13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 3408

ENERGY DISSIPATION CAPACITY FOR AN ACTIVE STRUCTURE MEMBER IN A BUILDING MODEL UNDER NONSTATIONARY RANDOM DISTURBANCES

Isao NISHIMURA[1]

SUMMARY

This paper discusses a vibration control algorithm for the Maxwell model that represents a building structure, under nonstationary random disturbances. First discussed is the optimum selection of passive mechanical parameters and active feedback gains for the system under stationary random process. Selecting an appropriate set of passive mechanical parameters under a given feedback gain minimizes the energy requirement for the controller. This optimum selection is probabilistically obtained under stationary random process. It is also proved that the same optimum parameter selection minimizes the force requirement for the controller. The stochastically expected optimum parameter set is numerically evaluated in the time domain under nonstationary random excitations. The deterministic analyses ascertained that the control energy converges to zero as the time goes to infinity under any nonstationary random disturbances. It is, then, theoretically proved that there exists an algorithm that could completely eliminate the energy requirement for the active controller under any random excitations.

INTRODUCTION

The structural control concept can be traced back to Yao [1] followed by theoretical investigation by several people such as Roorda [2], and Yang [3], just to mention a few. One of the pioneering age's application projects was the Tuned Mass Damper activated by hydraulic actuators that was placed on John Hancock Tower in Boston, 1977 [4]. Soong [5] was one of the consulting researchers participated in this project, which made him to proceed to the active control research in the following years. He also used an active tendon to be placed in a building model on a shaking table, and successfully controlled the response vibration of the system [6]. Nishimura and Masri [7] conducted an experimental study of active control, which used a steel model (5 degree of freedom model) with an active controller placed in the middle of it. The phase lag of the upper portion of the specimen with respect to the lower portion works as a damping augmentation, which was successfully observed in the laboratory test. Theoretically predicted control performance was experimentally observed for the 1st, 2nd and 3rd mode vibration simultaneously. Those early attempts, however, did not successfully grow into a feasible technology in the civil engineering, though they received much attention.

[1] Dep. of Architecture, Faculty of Engineering, Musashi Institute of Technology, Tokyo, Japan

On the other hand, Kobori [8,9,10] started a methodological research project and successfully completed the first application of AMD (Active Mass Driver) for an office building in Tokyo, 1989. This machine was composed of 4000kg mass, hydraulic actuator, sensing devices, signal processor and miscellaneous mechanical equipments. It could instantaneously activate itself to attenuate the response vibration of the structure as soon as its sensors detect the ground motion in case of earthquake.

Since this first application was successfully completed in 1989, there were numerous research groups attempting to refine this technology. Attention was paid to how to reduce the controller's force and power without degrading the control performance. These research activities resulted in what is now called hybrid mass dampers, which were implemented into several tall buildings in Japan. The theoretical formation of those projects was similar to what is shown by Yang and Samali [11] in the sense that they are both based on the modern control theory and its optimum algorithm. In retrospective view, Morison and Karnopp [12] are probably the first people who investigated hybrid mass dampers. They used modern control theory and came to the conclusion that numerically obtained optimum feedback gains could not be intuitively obtained by a simple physical analogy. In fact, it is impossible to constitute a system with active controller that requires no energy dissipation capacity. Because we could not determine that optimum feedback gains until we fix a system including a controller. Further discussion will be referred to the later part of this paper.

On the other hand, the author [13] found a unique control algorithm that made it possible to vanish the control energy completely under any nonstationary random disturbances. According to the algorithm, the control actuator generates either driving force or braking force according to the relative motion of the main structure to the auxiliary mass. Naturally, these two forces cancel each other when the system is under stationary condition. The physical meaning of the algorithm is clear and simple so that the extension of the same approach to another active structure formation such as active bracing seems possible and attractive.

In this paper, the integral of the power response from the beginning to the end of an event of earthquake is defined as an index for the active structural member's damage. Then, an algorithm is proposed and defined to reduce the system response. It has a unique feature that requires no energy dissipation capacity to achieve response reduction. The author conducted several numerical calculations according to this algorithm, which makes the control energy converge to zero as the time goes to infinity under any random disturbances regardless of their spectra and magnitude.

OPTIMIZATION AND LIMINATION OF PASSIVE DAMPING

An example multi-degree-of-structure model is shown in Fig.1, while partially strengthened model by means of bracing is shown in Fig.3. The model in Fig.2 represents a frame with linear dampers equipped inside. We wish to minimize the response of the frame in Fig.2 under earthquake disturbances by selecting the most appropriate damping coefficient. As the damping coefficient becomes smaller, the frame dynamics in Fig.2 comes closer to that of Fig.1. On the contrary the damping coefficient becomes larger, the dynamics in Fig.2 comes closer to that of Fig.3. There is no response reduction expected from either case. In other words, there is expected the optimum value that gives the structure the maximum damping performance.

The optimal damping coefficient is obtained in this section under stationary random excitation. Then, we evaluate the performance of the Maxwell model by the equivalent SDOF (Single Degree of Freedom) model. The equation of motion in Fig.2 is shown in below.

$$\begin{cases} m\ddot{x} + kx + k_d(x-y) = mf(t) & (1) \\ k_d(x-y) - c_d\dot{y} = 0 & (2) \end{cases}$$

The disturbance excitation is supposed to be (3) and several parameters are defined by (4).

$$f(t) = -\ddot{x}_G \tag{3}$$

$$\omega_o = \sqrt{\frac{k}{m}} \quad \omega_d = \sqrt{\frac{k_d}{m}} \quad c_d = 2m\omega_o\eta_d \tag{4}$$

We obtain (5) by considering (3) and (4) and taking the Laplace transforms of (1) and (2).

$$\begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} = \begin{bmatrix} s^2 + \omega_o{}^2 + \omega_d{}^2 & -\omega_d{}^2 \\ -\omega_d{}^2 & 2\omega_o\eta_d s + \omega_d{}^2 \end{bmatrix}^{-1} \begin{bmatrix} F(s) \\ 0 \end{bmatrix} \tag{5}$$

The transfer function representation is shown below.

$$\begin{cases} X(s) = H_X(s)F(s) \\ Y(s) = H_Y(s)F(s) \end{cases} \tag{6}$$

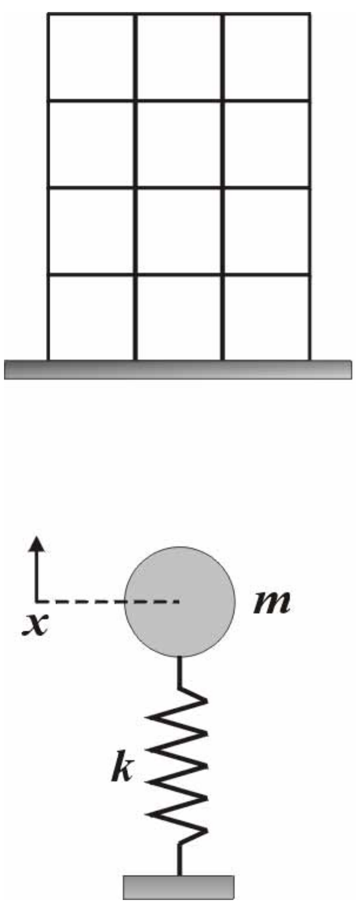

Fig.1 The Original Frame

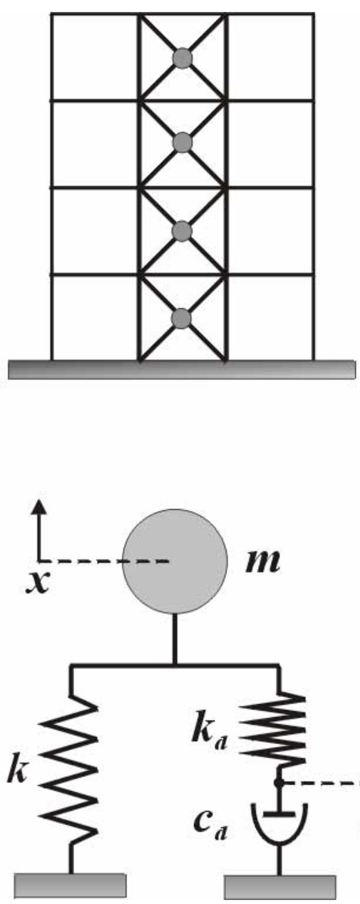

Fig.2 Maxwell Model

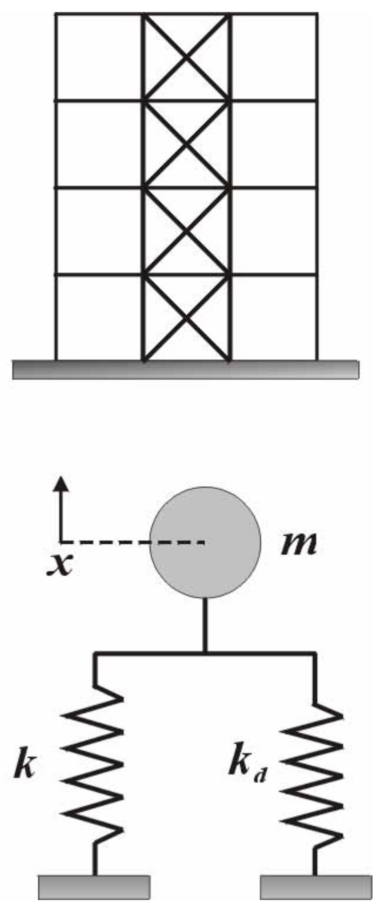

Fig.3 Brace Model

where

$$H_X(s) = \frac{\omega_d{}^2 + 2\omega_o\eta_d s}{2\omega_o\eta_d s^3 + \omega_d{}^2 s^2 + 2\omega_o\eta_d(\omega_o{}^2 + \omega_d{}^2)s + \omega_o{}^2\omega_d{}^2} \tag{7}$$

$$H_Y(s) = \frac{\omega_d{}^2}{2\omega_o\eta_d s^3 + \omega_d{}^2 s^2 + 2\omega_o\eta_d(\omega_o{}^2 + \omega_d{}^2)s + \omega_o{}^2\omega_d{}^2} \tag{8}$$

If the disturbance excitation is supposed to be a stationary random process, the expected mean square of random variable X is given by (9).

$$E[X^2] = \int_{-\infty}^{+\infty} H_X(i\omega)H_X(-i\omega)S_f(\omega)d\omega \tag{9}$$

Under a white noise disturbance whose spectrum $S_f(\omega)$ is equal to S_o, we find that (9) is equivalent to

$$E[X^2] = \frac{\pi S_o}{2\omega_o{}^3}\left(\frac{1}{\eta_d} + 4\frac{\omega_o{}^4}{\omega_d{}^4}\eta_d\right) \tag{10}$$

We can minimize $E[X^2]$ to (11) by selecting η_d to (12).

$$E[X^2] = \frac{2\pi S_o}{\omega_o\omega_d{}^2} \tag{11}$$

$$\eta_d = \eta_{opt} = \frac{1}{2}\frac{k_d}{k} \tag{12}$$

It will be convenient to evaluate the performance of the optimum Maxwell model by identifying the SDOF model that has the same response under the same disturbance excitation. The transfer function of a SDOF model with the frequency ω_{eq} and damping ratio η_{eq} is given by (13) and (14).

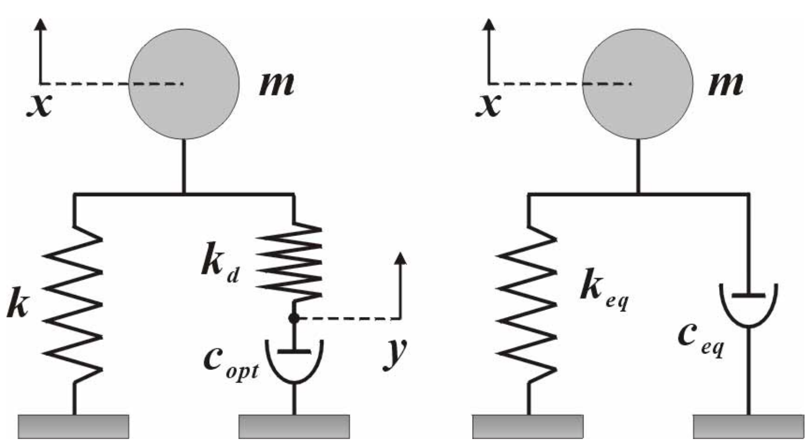

Fig. 4 The Optimum Maxwell Model and the Equivalent SDOF Model

$$X(s) = H_X(s)F(s) \tag{13}$$

$$H_X(s) = \frac{1}{s^2 + 2\omega_{eq}\eta_{eq}s + \omega_{eq}^2} \tag{14}$$

In the end, the mean square of the random variable X with respect to the white noise of the power spectrum S_o is given by

$$E[X^2] = \frac{\pi S_o}{2\omega_{eq}^3\eta_{eq}} \tag{15}$$

Comparing (11) with (15), we can evaluate the performance of the Maxwell model by means of equivalent damping ratio η_{eq}.

$$\eta_{eq} = \frac{1}{4}\left(\frac{\omega_o\omega_d^2}{\omega_{eq}^3}\right) \tag{16}$$

The natural frequency of the frame with the optimal damping coefficient c_{opt} can be found by solving the characteristic equation of (17).

$$2\omega_o\eta_{opt}s^3 + \omega_d^2 s^2 + 2\omega_o\eta_{opt}(\omega_o^2 + \omega_d^2)s + \omega_o^2\omega_d^2 = 0 \tag{17}$$

Although it is possible to obtain the rigorous solution of (17), it would be extremely complicated. When the optimal damping coefficient is selected, the complex solution of (17) represents the equivalent angular frequency ω_{eq} and damping ratio η_{eq}. From (12) we understand that the complex stiffness of the Maxwell model has the absolute value of $k + 0.5k_d$, therefore ω_{eq} can be approximated by (18)

$$\omega_{eq} = \omega_o\sqrt{1 + \frac{k_d}{2k}} \tag{18}$$

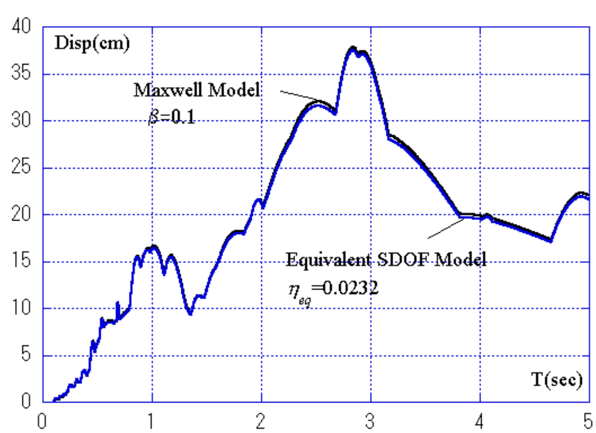

Fig.5 Displacement Response Spectra
El Centro(NS) 341 cm/sec^2

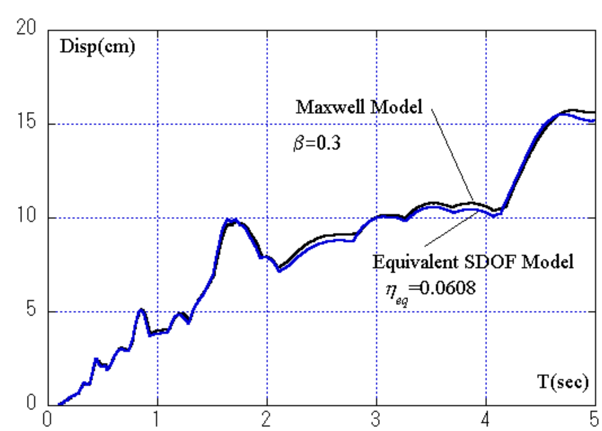

Fig.6 Displacement Response Spectra
Taft (EW) 176 cm/sec^2

Substitution of (18) into (16) yields (19), which represents the equivalent damping ratio of the SDOF model that has the same response power as the optimum Maxwell model in Fig.2.

$$\eta_{eq} = \frac{\beta}{2+\beta}\sqrt{\frac{1}{2(2+\beta)}} \quad or \quad c_{eq} = 2m\omega_{eq}\eta_{eq} \quad where \quad \beta = \frac{k_d}{k} \tag{19}$$

We understand from (19) that the equivalent damping ratio η_{eq} is directly linked with the ratio β between k_d and k. We understand from (18) that k_{eq} in Fig.4 is directly linked with β as well. These two equations have been obtained in the frequency domain under stationary random process. Hence, we numerically calculate the displacement response spectra of the optimum Maxwell model and the equivalent SDOF model in Fig.4 under nonstationary random earthquake disturbances. The results are shown in Fig.5 and Fig.6, where El Centro (NS) with peak acceleration 341cm/sec^2 and Taft (EW) with 176cm/sec^2 are used for the disturbances, respectively. The stiffness ratio β in Fig.5 is 0.1 and β in Fig.6 is 0.3. The difference between the lines of two models is so small that we can evaluate the response of a Maxwell model if we know the stiffness ratio β.

OPTIMIZATION OF ACTIVE BRACE (FUNDAMENTAL CONTROL LAW)

As is reviewed in the previous section, limitation does exist for passive damping installation. We try to find an active control law to improve the damping performance for the system in Fig.7, where we substitute the passive dampers by actuators or active controllers. We place those active controllers in series connection with the stiffness k_d, and we define the fundamental control law for a SDOF model that represents the building dynamics in Fig.7. Considering the equation of motion in (20) and (21), we understand that the primary response x is regarded as the input signal while the secondary response y can be viewed as the output signal.

As the output signal increases, the strain energy in the brace member also increases. Therefore, we need to increase the right side of (21) to push more strain energy into the brace member. At the same time the energy once accumulated in the brace member should be dissipated in the damping device as swift as possible. These two functions are requested for the active controller or the fundamental control law, which is shown in (22).

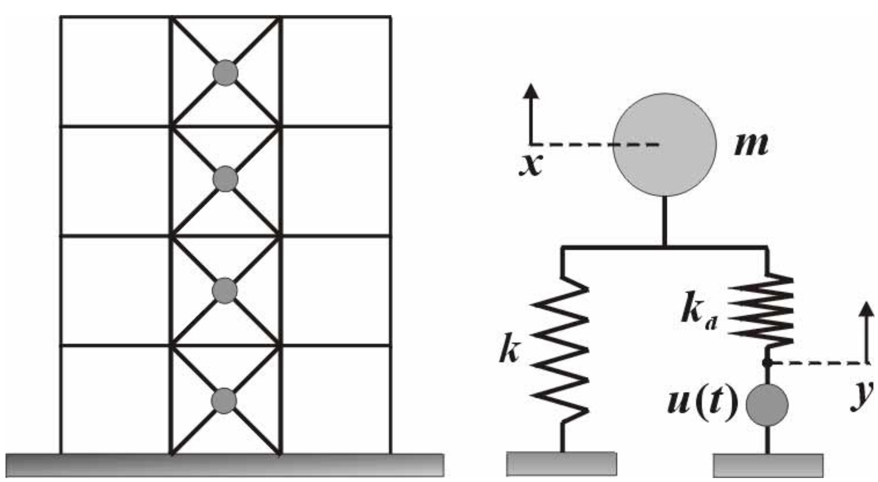

Fig.7 Fundamental control law for the active brace

$$\begin{cases} m\ddot{x} + kx + k_d x = k_d y + mf(t) & (20) \\ k_d y = k_d x + u(t) & (21) \end{cases}$$

$$u(t) = gkx - c_A \dot{y} \tag{22}$$

The first term of (22) works as an engine to push the vibration energy into the brace member, while the second term works as a braking force to dissipate the strain energy in the brace member. This is the physical meaning of the fundamental control law prescribed by (22). As we increase the feedback gain g, the energy dissipation per unit time linearly increases. Therefore we must select an appropriate c_A according to the feedback gain g. The following calculation is conducted to find the optimum $c_A (= c_{opt})$ for this purpose. The same procedure in the previous section is applied for the following calculation.

$$c_A = 2m\omega_o \eta_A \tag{23}$$

Substituting (22) and (23) into (21), we obtain the equation of motion in Laplace transform.

$$\begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} = \begin{bmatrix} s^2 + \omega_o{}^2 + \omega_d{}^2 & -\omega_d{}^2 \\ -\omega_o{}^2 g - \omega_d{}^2 & 2\omega_o \eta_A s + \omega_d{}^2 \end{bmatrix}^{-1} \begin{bmatrix} F(s) \\ 0 \end{bmatrix} \tag{24}$$

Therefore, the transfer function of X from F is given by (25).

$$H_X(s) = \frac{\omega_d{}^2 + 2\omega_o \eta_A s}{2\omega_o \eta_A s^3 + \omega_d{}^2 s^2 + 2\omega_o \eta_A (\omega_o{}^2 + \omega_d{}^2) s + \omega_o{}^2 \omega_d{}^2 (1-g)} \tag{25}$$

If we suppose that the external disturbance is a white noise whose power spectrum is S_o, the expected mean square of the random variable X is given by (26).

$$E[X^2] = \frac{\pi S_o}{\omega_o{}^3} \frac{4\eta_A{}^2 (1-g) + \beta^2}{2\beta \eta_A (1-g)(g+\beta)} \tag{26}$$

where the following substitution is introduced.

$$\beta = \frac{k_d}{k} = \left(\frac{\omega_d}{\omega_o} \right)^2 \tag{27}$$

The damping ratio η_A can be viewed as the main variable of the function (26) so that the optimal damping ratio can be obtained in the same procedure and given by (28).

$$\eta_A = \eta_{opt} = \frac{\beta}{2} \sqrt{\frac{1}{1-g}} \qquad where \quad (c_A = c_{opt} = 2m\omega_o \eta_{opt}) \tag{28}$$

The control performance of the above algorithm can be evaluated in terms of the equivalent damping ratio η_{eq} and frequency ω_{eq}. We can replace (26) by (29) when the optimum damping (28) is selected.

$$E[X^2] = \frac{2\pi S_o}{\omega_o{}^3(g+\beta)}\sqrt{\frac{1}{1-g}} \tag{29}$$

The equivalent frequency ω_{eq} can be obtained by the absolute value of the complex solution of (30).

$$2\omega_o\eta_{opt}s^3 + \omega_d{}^2 s^2 + 2\omega_o\eta_{opt}(\omega_o{}^2+\omega_d{}^2)s + \omega_o{}^2\omega_d{}^2(1-g) = 0 \tag{30}$$

According to the same analogy in the previous section, we can approximate the frequency ω_{eq} by (31).

$$\omega_{eq} = \omega_o\sqrt{1+\frac{(\beta-g)}{2}} \tag{31}$$

We can also estimate the equivalent damping ratio by comparing (15) with (29).

$$\eta_{eq} = \frac{(g+\beta)\omega_o{}^3}{4\omega_{eq}^3}\sqrt{1-g} \tag{32}$$

The feedback gain should be positive and satisfy the following equation to keep the system stable.

$$g < 1.0 \tag{33}$$

Analytically expected performance can be verified by response analyses under earthquake disturbances. The parameters for the fundamental model are shown in Table 2, where the optimum feedback gain c_A along with g, the equivalent SDOF model, and other necessary parameters are also indicated. These parameters are obtained from (28), (31) and (32).

Table 2 Parameters for the fundamental control law and the equivalent SDOF model

Parameters for the fundamental control law in Fig.7	$m = 1.0$ ω_o=6.32(rad/s) ω_d= 2.0 (rad/s) $k = 40.0$ $k_d = 4.0$ $g = 0.3$ $\beta = 0.1$ $\eta_A = \eta_{opt}$= 0.06 $c_A = c_{opt}$ = 0.756
The Equivalent SDOF model in Fig.7	m =1.0 ω_{eq}= 6.00 (rad/s) η_{eq} = 0.098 k_{eq} = 36.0 c_{eq} = 1.18

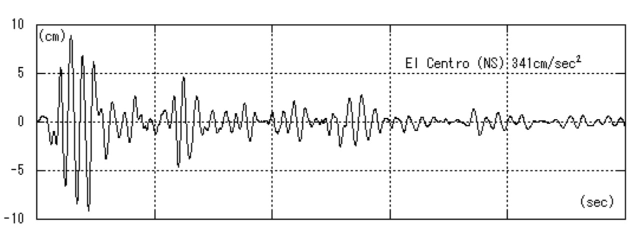

Fig.8 Fundamental Control Law

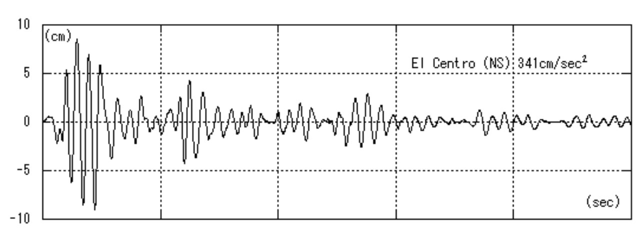

Fig.9 Equivalent SDOF model

The ground motion is supposed to be El Centro (NS) of which acceleration is 341cm/sec^2. The fundamental control law reduces the displacement response as isshown in Fig.8 that corresponds well to the equivalent SDOF model shown in Fig.9. The expected damping factor for the original Maxwell model would be 2.3% without active control, while the fundamental control law increases the effective damping ratio up to 9.8% shown in Table 2.

ENERGY DISSIPATION RESPONSE OF THE HYBRID CONTROL LAW

We have determined the optimum feedback gains for the fundamental control law to minimize the response of the Maxwell model. Is it possible to reduce the control force and energy without degrading the control performance that is once achieved by the fundamental control law? We try to find the positive answer to this question. This is the purpose of the following calculation in this section. We define the hybrid control as is shown in Fig.10, where active braces are installed into the structure along with passive damping devices. The equation of motion in Fig.10 is given below.

$$\begin{cases} m\ddot{x} + kx + k_d x = k_d y + mf(t) & (34) \\ k_d y + c_p \dot{y} = k_d x + u(t) & (35) \end{cases}$$

We select a control law that prescribes the actuator by (36) which has two feedback gains. You can select g arbitrary, but you must select another feedback gain c_A that should satisfy (37).

$$u(t) = gkx - c_A \dot{y} \tag{36}$$

$$c_{opt} = c_p + c_A \tag{37}$$

As long as we keep satisfying (37), there still remains one freedom left for us to select another feedback gain c_A. The purpose of hybrid control is to attribute some of the control force to the passive device so that the energy requirement for the control actuator is reduced without degrading the performance once achieved by active control. Indeed, we can completely eliminate the active control energy by selecting the optimum c_A. The following equation will be satisfied regardless of the active control algorithm.

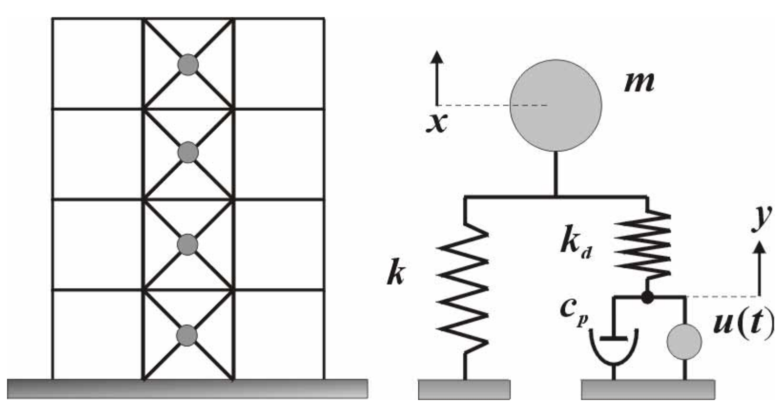

Fig.10 Hybrid control law for the active brace

$$\begin{bmatrix} m & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & c_p \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} k+k_d & -k_d \\ -k_d & k_d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ -u(t) \end{bmatrix} = \begin{bmatrix} mf(t) \\ 0 \end{bmatrix} \tag{38}$$

The power equilibrium can be obtained by multiplying the velocity vector to both sides of (38).

$$m\dot{x}\ddot{x} + c_p \dot{y}^2 + kx\dot{x} + k_d \dot{x}(x-y) - k_d \dot{y}(x-y) - \dot{y}u(t) = \dot{x}f(t) \tag{39}$$

Integrating both sides of (39) from the beginning to the end of an event of earthquake, we can evaluate the energy equilibrium, which is given by (40).

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 + \frac{1}{2}k_d(x-y)^2 + c_p\int_0^T \dot{y}^2 dt - \int_0^T \dot{y}u(t)dt = m\int_0^T \dot{x}f(t)dt \tag{40}$$

Provided that the system is stable, the vibration will be subdued gradually as the time goes to infinity. Hence, we expect that (40) will converge to (41).

$$c_p\int_0^{+\infty} \dot{y}^2 dt - \int_0^{+\infty} \dot{y}u(t)dt = m\int_0^{+\infty} \dot{x}f(t)dt \tag{41}$$

The first term of the left hand side of (41) represents dissipation energy, the second term means actuator’s control energy, while the right side of (41) is the total input energy due to the excitation ground motion. If we knew the ground motion in advance, we would be able to calculate both sides of (41) precisely. In addition to this we could select c_d to the optimum value according to (42) so that we could make the control energy converge to zero for this particular ground motion. Of course it is impossible to tell what should happen in the future, we could not select the optimum c_d until an earthquake actually takes place. This is the limitation for the deterministic approach.

$$c_p = \frac{m\int_0^{+\infty} \dot{x}f(t)dt}{\int_0^{+\infty} \dot{y}^2 dt} \tag{42}$$

On the other hand, if we replace the nonstationary random disturbance by a stationary random process, we will be able to predict the response and evaluate (43) instead of (42). This stochastic approach, however, necessitates us to suppose a spectrum for the disturbance excitation in the frequency domain. If we wish that any spectrum could satisfy the condition given by (43), a white noise should satisfy it as well. This is a necessary condition.

$$c_p = \frac{mE[\dot{X}F]}{E[\dot{Y}^2]} \tag{43}$$

If the white noise has a uniformly distributed spectrum S_o, the denominator of (43) is given by

$$E[\dot{Y}^2] = \frac{\pi S_o}{2\omega_o \eta_{opt}}\left(\frac{g+\beta}{\beta}\right) \tag{44}$$

The numerator of (43) is equivalent to the following equation.

$$E[\dot{X}F] = E\left[\int_{-\infty}^{+\infty} \dot{h}_x(\tau) f(t-\tau) d\tau f(t)\right] \tag{45}$$

where $\dot{h}_x(t)$ is the velocity impulse response function. Hence, (45) is equal to (46). If the auto-correlation function of the disturbance is defined by (47), (45) eventually equals to (48).

$$E[\dot{X}F] = \int_{-\infty}^{+\infty} \dot{h}_x(\tau) E[f(t-\tau) f(t)] d\tau \tag{46}$$

$$R_F(\tau) = E[f(t-\tau) f(t)] \tag{47}$$

$$E[\dot{X}F] = \int_{-\infty}^{+\infty} \dot{h}_x(\tau) R_F(\tau) d\tau \tag{48}$$

The auto-correlation function is the Delta function given by (49), because we suppose the disturbance is the white noise whose power spectrum is S_o.

$$R_F(\tau) = 2\pi S_o \delta(\tau) \tag{49}$$

The velocity impulse response satisfies (50) so that substitution of (49) and (50) into (48) yields (51).

$$\dot{h}_x(+\varepsilon) = 1 \qquad \dot{h}_x(-\varepsilon) = 0 \tag{50}$$

$$E[\dot{X}F] = 2\pi S_o \times \frac{1}{2} \dot{h}_x(+\varepsilon) = \pi S_o \tag{51}$$

We found the optimum feedback gain c_A and the passive damping coefficient c_p are given by (52) after substituting (44) and (51) into (43).

$$c_p = \frac{\beta}{\beta + g} c_{opt} \qquad c_A = \frac{g}{\beta + g} c_{opt} \tag{52}$$

In addition to this, we can prove that (52) is not only the necessary condition but also the satisfactory condition as well. If the condition by (52) is satisfied, the transfer functions of $X(s)$, $Y(s)$ and $U(s)$ from $F(s)$ are given by (53), (54) and (55), respectively.

$$X(s) = \frac{\omega_d^2 + 2\omega_o \eta_{opt} s}{\Delta(s)} F(s) \tag{53}$$

$$Y(s) = \frac{\omega_d^2 + \omega_o^2 g}{\Delta(s)} F(s) \tag{54}$$

$$U(s) = \frac{g k \omega_d^2}{\Delta(s)} F(s) \tag{55}$$

where

$$\Delta(s) = 2\omega_o \eta_{opt} s^3 + \omega_d{}^2 s^2 + 2\omega_o \eta_{opt} (\omega_o{}^2 + \omega_d{}^2) s + \omega_o{}^2 \omega_d{}^2 (1-g) \tag{56}$$

Once the hybrid control law and the associated parameters are determined according to (28), (36), (37) and (52), we can evaluate the power spectrum for the actuator control power by calculating the left hand side of (57). It is understood from (57) that the condition is always satisfied regardless of the spectrum of disturbances. Hence, we proved that (52) is not only the necessary condition but also the satisfactory condition for (43).

$$E[U\dot{Y}] = \frac{gk\omega_d}{\omega_o{}^2 g + \omega_d{}^2} E[Y\dot{Y}] = 0 \tag{57}$$

A small comment on the comparison with the modern control method is stated as follows. In general, we could not formulate a linear quadratic optimum analysis until we select and determine the target system model. According to the method in this section, we select c_p in Fig.10 after determining feedback gain g. Therefore, this optimum solution could not be achieved by the conventional modern control approach.

CONTROL FORCE RESPONSE OF THE HYBRID CONTROL LAW

We have successfully adjusted the damping coefficient and found the necessary and satisfactory condition that minimizes the control power in a stochastic sense for the active brace shown in Fig.10. In this section, we try to minimize the control force without degrading the damping performance in the same manner as before. We prescribe the control law that is identical to (35).

$$u(t) = k_d (y - x) + c_p \dot{y} \tag{58}$$

If the disturbance is supposed to be a stationary random process, the expected mean square of the control force is given by (59).

$$E[U^2] = c_p{}^2 E[\dot{Y}^2] - 2c_p k_d E[X\dot{Y}] + k_d{}^2 E[(Y - X)^2] \tag{59}$$

We can view the right side of (59) as a quadratic function with respect to c_p. When c_p takes the following value, the expected control force response will be minimum.

$$c_p = k_d \frac{E[X\dot{Y}]}{E[\dot{Y}^2]} \tag{60}$$

In the time domain, we have the following equation for the control power.

$$u(t)\dot{y} = c_p \dot{y}^2 + k_d \dot{y}(y - x) \tag{61}$$

If the disturbance is again supposed to be a stationary random process, the both sides of (61) can be viewed as power spectrum.

$$E[U\dot{Y}] = c_p E[\dot{Y}^2] + k_d E[\dot{Y}(Y - X)] \tag{62}$$

Hence, we found the passive damping coefficient c_{opt} that could vanish the control power in an ensemble sense, which is given by (63).

$$c_p = k_d \frac{E[X\dot{Y}]}{E[\dot{Y}^2]} \tag{63}$$

We found that (63) is identical to (60). As a result, c_p that minimizes the control force in a probabilistic sense also minimizes the control power at the same time. In the previous section, we proved that (52) is the necessary and satisfactory condition to minimize the control power under any stationary random process regardless of their spectrum. In the end, what minimizes the control force under any random process is also equal to (52).

In the long run, we proved that the optimum solution that minimizes the control power in an ensemble sense also minimizes the control force under any stationary random process. In addition to that this closed form solution has nothing to do with the spectrum of the disturbance excitation.

NUMERICAL ANALYSES

We have found the optimum feedback gains and passive parameters that could minimize the control force and power for the hybrid brace control law under stationary random process. This is a probabilistic method. In this section we set up an example model that was based on the previous formulas, then this model is used to check the energy convergence or (64) under some ground motions.

$$\lim_{T \to \infty} \int_0^T \dot{y}u(t)dt = 0 \tag{64}$$

Table 3 Parameters for hybrid control law for the active brace

Parameters for the analytical model in Fig.10	$m = 1.0$ ω_o=6.32(rad/s) ω_d= 2.00(rad/s) $k = 40.0$ $k_d = 4.0$
Hybrid control law in Fig.10	$g = 0.3$ β =0.1 $c_{opt} = 0.756$ c_p = 0.189 $c_A = 0.567$

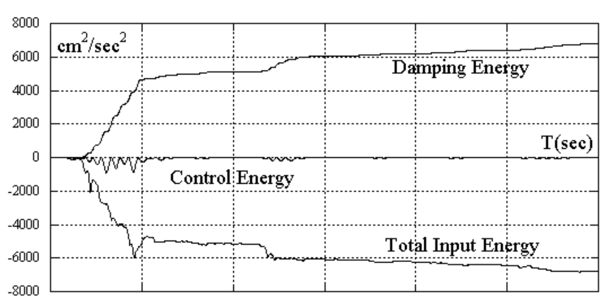

Fig.11 Energy Response under El Centro(NS)

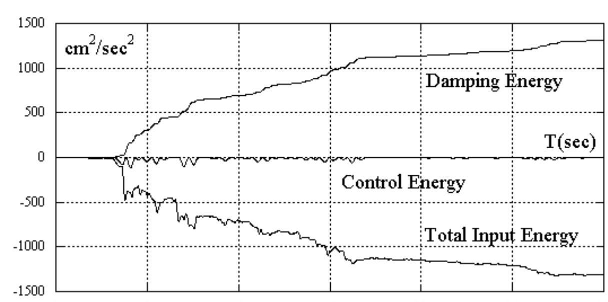

Fig.12 Energy Response under Taft(EW)

If we knew the whole ground motion in advance, we could calculate the whole system response and the optimum c_p that could satisfy (42). In other words, we could make (64) be satisfied. It is, of course, impossible, because we can't tell the future. Therefore, we obtained the most probable parameters by a stochastic method. Now we can calculate c_p and c_A in advance. Yet, there remains a suspicion if (64) is satisfied in a deterministic sense. Is (64) satisfied under any nonstationary random excitations? This is the motivation and purpose of the following calculation.

There is one example model given in Table 3. The optimum parameters are obtained by (28) and (52). The total input energy, the active control energy, and the passive dissipated energy are defined by (65), (66), and (67), respectively. The energy coming into the system is defined negative, and the energy dissipated in the system is positive.

$$E_t(T) = \int_0^T \dot{x}\ddot{x}_G(t)dt \tag{65}$$

$$E_c(T) = -\frac{1}{m}\int_0^T \dot{y}u(t)dt \tag{66}$$

$$E_d(T) = \frac{c_p}{m}\int_0^T \dot{y}^2(t)dt \tag{67}$$

The energy responses are shown in Fig.11 when the ground motion is supposed to be El Centro (NS) 341 cm/sec^2. The control energy converges to zero under the earthquake ground motion, which is a typical example of nonstationary random excitation. Another example is shown in Fig.12, where the control energy again converges to zero under Taft (EW) 176cm/sec^2 earthquake disturbance. Therefore, (64) is actually satisfied under nonstationary random disturbances. We can explain the reason for this interesting result. We found that (64) is equal to (68) by referring to (54) and (55).

$$\lim_{T\to\infty}\int_0^T \dot{y}u(t)dt = \frac{gk\omega_d^{\,2}}{\omega_o^{\,2}g + \omega_d^{\,2}}\lim_{T\to\infty}\int_0^T \dot{y}y(t)dt = \frac{gk\omega_d^{\,2}}{2(\omega_o^{\,2}g + \omega_d^{\,2})}\lim_{T\to\infty} y^2(T) = 0 \tag{68}$$

Thus, the control energy under any earthquake motion converges to zero, as long as the system is stable. It also is proved that the control energy is always negative by referring to (68). In fact, we found that the control energy responses are negative at any moment in Fig.11 and Fig.12.

CONCLUSIONS

We found that the damping augmentation expected from the passive devices in a building structure is relatively small. It is true that we could make passive damping devices affluent with energy dissipation capacity, but it is virtually impossible to put the whole input energy into a small portion of a structure. As a result, we could not make maximum use of the damping devices to attenuate the response vibration of the system. Under the same constraint condition, there is an active control law that could improve the control performance without any energy supply. According to this method, it is always possible to make the control energy converge to zero as the time goes to infinity under any non-stationary random disturbances.

The conventional definition of the damage on structure members depends on strength, deformation level, and accumulated energy dissipation. Especially, the energy integral over the whole length of excitation period has been recognized as the most reliable and general index that is least influenced by earthquake spectrum, structure systems, material properties, strength, and so on. But we have to admit that there exits an active structure member for which we could not apply the conventional energy based assessment approach as least theoretically.

REFERENCES

1. Yao, J.T.P., ”Concept of Structural Control,” ASCE, Journal of the Structural Division, Vol.98, No.7, pp.1567-1574, 1972
2. Roorda,J., “Tendon Control in Tall Structures,” ASCE, Journal of the Structural Division, Vol.101, No.3, pp.505-521, 1975
3. Yang, J., “Application of Optimal Control Theory to Civil Engineering Structures,” ASCE, Journal of the Engineering Mechanics Division, Vol.101, No.6, pp.819-838, 1975
4. McNamara,R.J., ”Tuned Mass Dampers for Buildings,” ASCE, Journal of Structural Division, Vol.103, No.9, pp.1785-1798,1977
5. Chang, C.H., Soong, T.T., “Structural Control Using Active Tuned Mass Dampers,” ASCE, Journal of the Engineering Mechanics Division, Vol.106, No.6, pp.1091-1098, 1980
6. Chung,L.L., Reinhorn,A.M., Soong,T.T., ”Experiments on Active Control of Seismic Structures,” Journal of Engineering Mechanics, Vol.114, No.2, pp.241-256, 1988
7. Nishimura,I., Abdel-Ghaffa,A.M., Masri,S.F., et al, ”An Experimental Study of the Active Control of a Building Model,” Journal of Intelligent Materials Systems and Structures, Vol.3, No.1, pp.134-164,1992
8. Kobori,T., et al., ”Seismic-response-controlled Structure with Active Mass Driver System (Part 1) Design,” Earthquake Engineering & Structure Dynamics, Vol.20, No.2, pp.133-143, 1991
9. Kobori,T., et al., “Seismic-response-controlled Structure with Active Mass Driver System (Part 2) Verification,” Earthquake Engineering & Structural Dynamics, Vol.20, No.2, pp.133-143, 1991
10. Kobori,T., “Shaking Table Experiment and Practical Application of Active Variable Stiffness (AVS) System,” Proc. of 2nd Conference on Tall Building in Seismic Regions, Council on Tall Buildings and Urban Habitat, pp.213-222, 1991
11. Samali,B., Yang,J.N., “Control of Lateral-Torsional Motion of Wind-Excited Buildings,” ASCE, Journal of the Engineering Mechanics, Vol.111, No.6, pp.777-796, 1985
12. Morison,J., Karnopp,D., “Comparison of Optimized Active and Passive Vibration Absorber,” 14th Annual Joint Automatic Control Conference, pp.932-938, 1973
13. Nishimura,I., Sakamoto,M., Yamada,T., et al, “The Optimal Active Tuned Mass Damper under Random Excitation,” Proc. of the 5th International Conference on Adaptive Structures, pp.334-343, 1994