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REVIEW OF CODE PROVISIONS TO ACCOUNT FOR EARTHQUAKE INDUCED TORSION

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SUMMARY

It is well known that torsional oscillations during an earthquake may cause severe distress in a building structure. Most seismic building codes therefore include some provisions for the design of structures to resist the forces induced by torsional vibrations. Provisions of several codes on design against torsion are reviewed. The review includes the National Building Code of Canada (NBCC), 1995; the Uniform Building Code (UBC), 1997; the National Earthquake Hazard Reduction Program (NEHRP), 1997; New Zealand Standard (NZS), 1992; Mexico Code, 1993; and a set of provisions being considered for NBCC 2005. A mono-symmetric single-story building model is analyzed for its elastic response to ground motions represented by idealized spectral shapes, and for its inelastic response to a set of 16 recorded earthquake ground motions. The results of the analytical studies are used to examine the effectiveness of various design provisions.

INTRODUCTION

Damage reports on recent earthquakes, including the 1985 Mexico earthquake, Esteva [1], the 1989 Loma Prieta earthquake, Mitchell [2], the 1994 Northridge earthquake, Mitchell [3], and the 1995 Kobe earthquake, Mitchell [4], have indicated that one major cause of distress in building structures is the torsional motion induced by the earthquake. This has renewed interest in the study of torsional response of buildings. A large number of research studies have been carried out in the past on elastic and inelastic torsional response of building models. However, perhaps due to the complexity of torsional behavior, particularly in the inelastic range, findings of various studies have not always been consistent, leading to widely differing torsional provisions in different building codes.

A recent study by Humar and Kumar [5, 6] has shown that certain parameters that govern the torsional response have not been given the attention they deserve. The most important of these is the torsional stiffness as measured by the ratio of uncoupled torsional frequency to the uncoupled lateral frequency, yet building codes generally do not contain any explicit provision in respect of the torsional stiffness, or of the frequency ratio. Based on their studies, the authors have proposed new torsion design provisions that represent some improvement, are simple to apply, and are not very different from the now familiar

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provisions of some of the existing codes. These proposals form the basis for the provisions in NBCC 2005.

The objective of this paper is to review the torsion design provisions in selected building codes, and compare them with the newly suggested provisions. Five building codes have been selected for this study: (1) National Building Code of Canada, NBCC 1995, (2) Uniform Building Code, UBC 1997, (3) National Earthquake Hazard Reduction Program Recommended Provisions, NEHRP 1997 (4) New Zealand Standards NZS 4203-1992, and (5) Mexico Code 1993. A mono-symmetric, single-story, shear type building model is studied for its elastic response to a design ground motion represented by an idealized spectrum, and inelastic response to a set of 16 ground motions. Studies on the elastic model are used to provide a comparison of the design eccentricity expressions in codes with the effective edge eccentricities obtained from response spectrum analyses, while studies on the inelastic model provide a comparison of the ductility demands at the edge elements of a torsionally unbalanced model, designed as per the torsional provisions of building codes, with those in the associated torsionally balanced building models. In all cases building models with a wide range of the values of eccentricity and frequency ratio are selected for study.

REFERENCE BUILDING MODEL

For comparative evaluation of the various code provisions, we use the simple single-story building model shown in Fig. 1. In this model, the building floor is assumed to be infinitely rigid in its own plane. The entire mass of the structure is uniformly distributed at the floor level. The origin of the coordinate axes considered in the analysis is at the mass center, denoted by CM. The mass center is located at the geometric center of the floor. Forces opposing the motion are provided by vertical in-plane resisting elements oriented along the two orthogonal axes. The in-plane resisting elements, referred to herein as resisting planes or simply planes, may comprise columns, shear walls, braced frames or a combination thereof. The *i*th plane parallel to the *x* axis has a stiffness k_{xi}, while the *i*th plane in the *y* direction has a stiffness k_{yi}. The distribution of stiffnesses is symmetrical about the *x* axis, but is unsymmetrical about the *y* axis. Thus the center of stiffness, or center of rigidity (CR), lies on the *x* axis at a distance *e* from the center of mass. In Fig. 1 Plane 1 is less stiff than Plane 3, hence CR is closer to Plane 3 than to Plane 1. In the following, Plane 1 is referred to as the flexible plane or the flexible edge plane, while Plane 3 is referred to as the stiff plane or the stiff edge plane. For translational motion in the *y* direction, the elastic force in a *y*-direction resisting plane is proportional to the plane's stiffness. Hence, the center of resistance coincides with the center of stiffness.

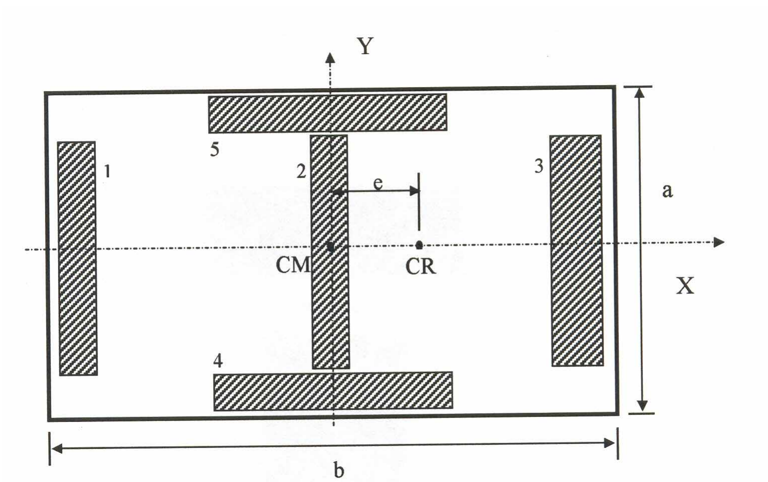

Fig. 1: Single story building model

It is assumed that earthquake ground motion is directed along the y axis. The dimension of the floor perpendicular to the direction of earthquake is b, and that parallel to the earthquake is a. The floor aspect ratio is given by a/b. The mass of the floor is m; r is the radius of gyration of the floor about CM; $K_y = \sum_{i=1}^{N} k_{yi}$ is the total stiffness in the y direction; and $K_{\theta R}$ is the torsional stiffness about CR.

We define an uncoupled translational frequency ω_y and an uncoupled rotational frequency ω_θ given by

$$\omega_y = \sqrt{\frac{K_y}{m}} \qquad \omega_\theta = \sqrt{\frac{K_{\theta R}}{mr^2}} \tag{1}$$

The ratio of the uncoupled frequencies is denoted by Ω_R, so that $\Omega_R = \omega_\theta / \omega_y$.

The building model described in the previous paragraphs is referred to as an asymmetric or torsionally unbalanced model. For the purpose of evaluating the effect of torsion, we also define an associated torsionally balanced model. Such a model has the same K_y and m as the unbalanced model but has coincident CR and CM.

CODE PROVISIONS

Most seismic codes specify a simple equivalent static load method for design against earthquake forces. The static load methods also include provisions for torsion induced in an asymmetric building. These provisions usually specify values of design eccentricities that are related to the static eccentricity between the center of stiffness and the center of mass. The earthquake-induced shears are applied through points located at the design eccentricities. A static analysis of the structure for such shears provides the design forces in the various elements of the structure. In some codes the design eccentricities include a multiplier on the static eccentricity to account for possible dynamic amplification of the torsion. Also, the design eccentricities often include an allowance for accidental torsion. Such torsion is supposed to be induced by the rotational component of the ground motion and by possible deviation of the centers of stiffness and mass from their calculated positions.

The design eccentricity formulae given in building codes can be written in the following form

$$e_{fc} = \alpha e + \beta b \tag{2a}$$

$$e_{sc} = \gamma e - \beta b \tag{2b}$$

where e_{fc} and e_{sc} are the design eccentricities, and α, β, and γ are coefficients, that have different values in different building codes.

The first term in the expressions for a design eccentricity represents natural torsion, while the second term is supposed to represent accidental torsion. Factors α and γ are applied to the static eccentricity e to take into account the effects of dynamic torque amplification. Accidental torsion, which can be assessed only in an indirect manner, is taken as a fraction of the plan dimension b.

In general, eccentricity e_{fc} applies to the flexible edge of the building, while e_{sc} applies to the stiff edge. It may, however, be noted that if the building is torsionally very flexible, Eq. 2a may lead to a higher force in the stiff edge plane than Eq. 2b. In such a case e_{fc} is the critical design eccentricity for the stiff edge plane as well.

NBCC 1995

The design eccentricities in NBCC 1995 are obtained from Eq. 2a and b with $\alpha = 1.5$, $\beta = 0.1$ and $\gamma = 0.5$. NBCC suggests that as an alternative to the use of floor torques equal to the product of floor forces and the corresponding design eccentricities, a 3-D dynamic analysis may be carried out to evaluate the effect of torsion. When a dynamic analysis procedure is used, accidental torsion can be accounted for by applying a torque equal to floor force times $0.1b$ at each floor. The forces produced by these torques should be added to or subtracted from the forces obtained from 3-D analysis to obtain the maximum design force for each resisting element. The results presented in this paper show that the NBCC provisions are too conservative for the design of flexible edge plane, but may be inadequate for the design of stiff edge plane in a torsionally flexible building.

UBC 1997

The design eccentricity coefficients specified in UBC 1997 are: $\alpha = 1.0$, $\beta = A_x(0.05b)$, and $\gamma = 1.0$. It will be noted that the accidental torsion has been amplified by factor A_x. In earlier versions of UBC such amplification was not present. This led to unconservative results in some cases. The introduction of amplification on accidental eccentricity has corrected the situation in most cases. However, for torsionally flexible buildings the design provisions for the stiff edge plane may still be unconservative. Factor A_x is determined from the following equation

$$A_x = \left(\frac{\delta_{\max}}{1.2\delta_{\text{avg}}} \right)^2 \tag{3}$$

where $\delta_{\max}$ is the maximum displacement of the floor produced by the equivalent static earthquake forces, and δ_{avg} is the average of the displacements of the extreme points of the structure. In calculating $\delta_{\max}$ the effect of accidental torsion must be accounted for. The code does not clarify how δ_{avg} is to be calculated. It is assumed here that accidental torsion need not be included while calculating δ_{avg}. With this assumption, two separate displacement calculations must be carried out to determine A_x.

To determine $\delta_{\max}$ shear V_0 is applied at a distance $e + 0.05b$ from CR, giving

$$\delta_{\max} = \frac{V_0}{K_y} + \frac{V_0(e + 0.05b)}{K_{\theta R}} \left(\frac{b}{2} + e \right) \tag{4}$$

For determining δ_{avg} shear V_0 is applied through the center of mass. This gives

$$\begin{aligned} \delta_{\text{avg}} &= \frac{1}{2} \left[\frac{V_0}{K_y} + \frac{V_0 e}{K_{\theta R}} \left(\frac{b}{2} + e \right) + \frac{V_0}{K_y} - \frac{V_0 e}{K_{\theta R}} \left(\frac{b}{2} - e \right) \right] \\ &= \frac{V_0}{K_y} + \frac{V_0 e^2}{K_{\theta R}} \end{aligned} \tag{5}$$

Substituting Eqs. 4 and 5 in Eq. 3 we get

$$A_x = \left[\frac{1 + \dfrac{1}{\Omega_R^2} \left(\dfrac{b}{r}\right)^2 \left(\dfrac{e}{b} + 0.05\right)\left(0.5 + \dfrac{e}{b}\right)}{1.2\left\{1 + \dfrac{1}{\Omega_R^2}\left(\dfrac{e}{b}\right)^2 \left(\dfrac{b}{r}\right)^2\right\}} \right]^2 \tag{6}$$

The UBC also provides that A_x may not be taken as less than 1 and need not be greater than 3.

NEHRP 1997

The torsion provisions of UBC 1997 were based on NEHRP 1994. The 1997 version of NEHRP contains slightly revised provisions for design against torsion. With this revision, the amplification factor A_x is applied to both the natural and the accidental torsion components of the design eccentricities, not just to the accidental torsion component. The design eccentricity coefficients thus become: $\alpha = A_x$, $\beta = 0.05A_x$ and $\gamma = A_x$.

It is of interest to note that the revised expression for the stiff edge design eccentricity, A_x (e-0.05b), leads to a smaller force in the stiff edge plane than does (e-0.05b), unless the eccentricity e is smaller than 0.05b. The provisions of NEHRP thus lead to a design that has a weak stiff edge plane. On the other hand, in most cases the amplification of both the natural and the accidental eccentricity leads to excessive conservatism in the design of flexible edge plane. It appears that the revised provisions in NEHRP 1997 do not represent an improvement. This will be evident from the results of elastic and inelastic response analyses presented later in this paper.

The variation of the ratio $\delta_{\max}/\delta_{\text{avg}}$ with e for several different values of Ω_R and two values of the floor aspect ratio is shown in Fig. 2. It may be noted that the UBC and NEHRP provisions discourage the use of structural layouts having $\delta_{\max}/\delta_{\text{avg}} > 1.4$ and, in fact, prohibit their use for seismic design categories E and F. Referring to Fig. 2a, it is seen that for an aspect ratio of 1.0, structures with $\Omega_R = 0.75$ are prohibited except when the eccentricity is quite small (e/b < 0.025). Structures with $\Omega_R = 1.0$ may be used provided e/b < 0.075. When $\Omega_R = 1.5$, structures with eccentricities up to 0.275 are acceptable. From Fig. 2b, it will be evident that for an aspect ratio of 0.5, structures with $\Omega_R = 0.75$ will be entirely prohibited. Even with $\Omega_R = 1.5$, the structural system is acceptable only when $e/b \le 0.15$

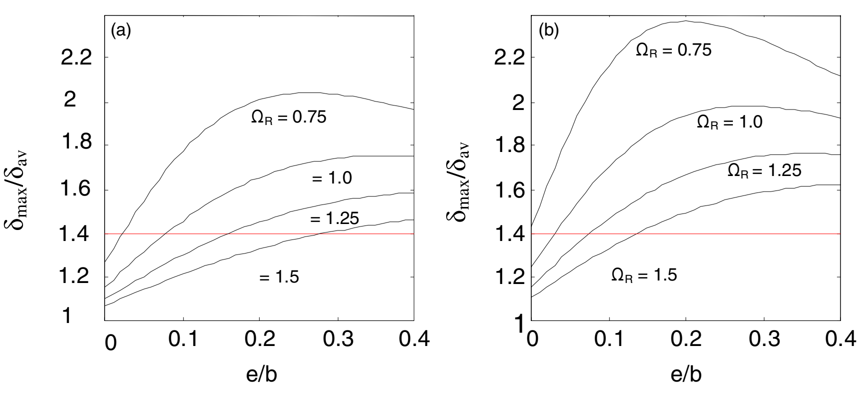

Fig. 2: Variation of ratio $\delta_{\max}/\delta_{\text{avg}}$ with the eccentricity ratio e/b; aspect ratio (a) 1.0, (b) 0.5

NEW ZEALAND STANDARD

The torsion design provisions of New Zealand Standard (NZS 1992) specify the use of design eccentricity expressions Eq. 4a and b with $\alpha = \gamma = 1$ and $\beta = 0.1$. However, NZS allows the use of an equivalent static lateral load method only when one of the following horizontal regularity criteria are satisfied: (1) $e \leq 0.3b$ and eccentricity does not change its sign over the height of the building; and (2) under the action of equivalent static loads applied at a distance $e_d = e \pm 0.1b$ from CR, the ratio of horizontal displacements at the ends of an axis at any horizontal plane transverse to the direction of forces is in the range of 3/7 to 7/3. The results presented here will show that the provisions of the New Zealand standard are satisfactory in most cases; however, they may be unconservative for the design of stiff edge plane in a torsionally flexible building.

MEXICO CODE

Mexico Code 1993 specifies the use of Eqs. 2a and b with $\alpha = 1.5$, $\gamma = 0.5$, and $\beta = 0.1$. The Code also requires that for buildings that do not possess geometrical and structural regularity, the force reduction factor Q that is applied to the elastic forces to obtain the design forces should be multiplied by 0.8. The requirements of regularity that are relevant to the current study are: (1) $e/b \leq 0.1$, and (2) $a/b \geq 0.4$. This implies that for building models with a static eccentricity greater than 0.1b, or an aspect ratio less than 0.4, the total design strength should be increased by 25%.

In addition to the above, the Mexico Code requires that for $Q \geq 3$, the centroid of the strength of lateral load-resisting elements should be located on the same side of the point of application of shear force as the center of twist, and that

$$\begin{aligned} e_p &\geq e - 0.2b \qquad \text{if } Q = 3 \\ e_p &\geq e - 0.1b \qquad \text{if } Q > 3 \end{aligned} \tag{7}$$

where e_p is strength eccentricity measured from CM.

In the present study Q has been taken as 4. Restrictions on strength eccentricity therefore imply that for the three-plane building model, comprising planes 1, 2 and 3 as shown in Fig. 1, the following steps be used to determine the strength of the stiff edge plane.

Design planes 1, 2 and 3 according to the provisions of Mexico Code without regard to the restrictions in the preceding paragraph. Calculate the strength eccentricity from

$$e_p = \frac{(V_3 - V_1)b}{2(V_1 + V_2 + V_3)} \tag{8}$$

where V_i is the strength of plane i. If $e - e_p > 0.1b$, assume $e_p = e - 0.1b$ and reevaluate V_3 as follows

$$V_3 = \frac{2(e_p/b)(V_1 + V_2) + V_1}{1 - 2(e_p/b)} \tag{9}$$

It will be seen that the provisions of the Mexico Code are conservative in most cases, at times excessively so.

NEW PROVISIONS

In a series of recent studies, Humar and Kumar [5, 6, 7] have observed that torsion provisions in codes are, in general, very conservative for the design of flexible edge plane, but, at times, may not be adequate for the design of stiff edge planes. Based on this observation, and a large number of elastic and inelastic response analyses they have suggested the following alternative provisions

$$e_{fc} = e + 0.1b \tag{10}$$

$$e_{sc} = e - 0.1b \qquad \text{for} \quad \Omega_R \geq 1.0 \tag{11a}$$

$$e_{sc} = -0.1b \qquad \text{for} \quad \Omega_R < 1.0 \tag{11b}$$

It may be noted that a positive eccentricity is measured in the same direction of the center of resistance as the center of mass is. Again, for torsionally flexible buildings Eq. 10 rather than Eq. 11 may govern the design of the stiff edge plane.

Equations 10 and 11 were derived by simple curve fitting to the results of the response analysis. In doing so it was recognized that no simplified code provisions could accurately reflect the highly complex phenomenon of torsion, particularly in a building expected to be strained into the inelastic range. Excessive refinement in curve fitting was therefore not justified. It was also recognized that given the simplification inherent in a code design provision it would be prudent not to depart radically from the format of the existing design provisions that had become familiar to the design community. It was found that the provisions of New Zealand code were fairly satisfactory except for two factors. First, there was the need to emphasize the provision of adequate torsional stiffness as a desirable design goal. Second, the New Zealand design provisions were found to be unconservative for the design of stiff edge planes in a torsionally flexible building. These two factors were addressed in a simple manner by the introduction of design eccentricity provisions given by Eqs. 11a and b.

Equations 10 and 11a form the basis for the seismic design provisions of NBCC 2005. The restrictions implied in Eqs. 11a and b are incorporated by requiring that a dynamic analysis be used for buildings that are torsionally flexible.

ANALYSIS OF ELASTIC MODELS

To assess the torsional response of asymmetric elastic models, response spectrum analyses of the building models are carried out for earthquake input represented by an idealized spectrum. It should be noted that for torsionally unbalanced models correlation may exist between the translational and torsional modes. In such cases the simple square root of the sum of squared modal responses may give inaccurate estimate of the response. In the analytical results presented here the complete quadratic combination was used instead to combine the modal responses. Details of the analysis procedure have been provided by Humar [5].

Two kinds of spectral shapes are used in the analysis: (1) a flat spectrum, and (2) a hyperbolic spectrum. The response spectrum analysis provides the maximum flexible edge displacement Δ_f and the maximum stiff edge displacement Δ_s. It is useful to normalize Δ_f and Δ_s by the displacement Δ_0 of the associated torsionally balanced structure when subjected to the same earthquake motion. Thus we have

$$\overline{\Delta}_f = \frac{\Delta_f}{\Delta_0} \qquad \overline{\Delta}_s = \frac{\Delta_s}{\Delta_0} \tag{12}$$

We now define an effective eccentricity e_f as the distance from CR at which the application of base shear V_0 would produce a flexible edge displacement Δ_f, and eccentricity e_s as the distance from CR at which the application of V_0 would produce a stiff edge displacement of Δ_s. It can be shown that

$$\frac{e_f}{b} = \left(\overline{\Delta}_f - 1\right) \frac{\Omega_R^2}{\left(\frac{b}{r}\right)^2 \left(0.5 + \frac{e}{b}\right)} \tag{13}$$

and

$$\frac{e_s}{b} = \left(1 - \overline{\Delta}_s\right) \frac{\Omega_R^2}{\left(\frac{b}{r}\right)^2 \left(0.5 - \frac{e}{b}\right)} \tag{14}$$

The effective eccentricities given by Eqs. 13 and 14 can be compared with the design eccentricities given in the various code provisions.

ACCIDENTAL TORSION

As stated earlier, the code provisions include an allowance for accidental torsion. Thus, for a proper comparison between the code specified eccentricities and the effective eccentricities derived from a dynamic analysis, the latter should also include the effect of accidental eccentricity. Recent studies by De La Llera and Chopra [8] have shown that the effect of ground rotational motion is quite small and may be neglected. Accidental torsion induced by uncertainties in the distribution of mass and/or stiffness may be accounted for by modifying the analytical model used in the dynamic analysis. In fact two different modified models are used, one in which the CM is shifted by +0.05*b* from its original position and the second in which the CM is shifted by -0.05*b* from its original position. The larger of the forces obtained in a resisting plane from the two sets of analysis is taken as the design force.

As stated above an eccentricity of 0.05*b* may be sufficient to account for accidental torsion. In this context the term 0.1*b* in Eqs. 10 and 11a should not be viewed as having been provided only to take care of the accidental torsion. Equations 10 and 11a should rather be considered as empirical expressions that account for the combined effect of the natural and the accidental torsion.

RESULTS OF ELASTIC ANALYSIS FOR A FLAT SPECTRUM

The results of analytical studies based on a flat spectrum and an aspect ratio of 1 are presented in Fig. 3, which shows the effective eccentricity e_f/b as a function of the static eccentricity e/b for various values of Ω_R. For purpose of comparison, the design eccentricities specified in the various codes are also shown. It may be noted that a design eccentricity that is larger than the effective eccentricity obtained from the dynamic analysis provides a conservative estimate of the design force in the flexible edge element. The results show that the provisions of NBCC and the Mexico Code are quite conservative. The provisions of NEHRP are also conservative for larger values of *e*/*b*, at times even more conservative than NBCC and Mexico Code, but may be somewhat unsafe for low *e*/*b* values. The provisions of UBC are close to the dynamic analysis results but may be somewhat unsafe for larger values of Ω_R (1.25 and 1.50). The new provisions and those of the New Zealand Standard are closest to the results of dynamic analysis and may be considered as being adequate for all values of *e*/*b* and Ω_R considered. Results for aspect ratios of 1/3 and 3, not presented here, show trends that are similar to those for an aspect ratio of 1.

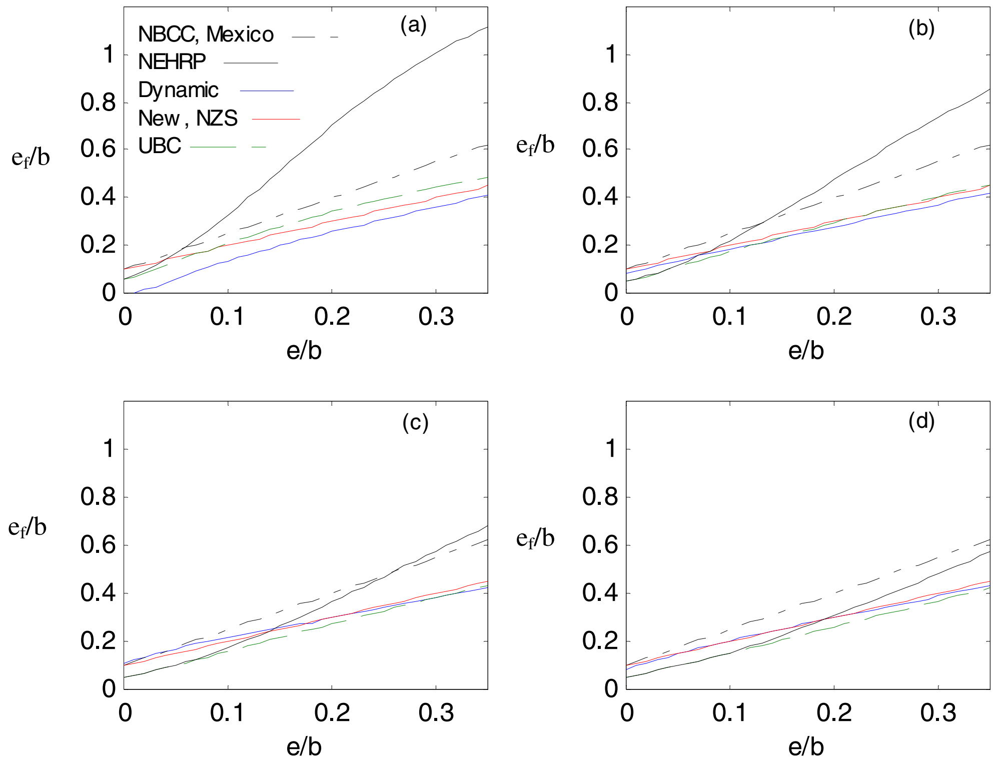

Fig. 3: Variation of effective flexible edge eccentricity with the static eccentricity for a flat spectrum and aspect ratio of 1: (a) Ω_R = 0.75, (b) Ω_R = 1.0, (c) Ω_R = 1.25, (d) Ω_R = 1.50.

Figure 4 shows the effective eccentricity e_s/b as a function of the static eccentricity e/b for various values of Ω_R and an aspect ratio of 1. For purpose of comparison, the design eccentricities specified in the various codes are also shown. It may be noted that in this case a design eccentricity that is smaller than the effective eccentricity obtained from the dynamic analysis provides a conservative estimate of the design force in the stiff edge element. The results show that for $\Omega_R = 0.75$ there are wide discrepancies in the effective eccentricities obtained from dynamic analyses and those specified in the codes. The provisions of NEHRP, UBC, NZS, Mexico Code and NBCC may be unsafe for a range of values of *e*/*b*. On the other hand, the new provisions are quite conservative for the entire range of values of *e*/*b*.

For $\Omega_R = 1.0$, the new provisions and those of NZS, UBC, Mexico Code, and NBCC are all safe, the NBCC provisions being the most conservative. The NEHRP provisions are inadequate. For higher values of Ω_R, all of the design provisions are quite conservative. Again, NBCC and Mexico Code provisions are the most conservative. Results for aspect ratios of 1/3 and 3 show trends that are similar to those for an aspect ratio of 1.

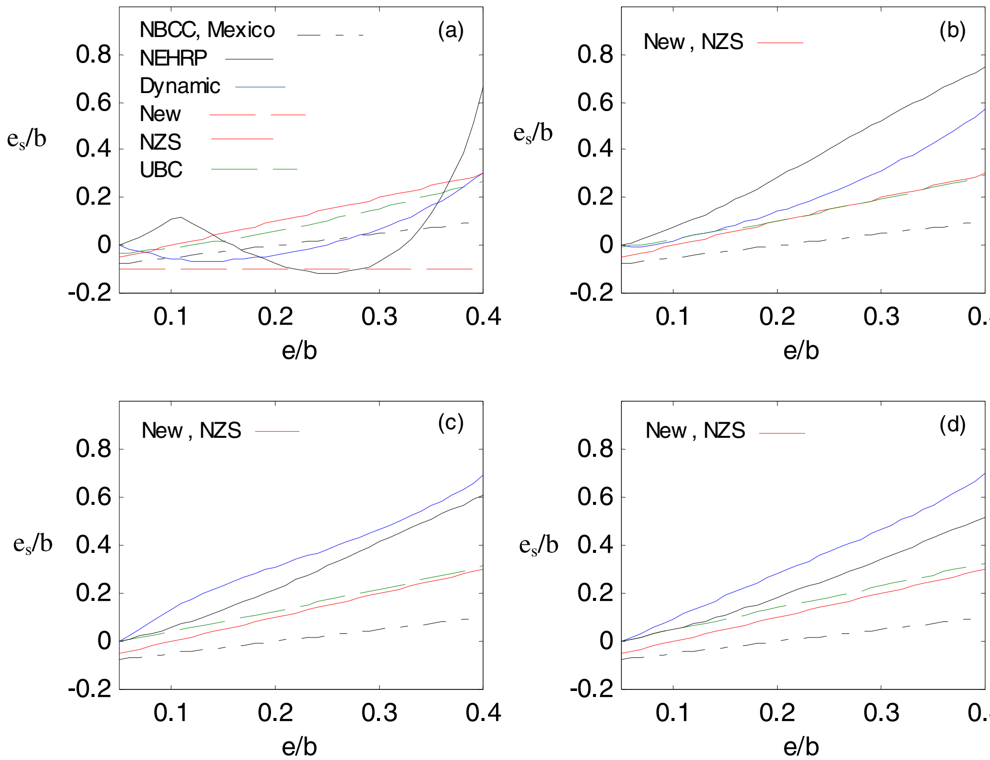

Fig. 4: Variation of effective stiff edge eccentricity with the static eccentricity for a flat spectrum and aspect ratio of 1: (a) Ω_R = 0.75, (b) Ω_R = 1.0, (c) Ω_R = 1.25, (d) Ω_R = 1.50.

RESULTS OF ELASTIC ANALYSIS FOR A HYPERBOLIC SPECTRUM

Analytical studies are repeated for a hyperbolic spectrum and various values of Ω_R and the aspect ratio. The effective flexible edge eccentricities for an aspect ratio of 1 are shown in Fig. 5. For Ω_R = 0.75 and 1.0 all of the design provisions are quite conservative. For $\Omega_R = 1.25$ and 1.50 the design provisions are fairly conservative, except that when *e*/*b* is small UBC and NEHRP provisions may be somewhat unsafe. Results for aspect ratios of 1/3 and 3 show trends that are similar to those noted for an aspect ratio of 1.

The effective stiff edge eccentricities for an aspect ratio of 1 are shown in Fig. 6. For $\Omega_R = 0.75$ the UBC and NZS provisions are unsafe. The NEHRP provisions are conservative for intermediate values of *e*/*b*, but may be unsafe for high and low *e*/*b*. The NBCC and Mexico Code provisions are adequate, but slightly unconservative for a range of eccentricities. The new provisions are quite conservative. For $\Omega_R = 1.0$, the UBC and NZS provisions as well as the new provisions are adequate, or slightly unconservative. The NEHRP provisions are unsafe, while the NBCC and Mexico Code provisions are quite conservative. For $\Omega_R = 1.25$ and 1.5 all of the provisions are conservative.

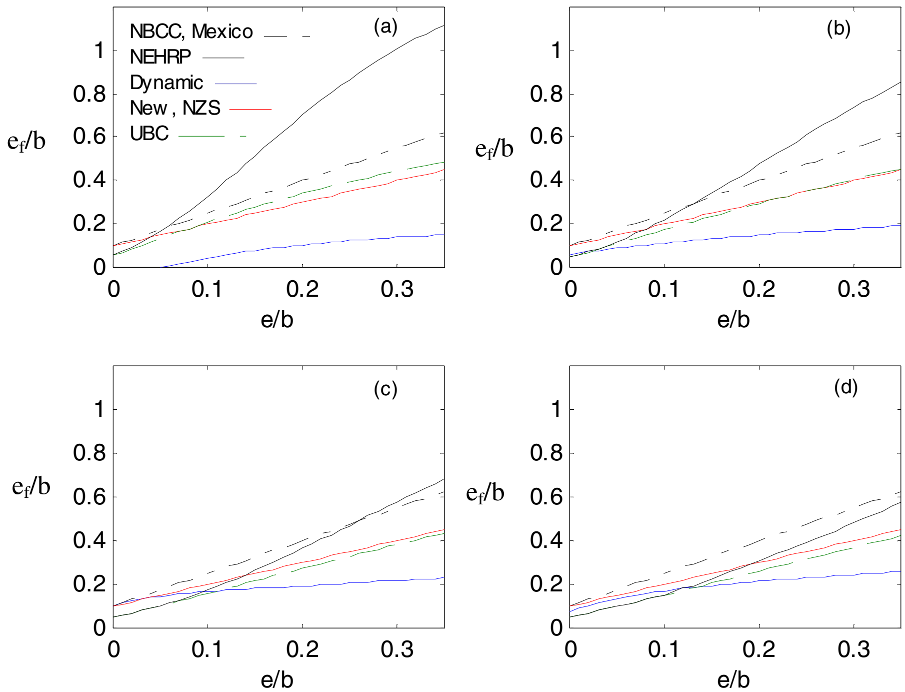

Fig. 5: Variation of effective flexible edge eccentricity with the static eccentricity for a hyperbolic spectrum and aspect ratio of 1: (a) Ω_R = 0.75, (b) Ω_R = 1.0, (c) Ω_R = 1.25, (d) Ω_R = 1.50

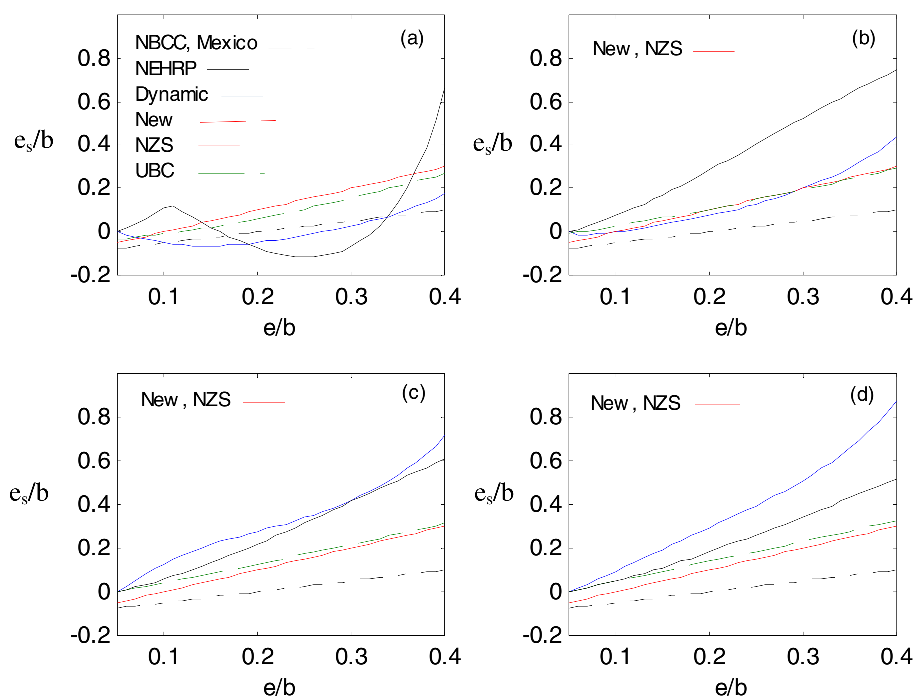

Fig. 6: Variation of effective stiff edge eccentricity with the static eccentricity for a hyperbolic spectrum and aspect ratio of 1: (a) Ω_R = 0.75, (b) Ω_R = 1.0, (c) Ω_R = 1.25, (d) Ω_R = 1.50

ANALYSIS OF INELASTIC MODELS

A single-story building similar to that shown in Fig. 1, and having three resisting planes in the y direction but only one central resisting plane along the x axis is studied for its inelastic response to a set of 16 recorded ground motions. Details of the ground motions used have been provided by Humar and Kumar [6]. The following numerical data is used in the study: mass of the building floor = 400 t; mass moment of inertia = 54,000 tm^2; aspect ratio a/b = 0.5; floor width b = 36 m; uncoupled translational period in y direction = 1.0 s. Strain hardening ratio of 5% is assumed for all planes and the damping ratio is taken as 5% of critical in each of the two coupled modes. The frequency ratio Ω_R and the eccentricity ratio e/b are varied over a range of physically admissible values. Specified values of Ω_R and e/b are achieved by adjusting the values of k_1, k_2 and k_3, the stiffnesses of the planes in y direction. Building models with eccentricity values e/b = 0.05, 0.1, 0.15, 0.2, 0.25 and 0.3 and frequency ratios 0.75, 1.0, 1.25 and 1.50, except those that would require a stiffness to be negative, are considered.

The yield strength of an individual plane is given by

$$V_1 = V_0 \frac{k_1}{K_y}\left[1 + \frac{1}{\Omega_R^2}\left(\frac{b}{r}\right)^2 \frac{e_{fc}}{b}\left(\frac{e}{b} + 0.5\right)\right] \tag{15}$$

$$V_2 = V_0 \frac{k_2}{K_y}\left[1 + \frac{1}{\Omega_R^2}\left(\frac{b}{r}\right)^2 \frac{e_{fc}}{b}\left(\frac{e}{b}\right)\right] \tag{16}$$

$$V_3 = V_0 \frac{k_3}{K_y}\left[1 - \frac{1}{\Omega_R^2}\left(\frac{b}{r}\right)^2 \frac{e_{fc}}{b}\left(0.5 - \frac{e}{b}\right)\right] \tag{17}$$

$$V_3 = V_0 \frac{k_3}{K_y}\left[1 - \frac{1}{\Omega_R^2}\left(\frac{b}{r}\right)^2 \frac{e_{sc}}{b}\left(0.5 - \frac{e}{b}\right)\right] \tag{18}$$

where V_0 is the design base shear in the associated balanced building model. In determining V_3 the larger of the absolute values obtained from Eqs. 17 and 18 is used.

Response analyses are carried out for the 16 earthquake records. One record is selected at a time and normalized so that its peak ground acceleration is 0.28g. An elastic response spectrum for 5% critical damping is obtained for the selected record. The total elastic strength V_e of the resisting planes in the y direction is obtained from the elastic response spectrum, corresponding to a period of 1.0 s. The total design strength for the torsionally balanced model is taken as $V_0 = V_e/4$. This strength is distributed among the individual planes of the balanced building in proportion to their stiffness. The strengths of planes in the unbalanced building are determined from Eqs. 15 through 18. The strength distribution in an unbalanced model is different for different codes, since the expressions for design eccentricities vary from code to code. Thus, six different unbalanced models, designed according to the proposed expressions, NBCC, UBC, NEHRP, NZS and the Mexico Code, are considered. It will be noted that the associated balanced model is same for all the above codes.

To account for the effect of accidental torsion, the center of mass CM is moved by $\pm 0.05b$ in the torsionally unbalanced buildings, to produce two modified unbalanced models corresponding to each set of e/b and Ω_R values. In the analytical results presented here the maximum of the response values obtained from the two modified models is considered. All of the modified unbalanced and associated torsionally balanced models are now analyzed for the selected and normalized earthquake record. The entire process is repeated for each of the 16 records.

The maximum ductility demand in a plane in any torsionally unbalanced model subjected to a given earthquake is denoted by μ_u while the maximum ductility demand for the associated torsionally balanced model is denoted by μ_b. The ratio of the two ductilities, $r_\mu = \mu_u/\mu_b$, provides a measure of the effect of torsional motion. The mean value of the ratio of ductilities for the flexible edge $\bar{r}_{\mu f}$, obtained for the set of 16 earthquakes, is plotted against *e/b* in Fig. 7 for selected values of Ω_R. The value of $\bar{r}_{\mu f}$ is less than 1 for all the codes and all cases, implying that the flexible edge ductility in a torsionally unbalanced model is less than that in the associated torsionally balanced model. The provisions of the Mexico Code are most conservative of all.

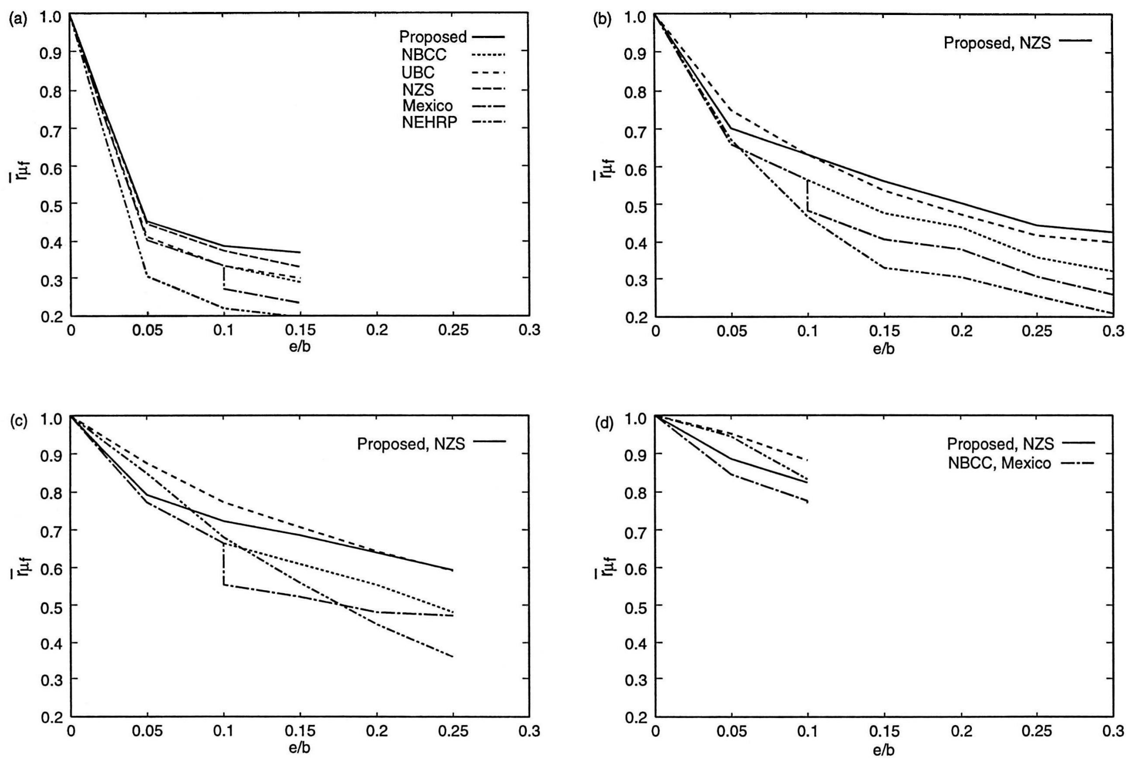

Fig. 7: Ratio of flexible edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean for 16 earthquakes, (a) Ω_R = 0.75, (b) Ω_R = 1.0, (c) Ω_R = 1.25, (d) Ω_R = 1.50

The mean value of the ratio of ductilities for the stiff edge, $\bar{r}_{\mu s}$, obtained for the set of 16 earthquakes, is plotted against *e/b* in Fig. 8 for several values of Ω_R. It is seen that when $\Omega_R = 0.75$, $\bar{r}_{\mu s}$ may be considerably larger than 1 for NZS. The UBC and NEHRP provisions may also lead to values of $\bar{r}_{\mu s}$ that are larger than 1, although not by a substantial amount. For $\Omega_R = 1.0$ the new provisions as well as those of NZS are adequate, although they lead to $\bar{r}_{\mu s}$ slightly larger than 1 for higher values of *e/b*. Provisions of UBC, NBCC and Mexico Code are quite conservative, the Mexico Code being the most conservative. The NEHRP provisions are unsafe for a range of values of e/b. For $\Omega_R = 1.25$ all provisions other than those of NEHRP are adequate. For $\Omega_R = 1.5$ all provisions are adequate, although NEHRP and UBC give values of $\bar{r}_{\mu s}$ that are somewhat larger than 1.

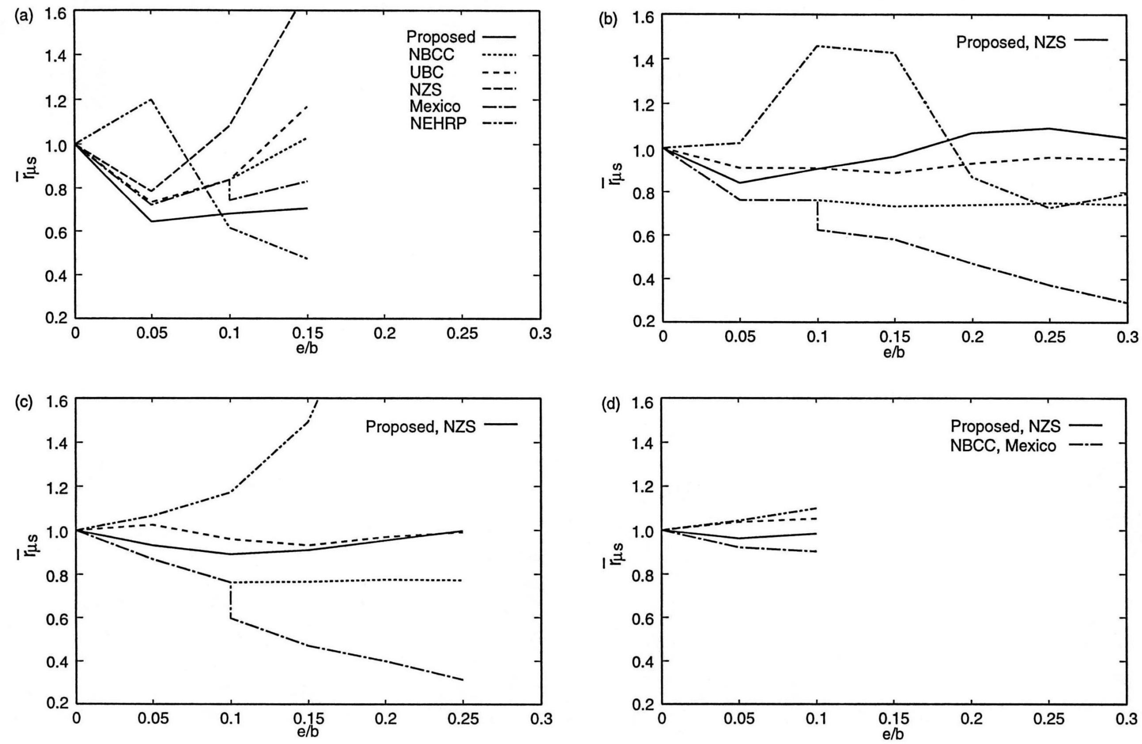

Fig. 8: Ratio of stiff edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean for 16 earthquakes, (a) Ω_R = 0.75, (b) Ω_R = 1.0, (c) Ω_R = 1.25, (d) Ω_R = 1.50

SUMMARY AND CONCLUSIONS

Analytical results are presented for the elastic and inelastic response of single-story torsionally unbalanced models. The results of elastic studies are compared with the design provisions of different building codes. The results presented here show that the provisions of NBCC, NEHRP and Mexico code are overly conservative for the design of elements on the flexible side of the building. The NEHRP provisions deviate most from the dynamic analysis results. For the design of the stiff edge plane, the torsional provisions of all the codes are unconservative when $\Omega_R = 0.75$. Only the proposed expressions provide an adequate assessment of the stiff edge responses. For $\Omega_R = 1.0$ all provisions, except those of NEHRP, are adequate. For higher values of Ω_R, NEHRP provisions are again unsafe. All other provisions are either adequate or conservative, the NBCC and the Mexico Code provisions being the most conservative.

The results of inelastic response to recorded motions indicate that the provisions of all the codes and of the proposed expressions are conservative for the design of flexible edge, the value of $\bar{r}_{\mu f}$ being less than 1 in all cases. Provisions of Mexico code and NEHRP are most conservative. These results also indicate that the provisions of certain codes may be unsafe for the elements on the stiff side of the building, in certain situations. The proposed expressions generally give an adequate design. For $\Omega_R = 1.0$, $\bar{r}_{\mu s}$ is somewhat more than 1 for the models designed according to the proposed expressions, NZS and UBC, particularly for large values of eccentricity. However, $\bar{r}_{\mu s}$ is only slightly greater than 1 and these provisions may be considered adequate even for $\Omega_R = 1.0$. The NEHRP provisions are unsafe for $\Omega_R = 1.0$ and 1.25 for a range of values of *e/b*.

Humar and Kumar [5, 6] have studied the application of the new provisions in the design of elastic as well as inelastic single- and multistory buildings. The design provisions work well with single-story buildings. They also work well with multistory buildings as long as the frequency ratio does not vary significantly across the height. Variation in the frequency ratio across the height can be treated as a sign of vertical irregularity. Additional studies are needed to develop a more precise definition of such irregularity.

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