

MODELING OF UNKNOWN SEISMIC RESPONSES OF PARTIALLY INSTRUMENTED BRIDGE STRUCTURE

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SUMMARY

Monitoring technique based on the analysis of responses recorded on the structure are becoming a widespread method to monitor structural health conditions. An effective monitoring system should be characterized by three main requirements: accuracy of the recorded data, cost-effectiveness of the sensors systems, availability of procedures to interpret the recorded data. Due to economic reasons, concerning the cost related to data acquisition and/or to the inaccessibility of some degrees of freedom, bridge structures are usually instrumented with a limited number of sensors. The economic advantage associated to the lower cost of the sensors system is counterbalanced by the lack of data in location where recording sensors are not available. In this paper a tentative solution to this problem is proposed through a method to reconstruct unknown responses from the ones recorded by a limited number of sensors. Unknown responses can be calculated by interpolation of recorded ones through an appropriate spline shape function. The method has been already applied with good results in the case of multistory buildings. In this paper bridge structures are considered and the procedures has been applied for the reconstruction of the seismic response of a symmetric multispan bridge, of a cable-stayed bridge and of an arch bridge. In the first two cases analysis have been carried out on the numerical models of the structures while for the arch bridge responses recorded during a real seismic event have been analyzed.

Both the accuracy of calculated responses and the cost of data acquisition increase with the number of recording sensors. In order to characterize the cost-effectiveness of the employed set of recording sensors an "effectiveness function" defined in terms of the number of recording sensor and of the achievable accuracy is proposed.

INTRODUCTION

The recording of structural response of bridge structures during seismic excitation is of fundamental importance for analysis, design and monitoring of earthquake resistant structures.

The analyses of recorded responses allows to persecute both design verification, through analyses of real structural response, and structural maintenance, through analysis of structural characteristics aimed to detect changes that may indicate damage or degradation.

The conventional visual inspection for bridges is not always able to timely detect deterioration allowing a prompt intervention. Moreover visual inspection requires a large amount of time and is prone to human

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errors. For these reasons and thank to the advance in sensor technology, data acquisition systems and data interpretation algorithms, monitoring technique based on the analysis of responses recorded on the structure are becoming a widespread method to monitor structural health conditions. Acceleration responses are commonly used to analyze structural behavior during both strong seismic events and the everyday life of the structure. These data can be used to perform the identification of the parameters of the dominant modes fo the structure from both ambient vibrations data and from strong motion records. The former characterize the elastic response of the structure while the latter allow investigating the possible state of damage of the structure. Furthermore recorded responses allow to check and update, if necessary, the analytical model used in the analyses of the structure and to estimate both the forces induced in the structural elements and the global structural displacements.

An effective monitoring system should be characterized by three main requirements: accuracy of the recorded data, cost-effectiveness of the sensors systems, availability of procedures to interpret the recorded data.

The cost-effectiveness of the sensors system could be improved using a number of sensors lower than the number of the degree of freedom of the structure hence recording responses only at selected location of the structure. The economic advantage associated to the lower cost of the sensors system is counterbalanced by the lack of data in location where recording sensors are not available.

In this paper a tentative solution to this problem is proposed through a method to reconstruct unknown responses from the ones recorded by a limited number of sensors. The method has been already applied with good results in the case of multistory buildings. In this paper bridge structures are considered. Two main differences exist in the application of the method to bridge structure and to multistory buildings: the boundary condition and the number of input excitation.

Furthermore in this paper an "effectiveness function" is proposed to quantify the cost-effectiveness of the employed set of recording sensors.

Three examples of bridge structures, characterized by an increasing level of complexity, have been considered. The first is the two-dimensional model of a symmetric multispan bridge, the second is the three-dimensional model of a dissymmetric cable-stayed bridge and the last one is an arch bridge for which responses recorded during a real seismic event are available. The method of reconstruction of unknown responses through a spline shape function has been applied and the effectiveness of the set of recording sensors has been investigated following the evolution of the effectiveness function at the increase of the number of sensor.

THE METHOD

Model formulation

The application of the method to reconstruct unknown responses is based on two assumptions:

- responses in terms of absolute acceleration are available in a limited number of locations along the bridge axis;
- a cubic spline shape function interpolates the function absolute acceleration $\ddot{u}(z,t)$ along the longitudinal axes of the bridge.

The location where responses are recorded by sensors are assumed as knots of the spline function and, for each time instant, the unknown coefficients of the function are determined from continuity, interpolation and boundary conditions using recorded responses.

The boundary conditions depend on the geometry of the bridge and are easily determined assuming a beam like behavior of the structure.

Interpolation of absolute acceleration

For a linear multidegree of freedom system, in the case of linear behavior, the response v(s,t) in the location defined by vector s can be expressed as a superposition of modal contribution as:

$$v(s,t) = \sum_{r=1}^{N} v^{r}(s,t) = \sum_{r=1}^{N} \alpha^{r} \phi^{r}(s) D^{r}(t) = \sum_{r=1}^{N} \beta^{r}(s) D^{r}(t)$$
(1)

where α^r is the *r*-th modal participation factor, $\phi^r(s)$ is the *r*-th modal shape, $\beta^r(s)$ is the effective participation factor of the *r*-th mode and $D^r(t)$ is the *r*-th modal response to the base excitation.

If the function relative acceleration is considered, its variation along one direction, say z, can be expressed as:

$$\ddot{u}(z,t) = \sum_{r=1}^{N} \ddot{u}^{r}(z,t) = \sum_{r=1}^{N} \alpha^{r} \phi^{r}(z) D^{r}(t) = \sum_{r=1}^{N} \beta^{r}(z) D^{r}(t)$$
(2)

In this paper a spline shape function is proposed to model function $\ddot{u}(z,t)$. A spline is a function composed by polynomial defined in subintervals joined together with certain smoothness conditions (see Boor [1]).

At a given time *t*, if the values of function $\ddot{u}(z,t)$, defined on the interval $[z_0, z_{n+1}]$, are known in *n* locations (knots) $z_1, z_2, ..., z_n$, the cubic spline interpolant to $\ddot{u}(z,t)$ is composed by *n*+1 cubic polynomials each defined in one subinterval $[z_i, z_{i+1}]$:

$$\ddot{u}(z,t) = \sum_{r=1}^{N} \beta^{r}(z) D^{r}(t) = \sum_{j=0}^{4} c_{j,i}(t) (z - z_{i})^{j} \qquad z \in [z_{i}, z_{i+1}]$$
(3)

For each one of the n+1 polynomials, 4 unknown coefficients (c_{0i} , c_{1i} , c_{2i} , c_{3i}) must be estimated hence a total number of 4(n+1) equation must be written to obtain a unique solution.

The continuity and interpolation conditions to be imposed in each one of the internal knots in order to obtain a continuous and smooth approximating function, give a total number of 4n equations (the spline function is assumed twice continuously differentiable so that it has also a continuous slope and a continuous curvature). The remaining 4 constraint are given by the boundary conditions to be imposed at the boundary of the interval of definition of the function (points z = a and z = b).

Expression (3) highlights the relationship between the effective participation factors of the structure and the spline function. In a former paper

The interpolation error

The accuracy of the method based on the spline interpolation, in reconstructing the entire set of unknown responses, depends both on the number of the available sensors and on their location throughout the structure. Furthermore it depends on the capability of the spline shape function in modeling the deformed shape of the structure.

In a former paper by Limongelli [3] the optimal location of a given number of recording sensor has been investigated and a function of the effective participation factors of the structure, able to identify such optimal locations, has been defined.

In this paper the cost-effectiveness of the employed number of recording sensors N_s is investigated: at the increase of N_s the error in the estimate of unknown responses decreases but, on the other hand, the cost of the instrumentation increases. The cost-effective number of sensors is the one opportunely counterbalancing the two circumstances. In order to study the relationship between the number of recording sensors and the error of the reconstructed response, in terms of absolute acceleration in a given location z of the structure, the following error function can be defined:

$$\varepsilon(z, N_s, z) = \sqrt{\sum_{j=1}^{N_T} \left(a_{r,i}(z, t_j) - a_{c,i}(z, t_j) \right)^2} / \sqrt{\sum_{j=1}^{N_T} a_{r,i}^2(z, t_j)}$$
(4)

being $a_r(z,t_j)$ and $a_c(z,t_j)$ the values of respectively the real and calculated absolute acceleration at the z location at the j^{th} instant of the time history. Vector $z(z_1,z_2,...,z_{Ns})$ collects the labels of the locations where the recording instruments are placed.

If a model of the structure is available, the interpolation error $\varepsilon(z, N_s, \overline{z})$ can be repeatedly calculated considering increasing values of N_s and, for each of them, all the possible distribution z of the recording sensors. In order to synthetically characterize the interpolation error two parameters have been used: the mean $\varepsilon_M(N_s, z)$ and the standard deviation $\sigma_{\varepsilon}(N_s, z)$ of the distribution of the errors $\varepsilon(z, N_s, \overline{z})$. The two parameters characterize the accuracy of the entire set of N responses reconstructed on the base of a given number of responses N_s recorded in locations defined by the vector z,

The effectiveness function

The accuracy achievable with distribution characterized by different numbers of recording sensors, can be measured by an 'effectiveness function' $e(N_s)$ defined as:

$$e(N_s,z) = \frac{\left[1 - \varepsilon_M(N_s,z)\right]}{\left[1 - \varepsilon_M(N)\right]} \cdot \frac{\left[1 - \varepsilon_M(N_s,z)\right]N_s}{\left[1 - \varepsilon_M(N)\right]/N} = \frac{\left[1 - \varepsilon_M(N_s,z)\right]^2}{\delta(N_s)} = \frac{\left[a_M(N_s,z)\right]^2}{\delta(N_s)}$$
(5)

where N is the total number of responses to be calculated and $\delta(N_s) = N_s / N$ is the density of the recording sensors. The maximum value of the mean error ε_M , reached for $N_s=0$, has been conventionally assumed equal to 1 hence the function $a_M(N_s, z) = 1 - \varepsilon_M(N_s, z)$ is a measure of the mean accuracy achievable with N_s sensors placed in locations defined by vector z. If the number of sensors is equal to the total number of responses N (that is all the responses are recorded hence the error $\varepsilon_M(N)$ vanishes) the accuracy $a_M(N)$ equals 1. The ratio between $a_M(N_s)$ and N_s can be interpreted as the 'amount of accuracy' due to the existence of each single sensor or as the 'exploitation' of each sensor; if N sensors are available the amount of accuracy 'contributed' by each one of them is equal to 1/N. If the number of sensors is equal to 40%; the corresponding accuracy is 60% and the 'amount of accuracy' required from each sensor is 10%. If the number of sensors is increased to 7 and correspondingly the error reduces to 20%, the amount of accuracy of the single sensor becomes 80/7=11.4% hence the exploitation of the single sensor reduces to 65/7=9.2%.

The expression (5) given for the effectiveness index takes into account this twofold effect of the increase of N_s : the first term equals the accuracy achievable with a given number N_s of recording sensors normalized to its maximum value $[1 - \varepsilon_M(N)]$. The second term is the exploitation of the single sensor normalized to its minimum value that is the one achieved in the condition of maximum accuracy $(N_s=N)$. For a given number of sensors N_s the maximum accuracy is achieved with the distribution \overline{z} leading to the minimum value of the error $\varepsilon_M(N_s, \overline{z})$ hence the maximum values of the effectiveness function for different values of the number of recording sensors are described by the function $e(N_s) = e(N_s, \overline{z})$.

APPLICATION TO THE NUMERICAL MODEL OF A MULTISPAN BRIDGE

The method has been checked with reference to the simple model of a four span bridge supported by abutments at each end and by three piers at intermediate locations as shown in figure 1. The four span measure 50 m and the three piers have identical section and height equal to 14m. The bridge model is derived from the text of Priestley et al. [5].



The frame model with distributed masses reported in figure 1 has been adopted for the structure. Modal analyses has been performed to calculate natural frequencies and mode shapes and an elastic time history analyses has been carried out for transversal ground motions assuming as input motion the North-South component of horizontal ground motion recorded at Imperial Valley, during the 1940 El Centro earthquake. The effect of the differential support ground motion has not been considered.





Responses in joints 1 to 17 (see figure 2) have been calculated and used for comparison with responses reconstructed via the spline interpolation considering a limited number of sensors N_s to be available. A minimum of two sensors is required to perform the interpolation hence the reconstruction of unknown responses has been carried out for N_s increasing from 2 to 16. For each value of N_s the distribution of sensors \bar{z} corresponding to the minimum value of the mean error ε_M has been considered for the evaluation of the effectiveness function $e(N_s)$

Table 1 reports for different values of N_s the values of $\varepsilon_M(\bar{z})$ and $\sigma_{\varepsilon}(\bar{z})$. In the last column of the table are reported the corresponding values of the 'effectiveness function' $e(N_s)$. The variation of $e(N_s)$ shows that the increase from 3 to 4 of the number of sensors reduces the mean error but does not increase the effectiveness of the recording system because, despite the increase of one unit of the sensors set, the error exhibits an almost insignificant reduction. On the contrary the increase from 4 to 5 sensors allows to sensibly reduce the mean error with a corresponding increase of the effectiveness. The most effective number of recording sensor is the one corresponding to the maximum value of function $e(N_s)$. In this case the function presents two local maxima for respectively 3 and 5 recording sensors.

N_s	$\mathcal{E}_{M}(\overline{z})$	$\sigma_{\epsilon}(\overline{z})$	$e(N_s)$
	[%]	[%]	
2	66	54	0.98
3	25	16	3.19
4	20	21	2.72
5	5.2	3.9	3.06
6	3.9	3.3	2.62
7	2.4	2.1	2.31
8	1.9	1.9	2.05
9	1.1	1.4	1.85
10	0.8	1.1	1.67
11	0.5	0.6	1.53
12	0.4	0.6	1.41
13	0.25	0.45	1.30
14	0.18	0.4	1.21
15	0.12	0.33	1.13
16	0.06	0.25	1.06

Table 1. Multispan bridge: interpolation error and effectiveness function

The difference of accuracy relevant to the two cases, quantified by the variation of ε_M reported in table 1, is evident in the comparison, between the real response in a given location and the ones calculated considering the two different values of N_s .

In order to visually show the quality of the fit obtained with the spline shape function figure 3 reports the comparison between real and calculated absolute acceleration relevant to location 13 for N_s = 3 and N_s =5. The effect of the increase in the number of sensors on the accuracy of the interpolated signal is more evident in the frequency domain since the match between the time histories is already very good with three recording sensors. The comparison between real and calculated function in terms of magnitude (see figure 4) and phase (see figure 5) of the transfer function shows an evident betterment of the agreement in the range of frequencies higher than the one of the first mode.



Fig. 3 Multispan bridge. Comparison between calculated and recorded absolute acceleration.

For N_s =3 the contribution of second mode of the structure is almost absent in the reconstructed transfer function while is accurately reproduced for N_s =5. The third mode contribution is badly reproduced both by 3 and by 5 sensors available. A further increment in the number of recording sensor and their proper placement would allow the contribution of the third mode to be interpolated. The consequent beneficial effect on the accuracy of reconstructed responses would be as much higher as much higher is the contribute of the third mode on the total response.



Fig. 4 Multispan bridge. Comparison between calculated and recorded magnitude of transfer function.



Fig. 5 Multispan bridge. Comparison between calculated and recorded phase of transfer function.

APPLICATION TO THE NUMERICAL MODEL OF A CABLE-STAYED BRIDGE

The second example herein considered is relevant to single tower cable stayed tramway bridge overcrossing a railway installation in Milan. Details on the bridge design are reported in Martinez et al. [3]. The bridge, reported in figure 6, spans 156m with a 3% longitudinal slope. The cast in place concrete tower is 23.20 m high and divides the bridge into two spans: the left one is 66m long and the right one is composed by four spans of variable length between 18.00m and 28.50 m. The deck is a two cells concrete box girder cast in place and post-tensioned. The width of the deck is 8.85m carrying a 7.20m tramway and two lateral walkways respectively 0.50m and 1.15m wide.

The model of the bridge used for the analyses has been validated using results of dynamic characterization tests carried out on the structure in the real of a research project funded by the Metropolitana Milanese. The model can thus reliably reproduce the effective behavior of the real structure characterized by an irregular geometry hence by an higher level of complexity with respect to the example shown in the previous section.



Fig. 6. The cable stayed bridge of via Palizzi, Milan (taken from reference [3])

A modal analysis and an elastic seismic analysis of the bridge, have been carried out calculating the modal frequencies and modal shapes and the transversal response of the bridge to the North-South component of the El Centro earthquake. The three dimensional model reported in figure 7 has been adopted for the analysis of the structure, neglecting the effect of the differential support ground motion.



Fig. 7. The three-dimensional model of the Palizzi bridge.

Responses in joints 1 to 14 (see figure 8) have been calculated from the three-dimensional finite element model and assumed to be the 'real responses'. Considering a limited number of responses to be available, that is considering a limited number of available sensors (gradually increasing from 2 to 14), for all the possible distribution of the given number of sensors, responses have been reconstructed in locations not provided with sensors and compared to the real ones.



Fig. 8. Sensors location in the Palizzi bridge

Figure 9 to 11 report, for a total number of recording sensors equal to five, the comparison between recorded and reconstructed absolute accelerations in joints 5 and 11. The comparison is carried out both in time and in frequency domain in terms of time history, magnitude and phase of the transfer function relevant to the base input. The two locations are placed respectively on the left (cable-stayed) and on the right (multispan) side of the tower.



Fig.9. Palizzi bridge. Comparison between calculated and recorded absolute acceleration. Left and right spans



Fig. 10. Palizzi bridge Comparison between calculated and recorded magnitude of transfer function. Left and right spans

The two locations exhibit a markedly different behavior. In the left span of the bridge the response is almost entirely due to the first transversal mode as shown by the relevant magnitude and phase of the transfer function. The right span of the bridge is characterized by a very high stiffness hence the absolute acceleration is almost equal to the ground acceleration: even with a very small number of sensors the differences between recorded and calculated signals in this location are very slight.



Fig. 11. Palizzi bridge. Comparison between calculated and recorded phase of transfer function. Left and right spans.

Table 2 reports, for increasing values of the number of recording sensors, the values of the mean error and of the standard deviation of the error function. In the last column the corresponding value of the effectiveness function is reported. The variation of $e(N_s)$ with N_s shows that the cost-effective number of sensors, corresponding to the maximum value of $e(N_s)$ is equal to 5.

N_s	$\mathcal{E}_{M}(\overline{z})$	$\sigma_{e}(ar{z})$	$e(N_s)$
	[%]	[%]	
3	30	35	2.22
4	17	22	2.41
5	5	7	2.53
6	0.97	1.2	2.29
7	0.46	0.76	1.98
8	0.36	0.71	1.74
9	0.26	0.46	1.55
10	0.13	0.27	1.40
11	0.10	0.20	1.27
12	0.04	0.09	1.17
13	0.01	0.05	1.08
14	0	0	1

Table 2. Palizzi bridge: interpolation error and effectiveness function

APPLICATION TO A REAL ARCH REINFORCED CONCRETE BRIDGE

The last example herein considered is relevant to a real arch bridge instrumented for monitoring and research purposes by the Italian Seismic Service (Osservatorio Sismico delle Strutture - Dipartimento della Protezione Civile - Ufficio Servizio Sismico Nazionale). Earthquake data recorded during the San

Leo-Novafeltria earthquake (August 1, 2000) on the Zingone bridge where used to check the spline function reconstruction technique by using real recorded data.

The Zingone Bridge, reported in figure 12 is a fixed arch reinforced concrete bridge located in Mercato Saraceno (Forli, Italy). The bridge features a reinforced concrete arch with open spandrels and one longitudinal rib. The rib is a hingeless arch fixed at the abutments with width of 6.5m and depth varying between 2.4m and 1.4m from abutments to crown. The arch spans 54.9m with a rise of 15.6m. The bridge deck is made up with six reinforced concrete beam supported by spandrel columns 6.5m wide with depth varying between 1.0 m and 0.6m, placed uniformly at 4.8m center to center. The out-out deck width is 7.9m, carrying a 6.8m roadway and two 1.5m sidewalks.



Fig. 12. The Zingone bridge (courtesy of Servizio Sismico Italiano)

Thirty-two seismic sensors were installed by the Seismic Observatory of Structures of the Italian Seismic Service to record the seismic behavior of the bridge. A total number of 26 sensors were placed on the structure and 6 sensors were located at two reference free field sites near the bridge. Figure 13 reports the sensors locations. Responses have been recorded in the vertical, longitudinal, and transversal direction of the bridge. In this paper only the analyses of the transversal response of the bridge has been reported.



Fig.13. Zingone bridge: sensors location (courtesy of Servizio Sismico Italiano)

In order to check the reliability of the spline function method in reconstructing unknown responses in the transversal direction, responses have been calculated by considering a number of recording sensors lower than the real one and gradually increasing from 3 to 6. 'Unknown' responses have been reconstructed through the spline shape function and compared to recorded ones.

Table 3 reports the values of the response mean error and of the standard deviation for different values of the recording sensors corresponding to different number of recording sensors. For each value of N_s the considered distribution of sensors is the one corresponding to the minimum value of $\mathcal{E}_M(\bar{z})$.

N_s	$\mathcal{E}_{M}(\overline{z})$	$\sigma_{\varepsilon}(\bar{z})$	$e(N_s)$
	[%]	[%]	
2	97	69	0.003
3	37	33	0.93
4	16	19	1.23
5	11	17	1.11
6	5	12	1.05

Table 3. Zingone bridge: interpolation error and effectiveness function



Fig.14 Zingone bridge. Comparison between calculated and recorded absolute acceleration.



Fig. 15. Zingone bridge Comparison between calculated and recorded magnitude of transfer function.



Fig. 16. Zingone bridge. Comparison between calculated and recorded phase of transfer function.

Figures 14 to 16 show the comparison between real and calculated time history, magnitude and phase of transfer function with respect to the ground motion, relevant to the absolute acceleration in the transversal direction. Two different cases characterized by a different total number of sensor are reported. In this case, a higher density of recording sensors with respect to the examples reported in the proceeding sections. is required to achieve a certain level of accuracy.

DISCUSSION

The variation of the function $e(N_s)$ with the density of recording sensors δ is reported in figure 17 for the three cases analyzed in this paper.



Fig. 17. Variation of the effectiveness function with the density of recording sensors.

The comparison shows that lower values of the effectiveness function for higher values of the density are achieved in the case of the arch bridge. This behavior is partly due to fact that interpolation of responses through the spline shape function is more accurate for one-dimensional structures with a beam-like behavior that is better approximated for the cases of multispan and cable-stayed bridge with respect to the arch bridge case. Furthermore inaccuracy deriving from an approximate measure of the sensors location in

this latter case and that do not affect the other two example were the sensors locations are directly derived from the numerical model may have affected results. Finally since responses of the arch bridge have been recorded on the structure during a real seismic event they are probably affected by noise which is not the case for the other two cases.

The differences between results relevant to the multispan and the cable-stayed bridge can be explained partly by the higher complexity of the second structure, mainly due to its dissymmetric geometry that requires a higher number of sensors to obtain a given level of accuracy from the spline interpolation method.

CONCLUSIONS

An effectiveness function has been proposed to chose the cost-effective number of recording sensors for monitoring purposes. The function is defined in conjunction with a method allowing the reconstruction of seismic responses of a structural system in locations where no recording sensors are available. The method is based on the interpolation of the available measurements of absolute acceleration along the axis of the structure by means of a spline shape function. The time dependent coefficients of the spline function are determined by the interpolation of the responses recorded by the available sensors. The influence of the number of sensors on the accuracy of the calculated responses has been investigated both for numerical models of multispan and cable-stayed bridges and for a real arch bridge structure subjected to seismic excitation. The comparison between the three cases has shown that a given number of recording sensors allows to achieve different levels of accuracy through interpolation of recording responses in dependence of the complexity of the structure. However for all the considered cases the proposed effectiveness function allowed to determine the cost-effective number of recording sensor to locate on the structure.

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