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AVERAGE STRESS-STRAIN RELATIONSHIP OF STEEL BARS EMBEDDED IN CONCRETE

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SUMMARY

In recent years, material property based modeling methods of reinforced concrete members are popularly adopted for nonlinear and dynamic frame analyses of structures, such as multi-spring model, fiber model and frame-element model for column and beam, also the panel model (macro-FEM) for shear wall. In these macro-models, concrete and steel bar are idealized as uniform and continuity materials, average stress-strain relations of concrete and steel bars should be used.

The average stress-strain relationships of concrete for reinforced concrete have been proposed in soften compression curve and stiffening tensile curve. By axial tension analysis of reinforced concrete bars with bond-slip relationship between steel bar and concrete, this study proposes an average stress-strain relationship of steel bars embedded in concrete. Also a bi-linear bond-slip model is proposed, the analytical approaches are stated. Using the average stress-strain relationship of steel bars for frame model of beam or column, a good correspondence between analytical and experimental results is obtained.

INTRODUCTION

Many average stress-strain relation models of steel bar embedded in concrete are proposed based on experimental methods and analytical methods. In this paper, we propose an average stress-strain relation model by analytical methods with a bi-linear bond-slip model between steel bar and concrete. Using the proposed bi-linear bond-slip model, reinforced concrete bars under tension loading are analyzed and good correspondence is got. Base on statistical analysis of analytical results, the average stress-strain relation model is proposed. And we also proposed the relation in compression region considering the bulking of steel bar. Using the proposed stress-strain model, we did analysis for frame-element model for beam, good precision is got.

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AVERAGE STRESS-STRAIN RELATION FOR CONCRETE

Average Stress-strain Relation of Concrete

Without using hysteretic models based on force-displacement relations, material properties based macro-element models are popularly proposed in recent studies for reinforced concrete members. In our study, average stress-strain relation of concrete shown in Figure.1 is adopted.

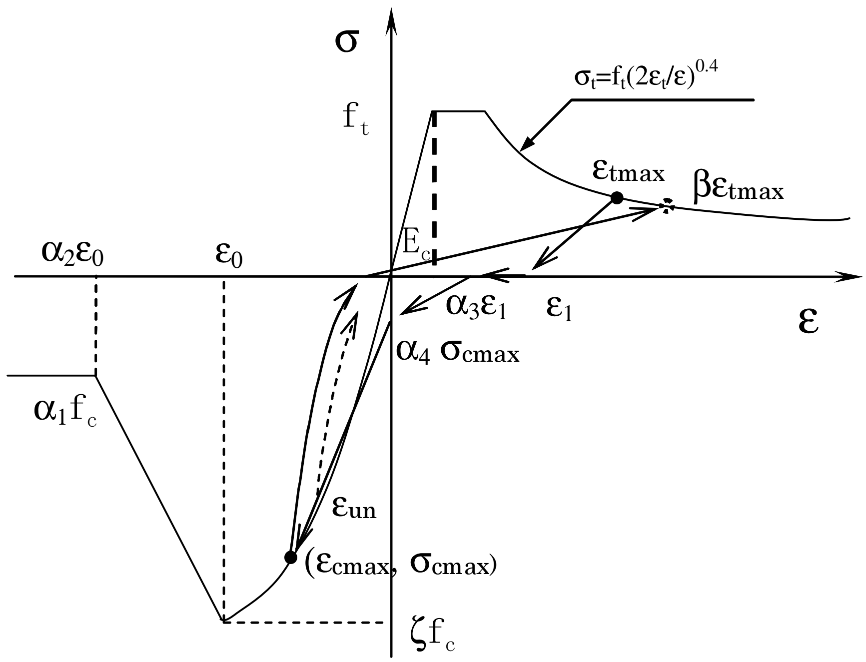


Figure.1: Average Stress-strain Relation of Concrete

Considering bond effect between steel bar and concrete, tension stiffening model is adopted for tension envelope [Izumo, Shima and Okamura, 1989]. But for plain concrete, tension strength is 0 after cracked like broken line. Under plane stress status for panel model of shear wall, stress softening model are adopted for compression envelope [Vecchio and Collins, 1982].

Bond Stress-slip Relation between Steel Bar and Concrete

In order to analysis reinforced concrete bars under tension loading, a bi-linear model is proposed for bond stress-slip relation (Figure.1).The bond strength (τ_u) can be calculated with next formula (1) proposed by [Fujii and Morita, 1982], τ_{c0} means the bond strength without stirrup bars, τ_{st} stand for the effect of stirrup bars. But in yielding region of steel bar, the bond strength is reduced to $0.2\tau_u$.

$$\begin{aligned}
\tau_u &= \tau_{c0} + \tau_{st} \\
\tau_{c0} &= (3.07b_i + 4.27)\cdot\sqrt{f_c} \\
\tau_{st} &= 249kA_{st}/sND\cdot\sqrt{f_c}
\end{aligned} \tag{1}$$

Where, b_i is decided by splitting patterns shown in Figure.2 (a), (b) and (c), takes the minimum value of next b_{vi}, b_{ci} and b_{si} calculated by formula (2). A_{st} stands for the sum of area of stirrup bars and s means the pitch of stirrup bars. The other parameters are shown in Figure.2.

$$
\begin{aligned}
b_{vi} &= \sqrt{3}\cdot(2c_{\min}/D+1), \quad k=0 \\
b_{ci} &= \sqrt{2}\cdot[(c_s+c_b)/D+1]-1, \quad k=\sqrt{2} \\
b_{si} &= b/ND-1, \quad k=1
\end{aligned}
\tag{2}
$$

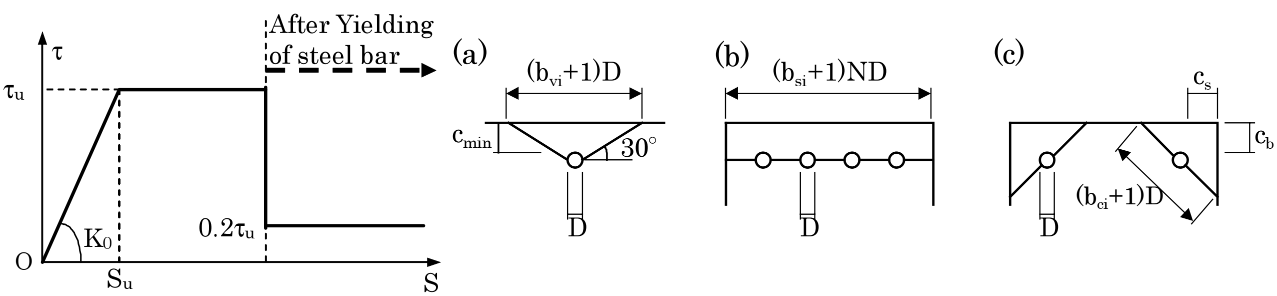


Figure.2: Bond Stress-slip Relation between Concrete and Steel Bar

The initial stiffness K_0 can be shown as formula (3) [Iwata, Li and Ohno, 1998].

$$K_0 = 1160000\times(f_c/2400)^{2/3}\times(13/D) \qquad (KN/m^3) \tag{3}$$

Using the tension skeleton of concrete and bond-slip relation, the tension skeleton cure of steel bar is derived at next paragraph.

THE AVERAGE STRESS-STRAIN RELATION FOR STEEL BAR

Tension skeleton curve

The skeleton curve of bare bar is often simplified by O-A-B-C, but the skeleton curve of bar embedded in concrete can be simplified as a bi-linear model like O-A'-C' (Figure.3). Here, ε_h=0.015, E_{sp}=0.025E_s.

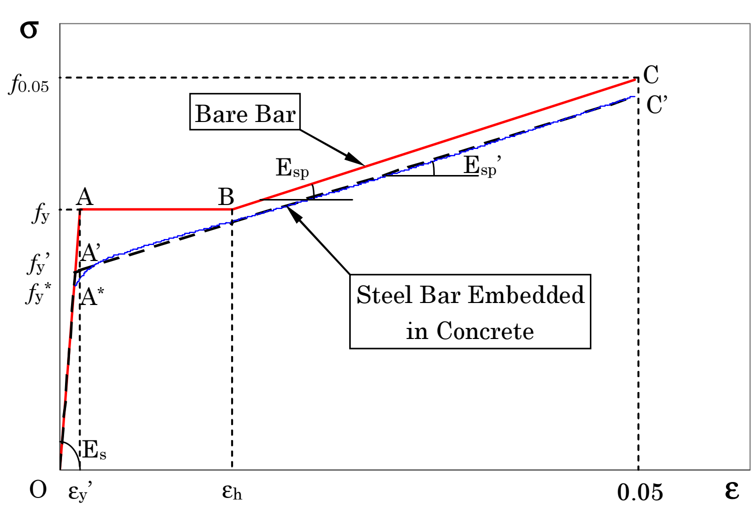


Figure.3: stress-strain relation of Steel Bar

The real yield strength of steel bar embedded in concrete can be calculated by next formula (4) [Hsu, 1993].

$$\frac{f_y^*}{f_y} = 1 - 1.5\frac{\sqrt{n}}{\rho}\left(\frac{f_t}{f_y}\right)^{1.5} \tag{4}$$

Where, ρ stands for reinforcement ratio, n stands for rate of E_s/E_c.
In order to decide the look yield strength $f_y^{'}$ and harden stiffness $E_{sp}^{'}$, reinforced concrete bars will be analyzed under increment tension loading (Figure.4).

The distributions of stress and strain are shown as Figure.4. In the analysis, next suppositions are made.

(1) Cracks have uniform distribution and same width.
(2) Stress in concrete section has uniform distribution. As soon as the stress comes to the tension strength of concrete, crack occurs.
(3) Concrete between cracks is elastic.
(4) Stress and strain of concrete near crack are 0.
(5) Bond stress distribution is symmetry between two close cracks.
(6) There is no slip at the middle between two close cracks.

The slip between steel bar and concrete can be calculated by next formula (5).

$$S(x) = \int_0^x (\varepsilon_s - \varepsilon_c)dx \tag{5}$$

Balance of steel bar can be shown as formula (6) or (6').

$$\tau \cdot \pi D \cdot dx = E \cdot d\varepsilon_s \cdot \pi D^2/4 \tag{6}$$

Or

$$\tau = C_s \frac{d\varepsilon_s}{dx} \qquad C_s = \begin{cases} E_s D/4 & \varepsilon_s \le \varepsilon_y \\ E_{sp} D/4 & \varepsilon_s > \varepsilon_h \end{cases} \tag{6'}$$

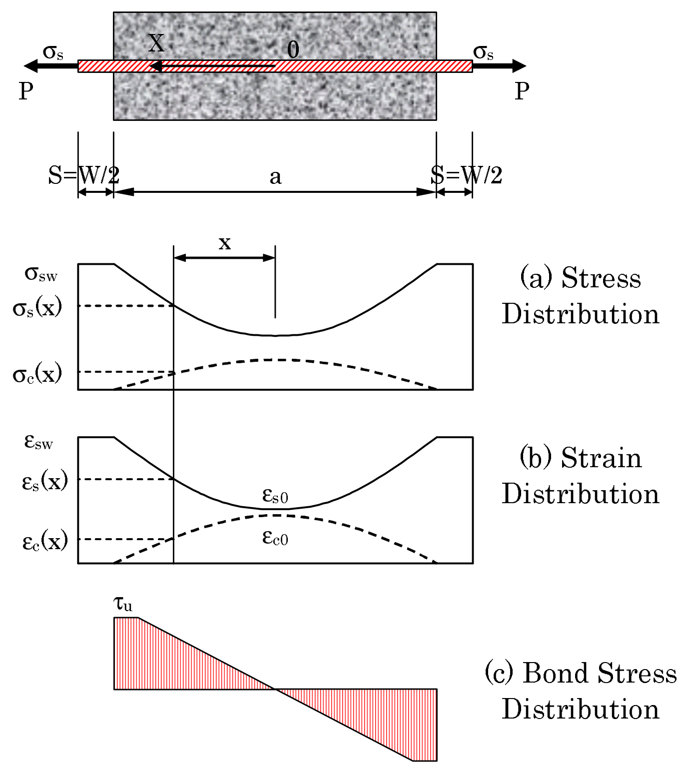


Figure.4 Bond Stress Distribution

Balance of concrete can be represented by formula (7) or (7').

$$\tau \cdot \pi D \cdot dx = -E_c \cdot d\varepsilon_c \cdot A_c \tag{7}$$

Or

$$\tau = -C_c \frac{d\varepsilon_c}{dx} \qquad C_c = E_c D/4 p_s \tag{7'}$$

The boundary conditions can be shown as formula (8).

$$\left.\frac{d\varepsilon_s}{dx}\right|_{x=0}=0,\qquad \left.\frac{d\varepsilon_c}{dx}\right|_{x=0}=0,\qquad \varepsilon_{c\,x=\pm a/2}=0 \tag{8}$$

Using above formula (5) – (8) and numerical method, average stress-strain relation of reinforced concrete bar can be solved. Figure.5 is a sample, the section is 125x125mm and the steel bar is deformed bar (D16). The analysis results (heavy line) reproduced experiment results (dotted line) with good precision; even the number of cracks had a good correspondence.

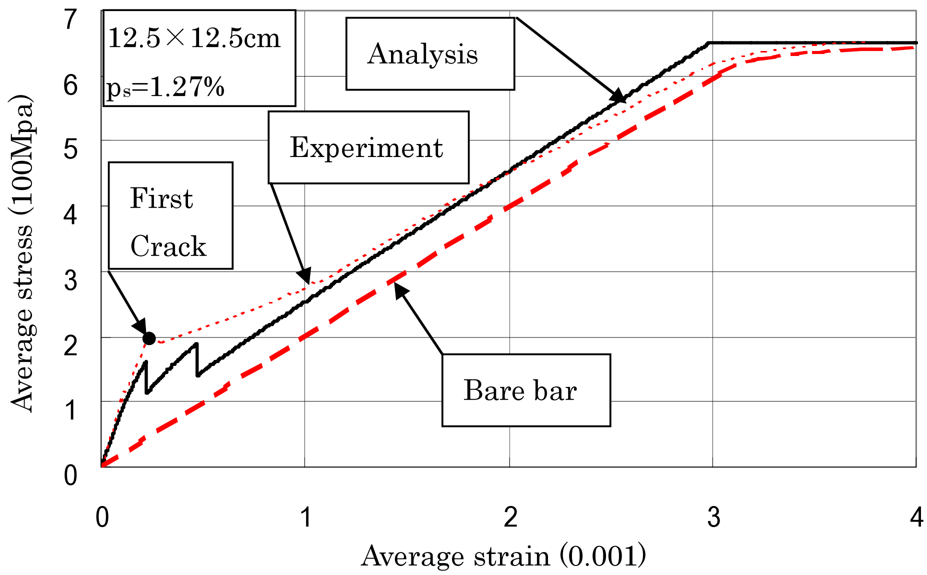
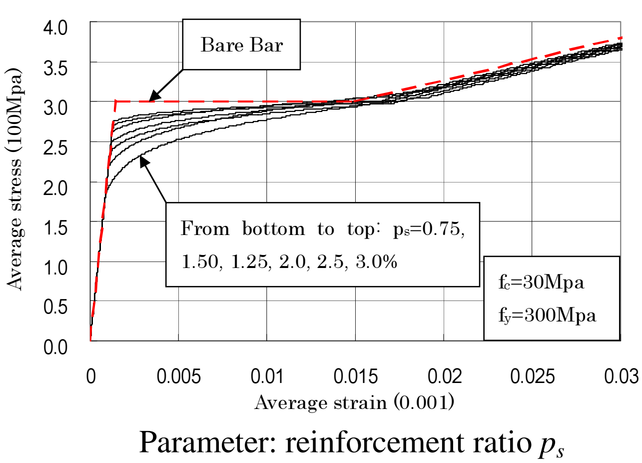


Figure.5 Average stress-strain relation of reinforced concrete bar

Cut out the tension contribution of concrete considered in tension stiffening model from the analysis results of reinforced concrete bar, average stress-strain relation of steel bar can be got. The left graph of Figure.6 shows the variation by parameter of reinforcement ratio p_s, and the right graph shows the variation by parameter of concrete strength f_c. In Figure.7, the influence of yield strength f_y of steel bar is shown.



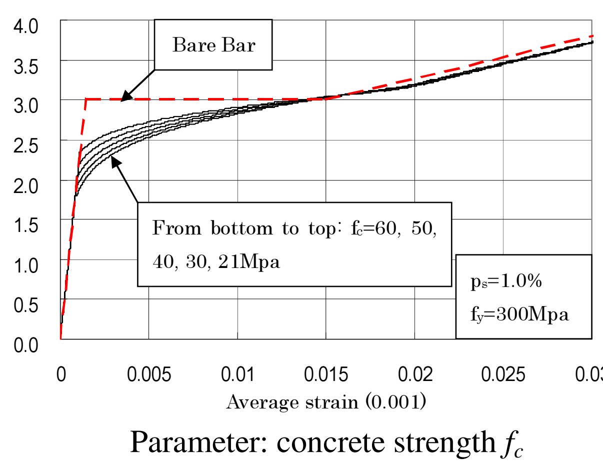


Figure.6 Average stress-strain relation of steel bar

Based on statistical analysis of analytical results with parameters of reinforcement ratio, concrete strength and steel yield strength, the look yield strength $f_y^{'}$ and harden stiffness $E_{sp}^{'}$ of bi-linear model can be shown as formula (9).

$$\frac{f_y^*}{f_y}=1-1.5\frac{\sqrt{n}}{\rho}\left(\frac{f_t}{f_y}\right)^{1.5}$$

$$\frac{f_y^{'}}{f_y}=0.51+0.42\frac{f_y^*}{f_y}$$

$$\frac{E_{sp}^{'}}{E_{sp}}=0.45+0.26\frac{f_y}{100}+\left(0.23-0.25\frac{f_y}{100}\right)\cdot\left(\frac{f_y^*}{f_y}\right) \qquad (9)$$

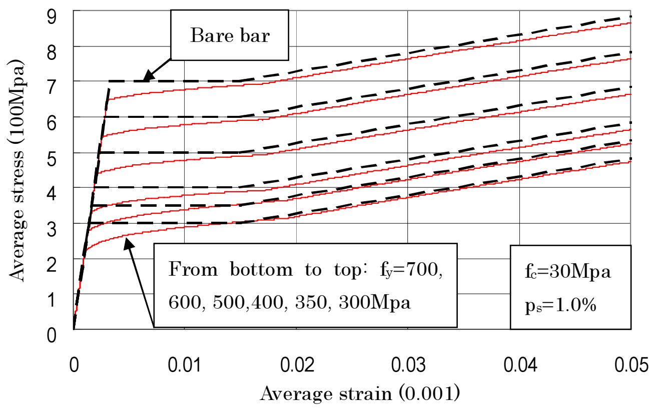


Figure.7 Average stress-strain relation of steel bar (Parameter: yield strength f_y)

Compression skeleton curve

In compression region of steel bar, bond can be neglected, so the skeleton of steel bar embedded in concrete is same as bare bar. The yield strain of steel bar is about 0.002, the strain at concrete compressive strength is also around 0.002. Because of expansion and collapse of concrete loading over concrete strength, bulking of steel bar will occur, so the decrease of strength is considered after $-1.5\varepsilon_y$. The inner rules are proposed like Figure.8.

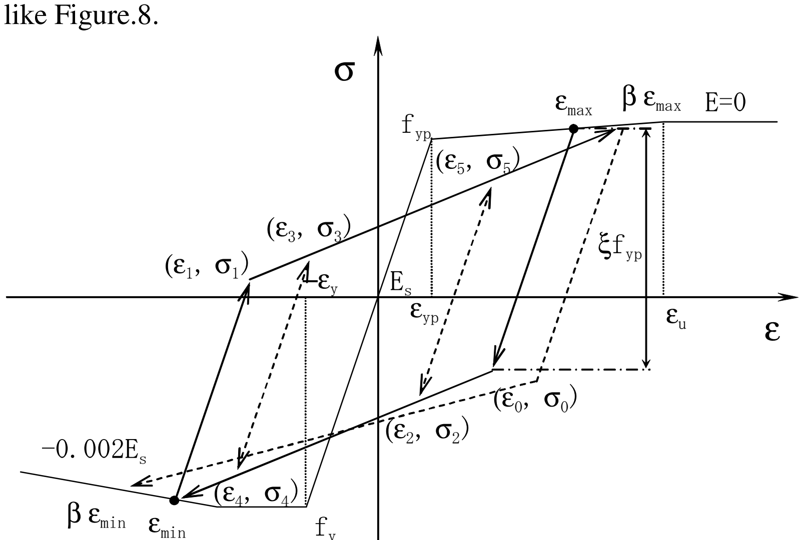


Figure.8 History model of steel bar

THE ANALYSIS FOR REINFOCED CONCRETE MEMBER

Experiment introduction

An experiment of cantilever is used to verify the proposed model, Figure.9 [Y. Kaneko, T. Kaneko and S. Otani, H. Shiohara, 2002]. In the experiment, only cyclic shear force is applied without axial force. The section of member is 200mm(width) x 300mm(height), and the shear span is 1150mm. The properties of concrete and steel bar are shown in Table.1.

Table.1 Properties of materials

Steel bar			
Type	Yield Strength [Mpa]	Tensile Strength [Mpa]	Modulus [Mpa]
D13	361	499	$186x10^3$
ϕ4	478	509	$208x10^3$
Concrete			
Age [Day]	Compressive Strength [Mpa]	Tensile Strength [Mpa]	Modulus [Mpa]
28	26.4	2.18	$30.4x10^3$
54	27.4	2.49	$29.1x10^3$
74	30.4	2.64	$27.2x10^3$

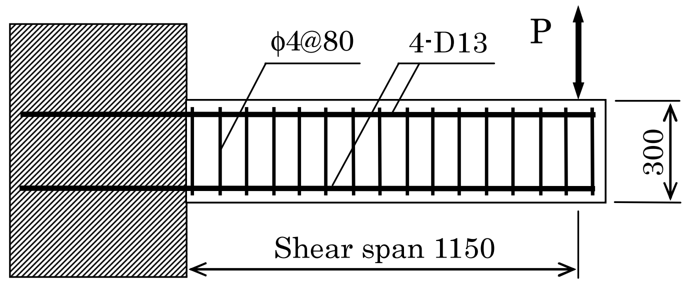


Figure.9 Sample of cantilever

Analytical model

The cantilever is modeled by 3 frame-elements (Figure.10). Considering pull out of steel at fix end, length of the analytical model is 50mm longer than that of the member.

The section is divided into core concrete, cover concrete and steel bar. Core concrete has same properties of plain concrete without tension stiffening effect, cover concrete looks as reinforced concrete considering tension stiffening effect.

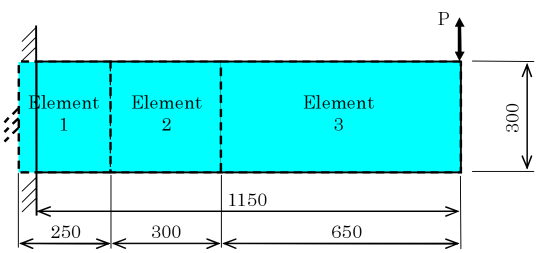


Figure.10 Analytical model of cantilever

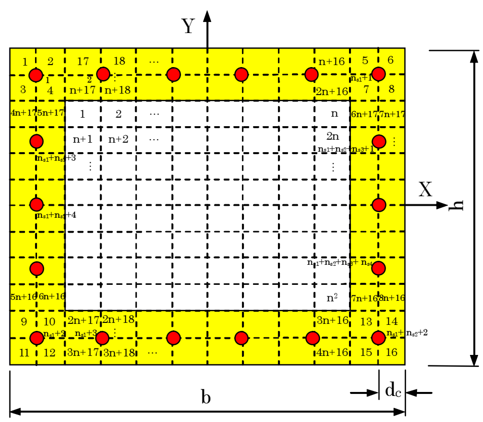


Figure.11 Cell division of section

Results of analysis

The load-displacement relation of test is shown in Figure.12; the analytical result is shown in Figure.13. In testing, displacement is shifted to negative direction.

The relation between axial extension and displacement is shown in Figure.14. The axial extension means the strain of 200mm region at the fixed end.

Testing results are reproduced about initial stiffness, yield load and axial extension with good accuracy.

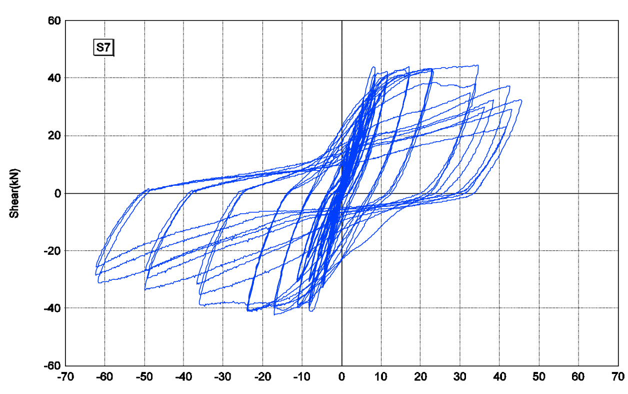


Figure.12 Load-displacement relation of test

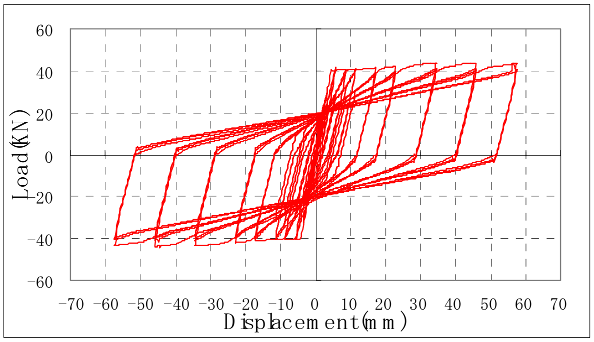


Figure.13 Load-displacement relation of analysis

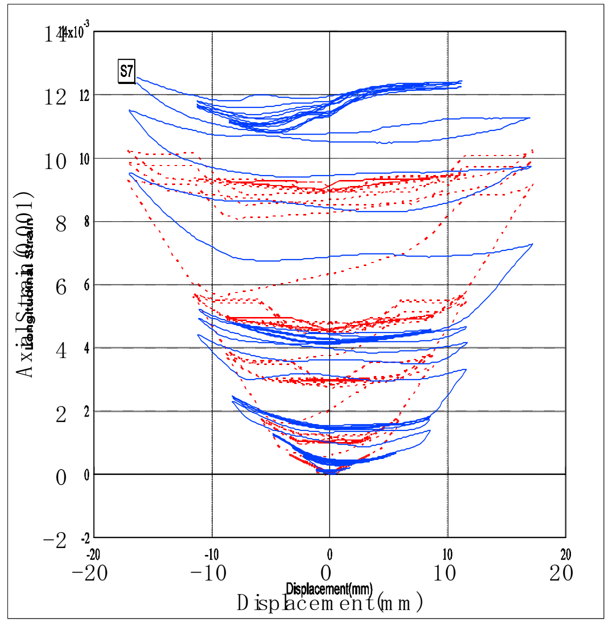


Figure.14 Axial extension to displacement

RESULTS

The results of this paper can be concluded by next points.

1. Proposed a method to solve bond-slip problem for reinforced concrete bar under tension loading.
2. Proposed a bilinear model for steel bar embedded in concrete.
3. Analyzed a cantilever with the proposed model and good correspondence was got.

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