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IN2 - A SIMPLE ALTERNATIVE FOR IDA

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SUMMARY

Simplified inelastic procedures used in seismic design and assessment combine the nonlinear static (pushover) analysis and the response spectrum approach. One of such procedures is the N2 method, which has been implemented into the Eurocode 8 standard. The N2 method can be employed also as a simple tool for the determination of the approximate summarized IDA (incremental dynamic analysis) curve. Such analysis is called the incremental N2 method (IN2). The IN2 curve can substitute the IDA curve in the probabilistic framework for seismic design and assessment of structures. In the paper, the IN2 method is summarized and applied to two test examples of infilled reinforced concrete (RC) frames, which are characterized by a substantial degradation of the strength after the infill fails. The approximate summarized IDA curves, determined by the IN2 method, and the data on dispersion due to randomness in displacement demand, determined in a previous study by the authors, were employed in the probabilistic risk analysis of test structures. The results were compared with the results obtained using the "exact" IDA curves. A fair correlation of results suggests that the IN2 method is a viable approach.

1. INTRODUCTION

Incremental dynamic analysis (IDA) is a parametric analysis method for the estimation of structural response under seismic loads [1]. A structural model is subjected to multiple levels of seismic intensity using one or more ground motion records. The objective of an IDA study is the understanding of structural behavior under different levels of seismic intensity. IDA is also a substantial part of a probabilistic framework for seismic performance assessment, developed at Stanford [2,3].

There is no doubt that IDA analysis provides the most thorough image of the seismic behavior of the structure among all analysis methods presently available. However, it is very time consuming and the question arises if it is possible to determine IDA curves with less input data and with less effort, but still with acceptable accuracy? A possible approach is to determine seismic demand for multiple levels of seismic intensity with the N2 method [4], which is based on pushover analysis and response spectrum approach. Such analysis, which was employed in our study, will be called the incremental N2 method (IN2). The IN2 method is a relatively simple tool for fast determination of approximate IDA curves and may represent a viable approach for seismic performance assessment, appropriate for example for

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parametric studies on influence of different uncertain ground motion and structural input data. In combination with predetermined data on dispersion typical for a specific structural system, it can be employed also in the probabilistic framework, as demonstrated in this paper.

The structural system, used in our study, is the infilled reinforced concrete (RC) frame. The pushover curve for an infilled RC frame is characterized by a substantial degradation of the strength after the infill fails. Thus, specific reduction factors, developed by the authors, have to be used for the determination of inelastic spectra. The approximate IDA curves, determined by the IN2 method, and the data on dispersion, determined in a previous study by the authors, were employed in the probabilistic risk analysis of two test structures. The results were compared with the results obtained using the “exact” IDA curves.

2. SUMMARY OF THE N2 METHOD AND ITS EXTENSION TO INFILLED FRAMES

The N2 method combines pushover analysis of a multi degree-of-freedom (MDOF) model with the response spectrum analysis of an equivalent single-degree-of-freedom model (SDOF). The formulation of the method in the acceleration – displacement format enables the visual interpretation of the procedure and of the relations between the basic quantities controlling the seismic response. Details about the basic version of the N2 method, limited to planar structural models, can be found in [4].

In the N2 method, the seismic demand for the equivalent SDOF system with a period T can be determined as follows: Elastic demand in terms of acceleration S_{ae} and displacement S_{de} is determined from the elastic spectrum. The inelastic acceleration demand S_a is equal to the yield acceleration S_{ay}, which represents the acceleration capacity of the inelastic system. The strength reduction factor R can be determined as the ratio between the accelerations corresponding to the elastic and inelastic system. The ductility demand μ is then calculated from inelastic spectra, which are defined by the period dependent relation between reduction factor and ductility ($R-\mu-T$ relation), and the inelastic displacement demand S_d is computed as $S_d = (\mu/R) S_{de}$.

In principle any $R-\mu-T$ relation can be used. A very simple and fairly accurate $R-\mu-T$ relation is based on the “equal displacement rule” in the medium- and long-period range [4]. It has been implemented in Eurocode 8 [5]. The application of the N2 method can be extended also to complex structural systems, for example to infilled frames, provided that an appropriate specific $R-\mu-T$ relation is known. In order to employ the N2 method for infilled RC frames, a multi-linear idealization of the pushover curve is needed instead of a simple elasto-plastic idealization, in addition to a specific $R-\mu-T$ relation. Both modifications are described in [6] and [7].

The basic characteristic of the pushover curve of infilled RC frames is a substantial decrease in strength after the infill has begun to degrade. The pushover curve can be modeled with four-linear force-displacement relationship as shown in Figure 1a. Four parameters are needed for the definition of the idealized pushover curve: yield displacement D_y and yield force F_y, ductility at the beginning of softening of infills μ_s, and the ratio between the force at which infills completely collapse and yielding force r_u.

The $R-\mu-T$ relation, which can be used for infilled frames, is described in [6]. The relation depends on the basic parameters of the pushover curve and the corner periods of the acceleration spectrum T_C and T_D. T_C represents the corner period between the constant acceleration and constant velocity part of the spectrum of the Newmark-Hall type, and T_D represents the corner period between the constant velocity and constant displacement part of the spectrum. As an example, an idealized force-displacement relation (pushover curve) and the $R-\mu-T$ relation for the given pushover curve are shown in Figure 1.

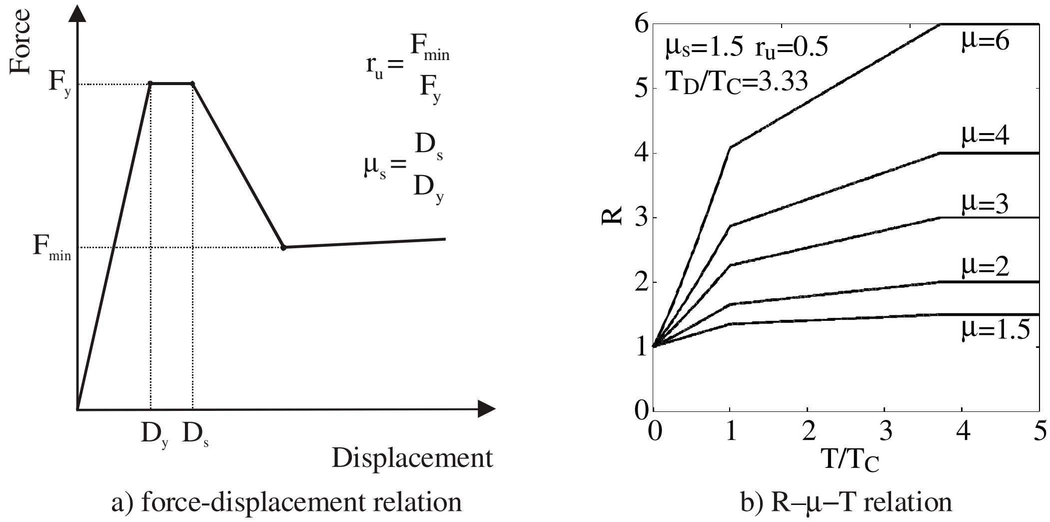

Figure 1. The idealized force-displacement relation for an infilled frame (a) and the R - μ - T relation (b) for the given force-displacement relation

3. IN2 METHOD

Incremental N2 (IN2) method is a relatively simple nonlinear method for determination of approximate IDA curves. IN2 method is, like the IDA analysis, a parametric analysis method. An IDA curve is determined with nonlinear dynamic analyses, while each point of an IN2 curve (approximate IDA curve), which corresponds to a given seismic intensity, is predicted with the N2 method. All limitations which apply to the N2 method [4] apply also to IN2 method.

In order to determine an IN2 curve, first the ground motion intensity measure and the demand measure have to be selected. The most appropriate pair of quantities is the spectral acceleration and the top (roof) displacement, which allow also the visualization of the procedure (Figure 2). Other relevant quantities, like maximum story drift, rotation at the column and beam end, shear force in a structural element and in a joint, and story acceleration, can be employed as secondary demand measures. They are related to roof displacement and can be uniquely determined if roof displacement is known. The secondary demand measures can be used, together with the main demand measure, for performance assessment at different performance levels.

Roof displacement and other relevant demand measures for a chosen series of spectral accelerations are determined by the N2 method. This step represents the main difference in comparison with IDA analysis because the N2 method is used for the determination of seismic response. Therefore the shape of the IN2 curve depends on the inelastic spectra applied in the N2 method, which are based on the relation between strength reduction factor, ductility and period (the $R-\mu-T$ relation). If a simple $R-\mu-T$ relation, based on equal displacement rule in the medium- and long-period range, is used, the IN2 curve is linear for structures with period higher than T_C and bilinear for structures with period lower than T_C. A more complex $R-\mu-T$ relation was proposed by authors for infilled RC frames [6,7]. In this case IN2 curve is four-linear (Figure 2). Considering the piecewise linearity of the IN2 curve, only a few points have to determined in order to obtain the complete N2 curve.

Usually the inelastic spectra, used in the N2 method, represent mean spectra and consequently the IN2 curve represents a mean curve. More specifically, the $R-\mu-T$ relation for infilled frames, used in this paper, represents an idealization of the $R-\mu-T$ relation, calculated for mean ductility given the reduction factor.

The schematic construction of the IN2 curve for a SDOF model in acceleration-displacement (AD) format is presented in Figure 2. The capacity diagram (multi-linear curve) shown in Figure 2 is characteristic for infilled RC frames and represents the idealized pushover curve of an equivalent SDOF model. As an example, two points (P_1 and P_2) of the IN2 curve, corresponding to two different ground motion intensities, are schematically constructed with the N2 method. The radial line from origin and crossing yield point represents the elastic system with period T. Elastic seismic demand in terms of elastic spectral acceleration ($S_{ae,1}$ or $S_{ae,2}$) and corresponding elastic spectral displacement ($S_{de,1}$ or $S_{de,2}$) is determined as the intersection of this line with the elastic spectrum for the appropriate ground motion intensity. The inelastic displacement demand ($S_{d,1}$ or $S_{d,2}$) is then determined with the N2 method. It corresponds to the point where the horizontal line, at the acceleration S_{ay}, intersects the appropriate inelastic spectrum. A point of the IN2 curve (e.g. the points P_1 and P_2) is defined with the pairs: elastic spectral acceleration on the Y-axis and the corresponding inelastic displacement demand on the X-axis (Figure 2). If inelastic displacements are determined for many levels of elastic spectral acceleration, the complete IN2 curve can be obtained.

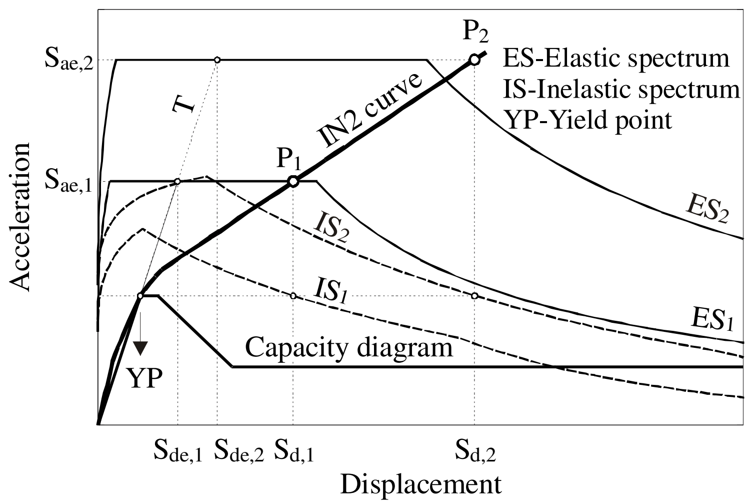

Figure 2. Schematic construction of an IN2 curve.

4. IMPLEMENTATION OF THE IN2 METHOD IN THE PROBABILISTIC SEISMIC PERFORMANCE ASSESSMENT ANALYSIS

The goal of the probabilistic framework adopted by the Pacific Earthquake Engineering Center [2] is to adequately predict earthquake losses and/or exceedance of one or more performance levels (or limit states), which can only be predicted probabilistically. The simplest form of the probabilistic framework is the estimation of the annual likelihood of the event that the demand exceeds the capacity at a chosen limit state, where all random elements of the problem are scalar values [3]. The random elements of this framework are the ground motion intensity measure, the demand D, and the capacity at a chosen limit state or performance level, denoted herein as C. The spectral acceleration S_a at the period of the idealized SDOF model is chosen to represent the intensity measure [3]. The hazard function $H(s_a)$ defines the annual probability that the spectral acceleration S_a is equal to or more than the selected level of spectral acceleration s_a. Both, the demand and the capacity are here characterized in terms of top displacement (see Section 3) instead of maximum story drift like in [3]. The top displacement capacity may be determined from different capacity measures like story drift, ultimate rotation, shear strength or others, which can be estimated from the pushover analysis. Thus the IDA analysis yields the relation between the spectral acceleration S_a and the top displacement D and provides the information needed to assess the structural performance at different limit states or performance levels.

Assuming that all random elements are lognormally distributed, the median hazard curve $\hat{H}(s_a)$ and the median IDA curve $\hat{D}(s_a)$ can be approximated by the forms

$$\hat{H}(s_a) = k_o s_a^{-k}, \tag{1}$$

$$\hat{D}(s_a) = a(s_a)^b \tag{2}$$

and the annual likelihood that the seismic demand exceeds the capacity at a limit state (performance level) P_{PL} can be expressed as

$$P_{PL} = \hat{H}\left(s_a^{\hat{C}}\right)\exp\left[\frac{k^2}{2b^2}\left(\beta_{DR}^2 + \beta_{CR}^2\right)\right] \tag{3}$$

where $s_a^{\hat{C}}$ is the spectral acceleration "corresponding" to the median displacement capacity $\hat{C}$, β_{DR} and β_{CR} are the dispersion measures for randomness in displacement demand and displacement capacity, respectively. The coefficients k and b are parameters of the hazard curve and IDA curve, respectively (Eqs. (1) and (2)). Expression (3) can be further developed if it is focused on the uncertainty of displacement demand and displacement capacity and if mean hazard curve is adopted [3]. In this case P_{PL} itself becomes an uncertain quantity and the x confidence level of P_{PL} is obtained as [3]

$$P_{PL}^x = \bar{H}\left(s_a^{\hat{C}}\right)\exp\left[\frac{k^2}{2b^2}\left(\beta_{DR}^2 + \beta_{CR}^2\right)\right]\exp\left[K_x\beta_{P_{PL}}\right],\ \beta_{P_{PL}} = \sqrt{\frac{k^2}{b^2}\left(\beta_{DU}^2 + \beta_{CU}^2\right)} \tag{4}$$

where $\bar{H}\left(s_a^{\hat{C}}\right)$ is the value of mean hazard at the spectral acceleration "corresponding" to the median displacement capacity, K_x is the standardized Gaussian variate associated with probability x of not being exceeded, β_{DU} is the dispersion measure for uncertainty of displacement demand, and β_{CU} is the dispersion measure for uncertainty of displacement capacity. The mean hazard curve can be calculated as

$$\bar{H}\left(s_a^C\right) = \hat{H}\left(s_a^C\right)\cdot\exp\left[\frac{1}{2}\beta_H^2\right] \tag{5}$$

where β_H is dispersion measure for hazard. The detailed explanation of the probabilistic framework and derivations can be found elsewhere [3].

To implement the results of the IN2 method into the described probabilistic framework, the IDA curve has to be replaced with the IN2 curve. In general, an IN2 curve is intended to approximate a summarized IDA curve (e.g. mean or median) and is not calculated for a single ground motion Therefore dispersion measures for randomness in displacement demand β_{DR} and displacement capacity β_{CR} can not be directly determined from the results of the IN2 method. Nonetheless the dispersion of randomness in displacement demand can be estimated from the coefficient of variation for displacement of the SDOF system, for which $R-\mu-T$ relation was determined, while β_{CR} has to be prescribed for typical structural systems in advance. It is also convenient and practical that dispersion measure for uncertainty in displacement demand β_{DU} and capacity β_{CU} are prescribed in advance. (Note that it is practical that these measures are prescribed in advance also in the original probabilistic seismic assessment with IDA curve). The determination of dispersion measures for typical structural systems is not within the scope of this paper. Additional studies are needed to determine the models for dispersions or to prescribe the appropriate values. For the presented examples dispersion measures β_{CR}, β_{CU} and β_{DU} were arbitrarily assumed.

In the N2 method inelastic spectra are used. Usually, they are intended to represent mean spectra and therefore the resulting IN2 curve represents the mean curve. In such a case, the mean IN2 curve has to be transformed to the median IN2 curve. This can be achieved with different methods if the dispersion is known. For example, if the method of moments is applied, then the standard deviation for natural logs β_{DR} can be calculated as

$$\beta_{DR} = \sqrt{\ln\left(V^2 + 1\right)}, \tag{6}$$

where V is the coefficient of variation for randomness in displacement demand, and the median values of the IN2 curve can be determined as

$$\hat{D}\left(s_a\right) = \bar{D}\left(s_a\right) \cdot \exp\left[-\frac{1}{2}\beta_{DR}^2\right] \tag{7}$$

where $\bar{D}$ stands for mean value of displacement determined with the N2 method.

So far, we have not yet developed the model for the dispersion measure for randomness in displacement demand. Based on our previous results [6,7,8], an upper limit of $V = 0.7$ and a lower limit of 0.4 can be assumed for coefficient of variation. The first value is appropriate for short- period structures, while the second value is reasonable for structures with moderate and long periods. Using Eqs. (6) and (7), the median IN2 curve can be estimated. Note that based on Eq. (7), the mean IN2 curve is more conservative (it yields larger displacements for a given spectral acceleration) than the median IN2 curve. For $V = 0.7$ and $V = 0.4$, the ratio between the median and mean value amounts to 0.82 and 0.93, respectively.

The median IN2 curve has to be fitted with the type of the function presented in Eq. (2). The least square fit method was applied to determine parameters a and b from the median IN2 curve. In the used probabilistic framework [3] there are no strict guidelines for the length of the interval which has to be used in the process of determination of parameters a and b. In our previous study [8] we realized that small length of the interval may cause qualitatively different conclusions if the results for probability (Eq. 4) and the results obtained with practical format for safety checking [3] are compared. In our analyses, the interval from the yield spectral acceleration, determined from the idealized SDOF system, to the spectral acceleration associated to the displacement capacity, was used in the least square fit procedure.

5. EXAMPLES

5.1 Test structures

The first test structure is a "four-story existing building" (Figure 3), for which the frame had been designed to reproduce the design practice in European and Mediterranean countries about forty to fifty years ago [9]. However, it may also be typical of buildings built more recently, but without the application of capacity design principles (especially the strong column - weak beam concept), and without up-to-date detailing. In such buildings a soft first story effect may occur even though the structure is uniformly infilled in its elevation [10].

The "four-story contemporary building" (Figure 3) was designed according to Eurocode 8, as a high-ductility class structure [11]. The design peak ground acceleration amounted to 0.3 g, which results in a base shear coefficient equal to 0.15.

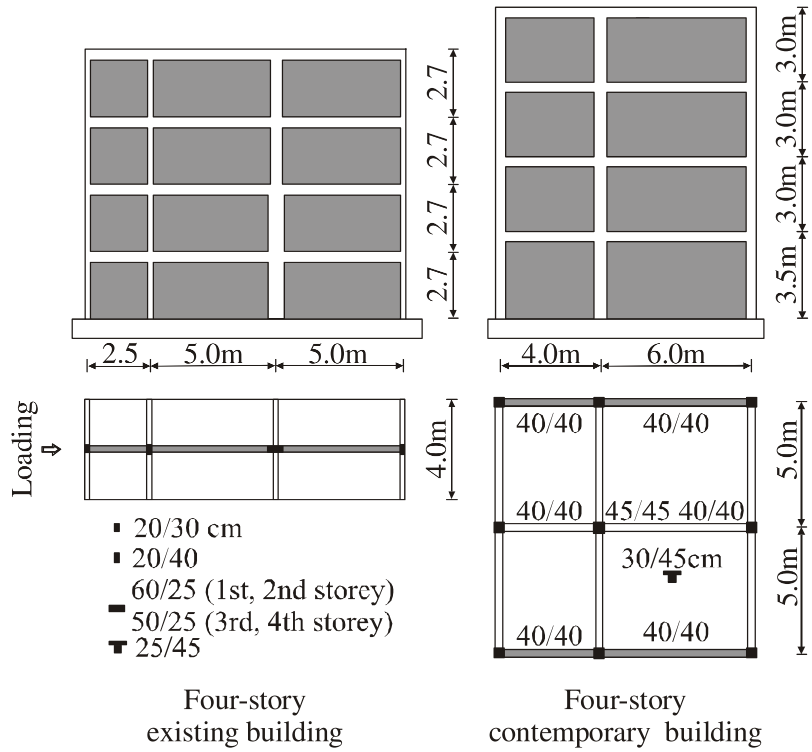

Figure 3. Test structures.

For both test structures full-scale pseudo-dynamic tests have been performed at the ELSA Laboratory, Ispra [9,11]. The technique proposed by the authors [12], which employs the results of pseudo-dynamic test, has been used to construct the mathematical models of the test structures. All the beams and columns were modeled by perfectly elastic, massless beam elements with two nonlinear rotational springs at each of the two ends. The moment – rotation relationship for each spring was defined by a trilinear envelope and Takeda's hysteretic rules. Asymmetric backbone curves were used for the beams. In addition to these elements, simple rotational connection elements were placed between the beams and joints to model the pinching behavior of the beams. The infill panels were modeled by equivalent diagonal struts, which carry loads only in compression. The shear-slip hysteretic model has been used for modeling the cyclic behavior of the infill panels. Strength deterioration was modeled only for the elements representing infills, whereas for the elements representing RC beams and columns unlimited ductility was assumed. All nonlinear analyses were performed using a modified version of the computer program DRAIN-2DX [13]. In all analyses 5 % damping was assumed. More details on the mathematical models of the test structures can be found elsewhere [8,12].

5.2 Ground motion

The seismic loading for the IN2 analysis is defined with the idealized spectrum presented in Figure 4. The parameters of the idealized Newmark-Hall type spectrum were obtained from the set of 20 accelerograms with the procedure explained in [6]. The characteristic periods T_B, T_C and T_D of the idealized spectrum are equal to 0.22, 0.55 and 1.76 s, respectively. The spectral amplification at the constant acceleration range amounts to 2.39. For the IDA analysis the same set of 20 accelerograms [6] was used. The mean spectrum and its standard deviation are presented in Figure 4. Note that the spectral acceleration at the initial period of the idealized equivalent SDOF system was used as the ground motion intensity measure. The periods are equal to 0.291 and 0.367 s for existing and contemporary building, respectively. Therefore different mean spectra are obtained for the two buildings (Figure 4). Both mean spectra fit very well the idealized target spectrum, which facilitates the comparison between the results of the IDA and the IN2 analysis.

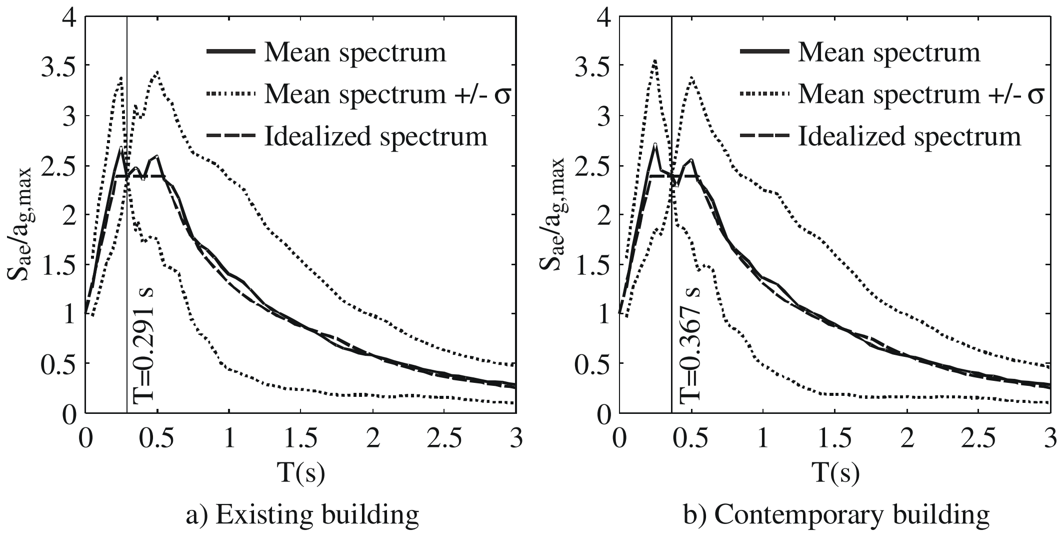

Figure 4. The mean and mean $\pm\sigma$ of 20 spectra, scaled to the same spectral acceleration at the period of the idealized equivalent SDOF system for existing and contemporary building, and the idealized Newmark-Hall type spectrum.

Seismic hazard for central Slovenia was employed in test examples. The spectral acceleration in the plateau is equal to 0.60 and 0.81 g for hazard levels 1/475 and 1/1000, respectively. The parameters k and k_o, which define the median hazard curve (Eq. 7) amount to 2.45 and 5.97E-04. For the dispersion of the median hazard curve β_H a value of 0.3 was assumed. For both examples the same hazard curve was used.

5.3 Seismic demand – Incremental N2 method

5.3.1 The Pushover curve and the equivalent SDOF system

The nonlinear static (pushover) analyses were performed with the force pattern, which was calculated from assumed inverted triangular displacement shape. Pushover curves for both test buildings are shown in Figure 5. It is obvious that after a certain deformation infills start to degrade. After the collapse of infills in first story for existing building and in the bottom two stories for contemporary building only frame resists the horizontal loading.

Pushover curves are then idealized using the procedure described in [7]. Absolute value of F_y, which is equal to maximum strength (Figure 1), is much higher for contemporary building (Table 1). For both buildings the maximum strength – total weight ratio F_y/W is rather high. It amounts to 0.67 and 0.40 for the contemporary and existing building, respectively. The high F_y/W ratio is a consequence of rather high strength of infills in comparison with the strength of bare frame. The minimum strength after the degradation of infills F_{min} (Figure 1) is also obtained from the idealization of pushover curve (Table 1, Figure 5). The ratio between F_{min} and F_y (parameter r_u) amounts to 0.46 and 0.61 for existing and contemporary building, respectively. Important parameters obtained from the idealized pushover curve are also yield displacement D_y and displacement at the start of strength degradation D_s (Figure 1). The ductility at the beginning of strength degradation μ_s is calculated as the ratio between D_s and D_y. Both parameters, r_u and μ_s, are important for the determination of ductility demand μ. The values of parameters are summarized in Table 1.

The constant Γ, which is used for the transformation from the MDOF to the equivalent SDOF system and vice versa (Eq. (17) in [4]) is obtained from the vector of story masses and assumed displacement shape. The equivalent mass of the SDOF system m^* is also calculated from the story masses and normalized displacement shape (Eq. 13 in [4]). Yield point of the equivalent SDOF system $\left(D_y^*, F_y^*\right)$ can be calculated simply by dividing D_y and F_y with the transformation constant Γ. The values for both buildings are presented in Table 1. The elastic period of the equivalent SDOF system T^* (Eq. 18 in [4]) are for both buildings smaller than ($T_C = 0.55\text{s}$ (Section 3.3.1), therefore the T^*/T_C ratio is less then one. The acceleration at the yield point is calculated as the ratio between F_y^* and m*.

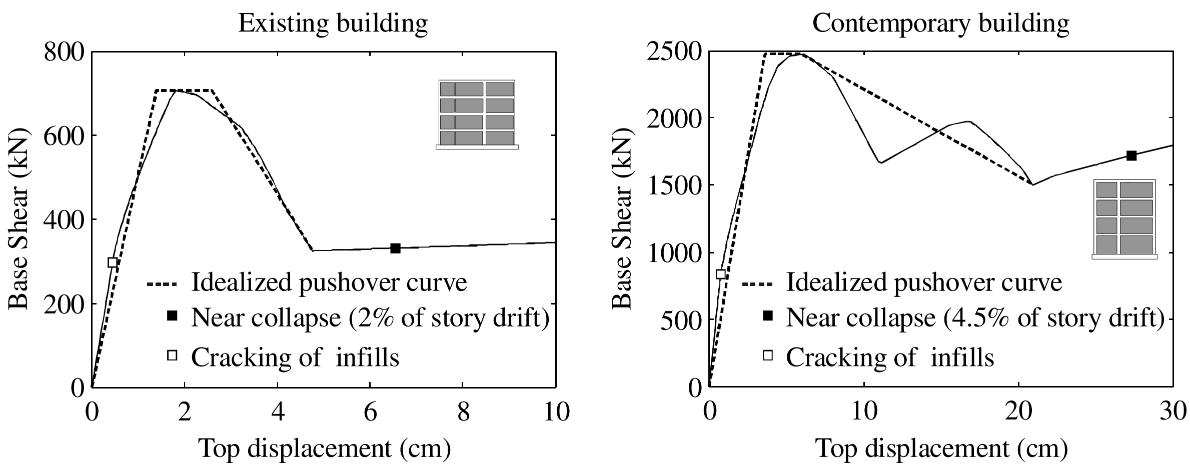

Figure 5. Base shear – top displacement relationship for existing and contemporary building and idealized pushover curves.

Table 1. Summary of parameters defining idealized pushover curves and equivalent SDOF systems for existing and contemporary building.

Parameter	Existing	Contemporary	Parameter	Existing	Contemporary
F_y (kN)	706	2472	Γ	1.358	1.344
$F_{\min}$ (kN)	324	1500	m^* (t)	109	236
F_y/W (%)	40	67	F_y^* (kN)	520	1839
D_y (cm)	1.39	3.57	D_y^* (cm)	1.02	2.66
D_s (cm)	2.59	5.99	T^* (s)	0.291	0.367
r_u, μ_s	0.46,1.87	0.61, 1.68	S_{ay} (g)	0.486	0.79

5.3.2 The seismic demand for a single level of spectral acceleration (N2 method vs. nonlinear dynamic analysis)

Knowing the parameters of the equivalent SDOF system (r_u, μ_s, T^*, S_{ay}) and the parameters of demand spectra ($T_C, T_D, S_{ae}(T^*)$), seismic demand can be easily computed. As an example, we will determine the seismic demand for existing building with a given reduction factor R=1.8. The reduction factor represents the ratio between the elastic spectral acceleration at the period of the equivalent SDOF system $S_{ae}(T^*)$ and the yield acceleration S_{ay}. The elastic spectral acceleration $S_{ae}(T^*)$ is then equal to

$1.8 \cdot 0.486\,g = 0.87\,g$. The ductility demand μ is calculated from $R-\mu-T$ relation [6] and is equal to 4.46. Multiplying the ductility demand μ by yield displacement $D_y^* = 1.02\,\text{cm}$ the displacement demand of the equivalent SDOF system $S_d = 4.55\,\text{cm}$ is obtained. Finally, top displacement of existing building $D_t = 6.2\,\text{cm}$ is obtained by multiplying $\Gamma = 1.358$ and $S_d = 4.55\,\text{cm}$. Story drifts and all local quantities of interest are obtained by performing pushover analysis with the assumed force pattern up to the calculated top displacement. Maximum drift occurs in the first story and amounts to 1.87% of story height. Note that all quantities obtained with the N2 method are in fact mean values (Section 4) [6].

Nonlinear dynamic analyses were performed with the set of 20 accelerograms (Section 5.2) each scaled to spectral acceleration at the period $T^* = 0.291\,\text{s}$ equal to 0.87 g. The mean value of the top displacement amounts to 7.0 cm, which is 13% more than the top displacement calculated with the N2 method. The maximum mean value of story drifts is obtained in the first story, like in the N2 method. It amounts to 1.62%, about 15% less than the story drift obtained in the N2 method.

The seismic demand was also calculated for the contemporary building with the same procedure as described above. The top displacement obtained by the N2 method amounts to 11.3 cm and it is about 10% smaller than the mean top displacement obtained with nonlinear dynamic analyses (12.6 cm). Maximum story drifts obtained with the N2 method and with nonlinear dynamic analysis are 2.2% and 1.9%, respectively.

5.3.3 The median IN2 curve versus the median IDA curve and parameters a and b

The median IN2 curve, herein defined as the relationship between median top displacement and spectral acceleration at the period of the equivalent SDOF system, is determined from the seismic demand for different levels of the seismic intensity. For the lowest seismic intensity a value of 0.2 was chosen for the ratio between elastic spectral acceleration at the period of equivalent SDOF system and yield spectral acceleration for particular building. The level of the ratio $S_{ae}(T^*)/S_{ay}$, which represents the reduction factor R, was then progressively increased, with the interval of 0.2, until the estimated median top displacement capacity (Section 3.4) was reached.

For example, a single point of the median IN2 curve is determined from the mean top displacement determined with the N2 method for a given seismic intensity (e.g. 6.2 cm for existing building and assumed $S_{ae}(T^*)/S_{ay} = 1.8$, Section 5.3.2). Assuming that the coefficient of variation for top displacement is equal to 0.7 (Section 3) and using Eqs. (6) and (7), then $\beta_{DR} = 0.63$ and median top displacement is equal to 5.1 cm.

In order to demonstrate the accuracy of the approximate approach, a single point of the median IN2 curve will be compared with a single point of the median IDA curve. The mean top displacement and the coefficient of variation, both calculated by nonlinear dynamic analyses for a given spectral acceleration 0.87g, are equal to 7.0 cm (Section 5.3.2) and 0.77, respectively. The corresponding median values, determined according to the method of moments and according to the definition of the median value, amount to 5.5 cm and 5.1 cm, respectively. The later median displacement is the same as that obtained from the IN2 method.

All points of the median IN2 curve were determined using the same coefficient of variation for randomness in top displacement $V = 0.7$, while for the median IDA curve the coefficient of variation for randomness in top displacement, as obtained from nonlinear dynamic analysis, was used. The values are presented in Figure 6. It can be seen that for both structures the coefficient of variation obtained from the IDA analysis first increases with increasing $S_{ae}(T^*)/S_{ay}$. Then, it becomes more or less constant and comparable with the assumed value of the coefficient of variation used in IN2 analysis. For comparison also the coefficient of variation due to randomness in maximum drift, calculated with the IDA analysis, is presented in Figure 6. For this coefficient high peaks are observed for both buildings in the range near to $S_{ae}(T^*)/S_{ay} = 1.5$. In this range the coefficient of variation for maximum drift exceeds the coefficient of variation for top displacement. This happens because in that range of $S_{ae}(T^*)/S_{ay}$ soft story effect occurs for only one or two ground motions, resulting in much higher maximum drift compared to maximum drifts obtained for other ground motions, whereas the differences in top displacement are not so large.

The median IN2 and IDA curves for both buildings are compared in Figure 7. A fair agreement can be observed, especially in the case of contemporary building. The median IN2 curves are slightly on the unsafe side.

Finally, median IDA and IN2 curves were fitted using the procedure described in Section 4. The values of a and b were obtained (Table 2). The fitted curves match the median IDA and IN2 curves very well as it can be seen in Figure 7.

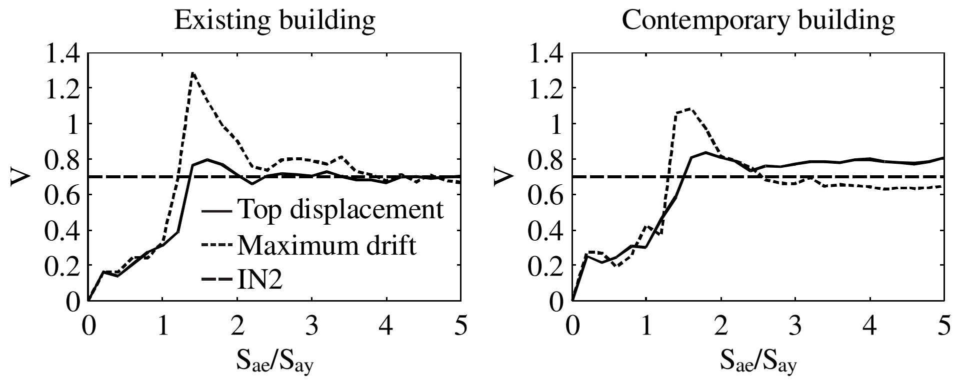

Figure 6. Coefficient of variation V due to randomness for top displacement and maximum drift calculated from IDA analysis and assumed value for IN2 analysis.

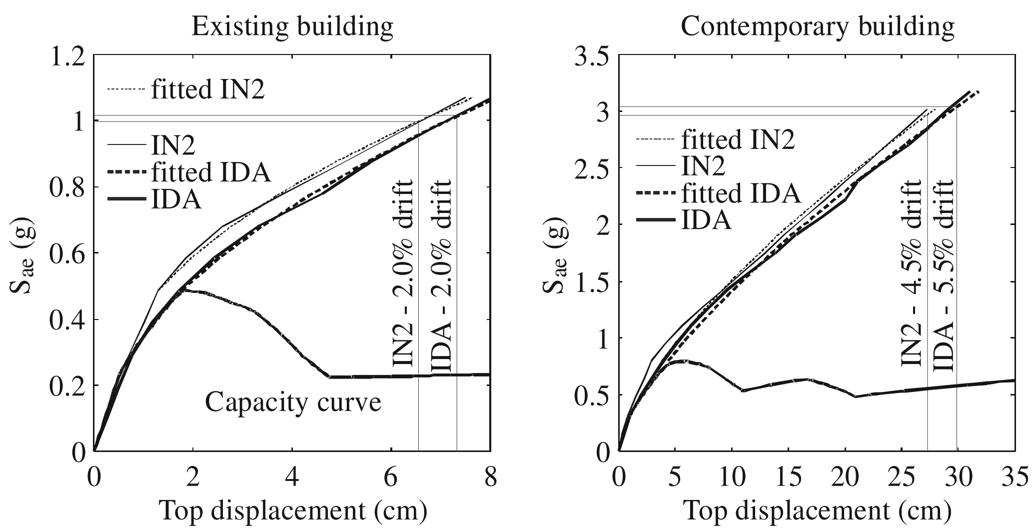

Figure 7. Comparison between median IN2 and IDA curves for existing and contemporary building. Fitted curves (Eq. 2) are also shown.

5.4 Estimation of the capacity of the building structures

In the test examples the so-called near collapse (NC) limit state is investigated. It has been conservatively assumed that the NC limit state is reached for the whole structure, when the NC limit state is reached for the first column. For beams and columns, the controlling quantities are chord rotation and shear forces. Shear resistance and ultimate chord rotation for structural elements were determined according to Eurocode 8 [14]. Beam column joints were also checked. However, the comparison of shear demands in joints, obtained from pushover analysis, and the corresponding shear capacities [14], indicated that joints do not represent a critical part of the test structures. Further, it was considered that the collapse of infills does not cause the NC limit state of building.

The capacities in terms of ultimate rotations for columns and beams of existing building are much lower than those of contemporary building. The minimum calculated value for the ultimate chord rotation amounts to 0.015 and corresponds to the strong column in the first story of existing building (Figure 3). Based on the assumption that NC limit state of the structure is reached when the rotation of the first column exceeds the calculated ultimate rotation, the NC limit state for existing building occurs due to exceeding ultimate rotation in the strong column in the first story. Based on pushover analysis, the first story drift at the ultimate chord rotation of the strong column in the first story is equal to 2.0% of the story height (5.4 cm), and the corresponding top displacement is equal to 6.55 cm (Figure 5). At this deformation all infills in the first story totally collapse and infills in the second story start to degrade. The soft story mechanism is formed in the first story at 1% story drift, much before the NC limit state. The shear demand exceeds the computed shear resistance before 2.0% maximum story drift is reached for two columns in the first story and at the top of one column in the second story. However, experimental results [9] suggest that this computed result is too conservative and not realistic, therefore it was not taken into account in further analyses. The experiment stopped at 1.5% first story drift to ensure the subsequent tests on retrofitted building. At this drift shear cracks and crushing of cover concrete occurred at the bottom of some columns. According to these results we consider that 2.0% story drift may be a reasonably conservative value defining the NC limit state.

The same drift was also used as NC limit state in nonlinear dynamic analyses. Based on these analyses, the median value of drift capacity (2.0%) is associated with the median top displacement equal to 7.34 cm.

The NC limit state for contemporary building was determined with a similar procedure. The critical structural element is the central column (Figure 3). The ultimate chord rotation at the bottom of this column, calculated according to Eurocode 8 (CEN 2003), amounts to 0.04. Based on pushover analysis, the corresponding story drift is equal to 4.5% and the top displacement is equal to 27.5 cm (Figure 5). This deformation stage was considered as NC limit state although shear demands of some beams and columns exceed the corresponding capacities determined according to Eurocode 8. Of course, infills collapse in the first and second story before 4.5% story drift is reached. The 4.5% story drift was also assumed as NC limit state for IDA analysis and it was associated with the median top displacement of 29.9 cm.

5.5 Risk evaluation

The annual likelihood ("probability") that the seismic demand exceeds seismic capacity was calculated with Eq. (3). The spectral acceleration at the median top displacement capacity $s_a^{\hat{C}}$ was obtained from Eq. (2), using the parameters a and b, which were determined as described in section 5.3.3 (Table 2). The median seismic hazard at $s_a^{\hat{C}}$ was obtained from Eq. (1), with parameters k_o and k determined in Section 5.2. The dispersion measure for randomness in top displacement capacity β_{CR} was assumed to be 0.25 for both IDA and IN2 analyses. The dispersion measure for randomness in displacement demand β_{DR} obtained from the IDA analysis amounts to 0.57 and 0.64 for existing and contemporary building, respectively. These values of dispersion measure do not differ significantly from $\beta_{DR} = 0.63$, which was used in the IN2 analysis (Section 4 and 5.3.2). Finally, the annual probability that demand exceeds NC limit state P_{PL} is obtained from Eq. (3). In Table 2 the results for P_{PL} and $P_{PL,50}$ as well as intermediate results are presented. $P_{PL,50}$ represents the probability of exceedance of NC limit state in fifty years. It is determined from the assumed binomial distribution of an event as $P_{PL,50} = 1-\left(1-P_{PL}\right)^{50}$.

A very good agreement can be observed between P_{PL} obtained by IN2 and IDA analyses. P_{PL} for contemporary building is about 10 times smaller than of existing building. A considerable probability of exceedance of NC limit state in 50 years (about 3.8 %) is obtained for existing building.

The probability $P_{PL,50}$ does not include uncertainty in displacement demand β_{DU} and uncertainty in displacement capacity β_{CU}. In order to include these uncertainties, Eq. (4) has to be used. The probability of exceedance of NC limit state in fifty years with the x=90 % confidence $P_{PL,50}^{x}$ was determined. For this analysis, dispersion measures for uncertainty in displacement demand and capacity β_{DU} and β_{CU} were assumed to be 0.25 for both IDA and IN2 analyses. More precise determination of these uncertainties is not within the scope of this paper. The summary of results is presented in Table 3. Similarly as in the previous case without uncertainty in displacement demand and capacity, very good agreement can be observed between probabilities obtained by IN2 and IDA analysis. A rather high value of dispersion measure for uncertainty in P_{PL} $\beta_{P_{PL}}$, is obtained. This is mainly a consequence of rather high values assumed for β_{CU} and β_{DU}. Note that substantially higher values are obtained for probabilities with a high confidence level (e.g. 90 % confidence level) in comparison with the 50 % confidence level. For example, the ratio between 90 and 50 % confidence level estimate of $P_{PL,50}^{x}$ amounts to about 2 to 2.5. Note that the results with 50 % confidence level are near to the results without uncertainty in displacement demand and capacity. The only source of difference is the different representation of hazard curves. In the case with uncertainties the mean hazard curve was employed instead of the median hazard curve [3].

According to [15] the structure is safe enough if there is less than 2 % chance in 50 years, with a 90 % confidence level, that the collapse prevention performance level would be exceeded. If we apply this criterion, existing building is not safe enough ($P_{PL,50}^{90}$ amounts to about 7%), while the contemporary building is safe ($P_{PL,50}^{90}$ amounts to about 1%).

Table 2. Summary of risk evaluation without uncertainties in displacement demand and capacity using median IN2 and IDA curves.

Parameter		Existing building		Contemporary building	
		IN2	IDA	IN2	IDA
$\hat{C}$: median displacement capacity (cm)		6.55	7.34	27.45	29.88
median drift capacity (%)		2.0	2.0	4.5	4.5
parameters of fitted IN2 and IDA curves	a	6.60	7.14	5.44	6.11
	b	2.30	1.93	1.49	1.43
$s_a^{\hat{C}}$ (g): spectral acc. “corresponding” to displ. capacity		1.00	1.01	2.96	3.04
$\hat{H}\left(s_a^{\hat{C}}\right)$: median estimate of spectral acceleration hazard		6.02E-04	5.77E-04	4.19E-05	3.94E-05
β_{DR} : dispersion measure of randomness in displ. demand		0.63	0.57	0.63	0.64
P_{PL} : annual probability exceedance of limit state		7.81E-04	7.87E-04	7.80E-05	7.90E-05
$P_{PL,50}$ (%): probab. of exceedance limit state in 50 years		3.83	3.86	0.39	0.39

Table 3. Summary of risk evaluation with 90 % confidence level using median IN2 and IDA curves.

Parameter	Existing building		Contemporary building	
	IN2	IDA	IN2	IDA
$\bar{H}\left(s_a^{\hat{C}}\right)$: mean estimate of spectral acc. hazard	6.99E-04	6.70E-04	4.87E-05	4.58E-05
$\beta_{P_{PL}}$: dispersion measure for uncertainty in P_{PL}	0.38	0.45	0.58	0.61
$\hat{P}_{PL}$: median estimate of P_{PL}	9.08E-04	9.14E-04	9.06E-5	9.18E-5
P_{PL}^{90} : 90% confidence level estimate of P_{PL}	1.47E-03	1.62E-03	1.91E-4	1.99E-4
$P_{PL,50}^{90}(\%)$: 90% confidence level estimate of	7.09	7.81	0.95	0.99

6. CONCLUSIONS

In the paper, the Incremental N2 (IN2) method, a simple alternative for IDA analysis, is introduced. The IN2 method, which represents an extension of the N2 method, can be used for determination of approximate summarized IDA curves (IN2 curves). The IN2 curve can substitute the IDA curve in the probabilistic framework for seismic design and assessment of structures.

The IN2 method has been applied to two test examples of infilled reinforced concrete frames, which are characterized by a substantial degradation of the strength after the infill fails. A specific $R-\mu-T$ relation, typical for infilled frames, and data on dispersion due to randomness in displacement demand developed by authors in a previous study, were employed in the probabilistic risk analysis of test structures.

A reasonable accuracy of the IN2 curve in comparison with the IDA curve has been demonstrated for both examples. It has been also shown that the dispersion for randomness in displacement demand for the MDOF system can be predicted from the dispersion for randomness in ductility demand obtained from the statistical study of $R-\mu-T$ relation. A fair correlation of results obtained by the approximate procedure with IN2 method and a more accurate analysis employing IDA curves suggests that the IN2 method is a viable approach.

The test examples indicate that the probability of the collapse for buildings designed according to modern standards is about 10 times smaller than for buildings built about 50 years ago. It has been also shown that a high confidence estimate for probability of exceeding a certain performance level, which includes uncertainties in displacement demand and capacity in addition to randomness, significantly increases the probability of exceeding this performance level compared to the case in which only randomness in displacement demand and capacity are taken into account.

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