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MODIFIED DESIGN FORMULAS FOR STRUCTURES WITH SUPPLEMENTAL VISCOUS DAMPERS

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SUMMARY

The current design formulas such as those provided by FEMA273/274 for building structures with supplemental viscous dampers were derived based on the shear building assumption. However, for medium-rise to high-rise buildings that deform in a combined form of shear and bending when subjected to seismic loading, the design using existing formulas may result into an actual damping ratio much lower than what is expected by the design. Therefore, the actual seismic performance of the structure may be worse than what is expected by the design. In this study, modified design formulas are derived considering both shear and flexural deformations of the building structures subjected to ground excitations. The modified formulas are derived for two often used installation schemes of viscous dampers including diagonal-brace-damper system and K-brace-damper system. Numerical verifications have indicated that the modified design formulas predict a more accurate viscous damping ratio contributed by linear viscous dampers and ensure a more conservative design for the structure with nonlinear viscous dampers.

INTRODUCTION

Supplemental viscous damping devices including linear and nonlinear viscous dampers are gaining more popularity in recent years. The attraction for adopting viscous dampers as energy dissipation devices may be attributed to the relative simplicity of the design formulas and procedure [1,2] (FEMA 273/274 1997; Constantinou and Symans 1992). Since the viscous dampers do not possess storage stiffness when the excitation frequency is within a low frequency range, e.g. 0 to 3 Hz, which is often sufficient to cover the first mode vibration frequencies of most building structures, the incorporation of a viscous dampers into a structure usually does not affect the fundamental natural period and mode shape of a structure.

This paper presents the modified design formulas of viscous dampers from the existing design specifications and research reports [1-3], Constantinou and Symans (1992) and Seleemah and

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Constantinou (1997)). The proposed formulas serve as a design tool for calculating the damping coefficients of linear and nonlinear viscous dampers corresponding to a desired additional damping ratio. The major contribution of the proposed design formulas is that the overturning effect (or flexural effect) in addition to the shear effect induced by seismic lateral force acting on a medium rise or a high rise building structure is taken into account. By so doing, the damping ratio contributed by the viscous dampers to the structure will not be over-estimated as in the current design formulas.

DAMPING RATIO CONTRIBUTED BY STRUCTURAL COMPONENTS

The design formulations for incorporating linear viscous dampers into traditional earthquake-resisting buildings can be derived based on the concept proposed by Raggett [4](1975). The current design formulas provided by NEHRP (1997) [4] were derived by Constantinou and Symans [2]. When deriving the design formula, a multi-story building structure was idealized as a shear building for which only the shear deformation of the building was considered. The energy dissipated by the dampers with a diagonal-brace installation configuration in one cycle of vibration of the k^{th} mode is expressed in modal coordinate by

$$\sum_j E_j = \sum_j \oint P_j \; d(\phi_{rj} \sin \frac{2\pi t}{T_k}) \tag{4}$$

in which

$$P_j = C_j \cos^2 \theta_j \; \phi_{rj} \; \frac{2\pi}{T_k} \; \cos(\frac{2\pi t}{T_k}) \tag{5}$$

where P_j = the horizontal component of the damper force of device *j*; ϕ_{rj} = the relative modal displacement between the ends of device *j* in the horizontal direction of the k^{th} mode of vibration; T_k = the natural period of the k^{th} mode of vibration; C_j = the damping coefficients of device *j*; and θ_j = the inclination angle of device *j*. The energy dissipated by the dampers is obtained as

$$\sum_j E_j = \frac{2\pi^2}{T_k} \sum_j C_j \cos^2 \theta_j \; \phi_{rj}^2 \tag{6}$$

The maximum strain energy of the structure is equal to the maximum kinetic energy

$$U_t = \frac{2\pi^2}{T_k^2} \sum_i m_i \; \phi_i^2 \tag{7}$$

where m_i = the mass of the i^{th} story; ϕ_i = the modal displacement of the i^{th} story in the k^{th} mode of vibration. Substituting Eqs. (6) and (7) into Eq. (3), the equivalent damping ratio contributed by the linear viscous dampers to the structure is given by

$$\xi_k = \frac{T_k \sum_j C_j \cos^2 \theta_j \phi_{rj}^2}{4\pi \sum_i m_i \; \phi_i^2} \tag{8}$$

Recognizing that the higher mode responses will be highly suppressed when sufficient dampers are incorporated into a building structure, in the design provisions of FEMA 273 the damping ratio of a building structure with added linear dampers is approximated by the first mode vibration.

Following the formulations of linear viscous dampers and considering only the first vibration mode, the damping ratio of a building structure contributed by supplemental nonlinear viscous dampers was derived by Soong and Constantinou [5], Soong, et al. [6] and Seleemah and Constantinou [3]. The work done by the nonlinear viscous dampers in one cycle of vibration is equated to the energy dissipated by a linear viscous damping system. Using Eq. (3), the equivalent viscous damping ratio contributed by the nonlinear viscous dampers is obtained by

$$\xi = \frac{T^{2-\alpha} \sum_j C_j \, \lambda \cos^{1+\alpha}\theta_j \, \phi_{rj}^{1+\alpha}}{(2\pi)^{3-\alpha} A^{1-\alpha} \sum_i m_i \, \phi_i^2} \tag{9}$$

where α = the damping exponent; A = the roof response amplitude corresponding to modal displacement ϕ_j normalized to a unit value at the roof; and λ is a parameter which can be calculated by

$$\lambda = 2^{2+\alpha} \frac{\Gamma^2(1+\alpha/2)}{\Gamma(2+\alpha)} \tag{10}$$

in which Γ is the gamma function. It should be noted that Eq. (9) is just an approximation due to the fact that, in the derivation of the formula, the energy dissipated by nonlinear viscous dampers is equated to the energy dissipated by a linear viscous damping system.

NEW DESIGN FORMULAS FOR STRUCTURES WITH VISCOUS DAMPERS

The aforementioned design formulas were derived based on the assumption of a shear building for which only the shear deformation of the building is considered. Therefore, the work done or the energy dissipated by the viscous dampers can be simplified as the sum of the integration of the horizontal component of damper force with respect to the horizontal modal displacement of each story, as depicted by Eq. (4). However, for a medium-rise or high-rise multi-story building structure, it is no longer appropriate to assume the structure as a shear building. This is due to the fact that the overturning moment effect or flexural effect may be significant and should not be neglected. For example, a twenty story building frame shown in Fig. 1 is designed with additional viscous dampers. The sectional properties of the member are given in Table 1. The seismic reactive weights are 1082 kN on each of second to fifth floors, 1014 kN on each of sixth to thirteenth floors, and 947 kN on each of fourteenth floor to the roof. From the computed first mode shape and the corresponding relative modal displacements shown in Fig. 1(a) and (b), it is found that the deformations of lower stories are primarily attributed to the shear deformation. However, for the medium to high stories, the flexural and/or overturning deformations are significant. This can also be seen by examining Table 2 in which the normalized first modal displacements are shown for the right span of the frame with dampers. The modal displacements are normalized corresponding to a unit value assigned for the lateral roof displacement. $(\phi_h)_i$ is the normalized lateral modal displacement at i^{th} story, $(\phi_v)_{i\_r}$ is the vertical normalized modal displacement on the right end of the damper corresponding to column line B in the i^{th} story and $(\phi_v)_{i\_l}$ the vertical normalized modal displacement on the left end of each damper corresponding to column line A in the i^{th} story. Based on the table, the relative vertical modal displacement is comparable in magnitude with the relative lateral modal displacement in each of the higher stories. For example, the horizontal relative modal displacement of the 20th story is 0.0261 and the vertical relative modal displacement at both ends of the damper at 20th story is -0.0320. As a consequence, the axial displacements of the dampers at medium to high stories will be significantly smaller than the horizontal story drifts. Therefore, it is not appropriate to assume the building deformation is in shear mode, and thus the energy dissipated by the viscous dampers should not be simplified as the sum of the integration of the horizontal component of damper force with respect to the horizontal modal displacement of each story. Instead, the energy dissipated by the damper should be calculated as the sum of the integration of damper axial force with respect to the damper axial deformation corresponding to the normalized mode shape.

Another illustration in given in the follows to explain that the existing design formula given by Eq. (8) may over-estimate the equivalent damping ratio for medium to high rise building structures. The average C value of the dampers calculated according to Eq. (8) corresponding to a 15% added damping ratio to the structure of Fig. 1 is $8429\, kN - \sec/m$. If the structure is subjected to a ground acceleration pulse shown in Fig. 2(a), the calculated response time history at the roof of the structure with linear viscous dampers is larger than what is obtained for the same structure without dampers but with an inherent viscous damping ratio of 15%, as shown in Fig. 2(b). Based on the logarithm decay of the free vibration part of the calculated displacement response time history, the identified damping ratio for the structure with dampers is approximately equal to 9% which is much smaller than what is expected by Eq. (8). Due to the flexural effect, the axial displacements of the dampers at the medium to high stories are not equal to the lateral story drifts multiplied by $\cos\theta$. An example is given in Fig. 2(c) in which the calculated axial displacement of the dampers at 20th story is much smaller than the calculated lateral story drift multiplied by $\cos\theta$ when the structure is assumed to remain elastic and is subjected to the N-S component of the 1949 El Centro earthquake. This result shows that the maximum axial force of the damper should not be calculated by $2\pi C_j \phi_{rj} \cos\theta / T$ of Eq. (5).

Based on the aforementioned discussions, the design formula given in Eq. (3) should be modified so that the added damping ratio to the building structures may be more accurately and conservatively predicted. The modified formula is derived corresponding to commonly used installation schemes of linear (α=1) and nonlinear (α< 1) viscous dampers including the diagonal-brace-damper system and K-brace-damper system [7]

$$\xi_d = \frac{T^{2-\alpha} \sum_j C_j \lambda_j \left| (f_h)_j (\phi_h)_{rj} - (f_v)_j (\phi_v)_{rj} \right|^{1+\alpha}}{(2\pi)^{3-\alpha} A^{1-\alpha} \sum_i m_i (\phi_h)_i^2} \tag{11}$$

In which $(f_h)_j$ and $(f_v)_j$ are respectively the magnification factors in the horizontal and vertical directions of the j^{th} viscous dampers with the diagonal brace and K-brace installation schemes shown in Table 3.

NUMERICAL VERIFICATION

For the numerical verification, the twenty story building frame of Fig. 1 is used for the design of viscous dampers and the analysis of seismic responses. The structure is assumed to remain elastic under ground excitations. Diagonal-brace-damper systems are designed based on Eqs. (8) and (11) for an expected supplemental damping ratio of 20% and a zero inherent viscous damping ratio. The average damping coefficient of each damper is $11239\, kN - \sec/m$ calculated using Eq. (8) and is $18182\, kN - \sec/m$ determined by Eq. (11). Calculated using SAP2000N for the structure subjected to the horizontal ground acceleration impulse of Fig. 2(a), the first mode displacement response time histories at the roof of the frame are shown in Fig 3. From the figure it can be seen that the response of the frame with the dampers designed using Eq. (11) agrees well with the response of the frame without viscous dampers but with an inherent viscous damping ratio of 20% , while the frame with the dampers designed with Eq. (8) reveals a much larger response.

Regarding the verification for the modified design formulas of the nonlinear viscous dampers, to ascertain the conservatism of the proposed formulas is of more interesting than to examine the agreement between the maximum structural responses obtained for the nonlinear viscous damping system and the equivalent linear viscous damping system. This is because an equivalent linear viscous damping system is just an approximation rather than an exact solution to the nonlinear viscous damping system. The linear viscous dampers of the diagonal-brace-damper system in the previous example are replaced by nonlinear viscous K-brace-damper system. The damper exponent is selected to be α =0.4 which will yield a λ value of 3.58. The average damping coefficients are determined respectively based on Eqs. (9) and (11) corresponding to the A values determined for the building frame which possesses an linear viscous damping ratio of 15% and is subjected to a few selected ground motions, as given in Table 4.The comparison between the calculated maximum displacement and acceleration responses are shown in Tables 5 and 6. From the tables, it is concluded that the design using the modified formula in general results into a better response control of the structure.

CONCLUSIONS

The existing design formulas for structures with supplemental viscous dampers have been modified in this study to account for the effects of combined shear and flexural deformation of the structures. The formulas are derived for the application of both linear and nonlinear viscous dampers for a few often used installation schemes such as diagonal brace and K-brace. According to the numerical verification, it is demonstrated that the modified formulas can predict the additional viscous damping ratio contributed by the linear viscous dampers more accurately than the existing design formula. The existing design formula provided by FEMA273 has over-estimated the added damping ratio by the linear viscous dampers, and as a consequence the seismic performance predicted by FEMA 273 is not conservative. Besides, the modified design formulas for the nonlinear viscous damper provide a more conservative design than the existing design formula.

ACKNOWLEDGEMENTS

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TABLE 1. Member sections of the 20-story frame

Story	Beam	Column
13~20	W600×300×25×25	?900×900×25×25
05~12	W700×350×25×25	?1100×1100×25×25
01~04	W800×400×25×25	?1300×1300×25×25

TABLE 2. The mass and the normalized mode shape of the 20-story frame

Story	Mass $(kN-s^2/m)$	$(\phi_h)_i$	$(\phi_{vl})_i$	$(\phi_{vr})_{i-1}$
20	96.6	1.0000	0.0029	-0.0291
19	96.6	0.9739	0.0029	-0.0291
18	96.6	0.9418	0.0030	-0.0288
17	96.6	0.9026	0.0030	-0.0286
16	96.6	0.8559	0.0031	-0.0283
15	96.6	0.8026	0.0032	-0.0277
14	96.6	0.7433	0.0033	-0.0270
13	96.6	0.6804	0.0033	-0.0261
12	103.5	0.6184	0.0033	-0.0251
11	103.5	0.5594	0.0033	-0.0240
10	103.5	0.4996	0.0032	-0.0225
9	103.5	0.4381	0.0031	-0.0209
8	103.5	0.3753	0.0030	-0.0190
7	103.5	0.3122	0.0028	-0.0168
6	103.5	0.2494	0.0026	-0.0144
5	103.5	0.1890	0.0023	-0.0117
4	110.4	0.1352	0.0019	-0.0092
3	110.4	0.0892	0.0015	-0.0064
2	110.4	0.0492	0.0011	-0.0038
1	110.4	0.0177	0.0006	0

TABLE 3 The magnification factors

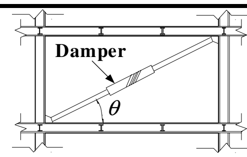
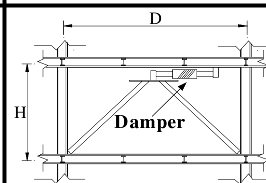
Installation type	Illustration	Magnification Factor	
		f_h	f_v
Diagonal Brace		$\cos\theta$	$\sin\theta$
K-Brace		1	H/D

TABLE 4. The nonlinear damping coefficient determined by Seleemah & Constantinou (1997) and the proposed formula corresponding to an added damping ratio of 15% with $\alpha = 0.4$

Earthquake	Inherent Damping Ratio = 15% ($cm/\sec^2$)	Constantinou & Seleemah ($cm/\sec^2$)	Proposed Formula ($cm/\sec^2$)
Kobe	635.3	741.8	671.2
New Hall	680.6	755.8	687.6
Mexico	617.6	695.7	516.5
TCU065	622.4	765.2	539.0
TCU074	581.7	701.8	620.5

TABLE 5. Comparison of the maximum roof displacement of the frame with the nonlinear damping coefficient determined by Seleemah & Constantinou (1997) and the proposed formula with $\alpha = 0.4$

Earthquake	Inherent Damping Ratio = 15% (cm)	Constantinou & Seleemah (cm)	Proposed Formula (cm)
Kobe	40.4	47.0	40.0
New Hall	44.4	48.8	41.7
Mexico	59.6	65.7	45.5
TCU065	59.6	68.9	49.3
TCU074	45.2	53.4	45.5

TABLE 6. Comparison of the maximum roof acceleration of the frame with the nonlinear damping coefficient determined by Seleemah & Constantinou (1997) and the proposed formula with $\alpha = 0.4$

$T = 1.919\,\sec$ ¡B $\lambda = 3.58$ ¡B $\alpha = 0.4$			
Earthquake	A (cm)	C Seleemah & Constantinou $(kN-(\sec/m)^{0.2})$	C Proposed Formula $(kN-(\sec/m)^{0.2})$
Kobe	40.4	1220	1728
New Hall	44.4	1291	1829
Mexico	59.6	1541	2182
Tcu065	59.6	1541	2182
Tcu074	45.2	1305	1849

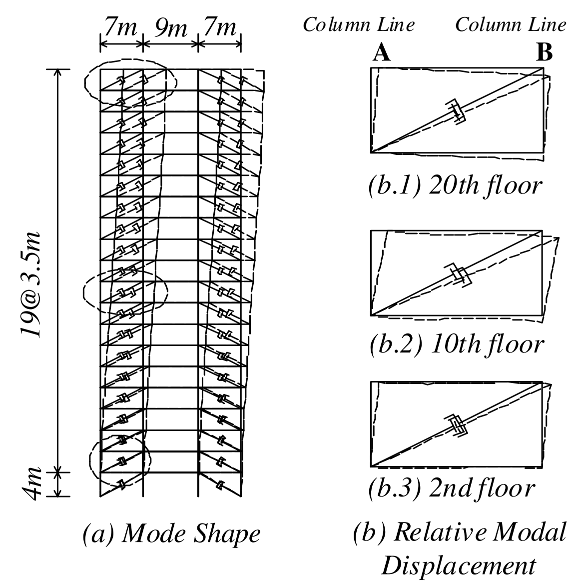


FIG. 1 Modal deformation shape of the 20-story frame with diagonal-brace-damper system

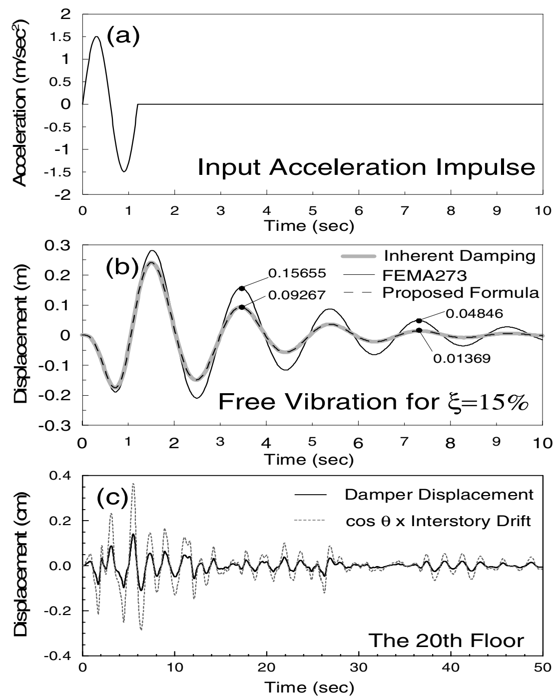


FIG. 2 Response comparison of the 20-story frame subjected to the acceleration impulse (K-brace-damper for added 15% damping ratio determined by FEMA273 and the modified formula)

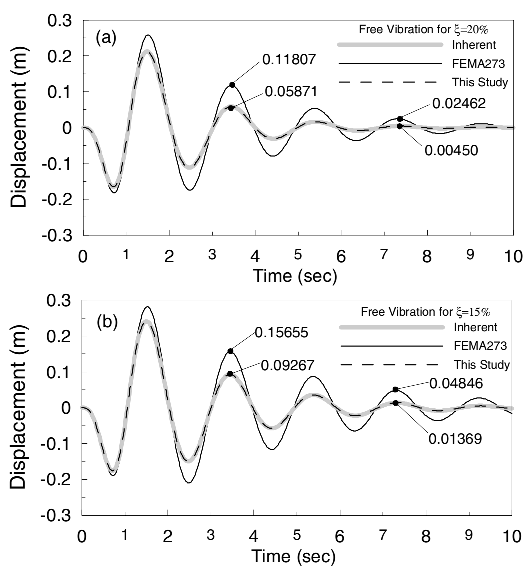


FIG. 3. Free vibration of the 20-story frame with linear dampers for adding (a) 20% damping ratio; (b) 15% damping ratio