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UNDERSTANDING THE DYNAMICS OF MULTI-DEGREE-OF FREEDOM STRUCTURES SUBJECT TO MULTIPLE SUPPORT EARTHQUAKE EXCITATION

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SUMMARY

Multiple support excitation during earthquakes is a significant hazard for structures with large distances between ground support points. Typically this applies to long-span bridges, but the hazard may also be significant for buildings and other structures with a large plan area, particularly if the geological/soil conditions under the structure are non-uniform. The current approach for modelling multiple support excitation of structures uses a linear superposition method based on response spectrum techniques. Little experimental testing has been carried out in order to verify the theory and to test the nonlinear behaviour of these types of structures under extreme loading cases. In this paper we present results for a series of experimental tests on a two degree of freedom system. There is good agreement between the experimental data and a simple numerical model analysed using direct integration methods. The results also show that, for a two degree of freedom model, if multiple support excitation is ignored and only synchronous load cases are considered, the peak response of the system may be significantly underestimated.

INTRODUCTION

When considering the effects of earthquakes on structures with small base area, generally the excitation at all support points is considered to be the same. This assumption is often used for larger structures but this does not take into account variations in excitation caused by one or more of the following effects:

1. Difference in arrival time due to finite wave speed (wave passage effect), Kiuregihan *et al* [1].
2. Loss of coherency of motion due to reflection and refraction in heterogeneous ground, as well as superposition of waves arriving from extended sources (incoherence effect), Kiuregihan *et al* [1].
3. Difference in soil conditions (local effect), Kiuregihan *et al* [1].
4. The foundation's relative flexibility compared to the soil, as the foundation is not always able to vibrate according to the displacement field that is imposed on it by the incoming waves (foundation effect), Sextos *et al* [2].
5. Rock either side of a valley swaying (sway effect), Yamamura *et al* [3].

Any of these effects can be considered to cause "multiple support excitation" (MSE) which effectively refers to the existence of different excitations at different supporting points of a structure.

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Early references to the theoretical existence of MSE include Bogdanoff *et al* [4] and Clough and Penzien [5] who discussed the implications of MSE for multi-degree-of-freedom structures. More recently the existence of MSE has been substantiated through measurements of excitations at the Smart 1 array Loh *et al* [6]. Clough and Penzien [5] recommended analysing MSE by separately considering the dynamic response and the pseudo-static response, due to the differential movements of the inputs. The individual responses are summed to calculate the overall response of the structure. This method has been adopted by many and has been applied to various bridge problems by, among others, Dumanoglu *et al* [7-9] and Hao [10, 11]. With the advent of increased computing power a more direct method has been used by Wagg *et al* [12] and DebChaudhury *et al* [13] which analyses the equations of motion of the system without first breaking the response down into static and dynamic components of response. However, very little experimental work has been performed to validate these methods, particularly in the nonlinear range and this research is particularly focussed on experimental studies of the effects of MSE.

In this work, the direct integration method has been used to analyse a simple two degree of freedom model subjected to various types of MSE. The numerical results have been compared with the physical response of a comparable experimental model in order both to validate the numerical model and to assess how significant the effects of MSE may be in the design of long span structures.

The system under consideration can be considered to have two different types of degrees of freedom. Each input into the system, typically the ground, shall be referred to as a “ground degree of freedom” (GDOF) whilst the response of the system, in this case each of the masses which will be excited, shall be referred to as a “response degree of freedom” (RDOF). The following terminology will be used in this paper to describe the problem and different types of input motion. An input into a structure, which is identical at each support with no time delay between each support, will be referred to as “synchronous” excitation. This is the typical design case for small structures although it is also often used in the design of long span structures. Two different varieties of MSE can then be considered. In the first case, an identical time history is applied to each of the support points but with a variable time delay between the start of the input motions. This is equivalent to the “wave passage effect” outlined earlier and will be referred to as “asynchronous” excitation. This type of loading is a subset of the second type of MSE that can be considered. In this case two or more different time histories are applied to the support points and this will be referred to as MSE. This case covers the “incoherence”, “local”, “foundation” and “sway” effects discussed earlier and is the most general type of loading that can be applied to a structure.

EXPERIMENTAL MODEL

In developing an approach for numerically and physically modelling the effects of MSE, models with many different levels of complexity have been considered. This paper will focus on initial studies performed on a simple two RDOF model (schematic diagram in figure 1). Whilst this model does not represent the complexities of a real long span bridge it is able to capture the essence of the problem and has been an effective way to develop the testing techniques that are essential when performing this type of physical test.

Prior to looking at this two RDOF model a simpler single RDOF system with MSE inputs was investigated. This simpler model still had two GDOFs as this is the minimum number required to produce a system with MSE. However, initial research into this single RDOF showed that the case of synchronous excitation always generated the peak system response. The two RDOF model, however, is more interesting and as shown later, can experience larger response at the second mode, when considering MSE.

The physical model, equivalent to the system shown in figure 1, had two actuators, one at either end of the structure, applying input motions GDOF1 and GDOF2. Two masses, RDOF1 and RDOF2, on linear bearings between the actuators were linked by three springs of similar stiffness. The dampers

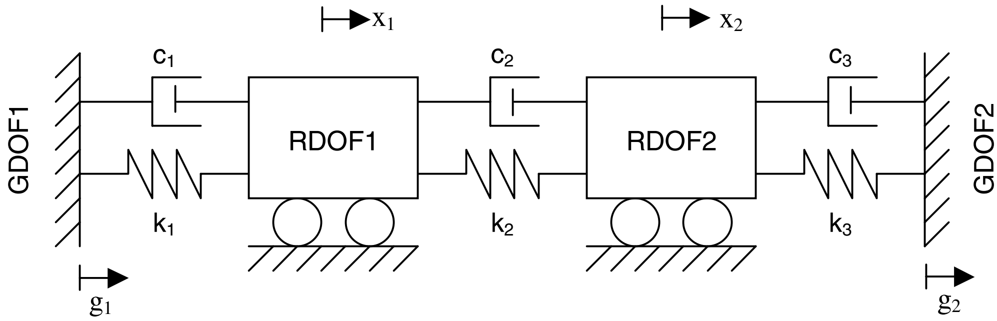

Figure 1. Model with two GDOF and two RDOF

shown in figure 1 did not exist physically, but are included in the schematic as the equivalent numerical model reproduced the damping in the system by incorporating dampers at these points between the masses. The experimental set-up can be seen in figure 2.

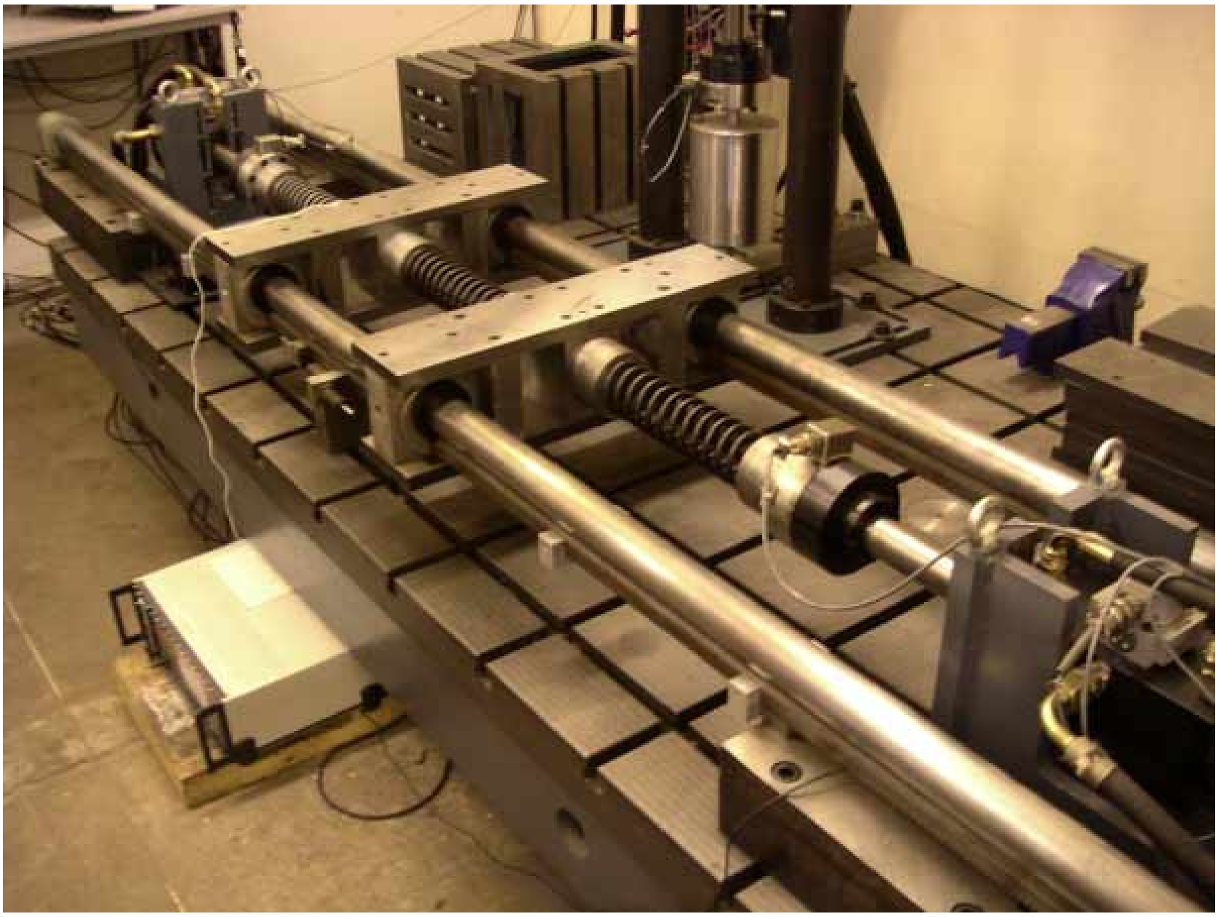

Figure 2. The experimental model with two GDOF and two RDOF

The two RDOF masses weighted 87kg each and were mounted on two parallel rails which constrained their motion to one direction. The three springs used to couple the masses and actuators k_1, k_2 and k_3 had stiffness of 16042, 16923 and 16080N/m respectively. The damping c from viscous friction was estimated as 10Ns/m.

Whilst this model is not apparently bridge-like, this model is mathematically equivalent to a simple two RDOF bridge subjected to out-of-plane vibration. For example, this model could be used to represent a simplistic model of a bridge with two supports where all the input energy results in responses of just the first two modes, one symmetrical and one asymmetrical. This method of creating equivalent models was first suggested by Gulkan *et al* [14] who approximates a complex structure to a simple single degree of freedom system. The findings of this set of tests could therefore be applied to a simple single span bridge with two strongly dominant degrees of freedom, one being symmetrical and one asymmetrical.

To investigate the effect of MSE on this two RDOF system various different input motions were applied to the model via the hydraulic actuators. The drive signals for the actuators were generated using SIMULINK running on dSPACE hardware. This hardware also incorporated several data acquisition channels that were used to record the displacements of the two masses using external linear variable displacement transducers (LVDTs) mounted on the test bed. The data was post-processed using MATLAB and GnuPlot.

The control of this type of physical system is inherently difficult because of the potential for significant interaction between the specimen and the actuators driving the system. This tends to mean that a real time adaptive control has to be used to accurately control the motion of the two actuators, especially if the model is relatively stiff compared to the capacity of the actuators. In the initial tests, the stiffness of the springs was limited so that a simple proportional controller could be used to simplify the design of the experiment. The actuators incorporated internal LVDTs and these were used to provide the feedback for a simple proportional controller. However, in parallel with these experiments, several more complex control strategies have also been developed that can deal with models that are much stiffer and exhibit significant interaction between the model and the actuators.

NUMERICAL MODEL

In parallel with the experimental testing, a numerical model of the two RDOF system shown in figure 2 was developed. The motion of the RDOFs, x_1 and x_2, with GDOFs, g_1 and g_2 as shown in figure 1, can be described using the following equations:

$$m_1\ddot{x}_1 + \hat{c}_a\dot{x}_1 + \hat{k}_a x_1 = \tilde{c}_a\begin{pmatrix}\dot{g}_1\\ \dot{x}_2\end{pmatrix} + \tilde{k}_a\begin{pmatrix}g_1\\ x_2\end{pmatrix} \qquad \text{eqn 1}$$

$$m_2\ddot{x}_2 + \hat{c}_b\dot{x}_2 + \hat{k}_b x_2 = \tilde{c}_b\begin{pmatrix}\dot{x}_1\\ \dot{g}_2\end{pmatrix} + \tilde{k}_b\begin{pmatrix}x_1\\ g_2\end{pmatrix} \qquad \text{eqn 2}$$

where

$$\hat{c}_a = c_1 + c_2, \hat{c}_b = c_2 + c_3, \hat{k}_a = k_1 + k_2, \hat{k}_b = k_2 + k_3 \qquad \text{eqn 3}$$

$$\tilde{c}_a = (c_1, c_2), \tilde{c}_b = (c_2, c_3), \tilde{k}_a = (k_1, k_2), \tilde{k}_b = (k_2, k_3) \qquad \text{eqn 4}$$

These equations were solved directly using a Runge Kutta 4^{th} order method of numerical integration with a time step of 0.01secs following the methodology outlined in Wagg *et al* [12]. This states that if only the steady state system response is considered, then the pseudo-static method of analysing systems subjected to MSE produces the same results as this direct integration method. The direct method and pseudo-static methods, however, produce slightly different solutions particularly at the start of the excitation, where the response is transient.

EXPERIMENTAL RESULTS

The behaviour of the experiment model, shown in figure 2, under MSE was investigated by applying different frequency sine wave motions with constant displacement to the two ends of the model to create MSE. By holding the frequency of motion at one end of the model constant and varying the frequency of motion at the other end it was possible to determine the frequencies of motion that would result in the largest response of the system. The frequency at GDOF2 was maintained at a constant frequency close to either the first or second natural frequency of the system and in each case the frequency at GDOF1 was increased in 0.1Hz steps from 1Hz to around 5Hz. The duration of excitation for each frequency step was sixty seconds and in this time the model had time to settle down to a steady state response. The peak displacements of the two RDOFs for this steady state were then measured from the time history displacement values for each frequency increment of excitation.

In the first series of experimental tests the input at GDOF2 was fixed at 1.95Hz whilst the frequency of GDOF1 was varied. The frequency of GDOF2 was close to the first natural frequency of the system (2.16Hz), but was not set at the first natural frequency in case the displacements of the two masses was too large. The input displacement was kept constant at 0.001m.

In the second series tests the frequency of GDOF2 was fixed at 3.85Hz whilst the frequency of GDOF1 was varied. In this case, the frequency of GDOF2 was close to but not at the second mode natural frequency (3.80Hz). It was important that in both of these series of fixed frequency experiments the fixed frequency was at no point the same as the varying frequency. In the case when the frequencies are the same the phase difference is fixed, typically at zero. This does not allow the full spectrum (between 0 and 2π) of phase differences to occur. However, having a slight difference in the frequency allowed the excitation at the two supports to go in and out of phase, leading to a more accurate representation of MSE rather than purely a fixed phase difference excitation. The effects of the simpler asynchronous excitation (same input frequency but variable phase difference or same input motion but variable time delay) were investigated numerically and the key results are presented later.

The peak experimental responses of RDOF1 under MSE are presented in figures 3 and 4 with the frequency of GDOF2 fixed at 1.95Hz and fixed at 3.85Hz respectively. The RDOF2 results are almost identical and so have not been included to avoid repetition. The experimental results are represented with asterisk and are compared with the numerical results described in the next section.

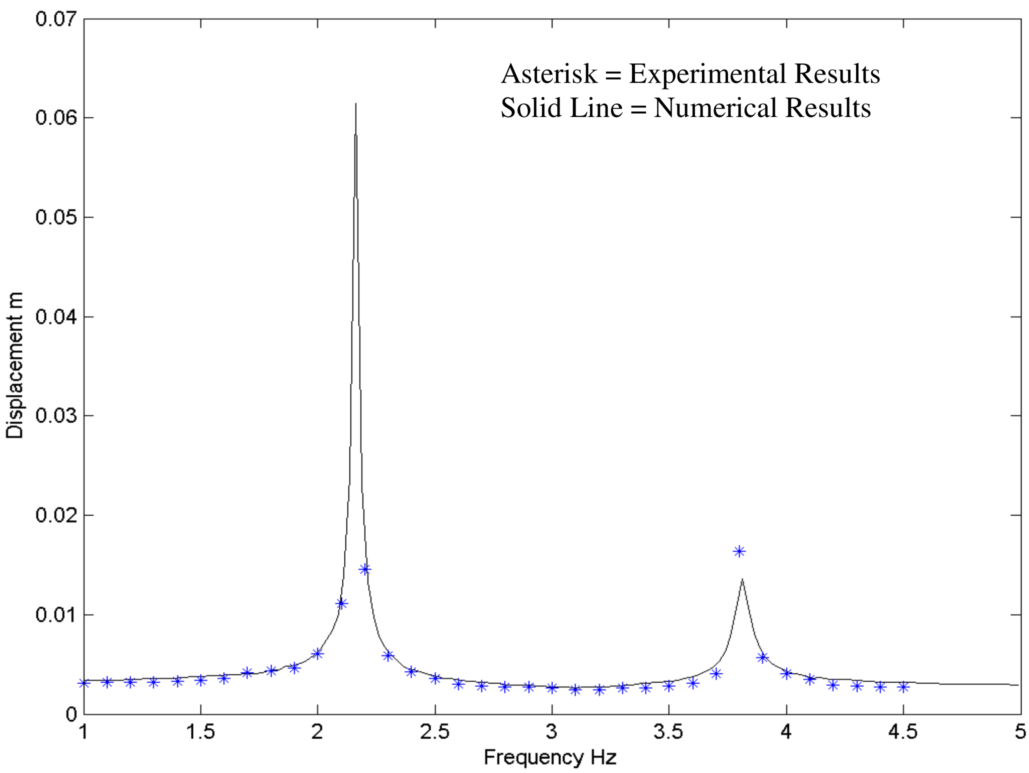

Figure 3. Peak steady state response of the model with a variable frequency input at GDOF1 and fixed frequency input of 1.95Hz at GDOF2

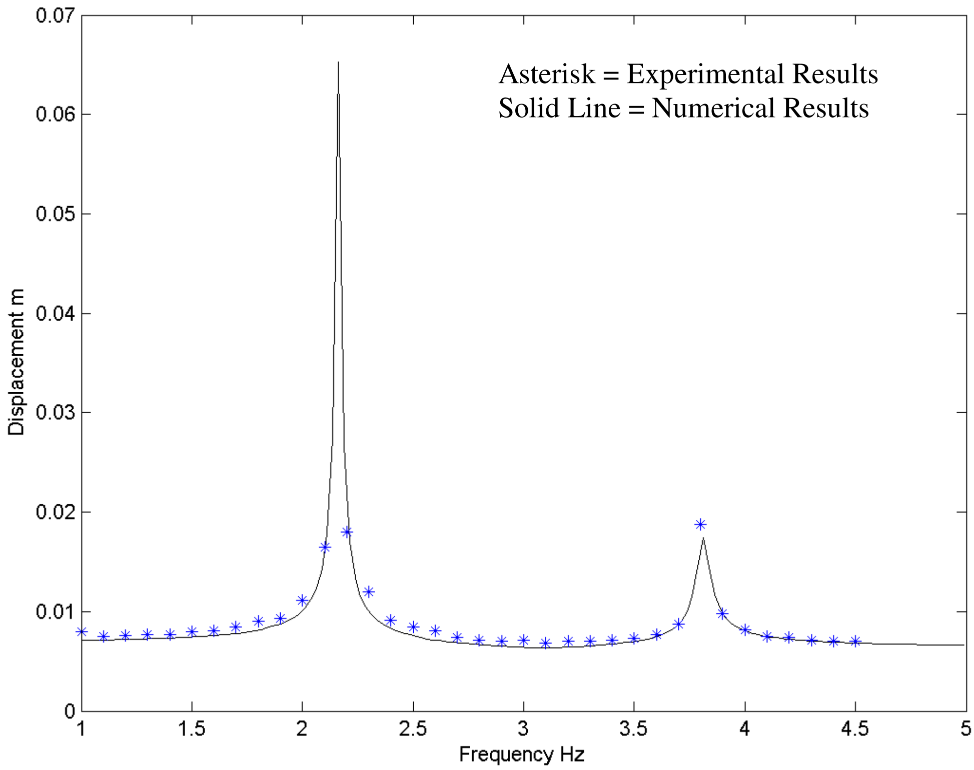

Figure 4. Peak steady state response of the model with variable frequency input at GDOF1 and a fixed frequency input of 3.85Hz at GDOF2

NUMERICAL RESULTS

In order to provide a direct comparison between the experimental data and the numerical model, the physical parameters of the experimental model were used when defining the numerical model. The numerical model was analysed using both sinusoidal inputs and a scaled El Centro earthquake time history. The sinusoidal inputs were used to allow a direct comparison with the experimental data.

Initially, the same types of excitation as used in the experimental model were used in the numerical simulation. For both sets of numerical analysis the starting frequency was taken as 0.9975Hz so that the variable input frequency was never the same as the fixed frequency. Therefore the case with only a synchronous input was not able to occur for the reasons discussed previously. The frequency step for the numerical simulations was reduced to 0.025Hz to give a more accurate picture of the peak response. An analysis time step of 0.01s was used and the response of the system was calculated for a duration of 200s to allow the response of the RDOFs to reach steady state. The duration for the numerical model was longer than that for the experimental model as the frequency difference between the variable frequency of GDOF1 and the fixed frequency of GDOF2 was much smaller at the point when they were closest. Therefore it took longer for the system to go in and out of phase and for a steady state response to occur. The peak steady state values for RDOF1 are plotted on figures 3 and 4 as a solid line to provide a direct comparison with the experimental data.

Once the numerical model had been shown to be an effective predictor of the system response a second series of analyses were performed looking at the effect of asynchronous excitation of the system. For these analyses, a sine wave at the same frequency was used to excite both GDOFs of the model, but the phase difference between the two input motions was varied between 0 and 2π. This therefore covered synchronous excitation (phase difference of 0), and fully asymmetric excitation (phase difference of π), as well as all other possible asynchronous cases, generating a peak response envelope.

The peak response envelope is shown in figure 5. The solid line shows the peak response of the system for synchronous excitation. The dashed line shows the maximum asynchronous peak displacement. This value is calculated from the maximum system response with the two GDOF inputs at the same frequency but with a phase difference between the inputs having any value between 0 and 2π. The dash-dot line shows the minimum peak displacement, again calculated with the two GDOF inputs at the same frequency and with any phase difference between the inputs between 0 and 2π. The dashed and dash-dot lines therefore provide the upper and lower bounds for the response envelope for sine wave excitation of the system with any possible phase difference between the inputs. The case of synchronous excitation must fall on, or between, these two lines but in this case it follows either the maximum or minimum response line.

Another useful tool in understanding what is happening in the model is the relative displacement across the springs. The relative displacement is the difference in displacement of two consecutive degrees of freedom. In this paper we will consider the relative displacement of spring 2, which is the difference in displacement between RDOF1 and RDOF2. The relative displacement provides us with two important pieces of information. Firstly it tells us whether the first mode is being excited, in this model the first mode involves both masses moving together and therefore the relative displacement is zero. When the relative displacement is non zero the second mode must also be involved although this does not mean that the first mode is not also involved. Secondly the relative displacement can be used to calculate the force in the spring, therefore the larger the relative displacement the larger the force in the spring. The peak response envelope for the relative displacement of spring 2 is given in figure 5.

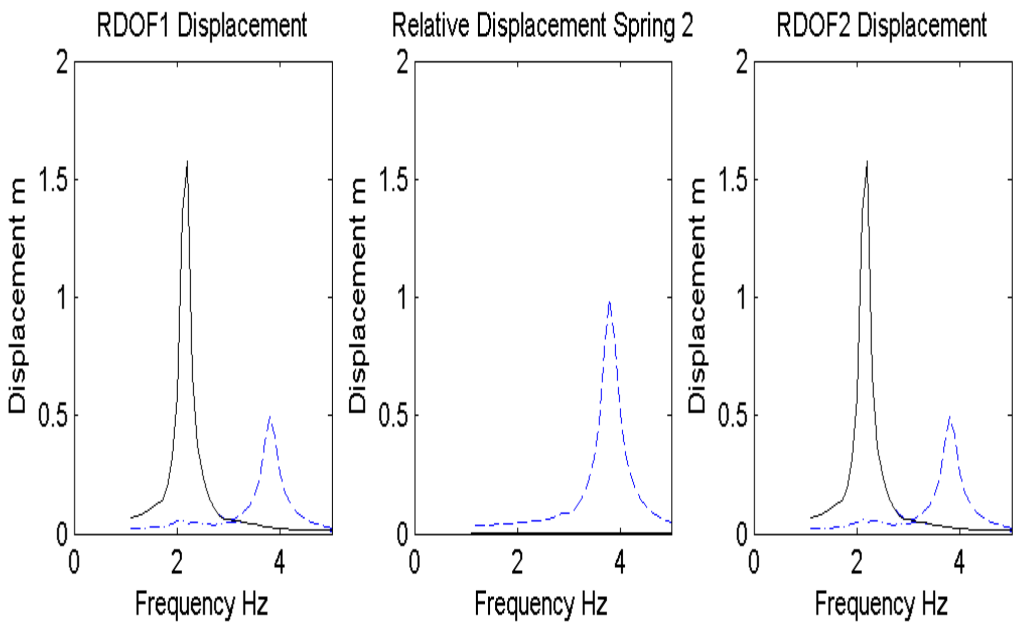

Figure 5. Peak response envelope: Solid line is synchronous case,
dashed line is maximum asynchronous case, dot-dash line is minimum asynchronous case

In the third series of numerical analyses the same numerical model was subjected to earthquake excitation. A scaled El Centro earthquake was used as input to the model and two cases were studied. In the first case the excitation was synchronous and in the second case a time delay of 1 second was used to create an asynchronous excitation of the two supports. So that the energy content of the time history more closely matched the natural frequencies of the numerical model the time step of the El Centro record was scaled down by 50%. The results of the analyses are shown in figures 6 and 7 for synchronous and asynchronous cases respectively.

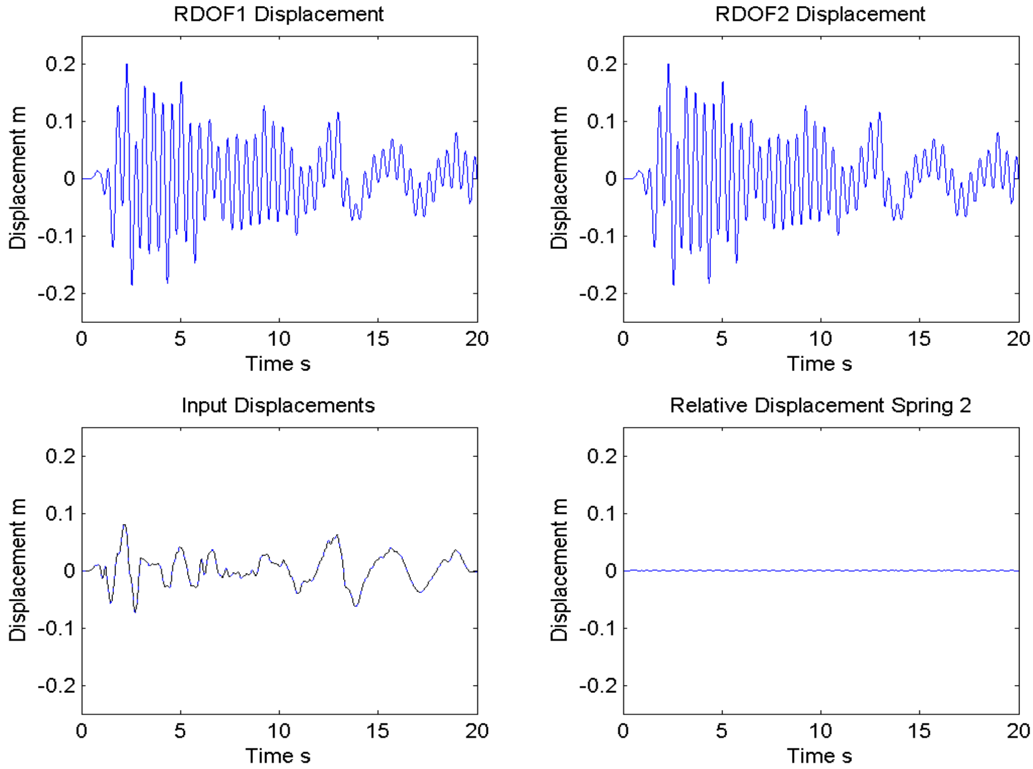

Figure 6. Response of a two GDOF, two RDOF model to the El Centro earthquake time history with synchronous excitation (both inputs to model are identical)

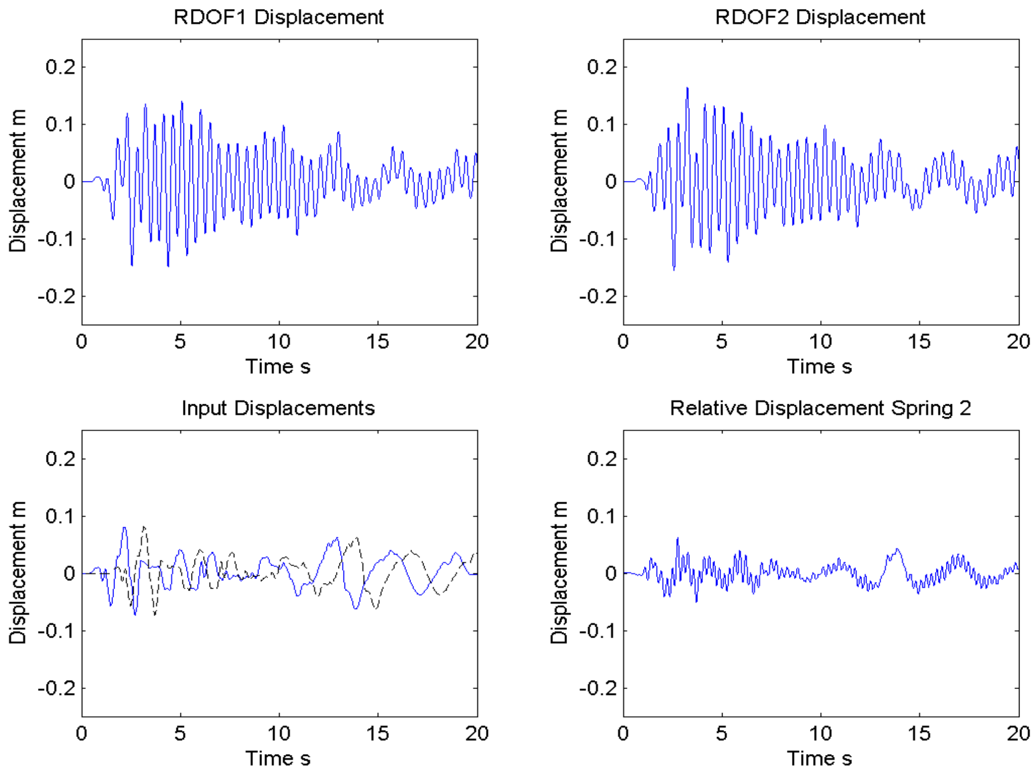

Figure 7. Response of a two GDOF, two RDOF model to the El Centro earthquake time history with asynchronous excitation (delay of 1 sec. between input motions, see bottom left plot)

DISCUSSION

The behaviour of a simple two RDOF system subjected to several different types of MSE excitation has been studied. Figures 3 and 4 show that the simple numerical model developed, accurately predicted the response of the physical model. This also means that the control methodology used inthe experimental tests worked well and was sufficient to overcome any actuator-specimen interaction. This is particularly important as we can be more confident that we can now perform MSE experiments at a larger scale on stiffer models. This opens up the possibility of performing MSE tests on more realistic physical models that also incorporate non-linear elements.

When the model was subjected to earthquake excitation, by comparing the top two plots in both figures 6 and 7 we can see that the greatest RDOF displacement (0.2m) occurred when the excitation was synchronous. We can also see that for the synchronous case, the masses were vibrating together at the first modal frequency, seen from the fact that the relative displacement across spring 2 is approximately equal to zero. This is somewhat counter intuitive since the main frequency of excitation was closer to the second modal frequency of the system. However, from the relative displacement of spring 2 plot in figure 7, we can see that both modes were excited. This is due to the fact that for a symmetrical system being excited synchronously, the second mode cannot be excited, even when excitation occurs at the frequency of the second mode. This is a result of the way the excitation forces propagates through the system that causes the system to lock into the first mode shape.

These effects can be also seen in figure 5. The maximum RDOF displacement (1.58m) occurs when the input motions are synchronous (solid line) and at the first natural frequency of the system (2.16Hz). The peak displacement response in the second mode (3.80Hz) only occurs if the input excitation is asynchronous (dashed line) and for this model the maximum displacement in the second model shape (0.5m) never exceeds the peak displacement in the first mode regardless of the phase difference between the input motions.

The relative displacement between the two RDOFs are shown bottom right in figures 6 and 7. For the synchronous case the relative displacement is small, this is due to the RDOF locking into the first mode regardless of the input frequency as discussed above. As a result, the peak relative displacement for the asynchronous case (0.06m) greatly exceeds that for the synchronous case (<0.01m). Again this can been seen in the centre plot in figure 5 where the relative displacement of the middle spring only occurs at the second natural frequency and greatly exceeds the displacement generated by synchronous excitation.

APPLICATION

There are several significant implications that can be drawn from these results that may have an impact on the consideration of MSE in design.

For a simple system with just two modes, if we only consider synchronous excitation, we could be significantly underestimating the design requirements of a structure. Work on the use of single degree of freedom models to represent structures, where all the excitation energy is absorbed by the first mode, is common, especially in the field of displacement based design Calvi *et al* [15] Kowalsky *et al* [16, 17]. Therefore, we can make a direct link between a bridge where the first two modes dominate, and our two mode model. As the relative displacement between the two RDOFs can be considered to be related to the design forces in a bridge deck, the results demonstrate that the design forces may be underestimated if only synchronous excitation, as is typical, is considered.

This problem is made worse by the symmetry of the model, as the more asymmetrical a model is, the greater the level of excitation of the second mode in the synchronous case. Current research on asymmetric systems has shown that although the second modal response does occur with synchronous excitation, for these models, asynchronous excitation still produces greater responses under certain conditions.

CONCLUSIONS

Currently the effects of Multiple Support Excitation (MSE) are rarely taken into account when analysing the behaviour of structures under earthquake excitation. A series of experimental tests and numerical analyses have shown that, in certain circumstances, the peak response of even simple systems may be significantly underestimated if only synchronous load cases are considered. Solution of the equations of motion of the system by direct numerical integration method produces responses that agree well with experimental tests as long as suitable control methodologies are used to control the experimental tests.

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