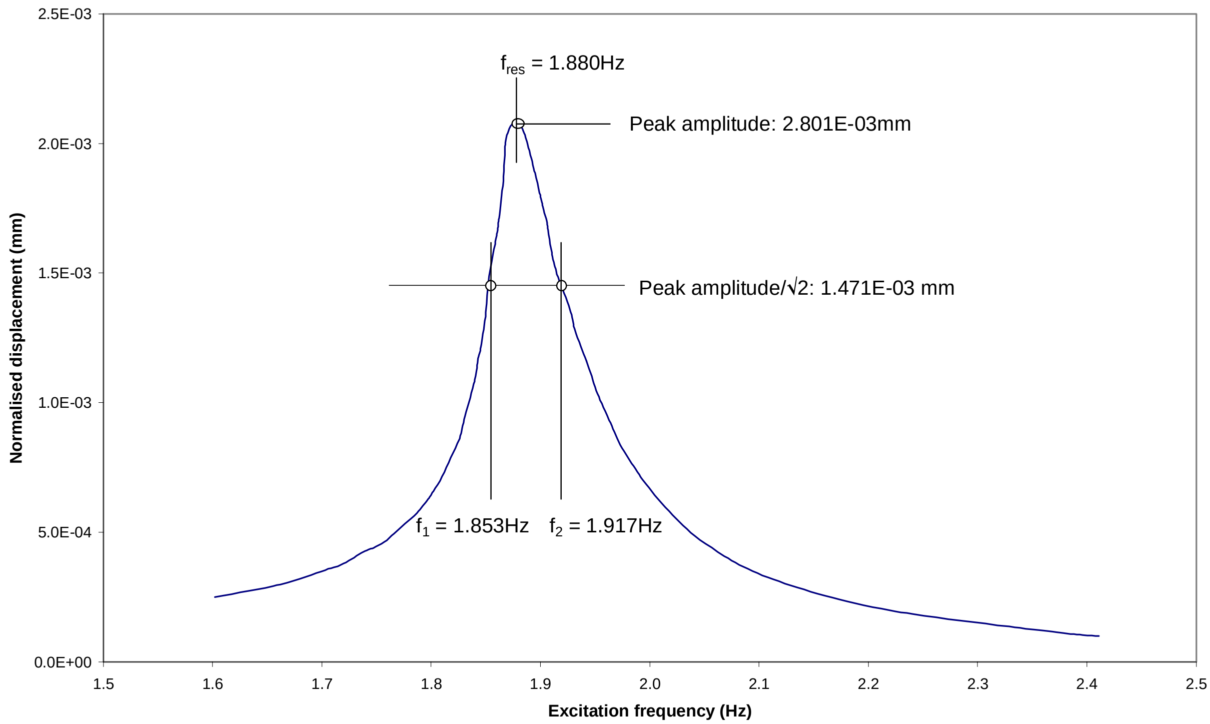

Figure 6 Spectrum for 1st N-S mode showing half-power bandwidth frequencies

The method used here is the well-known half-power bandwidth, which although strictly applicable only to lightly damped single degree of freedom systems is frequently applied to well-separated modes of multi degree of freedom systems. Figure 6 illustrates the procedure for the particular case of the 1st N-S mode. It is then assumed that half the total power dissipation in this mode occurs in the frequency band between f_1 and f_2, where f_1 and f_2 are the frequencies corresponding to an amplitude of $f_{res}/\sqrt{2}$. It is shown in standard texts (Chopra [6]) that the damping ratio, ξ, for small ξ is approximately

$$\xi = \frac{f_2 - f_1}{2f_{res}} \quad \textbf{(Eq 1)}$$

Alternatively if ξ is not assumed small, the relationship becomes

$$\xi = \sqrt{0.5 - \sqrt{0.25 - 0.0625\left(\frac{f_2 - f_1}{f_{res}}\right)^2\left(\frac{f_2 + f_1}{f_{res}}\right)^2}} \quad \textbf{(Eq 2)}$$

The resulting damping ratios for the various modes and excitation amplitudes are shown in Table 1.

Table 1 Modal damping ratios based on half-power bandwidth

Mode	Eccentric mass (kg)	Resonant frequency, f_{res} (Hz)	Damping ratio, ξ Equation 1 (%)	Damping ratio, ξ Equation 2 (%)
1st translational	10	1.911	1.518	1.525
	20	1.900	1.553	1.556
	40	1.880	1.702	1.707
1st torsional	10	2.480	1.391	1.394
	20	2.463	1.462	1.466
	40	2.447	1.635	1.640
2nd torsional	10	6.632	1.34	1.33
2nd translational	10	8.120	4.57	4.60

Results for the last two modes listed are considered less reliable than the first two due to the non-ideal, distorted form of that part of the response spectrum.

Damping from un-normalised spectra

The excitation from an eccentric mass rotating at an angular velocity of $\overline{\omega}$ rad/s is proportional to $\overline{\omega}^2$ as noted earlier, and since the half-power method presumes a sinusoidal exciting force of constant amplitude it is usual to normalise the response by scaling by $1/\overline{\omega}^2$. However, if the response of the system is nonlinear the normalised response will differ from that which would be obtained by excitation with a constant amplitude force. An alternative is to use the un-normalised spectrum directly in the modified half-power formula shown in Equation 3,

$$\xi = \frac{f_{res}(\bar{f}_2 - \bar{f}_1)}{\bar{f}_1^2 + \bar{f}_2^2} \qquad \textbf{(Eq 3)}$$

where $\bar{f}_1$ and $\bar{f}_2$ denote the frequencies corresponding to an un-normalised response amplitude of $1/\sqrt{2}$ of the peak un-normalised amplitude, and f_{res} is the resonant frequency. Equation 3 was obtained by numerically simulating the response of a viscously damped single degree of freedom system to eccentric mass type excitation and then working backwards to re-calculate the damping ratio. Equation 1 can also be used directly with the un-normalised spectrum, resulting in only small errors – about 1% at a damping ratio of 0.05, increasing to 3.7% at ratio of 0.10.

Although the test results showed that the response of the target building was slightly nonlinear, even at the very small test amplitudes used, the differences resulting from applying Equations 1, 2 or 3 were negligible.

Damping from resonant response amplitude

A method for deducing the damping ratio from measured resonant response is described below. It could be called a hybrid method in that it depends on both measured experimental data and information derived from a free vibration analysis of the target structure.

Consider a multi degree of freedom system subject to a harmonic exciting force $f_e \sin \overline{\omega} t$ acting at displacement degree of freedom u_e. The modal response equation for mode n will be

$$\ddot{Y}_n + 2\xi_n\omega_n\dot{Y}_n + \omega_n^2 Y_n = \frac{\boldsymbol{\varphi}_n^T \mathbf{F}(t)}{M_n}$$

where Y_n denotes the modal amplitude, $\boldsymbol{\varphi}_n$ is the mode shape vector, ω_n is the corresponding resonant frequency, $\mathbf{F}(t)$ is the external force vector and M_n is the modal mass.

Since $\mathbf{F}(t)$ is empty except for element e which contains $f_e \sin\overline{\omega}t$, it follows that the response will be

$$\ddot{Y}_n + 2\xi_n\omega_n\dot{Y}_n + \omega_n^2 Y_n = \frac{u_e f_e \sin\overline{\omega}t}{M_n},$$

so that $$Y_n = Ge^{-\xi_n\omega_n t}\cos(\omega_n t - \alpha) + A\cos(\overline{\omega}t - \phi)$$

where $$A_n = \frac{u_e f_e / K_n}{\sqrt{(1-\beta_n^2)^2 + (2\xi_n\beta_n)^2}}$$

K_n denotes the modal stiffness, and β_n is the frequency ratio $\overline{\omega}/\omega_n$.

If $\overline{\omega} = \omega_n$ (i.e. structure is excited at its resonant frequency), $\beta_n = 1$ and $\omega_n^2 = K_n / M_n$ giving

$$A_n = \frac{u_e f_e}{2\xi_n\omega_n M_n}$$

where $u_e = \text{displacement at d.o.f. e in mode n (from natural mode } \boldsymbol{\varphi}_n)$

$M_n = \boldsymbol{\varphi}_n^T \mathbf{M}\boldsymbol{\varphi}_n$ ($\mathbf{M}$ is the mass matrix)

$f_e = \text{force amplitude}$

$\omega_n =$ excitation <u>and</u> natural frequency

and A is the steady state modal amplitude, $Y_{n(max)}$.

The peak displacements at all degrees of freedom will be given by

$\mathbf{u}_{max} \approx A_n\boldsymbol{\varphi}_n$ (assuming that for small ξ contribution of other modes is negligible),

and if we choose another displacement degree of freedom, say u_m, then the peak displacement at u_m

$u_{m(max)} = A_n u_m$ (where u_m is taken from the mode $\boldsymbol{\varphi}_n$)

It follows that if resonant excitation is applied at degree of freedom e and the peak response $u_{m(max)}$ is measured at degree of freedom m, the expressions above can be rearranged to give the damping ratio for mode n as

$$\xi_n = \frac{u_e f_e u_m}{2M_n\omega_n^2 u_{m(max)}} \qquad \textbf{(Eq 4)}$$

where u_e and u_m are obtained from the natural mode $\boldsymbol{\varphi}_n$, and $u_{m(max)}$ is the measured peak response at degree of freedom m. Modal mass M_n is also obtained from a free vibration analysis.

The procedure is illustrated by applying it to the two measured translational modes. The free vibration data was taken from a SAP2000 model that was constructed for comparison with the experimental observations. The SAP code scales the modal vectors such that all modal masses are unity (1 tonne in the kN – metre – tonne units adopted). u_e was the displacement degree of freedom of the eccentric mass exciter and u_m was an accelerometer location close to the geometric centre of the level 14 floor slab (so the two points were very close to each other).

Table 2 Hybrid damping determination from resonant response

Mode	Resonant frequency (Hz)	Exciter force (kN)	Modal displacement (from SAP) (m)		Measured displacement $u_{m(max)}$ (m)	Damping, ξ (%)
			u_e	u_m		
1st N-S	1.911	0.47	0.0299	0.0314	8.79E-5	1.740
2nd N-S	8.160	8.5	0.0296	0.0286	2.05E-5	6.840

Note that the 1st mode result (1.74%) agrees closely with the half-power result (1.707%) at the same level of excitation (40kg eccentric mass). The 2nd translational mode result differs somewhat from the half-power result (6.84% vs. 4.6%) however, tending to confirm that the distorted spectral shape has reduced the accuracy of the half-power result in this case. The hybrid method depends only on data recorded *at the resonant frequency* and so is relatively immune to interaction with neighbouring modes (unlike the half-power method).

Damping from free vibration decay

Free vibration decay data was obtained for the lowest two modes (1st translational and 1st torsional) by operating the exciter at the relevant natural frequency until steady state vibration in the chosen mode was achieved. The exciter was then halted and the ensuing decaying response recorded.

Logarithmic decrement method

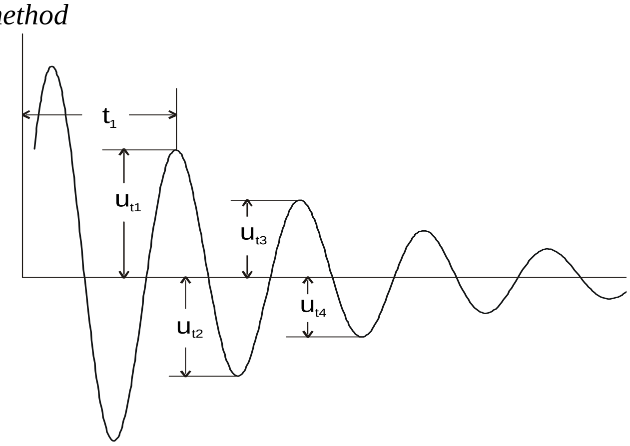

Figure 7 Illustrative decaying displacement time-history

A common method of acquiring a damping coefficient from free vibration decay data is the log-decrement method. Figure 7 represents a typical decaying time-history trace for a single degree of freedom oscillator in which u_{t1}, u_{t2}, etc, are successive amplitude peaks at times t_1, t_2, etc, where $t_2 - t_1 = T_d / 2 = \pi / \omega_d$. T_d and ω_d are the damped natural period and frequency respectively. It is shown in standard texts (Chopra, [6]) that the corresponding damping ratio is given by

$$\xi = \frac{1}{2\pi} \ln \frac{u_{t1}}{u_{t3}} \qquad \textbf{(Eq 5)}$$

To avoid the possibility of a zero off-set influencing the result it is advisable to base the calculation on peak-to-peak values. The formula is readily shown to apply equally well to peak-to-peak measurement as follows.

From Equation 5, $u_{t1} = u_{t3}e^{2\pi\xi}$, and since ξ also $= \frac{1}{2\pi}\ln\frac{u_{t2}}{u_{t4}}$, it follows that $u_{t2} = u_{t4}e^{2\pi\xi}$.

Subtracting, $u_{t1} - u_{t2} = e^{2\pi\xi}(u_{t3} - u_{t4})$,

so that $$\xi = \frac{1}{2\pi}\ln\frac{u_{t1} - u_{t2}}{u_{t3} - u_{t4}} \quad \textbf{(Eq 6)}$$

Equation 6 expresses the log-decrement result in terms of peak-to-peak amplitudes.

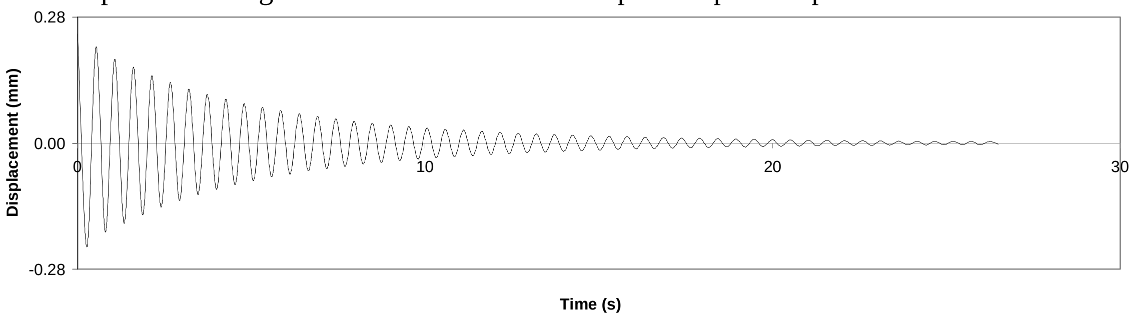

Figure 8 Free vibration decay – 1st translational mode

Figure 8 shows a typical free vibration time-history of level 14 displacement following excitation with 40kg eccentric mass in the 1st translational mode (1.88Hz). The first few cycles following cessation of excitation were discarded to allow time for attenuation of any transients induced by the change. As expected, the response showed increasing frequency and decreasing damping ratio as the amplitude decreased. Similar records were made using smaller eccentric masses (and hence smaller initial amplitudes). Close agreement of all records was observed, with the smaller starting amplitude cases closely matching the corresponding amplitude ranges of the largest mass case.

Log-decrement calculations for groups of oscillations at different amplitudes of free vibration gave the values of natural frequency and damping ratio shown in Table 3.

Table 3 Damping by log-decrement – 1st translational mode

Approx. amplitude (mm)	Frequency (Hz)	Damping ratio (%)
0.15	1.880	1.72
0.08	1.900	1.58
0.05	1.911	1.49

Damping ratios for other modes could not be obtained in the same way with any accuracy due to the irregular decay of their time-histories. For example, Figure 9. shows the result for the 1st torsional mode.

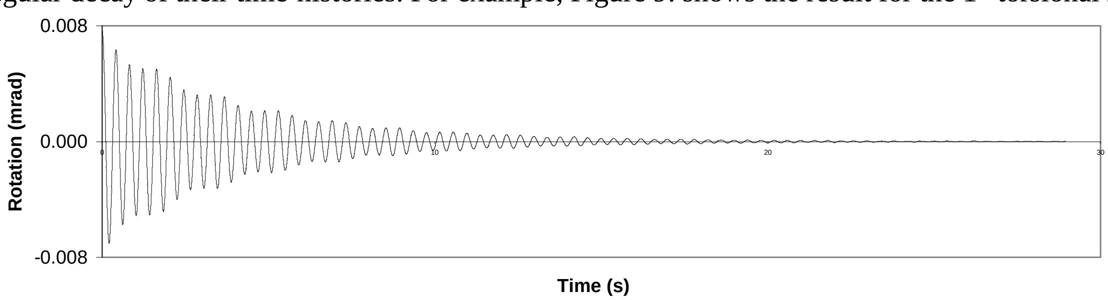

Figure 9 Free vibration decay – 1st torsional mode

Exponential envelope to free vibration response

Free vibration decay of an ideal viscously damped system lies within an envelope defined by $u(t) = \pm A e^{-\xi\omega t}$, where u(t) defines the amplitude of either a single degree of freedom system or a modal amplitude. By fitting an exponential curve of the form $u(t) = a e^{-bt}$ to the peaks of the experimental response data, over part or all of the decay duration, it is possible, by equating b to $\xi\omega$ and using the known value of ω, to deduce a value for ξ. Figure 10 shows such envelopes fitted to the data for the 1st translational modes.

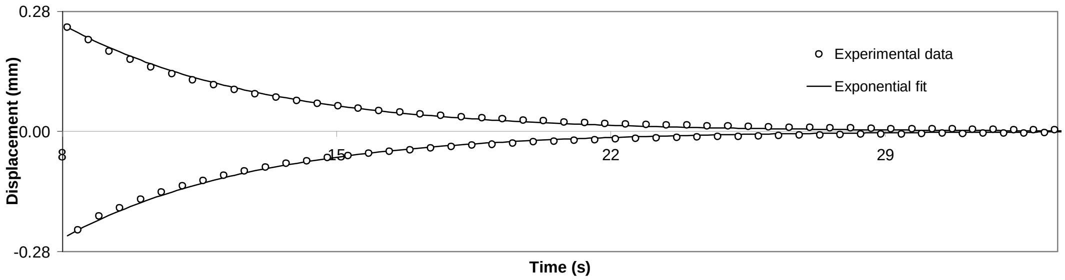

Figure 10 Exponential envelope fitted to 1st translational mode data

Applying a single envelope to the entire decay record implies an averaging process over this period with consequent loss of detail. By fitting envelopes to shorter sections of the decay trace the variations in the damping ratio can be preserved. This was done for both the 1st translational and torsional modes by selecting three consecutive sections of 5 to 10 seconds duration from each decay trace and fitting exponential envelopes to each. Figure 11(a) and (b) show the resulting envelopes for the positive peaks together with their defining equations. Similar envelopes can be found for the negative peaks. Damping values deduced from the average of the positive and negative envelope branches are shown in Table 4.

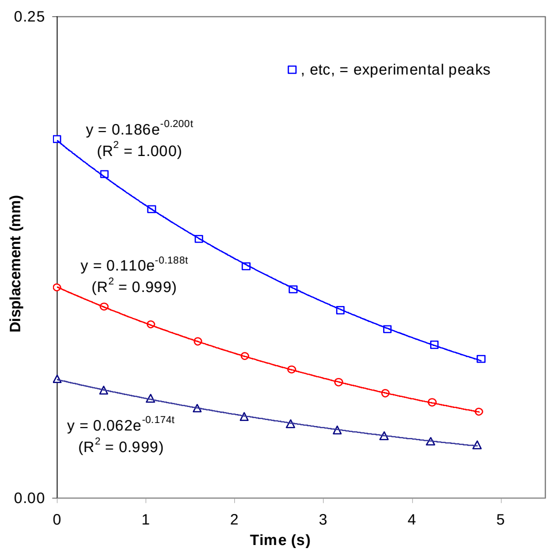

Figure 11 (a) 1st translational mode

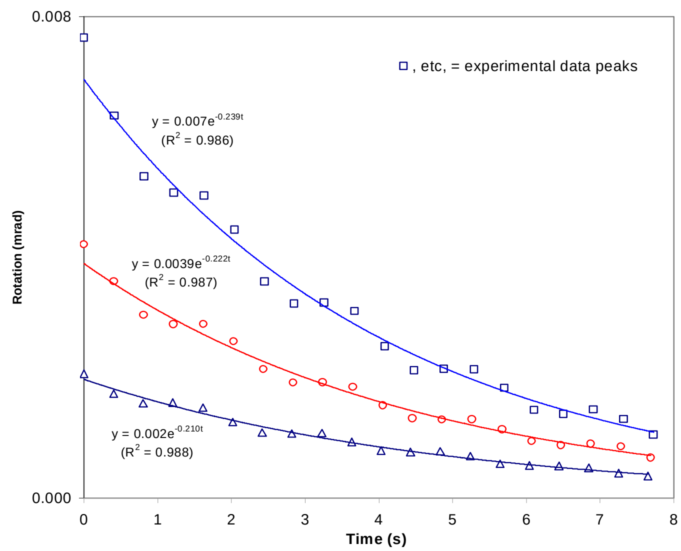

Figure 12 (b) 1st torsional mode

Table 4 Damping values deduced from fitted exponential envelopes

Mode	Amplitude (mm or mrad)	Resonant Frequency (Hz)	Damping Ratio (%)
1st translational	0.20	1.880	1.70
	0.11	1.900	1.56
	0.06	1.911	1.46
1st torsional	0.007	2.447	1.56
	0.0039	2.463	1.43
	0.0020	2.480	1.36

FINITE ELEMENT MODEL

A detailed FE model was constructed using the well-known package SAP 2000, mainly as an aid to interpreting and guiding the experimental procedures. Floor slabs, shear core and shear walls were broken down into shell and some beam elements. Horizontal stiffness of foundations were carefully modelled using equivalent linear springs at points of contact with pile tops and foundation walls, Lee [7]. Section properties were based on the gross, uncracked section. The elastic modulus of the concrete was increased by 10% to allow for dynamic conditions. Mass was based on normal specific weight values of the building fabric with in-service mass based on updated estimates of floor loading determined by inspection. The final model had over 5000 displacement degrees of freedom. Damping values based on experimental measurement were assigned to the first 6 corresponding modes (E-W translational modes were assumed to have the same damping values as the corresponding N-S modes).

Natural frequencies

Table 5 lists the first seven natural frequencies and mass participation percentages from the SAP model alongside the experimental values. It can be seen that agreement is closer for the translational modes, where the theoretical values differ by no more than 4% from the experimental values, than for the torsional modes where the difference averaged 7%. Both experimental torsional frequencies were higher than the predicted theoretical value, whereas the translational frequencies ranged from 1.2% higher to 4% lower.

Table 5 Natural frequencies – SAP model and experimental values

Mode No.	Results obtained from SAP model			Measured resonant frequency (Hz)	Mode description
	Modal mass participation (%)		Natural frequency (Hz)		
	(N-S dir.)	(E-W dir.)			
1	0.3	60.5	1.708	-	1st E-W
2	55.5	0.7	1.942	1.911	1st N-S
3	5.5	0.6	2.262	2.485	1st torsional
4	1.0	1.7	6.310	6.632	2nd torsional
5	0.5	16.7	7.384	-	2nd E-W
6	17.9	0.2	8.064	8.160	2nd N-S
7	1.1	0.0	8.709	8.370	3rd N-S

Mode shapes
As noted earlier, all mode shapes contained a mix of torsional and translational components due to a small, natural offset between the shear and mass centres. However, all the lower modes inspected were very recognisably dominated by either torsional or translational components and were accordingly named 1st N-S mode, 1st torsional mode, etc. A comparison between the calculated (SAP) modes and measured displacements and twists is shown in Figure 13. The displacements shown are translational for translation-dominated modes and twists for torsion-dominated modes. It can be seen that there was close agreement between the SAP model and experimental results.

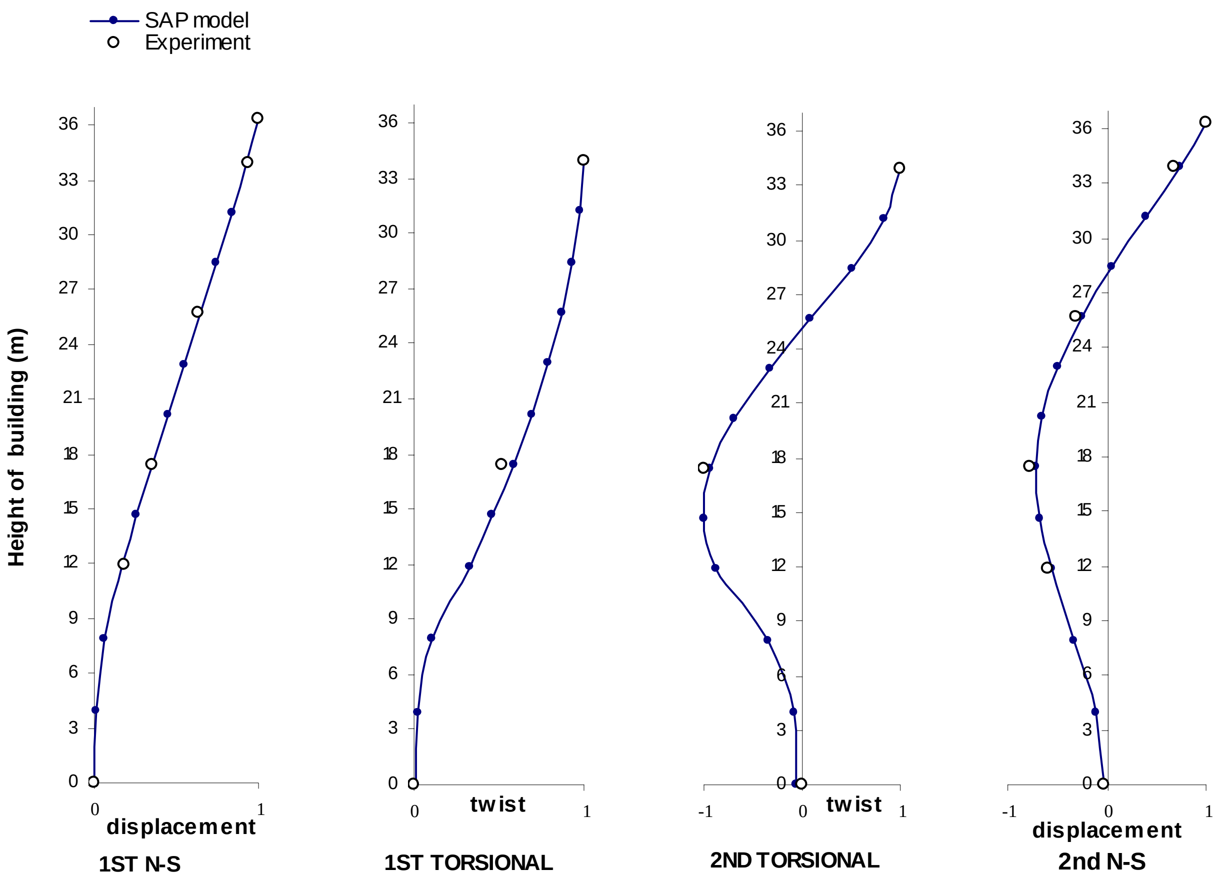

Figure 13 Comparison of some experimental and theoretical mode shapes

CONCLUSIONS

Forced vibration tests on an 11-storey reinforced concrete using an eccentric mass exciter were able to excite 4 distinct modes with frequencies and mode shapes closely matching those of a finite element model. Frequency sweep and free vibration decay tests were conducted with varying excitation amplitudes, revealing a small, approximately linear, reduction in natural frequency with increasing amplitude (Table 1).

Deduction of equivalent viscous modal damping ratios by applying the half-power bandwidth method to the normalised displacement spectrum gave values ranging from 1.5% for the 1st two modes up to 4.6% for the 2nd translational mode. Damping values also increased slightly with amplitude (Table 1). A revised

formula for applying the half-power method to un-normalised experimental results is suggested (Equation 3).
A hybrid method for deducing damping ratio from experimental data gathered at the resonant excitation frequency, together with modal data from a free vibration analysis, is proposed, avoiding the difficulties of applying the half-power method to skewed or distorted spectra (such as arise in multi degree of freedom systems). The method agreed closely with the half-power method for the well-separated 1st translational mode but gave a noticeably different, presumably more accurate, value for the 2nd translational mode, 6.8% compared with 4.6% from the half-power method (Table 2).

Log-decrement deduction of damping ratio (modified to use peak-to-peak amplitude) was applied to the first two modes giving results in close agreement with other methods, such as the half-power bandwidth, and again showing the previously reported increase in damping with amplitude (Table 3). Least squares fitting of exponential asymptotes to the free vibration decay records was helpful in overcoming the slightly erratic decay data recorded for the 2nd mode and provided further confirmation of the damping values deduced by other methods (Table 4). Useful decay records could be obtained only for the 1st two modes. More detail on this and other aspects can be found in Lee [7].

REFERENCES

1. Early, M.D., “Determination of damping in tall buildings”, Report 89-B9203, Central Labs, Works and Development Services Corporation (New Zealand) Ltd: 117p, 1989.
2. Kareem, A. and Gurley, K., “Damping in structures: its evaluation and treatment of uncertainty”, Journal of Wind Engineering and Industrial Aerodynamics, 59(2-3): 131-157, 1996
3. Littler, J. D., “An assessment of some of the different methods for estimating damping from full-scale testing.” Journal of Wind Engineering and Industrial Aerodynamics, 57(2-3): 179-189, 1995.
4. Reay, A. M., “Dynamic characteristics of civil engineering structures.” PhD thesis University of Canterbury, New Zealand, 1970.
5. Ellis, B. R., “Full-scale measurements of the dynamic characteristics of buildings in the UK”, Journal of Wind Engineering and Industrial Aerodynamics 59(2-3): 365-382, 1996.
6. Chopra, A. K., “Dynamics of structures.”, Prentice Hall Inc.,1995.
7. Lee, J., “Assessment of modal damping from full scale structural testing”, ME thesis, School of Engineering, University of Auckland, 2003.