13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 3203

APPLICATION OF TREFFTZ BOUNDARY METHOD TO COMPUTE THE HYDRODYNAMIC PRESSURE ON SUBMERGED STRUCTURE SUBJECTED TO GROUND MOTION

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SUMMARY

In the present paper the dynamic behavior of different types of submerged structures like vertical shafts of morning glory spillways, submerged tower intake, and oil tanks is studied. Then, the limitations of various approximated formula are demonstrated.

Due to approximation of satisfying radiation boundary conditions (e.g. Sommerfeld), solutions based on domain wise discretization are not accurate. Among the boundary wise discretization methods, the Trefftz method is adopted in this research. Assuming the fluid as linear compressible non-viscous material and considering the effect of surface waves, the dynamic behavior of submerged rigid structure subjected to ground vibration is investigated. The effect of the shape of embedded structure on the hydrodynamic pressure is presented and discussed. The proposed method is verified for a semi-circular cross section with the available analytical solution.

INTRODUCTION

Although a number of different methods are available for analysis of non-submerged structures [1, 2, 3, 4], however, there are few findings for submerged structures. From the practical point of view, there are some structures that are submerged like the vertical shaft of morning glory spillways, submerged tower intake, oil tanks which are installed on seabed and subsurface topographies in seas.
Due to the difficulty of satisfying radiating boundary conditions (Sommerfeld), solutions based on domain wise discretizations, for example the finite element method (FEM) and finite difference method are not accurate methods as they need a lot of elements which makes the calculation time consuming.

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By reducing the dimensions by one and satisfying the radiating boundary conditions, the solutions based on boundary wise discretizations are efficient and more accurate for fluid-structure interaction. Among boundary element methods, the Trefftz method [5] is utilized for analysis of the submerged structures. The main reason to select this method is due to non-singularity behavior of weight functions satisfying both the radiation condition and the governing differential equation making the method rather simple. This is in contrast to the conventional boundary element method (BEM) that uses fundamental solutions or the Green function which has singular behavior, making it a rather complicated method. Probably, for this reason, the conventional BEM is not popular for such cases.

Assuming water as a linear compressible non-viscous fluid and the structure as a rigid solid, and considering the effect of surface waves due to ground excitation and also the complicated shape of the structure, the well-known Trefftz method can not be used directly without any modification. Therefore, the domain is divided into interior and exterior domains with a virtual boundary. The boundaries between fluid and structure and the rigid floor are satisfied by the Trefftz method in the interior domain. For exterior domain, the Trefftz method is used to satisfy the radiation conditions and the effect of water surface waves.

The two solutions based on the exterior and interior domains with a virtual boundary meet the compatibility of velocity and equilibrium of pressure conditions minimizing the error in the sense of least square method.

To obtain the final results, a linear simultaneous system of equations should be solved. The coefficients of equations are integrals around discretized boundaries of structures, water surfaces, rigid bases and virtual boundaries between the interior and the exterior of the structures. The analytical solution of submerged rigid structures under harmonic base excitation is obtained for the semi-circular cross section case. The solution is used as a benchmark to verify the proposed method. Finally, a parametric study is carried out to evaluate the influence of a submerged structure's shape on the hydrodynamic pressures due to base excitation.

THEORETICAL FORMULATION

Considering a submerged structure shown in Figure 1 in fluid domain, its response is calculated under base excitation. For simplicity, it is assumed that the shape of structure and base excitation in the y direction is uniform. Hence its response may be obtained in 2D modeling. The findings are believed to have quite wide ranging validity in the context of 3D modeling for case of non-uniform cross section or base excitation.

The following assumptions are used in the case of fluid domain:

- The fluid is linear and non-viscous;
- The flow of fluid is irrotational;
- The base of fluid is assumed as rigid base.

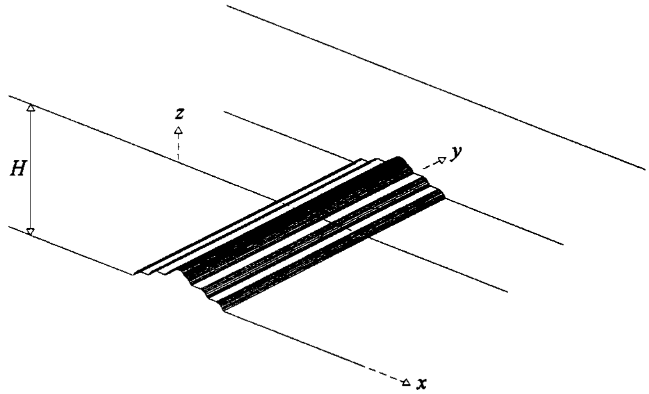

Fig. 1 - 3D description of submerged axisymmetric cylindrical structure

The hydro dynamical pressure satisfy the following equation in time domain

$$\nabla^2 \overline{p}(\xi,t) = \frac{1}{c^2}\frac{\partial^2 \overline{p}(\xi,t)}{\partial t^2} \tag{1}$$

where

∇^2 : Laplace operator

$\overline{p}(\xi, t)$: Hydrodynamic pressure

c : Sound velocity in fluid

ξ : Coordination of a general point in fluid domain

The above equation can be rewritten in terms of frequency domain that is expressed as Helmholtz equation.

$$\nabla^2 p(\xi,\omega) + k^2 p(\xi,\omega) = 0 \tag{2}$$

That $p(\xi,\omega)$ represents the complex hydrodynamic pressure in frequency domain and $k = \omega / c$, where ω is one frequency component of base excitation.

The boundary conditions which should be satisfied simultaneously with Equation 2 are:

$$\frac{\partial p(\omega,\xi)}{\partial n} = -\rho_w \ddot{u}_n(\omega,\xi) \qquad \text{on } \Gamma_1 \tag{3}$$

$$\frac{\partial p(\omega,\xi)}{\partial n} = 0 \qquad \text{on } \Gamma_2 \text{ and } \Gamma_3 \tag{4}$$

$\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4$ and Γ_υ are defined in Figure 2.

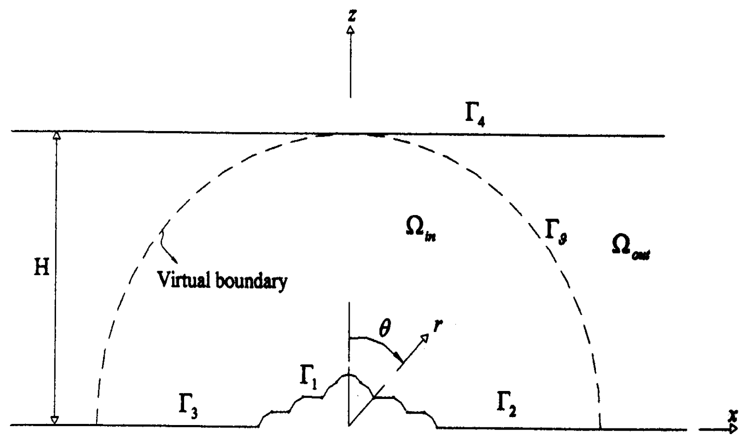

Fig. 2 - The definition of different boundary conditions

$$\frac{\partial p(\omega,\xi)}{\partial n}=\frac{\omega^2}{g}p(\omega,\xi) \qquad \text{on } \Gamma_4 \tag{5}$$

In the absence of surface wave, the above equation takes the form:

$$p(\omega,\xi)=0 \qquad \text{on } \Gamma_4 \tag{6}$$

where n is normal to structure in the outward direction of structure, $\ddot{u}_n(\xi,\omega)$ is normal acceleration component and ρ_w represents the water density and g is the gravitation acceleration. In the exterior domain, the solution should satisfy radiation boundary condition.

As shown in Figure 2, the polar coordinate system is chosen. Thus, the Equation 2 can be expressed in interior (in) domain:

$$\frac{\partial^2 p_{in}(\omega,\xi)}{\partial r^2}+\frac{1}{r}\frac{\partial p_{in}(\omega,\xi)}{\partial r}+\frac{1}{r^2}\frac{\partial^2 p_{in}(\omega,\xi)}{\partial \theta^2}+k^2 p_{in}(\omega,\xi)=0 \tag{7}$$

By satisfying the boundary condition on Γ_2 and Γ_3 and excitation condition as antisymmetric case, the solution yields to:

$$p_{in}(\omega,\xi)=\sum_{j}^{m}\left(A_j J_{\alpha j}(kr)+B_j y_{\alpha j}(kr)\right)\sin\alpha_j\theta \tag{8}$$

where $\alpha_j=2j-1$, $J_{\alpha j}(kr)$ and $y_{\alpha j}(kr)$ are the first and second kind of Bessel function respectively, with α_j as the order.

In exterior domain (out), the Equation 2 is written in Cartesian coordinate system that is also illustrated in Figure 2:

$$\frac{\partial^2 p_{out}(\omega,\xi)}{\partial x^2}+\frac{\partial^2 p_{out}(\omega,\xi)}{\partial z^2}+k^2 p_{out}(\omega,\xi)=0 \tag{9}$$

By satisfying radiation boundary conditions, Γ_2, Γ_3 and Γ_4, the solution of the above equation is given by the equation:

$$p_{out}(\omega,\xi)=\sum_{n=1}^{L} c_n \exp(-\mu_n x)\cos(\lambda_n z) \tag{10}$$

where $\lambda_n=(2n-1)\pi/2H$, $\mu_n=\sqrt{\lambda_n^2-k^2}$ and H is the depth of water. To find out the unknown coefficients $(A_n, B_n \text{ and } C_n)$, the compatibility of velocity components and equilibrium conditions are satisfied on the boundary between interior and exterior domains (Γ_υ in Figure 2) in the sense of least square method.

If $\varepsilon_1, \varepsilon_2$ and ε_3 are the errors pertinent to equilibrium and compatibility conditions in the least square method:

$$\varepsilon_1=\int_{\Gamma_\upsilon}\left(p_{out}(\xi,\omega)-p_{in}(\xi,\omega)^2\right)d\Gamma \tag{11}$$

$$\varepsilon_2=\int_{\Gamma_\upsilon}\left(\frac{\partial p_{out}(\xi,\omega)}{\partial r}-\frac{\partial p_{in}(\xi,\omega)}{\partial r}\right)^2 d\Gamma \tag{12}$$

$$\varepsilon_3=\int_{\Gamma_\upsilon}\left(\frac{\partial p_{out}(\xi,\omega)}{\partial \theta}-\frac{\partial p_{in}(\xi,\omega)}{\partial \theta}\right)^2 d\Gamma \tag{13}$$

By minimizing the errors in terms of unknown coefficients, the following equations are derived:

$$\sum_{n=1}^{L} c_n \int_0^{\frac{\pi}{2}} \exp(-\mu_n x')\cos(\lambda_n z')\sin(\alpha_i\theta)d\theta-\frac{\pi}{4}J_{\alpha i}(kH)A_i-\frac{\pi}{4}Y_{\alpha i}(kH)B_i=0 \quad i=1,2,\ldots,m \tag{14}$$

$$\sum_{n=1}^{L} c_n \int_0^{\frac{\pi}{2}} \exp(-\mu_n x')(\mu_n\cos(\lambda_n z')\sin\theta+\lambda_n\sin(\lambda_n z')\cos\theta)\sin(\alpha_i\theta)d\theta+\frac{\pi}{4}kJ'_{\alpha i}(kH)A_i+\frac{\pi}{4}kY'_{\alpha i}(kH)B_i=0$$
$$i=1,2,\ldots,m \tag{15}$$

$$\sum_{n=1}^{L} c_n \int_0^{\frac{\pi}{2}} \exp(-\mu_n x')(-\mu_n\cos(\lambda_n z')\cos\theta+\lambda_n\sin(\lambda_n z')\sin\theta)\cos(\alpha_i\theta)d\theta-\frac{\pi\alpha_i}{4H}J_{\alpha i}(kH)A_i-\frac{\pi\alpha_i}{4H}Y_{\alpha i}(kH)B_i=0$$
$$i=1,2,\ldots,m \tag{16}$$

If *L* is considered to *2m*, then the total number of equations becomes *3m* which the unknown coefficients will be *4m*. By satisfying the boundary condition on Γ_1, the *m* additional equations can be obtained as:

$$\sum_{j=1}^{m}\left(A_j\int_{\Gamma_1}S_j(r,\theta)S_i(r,\theta)d\Gamma+B_j\int_{\Gamma_1}N_j(r,\theta)S_i(r,\theta)d\Gamma\right)+\int_{\Gamma_1}\rho_w\ddot{u}_n(r,\theta)S_i(r,\theta)d\Gamma=0 \tag{17}$$

where

$$S_i(r,\theta)=kJ'_{\alpha i}(kr)\sin(\alpha_i\theta)L_r+\alpha_i J_{\alpha_i}(kr)S(\alpha_i\theta)L_\theta \tag{18}$$

$$N_i(r,\theta)=kY'_{\alpha i}(kr)\sin(\alpha_i\theta)L_r+\alpha_i Y_{\alpha_i}(kr)S(\alpha_i\theta)L_\theta \tag{19}$$

The prime indicates the derivation with regard to its argument. L_r *and* L_θ are cosine of normal direction with respect to *r* and θ coordinate system respectively. A two-node curve element is used to numerically calculate the integrals which appear in Equations 17, 18 and 19 for the arbitrary cross-section. This element is depicted in Figure 3.

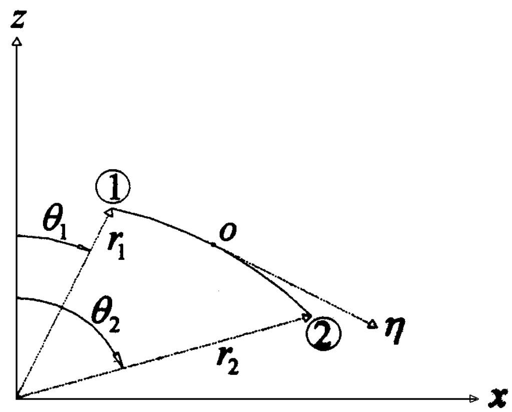

Fig. 3 - The geometrical description of a two-node curve element

The *r* and θ of each node on the element are given by

$$r = N_1 r_1 + N_2 r_2 \tag{20}$$

$$\theta = N_1 \theta_1 + N_2 \theta_2 \tag{21}$$

where $N_1 = (1-\eta)/2$, $N_2 = (1+\eta)/2$. (r_1, θ_1) and (r_2, θ_2) are the coordinates of node 1 and 2, respectively.

The normal component of ground acceleration is expressed as:

$$\ddot{u}_n(\omega, \xi) = \left(L_r\left(N_1 \sin\theta_1 + N_2 \sin\theta_2\right) + L_\theta\left(N_1 \cos\theta_1 + N_2 \cos\theta_2\right)\right)\ddot{u}_g(\omega) \tag{22}$$

where $\ddot{u}_g(\omega)$ is the ground acceleration in frequency domain. The values of L_r and L_θ at each point of the element are:

$$L_r = \frac{J_r}{|J|} \tag{23}$$

$$L_\theta = \frac{J_\theta}{|J|} \tag{24}$$

where $|J| = \sqrt{J_r^2 + J_\theta^2}$, $J_r = \left(N_1 r_1 + N_2 r_2\right)\left(-\theta_1/2 + \theta_2/2\right)$ and $J_\theta = r_1/2 - r_2/2$.

VERIFICATION OF THE FORMULATION

Considering the cross section of structure as semi-circular shown in Figure 4, the problem can be solved analytically. Figure 5 illustrates the vertical distribution of hydrodynamic pressure for $r_0/H=0.1$ and $\omega H/c=0.1$.

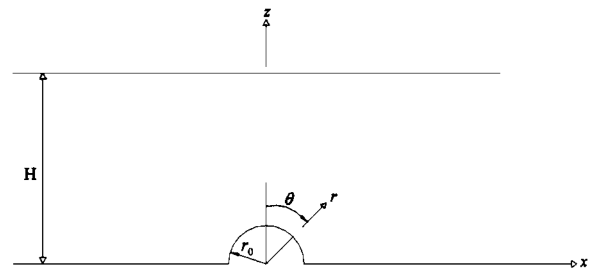

Fig. 4 - The geometrical description of a semi-circular structure

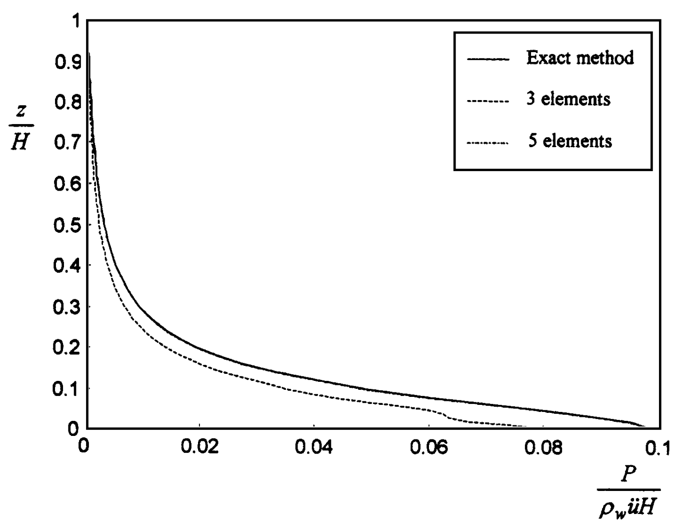

Fig. 5 - The vertical distribution of a semi-circular structure for $r_0/H=0.1$ and $\omega H/c=0.1$

It can be seen that the results appear to be consistent with the exact solution when 5 elements are used. The high rate of convergency is observed by comparing the results of 3 elements modeling with 5 element modeling.

PARAMETRIC STUDY

The purpose of the parametric study is to examine the influence of different parameters on the behavior of submerged structures in the case of base motion. Three factors are investigated including the r_0/H, dimensionless frequency ($\omega H/c$) and different cross section types on hydrodynamic pressure.

Effect of r_0/H

Figure 6 shows the effect of r_0/H on the vertical distribution of hydrodynamic pressure (HP) of semi-circular cross section structure for different r_0/H in the case of $\omega H/c=0.1$.

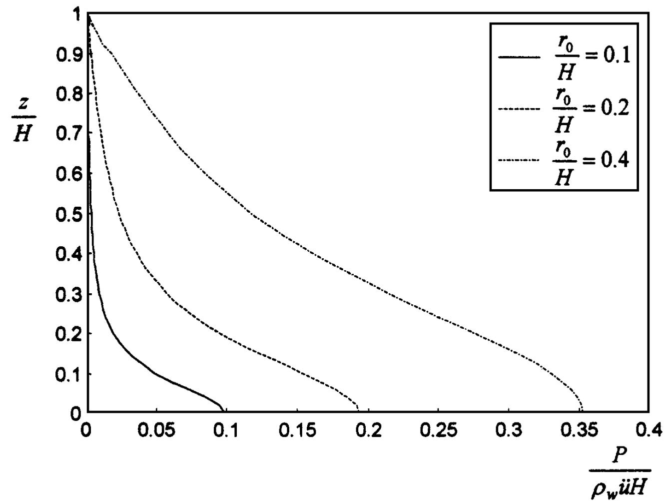

Fig. 6 - The vertical distribution of HP for ωH/c=0.1 for different r_0/H

It is obvious that by increasing r_0/H, the magnitude of hydrodynamic pressure also increases. However, when $\omega H/c< \pi/2$, where $\pi/2$ is dimensionless cutoff frequency, there is no imaginary part of HP. This fact is shown in Figures 7a and 7b.

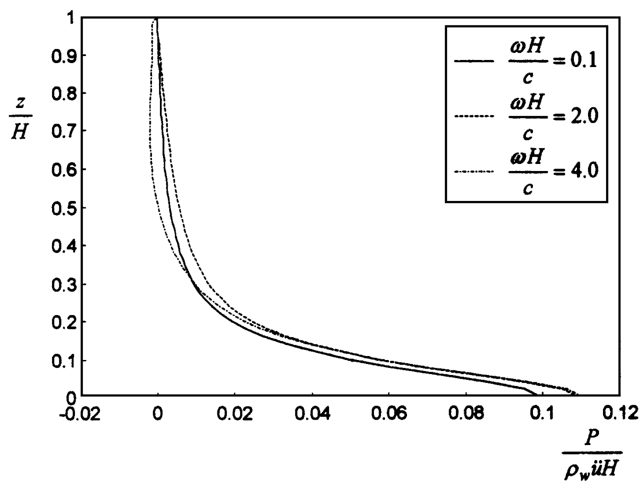

Fig. 7a - The real part of vertical distribution of HP for r_0/H=0.1 and different ωH/c

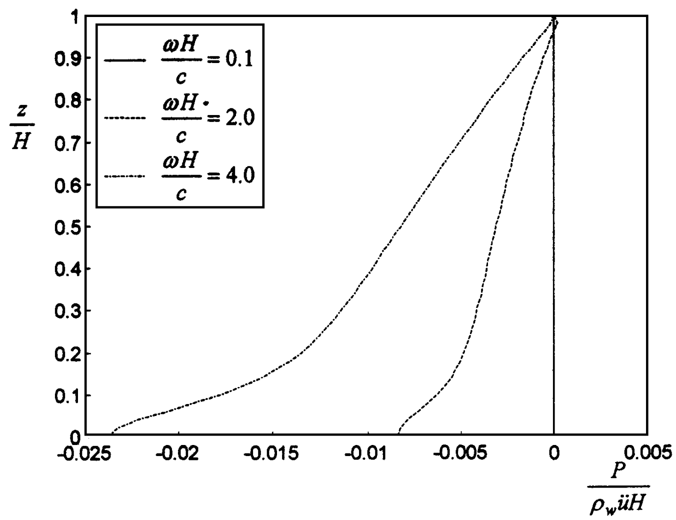

Fig. 7b - The imaginary part of vertical distribution of HP for $r_0/H=0.1$ for different $\omega H/c$

Effect of dimensionless frequency

To take into account of the dimensionless frequency ωH/c on hydrodynamic pressure of semi-circular cross-section, different values of r_0/H are selected. The real and the imaginary parts of vertical distribution of hydrodynamic pressure are presented in Figures 8a and 8b, respectively. The results show that the resonant frequency is equal to π/2. From these figures, it is noted that at the lower rate of ωH/c, the difference between the corresponding hydrodynamic pressures is high; however, with the increase of ωH/c, the variation becomes negligible.

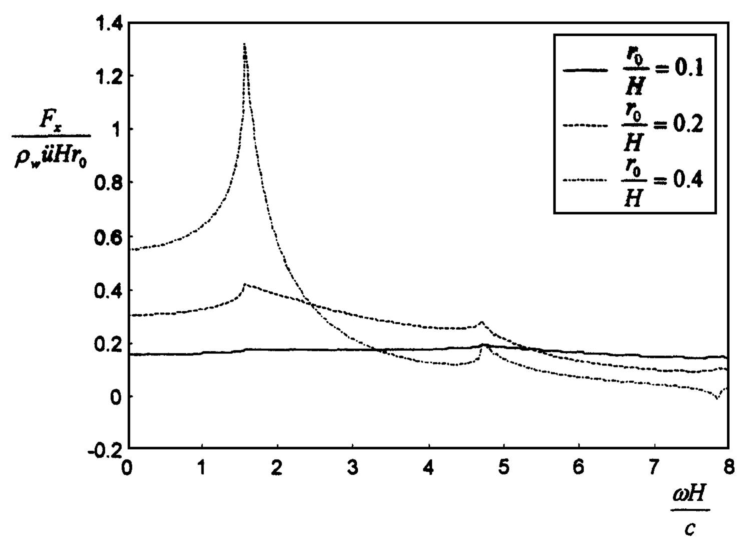

Fig 8a - The real part of vertical distribution of HP in terms of dimensionless frequency for different r_0/H

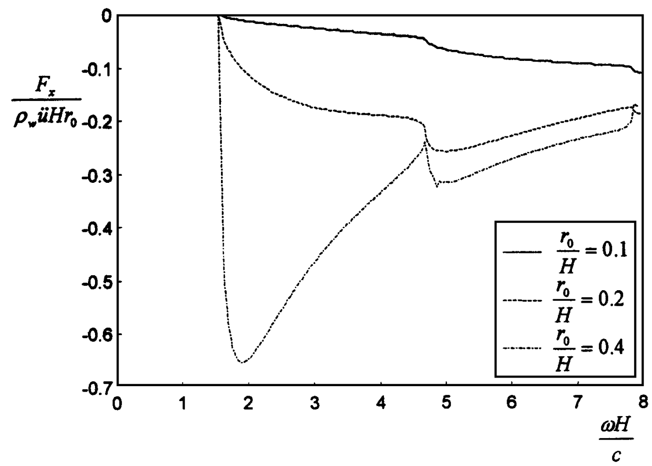

Fig 8b - The imaginary part of vertical distribution of HP in terms of dimensionless frequency for different r_0/H

Effect of different cross-section shape on HP

In order to assess the effect of cross-section shape on hydrodynamic pressure, the semi-elliptic submerged structure is considered as shown in Figure 9.

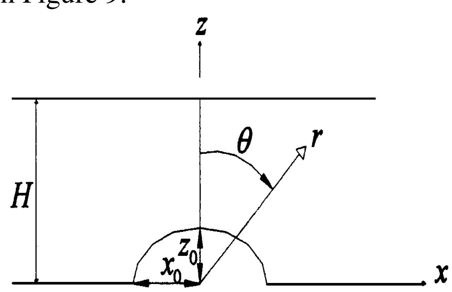

Fig. 9 - Semi-elliptic submerged structure

Figure 10a presents the isopressure for $x_0/H=0.15$, $z_0/H=0.1$and $\omega H/c=0.1$for real part of hydrodynamic pressure of elliptic cross-section. The obtained results can be compared with the results of semi-circular cross section as plotted in Figure 10b.

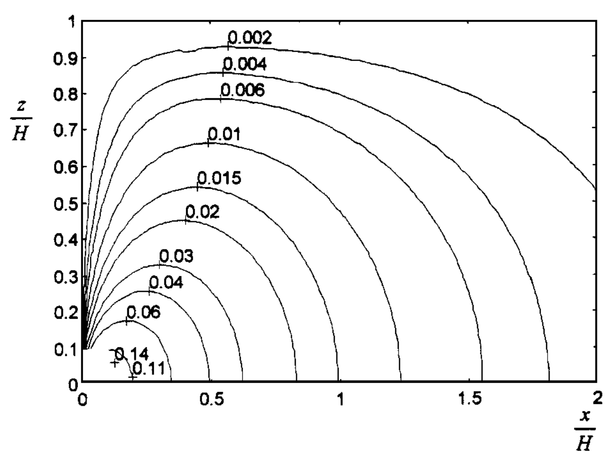

Fig. 10a - The real part of vertical distribution HP for semi-elliptic cross section with $x_0/H=0.15$, $z_0/H=0.1$ and $\omega H/c=0.1$

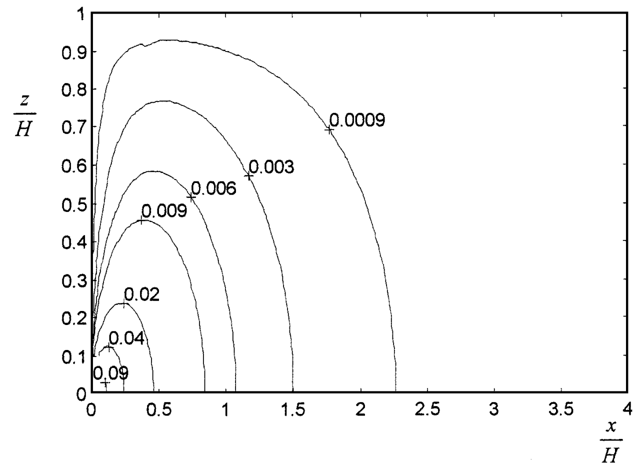

Fig. 10b - The vertical distribution HP for semi-circular cross section with $r_0/H=0.1$ and $\omega H/c=0.1$

CONCLUSION

On the basis of the proposed method through this study, the following points can be drawn:

(a) The performance of this approach has been assessed by comparison with results obtained from analytical solution. It is concluded that for the practical problem in hand, the approach is adequate for the range of different types of submerged structures under consideration;

(b) It has been shown that the Trefftz method produces an increase of the convergence in the analysis of dynamic behavior of submerged structures;

(c) It is found that the resonant dimensionless frequency namely, cutoff frequency for different ratios of r_0 /H is the same and equal to $\pi/2$;

(d) For semi-elliptic cross-section, similar to semi-circular cross-section, the maximum pressure occurs in the middle of fluid and structure boundaries in both cases.

REFERENCES

1. Szczesiak, T., Weber, B. "Hydrodynamic effects in a reservoir with semi-circular cross-section and absorptive bottom." Soil Dynamics and Earthquake Engineering 1992; 11:203-212.
2. Sharan, S. K. "Finite element modeling of infinite reservoirs." Journal of Engineering Mechanics, ASCE, 1985; 111 (8): 1457-1469.
3. Westergard, H. M. "Water pressure on dams during earthquakes." Transactions ASCE, 1933; 98: 418-472.
4. Noorzad, A., Noorzad, Ali and Mortezazadeh, A. A. "Hydrodynamic pressure of compressible fluid on unsubmerged structures under earthquake loading utilizing Trefftz boundary method." Proceedings of the 4th International Conference on Seismology and Earthquake Engineering (SEE4), 2003; 455-462.
5. Trefftz, E. "Ein gegenstruck zum ritzschem verfohren." Proceedings of the Second International Congress in Applied Mechanics, Zurich, 1956.