

# IMPROVED DISPLACEMENT MODIFICATION FACTOR TO ESTIMATE MAXIMUM DEFORMATIONS OF SHORT PERIOD STRUCTURES

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### SUMMARY

An improved procedure to estimate maximum inelastic deformation demands of short period structural systems is investigated. The procedure makes use of inelastic displacement ratios that permit the estimate of the maximum inelastic deformation from the maximum elastic deformation. It is shown that the ratio of peak inelastic to peak elastic displacement is strongly correlated to the ratio of peak ground velocity and peak spectral velocity for structures with short periods of vibration. This interesting observation is then used to propose a simplified displacement modification factor that is a function of the period of the system, its lateral strength and the ratio of spectral velocity to peak ground velocity. A large set of earthquake ground motions recorded on sites having average shear wave velocities larger than 180 m/s are used to compute the error statistics on the inelastic deformation estimations of this new equation. It is shown that the new procedure leads to significant reductions in the variability of the results, and hence provides a more reliable tool for estimating inelastic deformation demands to be used in displacement-based procedures.

## **INTRODUCTION**

The essence in the recent recommendations for seismic rehabilitation and evaluation of existing structures is the accurate deformation estimation to define the expected structural/non-structural building damage for a given seismic hazard level. The implementation of these recommendations requires the use of simplified analysis procedures for daily engineering practice. In the general context, these simplified procedures approximate the inelastic deformation of a structure by using the maximum inelastic deformation demand of an equivalent single-degree-of-freedom (SDOF) system that is expected to behave nonlinearly under a severe seismic hazard level. The incremental pushover analysis is the most commonly applied procedure to represent the multi-degree-of-freedom (MDOF) building system as an equivalent SDOF system. Thus,

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one of the crucial points that would determine the accuracy of simplified procedures is the estimation of maximum inelastic deformation demands on SDOF systems subjected to earthquake ground motions.

Of the various proposed procedures for estimating the SDOF displacement demand, the equivalent linear methods and displacement modification factors are of common interest for practicing engineers. The equivalent linear methods approximate the maximum inelastic SDOF deformation through an equivalent elastic SDOF system that has longer period (equivalent period) and higher damping ratio (equivalent damping) than that of the actual nonlinear oscillator. Based on the theoretical development of equivalent linear methods, the equivalent period and equivalent damping are functions of displacement ductility ratio that is defined as the maximum inelastic deformation divided by the yield displacement of the system. Methods based on displacement modification factors approximate the maximum deformation of the nonlinear oscillator by multiplying the maximum deformation of the corresponding elastic oscillator with a set of constants. As stated in the above paragraph, the prime interest for the seismic rehabilitation of existing buildings is to determine the likely structural deformation of a structure with known lateral strength which clearly indicates that the displacement ductility capacity is unknown. This requires the equivalent linear methods to be implemented in an iterative manner as the fundamental equations used in these procedures are functions of displacement ductility. On the other hand, the displacement modification factors that are functions of lateral strength ratio R (lateral yielding strength relative to elastic strength capacity) are non-iterative methods.

The practical engineering use of simplified analyses procedures postulates them to be easy in application but at the same time demands for reliable results that account for the uncertainties inherent both in earthquake ground motions and in nonlinear structural behavior. The accurate maximum inelastic deformation estimations should not only follow a close trend to the mean variation of exact nonlinear behavior but also provide small departures (dispersions) with respect to the mean exact nonlinear behavior. Controversially, the oversimplifications in these procedures often bring forward the difficulties in considering the random nature of earthquake ground motion and corresponding structural response. This compromise results a great variability in the deformation estimations of different simplified procedures. In fact, Aschheim [1] and MacRae [2] are among the few researchers who pointed that the approximate methods based on equivalent linear and displacement modification factors may yield substantially different maximum inelastic deformation estimations for a given SDOF system subjected to the same earthquake ground motion. A recent study conducted by Miranda [3] addressed in detail the problems associated with the recommended simplified nonlinear analysis procedures of ATC-40 [4] and FEMA-356 [5]. The inelastic deformation estimations of the ATC-40 document are based on an equivalent linear method, whereas the FEMA-356 report describes the same objective via displacement modification factors. Of particular interest, Miranda [3] computed the mean error statistics of these methods for known lateral strength ratios, R to see their performance on existing building systems. A suite of earthquake ground motions recorded on different site conditions with average shear wave velocities ranging from 1500m/s to 180m/s were used during the statistical evaluations. The short period ( $T \le 0.5$ s) statistics of these methods indicated that ATC-40 equivalent linear method yielded significantly conservative mean inelastic deformation estimations with very large dispersions whereas the FEMA-356 displacement modification factor produced significantly non-conservative mean inelastic deformation estimations with relatively small dispersions. The performance of long period inelastic deformation estimations of FEMA-356 method was fairly good which is not the case for the ATC-40 equivalent linear method as the mean deformation estimations varied significantly from conservative to non-conservative values depending on the choice of structural behavior type. As a follow up of this study, Akkar [6] investigated the accuracy of five approximate methods that are used for estimating the maximum inelastic deformations of non-degrading SDOF systems. Using known lateral strength values (i.e. R values), the statistical computations were realized from 216 acceleration time histories recorded on site classes B, C

and D that are described as rock to firm soil sites in the current building codes of the U.S. [7]. Among the five approximate methods, the short period error statistics of the three equivalent linear methods proposed by Iwan [8, 9] and Kowalsky [10] showed similarities to the ones obtained from the ATC-40 equivalent linear method. As the structural systems impose weaker lateral strength (i.e. large R), these equivalent linear methods tend to overestimate the mean maximum inelastic deformations with large dispersions especially for periods of vibration less than 0.5s. The statistics of other two methods proposed by Newmark [11] and Ruiz-García [12] that are based on displacement modification factors also revealed serious dispersion problems on the mean maximum inelastic deformation estimates in the periods of vibration less than 0.5s. The statistics of all these five methods yielded a reasonable accuracy on the mean inelastic deformation estimations whereas the method proposed by Ruiz-García [12] led to significantly better mean maximum inelastic deformation estimates in the short period ranges with respect to the other approximate procedures.

The common conclusion derived from the above studies highlight that the approximate procedures can lead to large errors in maximum inelastic deformation estimations of SDOF systems in the short period range particularly for periods of vibration less than 0.5s. The large dispersion on the mean estimates is the major contributor of their poor performance. Even if the mean estimate is good enough, the large dispersion superimposes a very high level of uncertainty on the predictions. The inadequate consideration of uncertainty in earthquake ground motions together with the rigorous nonlinear structural behavior in the short periods of vibration are dominant contributors in simplified procedures that yield significant dispersion on the mean maximum inelastic deformation estimations. This observation is an important drawback for such methods, as the seismic performance of short period structures constitute one of the major concerns in daily engineering practice.

The objective of this paper is to improve the maximum inelastic deformation estimations of the approximate procedure proposed by Ruiz-García [12] for short periods of vibration. This equation is combined with a simple multiplicative function to account for the earthquake ground motion uncertainty. This new function uses spectral velocity ( $S_v$ ) and peak ground velocity (PGV) parameters to reduce the bias in the short periods introduced by the method of Ruiz-García [12] due to large dispersions on the mean inelastic deformation estimations. Reduction of dispersion on the mean estimations via this new function increases the accuracy of the maximum inelastic deformation estimations significantly. Using the same earthquake ground motion data set presented in Ruiz-García [12], the paper derives the new function by describing the variation of maximum inelastic deformations with respect to PGV and  $S_v$  parameters. The performance of the proposed expression is evaluated through the error statistics including the mean error and standard deviation of the error. Comparisons are presented between the improved equation and its predecessor.

## DISPERSION ON THE MEAN INELASTIC DEFORMATIONS OF SHORT PERIOD SDOF SYSTEMS

The inherent dispersive character of maximum inelastic to elastic SDOF deformation ratio  $(\Delta_i/\Delta_e)$  tends to increase significantly towards shorter periods of vibration. This phenomenon is one of the most prominent contributors to the poor deformation estimates of approximate methods in short periods. Figure 1 illustrates an example case for the degree of dispersion that can be experienced in the short periods of vibration. The gray curves in Figures 1a and 1b are the results of elasto-perfectly plastic 0.1s and 0.25s oscillators subjected to 72 earthquake ground motion records of site class C soils and show the variation of  $\Delta_i/\Delta_e$  for different levels of yielding lateral strength that are defined by a fraction of elastic strength,  $F_e$ . The relationship between the elastic strength  $F_e$ , the yielding strength  $F_y$ , and the lateral strength ratio R is given by

$$R = \frac{F_e}{F_y} = \frac{mPSA}{F_y}$$
(1)

where *m* is the mass of the oscillator and *PSA* represents the pseudo-spectral acceleration. The thick dark lines are the 16 percent, mean and 84 percent percentile values computed from these curves. As the yielding strength of the system takes smaller values with respect to elastic strength (i.e. as the system becomes weaker) the gap between the mean and the bounding 16 percent and 84 percent percentile curves widens indicating the increase in the tendency of dispersion on  $\Delta_i/\Delta_e$  for large values of *R*. The dispersion on the mean inelastic deformation decreases as the period of vibration shifts towards longer period range. The comparison of Figure 1a and 1b gives a good sight for the diminishing character of scatter in  $\Delta_i/\Delta_e$ when the period of vibration is slightly shifted towards longer periods of vibration (for this example the period changes from 0.1s to 0.25s). It is noteworthy that even a gradual increase in the period of vibration results a significant tapering in the dispersion on  $\Delta_i/\Delta_e$  especially for weak systems.



Figure 1. Dispersion on  $\Delta_i/\Delta_e$  for short periods of vibration and different yielding strength levels

An alternative representation of dispersive nature in  $\Delta_i/\Delta_e$  and its significance on the maximum inelastic deformation estimations of SDOF systems are shown in Figures 2 and 3 that are reproduced from Ruiz-García [12]. Figure 2 describes the period dependent mean  $\Delta_i/\Delta_e$  for elasto-perfectly plastic oscillators with lateral strength ratios ranging from 1.5 to 6. The mean  $\Delta_i/\Delta_e$  curves were computed from a set of 72 earthquake ground motions recorded on site class B soils for periods of vibration between 0.05s and 3.0s. On average, the maximum inelastic deformations are larger than the corresponding elastic deformations for periods of vibration less than 0.85s. The maximum inelastic to elastic deformation ratios start to grow significantly with increasing *R* towards shorter periods of vibration highlighting the essence of accurate inelastic deformation structure in Figure 3 shows the plots of coefficient of variation (*COV*) for the mean  $\Delta_i/\Delta_e$  curves presented in Figure 2. The coefficient of variation is a measure of dispersion on the mean and it is the ratio of standard deviation,  $\sigma$  to mean,  $\mu$ .

$$COV = \frac{\sigma}{\mu} \tag{2}$$

The plots in Figure 3 show that *COV* increases as the system gets weaker and it departs towards very large values for periods of vibration less than approximately 0.5s. It should be noted that the descending branch in the *COV* for T < 0.25s and R > 2 gives a false picture of decrease in dispersion. As presented in Figure

2, high mean values of  $\Delta_i/\Delta_e$  for weak systems with short periods of vibration mask the inherently large standard deviations in this spectral region.



Figure 2. Exact variation of mean  $\Delta_t/\Delta_e$  for 72 site class B earthquake ground motions



Figure 3. Covariance statistics of  $\Delta I / \Delta_e$  for 72 site class B earthquake ground motions

The observations highlighted in Figures 1 to 3 indicate that the record-to-record variability (i.e. the random nature of earthquake ground motions) significantly dominate the inelastic deformation demands on short period structural systems. The consideration of ground motion parameters is essential to reduce the uncertainty due to record-to-record variability that is particularly beneficial for a short period building in which  $\Delta_i/\Delta_e$  amplifies significantly. The studies by Cordoba [13] and Ordaz [14] provided valuable information about the use of strong ground motion related parameters to achieve improved estimations on demand measures used in the performance evaluation of structures.

## EVALUATION OF THE RUIZ-GARCÍA AND MIRANDA METHOD FOR SHORT PERIOD SDOF SYSTEMS

Ruiz-García [12] studied the statistical variation of inelastic deformation demands on elasto-perfectly plastic SDOF systems subjected to a total number of 216 strong ground motions recorded on site classes

B, C and D. The study considered 12 California earthquakes with moment magnitudes ranging from 5.8 to 7.2. Ruiz-García [12] evaluated the magnitude, distance and site geology effects on  $\Delta_{e}/\Delta_{e}$  ratio and proposed a displacement modification factor to estimate the maximum inelastic deformations of SDOF systems given the lateral strength ratio and periods of vibration. The fundamental relation proposed by Ruiz-García [12] is

$$\Delta_i = C_R \Delta_e \tag{3}$$

In Equation (3) the term  $C_R$  is the proposed modifying constant that is multiplied by the elastic displacement  $\Delta_e$  to approximate the maximum inelastic deformation demand  $\Delta_i$  on SDOF systems subjected to earthquake ground motions. The  $C_R$  expression was derived from two-stage nonlinear regression analysis by using the mean variation of exact  $\Delta_i/\Delta_e$  with respect to period of vibration and lateral strength ratio. The proposed expression for  $C_R$  is given in Equation (4).

$$C_{R} = 1 + \left[\frac{1}{a(T/T_{s})^{b}} - \frac{1}{c}\right](R-1)$$
(4)

The variables a, b and c are the nonlinear regression constants to account for the local site conditions. The site dependent characteristic period is designated as  $T_s$ . The constants R and T define the lateral strength ratio and period of vibration, respectively. Table 1 lists the values of a, b, c and  $T_s$  computed from the regression analysis. The regression constants presented in the last raw of Table 1 are used whenever the information about site conditions is unreliable.

| Site Class | а  | b    | С  | T₅(s) |
|------------|----|------|----|-------|
| В          | 42 | 1.60 | 45 | 0.75  |
| С          | 48 | 1.80 | 50 | 0.85  |
| D          | 57 | 1.85 | 60 | 1.05  |
| B,C,D      | 54 | 1.82 | 55 | 0.85  |

Table 1. Site dependent coefficients used in Equation (4)

Figure 4 shows the  $C_R$  values computed from Equation (4) that are used to approximate the mean maximum deformation demands of earthquake ground motions recorded on site class B. These smooth curves were obtained by applying the nonlinear regression analysis on the exact variation of mean  $\Delta t / \Delta_e$  plots that are shown in Figure 2. A comparison between Figures 2 and 4 indicate that the regressed  $C_R$  values can adequately capture the exact mean variation of maximum deformation demands on SDOF systems.

Figure 5 shows a typical case for the statistical evaluation of Equation (4). Figure 5a displays the mean error  $(\vec{E}_{T,R})$  for the maximum deformation estimates of Equation (4) that is computed from the 72 earthquake ground motions recorded on site class C soils. Figure 5b presents the corresponding standard deviation  $(\sigma_{T,R})$  of the mean error. The mean error was computed by averaging the approximate  $(\Delta_{ap})$  to exact  $(\Delta_{ex})$  maximum deformation ratios for each specific period of vibration and lateral strength ratio over a family of earthquake ground motion records, *N*. The approximate deformations are computed by using Equations (3) and (4), whereas the exact deformations are the results of nonlinear response history results. A value of  $\vec{E}_{T,R}$  close to one is the indication of accurate estimations with respect to the mean variation of exact maximum deformations. The standard deviation defines the general tendency of dispersion of Equation (4) in estimating the maximum deformations. As the standard deviation takes larger values, the uncertainty in the accuracy of maximum deformation estimates increases. The mathematical expressions of these error measures are shown in Equations (5a) and (5b).

$$\overline{E}_{T,R} = \frac{1}{N} \sum_{i=1}^{N} \left[ \frac{(\Delta_{ap})_i}{(\Delta_{ex})_i} \right]_{T,R}$$
(5a)

$$\sigma_{T,R} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left[ (E_{T,R})_i - (\overline{E}_{T,R}) \right]^2}$$
(5b)



Figure 4. C<sub>R</sub> values for site class B earthquake ground motions proposed by Ruiz-García [12]



Figure 5. Mean error and dispersion of the maximum inelastic deformation estimations by Ruiz-García [12] for site class C earthquake ground motions

The plots in Figure 5 exhibit the general performance of Equation (4) in estimating the maximum deformation demands on nonlinear oscillators. On average, this modifying factor yields close estimates to the mean variation of exact maximum deformation demands on SDOF systems. However, as the period of vibration takes smaller values, the uncertainty in earthquake ground motions results biased approximate maximum deformations of Equation (4) as denoted by the large standard deviation values in Figure 5b. Figure 6 presents a good example for the lack of record-to-record variability in Equation (4) and how the ground motion features dominate the short period SDOF nonlinear response. This figure displays the exact variation of  $\Delta_{t}/\Delta_{e}$  with respect to spectral velocity ( $S_{\nu}$ ) normalized by the peak ground velocity (*PGV*). The plots are for a particular case and show the actual tendency in maximum response for a 0.2s nonlinear

oscillator with a yielding strength 1/6th of the elastic strength when subjected to 72 earthquake ground motions recorded on site class D soils. Also shown in this figure is the  $C_R$  value computed by Equation (4) when T=0.2s and R = 6 using the regression constants derived for site class D earthquake ground motions. The exact variation of  $\Delta_{e}/\Delta_{e}$  ratio asymptotically goes to infinity and zero as  $S_{e}/PGV$  attains small and relatively very large values, respectively. This hyperbolic tendency in the exact behavior is not observed in Equation (4) and yields a constant modification factor regardless the actual variation of  $\Delta_{e}/\Delta_{e}$  with respect to  $S_{v}/PGV$ . This is the expected behavior for Equation (4) as it lacks the ground motion uncertainty in its derivation.



Figure 6. Exact variation of  $\Delta_i / \Delta_e$  vs.  $S_v / PGV$  for T=0.2s and R=6 and the corresponding  $\Delta_i / \Delta_e$  estimation by Ruiz García [12]

Figure 7 gives a broader view for the variation of exact  $\Delta / \Delta_e$  with respect to PGV/S<sub>v</sub> computed for different periods of vibration and lateral strength ratios. Similarly to Figure 6, these plots are derived for 72 ground motions recorded on site class D soils and present the modifying constant of Equation (4) to have a better sight on the bias introduced by this equation in the short periods of vibration. The conspicuous linearly increasing variation of  $\Delta / \Delta_e$  with respect to PGV/S<sub>v</sub> is evident for SDOF systems that have short periods of vibration and high lateral strength ratios (e.g. T=0.1s-R=6.0 pair, T=0.3s-R=6.0 pair). The tendency in linear increase loosens, as the period of vibration takes relatively larger values or the lateral strength ratio R is relatively low (e.g. T=0.1s-R=4.0 pair, T=0.3s-R=4.0 pair). For small values of R, the increasing linear trend of  $\Delta_{t}/\Delta_{e}$  with respect to  $PGV/S_{v}$  diminishes almost independently from the variation in the period of vibration value (e.g. T=0.1s-R=1.5 pair, T=0.3s-R=1.5 pair). This relationship is explained statistically in Figure 8 that shows the correlation coefficient ( $\rho$ ) between the exact  $\Delta t/\Delta_e$  and  $PGV/S_{\nu}$  as functions of normalized lateral strength and period of vibration. Figure 8a confirms that the linear correlation between  $\Delta_i / \Delta_e$  and PGV/S<sub>v</sub> is high when R takes large values and period of vibration is small. The correlation weakens for increasing periods and decreasing R. Figure 8b is the verification of high correlation between  $\Delta_t/\Delta_e$  and  $PGV/S_v$  for short periods and its decreasing trend towards longer periods of vibration associated with small R values. The modifying constant of Equation (4) captures this exact trend only for small values of R and relatively longer periods. It starts to introduce large errors as the periods of vibration take smaller values and normalized lateral strength becomes large. The consequential effects of these observations are reflected in the error statistics similar to the ones as displayed in Figure 5. The reader is referred to Akkar [6] for a detailed discussion of the statistical performance of Ruiz-García [12] displacement modification equation.



Figure 7. Relationship between exact  $\Delta / \Delta_e$  and *PGV/S<sub>v</sub>* as a function of *T* and *R* for site class D earthquake ground motions



Figure 8. Correlation between inelastic deformation ratio and  $PGV/S_{\nu}$  with changes in strength (left) and period of vibration (right)

#### IMPROVED MAXIMUM DEFORMATION ESTIMATES FOR SHORT PERIOD SDOF SYSTEMS

The observations in the previous section have demonstrated the existence of a strong relationship between the maximum inelastic deformations and the ground motion parameters  $S_v$  and PGV especially in the short periods of vibration. The comparisons between the exact  $\Delta_i/\Delta_e$  as a function of  $S_v/PGV$  and Equation (4) also revealed that the short period maximum inelastic deformation estimations could be improved significantly by applying a correction term to the  $C_R$  described in Equation (4). This new term will consider the dominant random behavior of earthquake ground motions on the maximum nonlinear deformation response of SDOF systems by using  $S_v$  and PGV terms. This concept is applied to Equation (3) in the following form:

$$\Delta_i = \frac{1}{C_f} C_R \Delta_e \tag{6}$$

The term  $C_f$  is the correction term and it is computed by using the ratio of maximum approximate inelastic deformations  $(\Delta_{ap})$  to exact inelastic deformations  $(\Delta_{ex})$ . The approximate inelastic deformations were computed from Equation (3) whereas the corresponding exact deformations were found from the nonlinear response history analyses. The nonlinear regression analysis was applied to  $\Delta_{ap}/\Delta_{ex}$  ratios considering each period of vibration and lateral strength ratio. The earthquake ground motion data set presented by Ruiz-García [12] for site classes B, C and D were used during the regression analysis. The following equation is proposed for the correction factor after these calculations:

$$C_f(T,R) = A + B \frac{S_v}{PGV}$$
<sup>(7)</sup>

where

$$A(T, R) = 1 - exp(-\frac{6.2T}{R-1})$$
(8a)

$$B(T,R) = \frac{R-1}{36T^{-0.13(R-1.5)+d}}$$
(8b)

The regression constant d in Equation (8b) represents the geological conditions in different site classes and its values are listed in Table 2.

| Table 2. S | Site depe | ndent coeffic | cient used | in Ec | quation ( | 8b | ) |
|------------|-----------|---------------|------------|-------|-----------|----|---|
|------------|-----------|---------------|------------|-------|-----------|----|---|

| D    |  |  |
|------|--|--|
| 1.55 |  |  |
| 1.65 |  |  |
| 1.82 |  |  |
|      |  |  |

Figure 9 gives a comparative performance of Equations (3) and (6) for two particular short period SDOF systems (i.e. T=0.1s and T=0.2s, respectively). The plots include the exact variation of  $\Delta_t/\Delta_e$  with respect to  $S_v/PGV$  for 72 earthquake ground motions recorded on site class D soils. These comparisons show that at the expense of adding the correction term, Equation (6) adequately describes the uncertainty of ground motion features on the maximum inelastic deformations of SDOF systems. The maximum deformation estimates computed from Equation (6) yields a very good match with respect to the exact trend observed in  $\Delta_t/\Delta_e$ . Equation (3) does not reflect the variation of short period deformation demand on SDOF systems, as it lacks the evaluation of uncertainty in earthquake ground motions that affects the nonlinear behavior in great extend for short periods of vibration.

Figure 10 exhibits the mean error and standard deviation plots of Equation (6) in a way similar to Figure 5 that is displayed for Equation (3). The plots present the results for 72 earthquake ground motions recorded

on site class C soils. The mean error curves of Figure 10a point that the mean approximate deformation estimations of Equation (6) are very close to the mean exact maximum inelastic deformations. The standard deviations in Figure 10b marks the improvements brought by Equation (6) to the mean maximum deformation estimates. These curves imply that the use of correction term reduces the departures from the mean inelastic deformation estimations significantly for short period SDOF systems having large values of R.



Figure 9. Exact variation of  $\Delta / \Delta_e$  for site class D ground motions and comparison of maximum inelastic deformation estimations by Ruiz-García [12] and this study



Figure 10. Mean error and dispersion statistics of the proposed procedure for the maximum inelastic deformation estimations of site class C ground motions

A close up view for the enhanced deformation estimations of Equation (6) is presented in Figure 11. The standard deviation of mean errors for the 72 earthquake ground motions of site class D records are shown in Figure 11a when the SDOF system has a lateral strength ratio of 6.0. The standard deviation values computed from Equation (6) starts showing improvements with respect to Equation (3) for periods of vibration less than 1.0s. The dispersion in Equation (3) is reduced by 50 percent for periods of vibration at

about 0.5s. This reduction is larger than 100 percent at the period of 0.1s indicating a significant increase in the reliability of short period maximum deformation estimations. Figure 11b gives the variation of 10 percent and 90 percent percentile curves of Equations (3) and (6), for the 72 records from site class C soils, respectively. These curves represent that 80 percent of the individual errors (i.e. the ratio  $\Delta_{ap}/\Delta_{ex}$ computed from each record) lay in between these two lines. Thus, closer these two curves, more accurate the maximum deformation estimations are. Similarly to the results driven in Figure 11a, the 10 percent and 90 percent percentile curves describe a fairly good performance for the maximum deformation estimates of Equation (6) in the short periods of vibration. Especially, the comparisons between the 90 percent percentile curves show a considerable decrease in dispersion when Equation (6) is used for the maximum deformation estimations of SDOF systems for  $T \le 0.5s$ .



Figure 11. Improvement in dispersion on the estimation of expected  $\Delta_i$  of SDOF systems for earthquake ground motions recorded on site class D and C

#### SUMMARY AND CONCLUSIONS

An improved displacement modification factor to estimate the maximum inelastic deformation demands of short period, non-degrading SDOF systems has been presented. The method is based on the interesting observation that for short period structures the ratio of peak inelastic deformation to peak elastic deformation is strongly correlated to the ratio of peak ground velocity to spectral velocity. This correlation increases as the system becomes weaker, however it is shown that the correlation decreases as the period of vibration increases. Using the peak deformations of SDOF systems subjected to 216 recorded earthquake ground motions a new displacement modification factor is obtained through nonlinear regression analyses. It is shown that the proposed method leads to significant reductions in dispersion in inelastic deformation estimates by incorporating the ratio of spectral velocity to peak ground velocity. The decrease in dispersion is notable especially for weak systems that have lateral strength ratios greater than 4. In general, the reduction in dispersion varies from 50 percent to 100 percent with respect to the statistics of the predecessor method for periods of vibration between 0.6s and 0.1s, respectively. Hence the proposed method is more reliable than other simplified methods to estimate peak deformation demands of short period structures.

#### ACKNOWLEDGEMENTS

The first author was partially supported by NATO B1 grant provided by The Scientific Research and Technical Council of Turkey during the conduct of this investigation at Stanford University. The financial support is gratefully acknowledged. The authors extend their sincere thanks to Mr. Jorge Ruiz-García who computed the nonlinear time history response results.

#### REFERENCES

- 1. Aschheim MA, Maffei J, Black EF. "Nonlinear static procedures and earthquake displacement demands." Proceedings of the 6th US National Conference on Earthquake Engineering, Seattle, Washington, Paper No. 167, 1998.
- 2. MacRae G, Tagawa H. "Methods to estimate displacements of PG&E structures." Report PGE/PEER Task No. 505, University of Washington, Seattle, Washington, 2002.
- 3. Miranda E, Akkar SD. "Evaluation of approximate methods to estimate target displacements in PBEE." The Fourth U.S. Japan Workshop on Performance–Based Earthquake Engineering Methodology for Reinforced Concrete Building Structures, Pacific Earthquake Engineering Center Report No. PEER 2002/21, University of California at Berkely, Berkeley, CA, 75-86, 2002.
- 4. Applied Technology Council (ATC). "Seismic evaluation and retrofit of concrete buildings." Report ATC-40, Applied Technology Council, Redwood City, CA, 1996.
- 5. Building Seismic Safety Council (BSSC). "Prestandard and commentary for the seismic rehabilitation of buildings." Report FEMA-356, Washington, D.C., 2000.
- 6. Akkar SD, Miranda E. "Statistical evaluation of approximate methods for estimating maximum deformation demands on existing structures." Submitted for possible publication in Journal of Structural Engineering, ASCE.
- 7. International Code Council (ICC). "International building code." Birmingham, Alabama, 2000.
- 8. Iwan WD. "Estimating inelastic spectra from elastic spectra." Earthquake Engineering and Structural Dynamics 1980; 8(4): 375-388.
- 9. Iwan WD, Guyader AC. "An improved equivalent linearization procedure for the capacity spectrum method." Proceedings of the International Conference on Advanced and New Challenges in Earthquake Engineering Research, Harbin, China, 2002.
- 10. Kowalsky MJ. "Displacement-based design a methodology for seismic design applied to RC bridge columns." Master's Thesis, University of California at San Diego, La Jolla, CA, 1994.
- 11. Newmark NM, Hall WJ. "Earthquake spectra and design." Earthquake Engineering Research Institute, Berkeley, CA, 1982.
- 12. Ruiz-García J, Miranda E. "Inelastic displacement ratios for evaluation of existing structures." Earthquake Engineering and Structural Dynamics 2003; 32(8): 1237-1258.
- Cordova PP, Deierlein CG, Mehanny, SF, Cornell, CA. "Development of a two-parameter seismic intensity measure and probabilistic assessment procedure." The Second U.S. – Japan Workshop on Performance–Based Earthquake Engineering for Reinforced Concrete Structures, Pacific Earthquake Engineering Center Report No. PEER 2000/10, University of California Berkeley, CA, 195-214, 2000.
- 14. Ordaz M, Pérez-Rocha LE. "Estimation of strength reduction factors for elastoplastic systems: new approach." Earthquake Engineering and Structural Dynamics 1998; 27(10): 889-901.