

SENSITIVITY OF ANALYTICAL VULNERABILITY FUNCTIONS TO INPUT AND RESPONSE PARAMETER RANDOMNESS

Oh-Sung KWON¹ and Amr ELNASHAI²

SUMMARY

There are many approaches to derive vulnerability functions that are used to estimate the seismic risk of populations of structural systems. Previous research on the derivation of vulnerability curves employs various assumptions, aimed mainly at reducing the computational effort. Few studies use detailed fiber modeling approaches and accurate stiffness and strength degrading models. The end result is a proliferation of vulnerability curves that differ by large factors and for which the level of uncertainty is not always quantified. In this study, analyses are performed using the most detailed models available. These were verified by comparing the analytical model to the shaking table test results. The influence of the ground motions and material properties are investigated by isolating the effect of each variable through sensitivity analysis and detailed simulation. A statistical method to handle structural response in the vicinity of the collapse limit state is presented alongside criteria for selection of response limit states, ground motion duration and scaling factors. It is observed that the effect of ground motion variation is overwhelming, in comparison with the effect of representation of other random fields.

INTRODUCTION

Vulnerability curves are relationships between strong-motion shaking severity and the probability of reaching or exceeding a specified limit state. The curves can be classified into four categories [Rossetto and Elnashai, 2003]. Some are based on observational data from post-earthquake surveys [Orsini, 1999; Rossetto and Elnashai, 2003] while others are based on analytical studies [Mosalam et al., 1997; Reinhorn, 2001; Chryssanthopoulos, 2000]. There are vulnerability curves based mainly on expert opinions [ATC-13] or derived by a combination of these three methods. Empirical vulnerability curves are more realistic as they are based on actual structures subjected to real strong-motion. However, they do have limitations in general application since the curves are derived for a specific seismic region and a sample that is not necessarily similar to that sought. On the other hand, analytical vulnerability curves can be derived for general purposes, but the choice of analytical model, simulation method, and the lack of

¹ Graduate Research Assistant, University of Illinois at Urbana Champaign, Urbana, IL, USA. Email: okwon2@uiuc.edu

² Willett Professor of Engineering, Acting Director, Mid-America Earthquake Center, University of Illinois at Urbana Champaign, Urbana, IL, USA. Email: aelnash@uiuc.edu

computational power pose challenges for the development of the required relationship. The main restrictions on selection of the method are the availability of data which come from observation of postearthquake losses or analytical simulation. The data from simulation is again restricted by computational power and application which can represent the realistic nonlinear model. The limitation, in the analytical derivation of vulnerability curves using inelastic time history analysis, is diminishing with the expansion of computational power and the development of reliable analysis tools.

The SDOF system, which represents the pushover curve of an infilled and bare frames, was used for the analytical derivation of vulnerability curves [Mosalam et al, 1997]. In HAZUS [NIBS 1999], the variability in seismic demand is provided without explicit consideration of the influence of the structural parameters such as damping, period, and yield strength level. Reinhorn et al. [2001] used constant yield-reduction-factor inelastic spectra with capacity spectrum to evaluate inelastic response. All of these approaches used simplified methods since the derivation of vulnerability curves requires a large amount of simulation. Consequently, the results are approximate as these methods neglect the effect of higher modes, hysteretic damping, and element failure. This study derives vulnerability for the sample building from analytical methods using ZEUS-NL [2002]. Personal computers and a supercomputer in the National Center for Supercomputing Applications (NCSA) are utilized for the large amount of simulation

Since vulnerability curves give a probability of attaining a certain damage state when a structure is subjected to a specified seismic hazard, they play a critical role in regional seismic risk and loss estimation. These loss estimations are essential for the important purposes of disaster planning and formulating risk reduction policies. The driving technical engines of a regional seismic risk and loss estimation system are:

- Seismic hazard maps (i.e. peak ground parameters or spectral ordinates)
- Vulnerability functions (i.e. relationships of conditional probability of reaching or exceeding a performance limit state given the measure of earthquake shaking)
- Inventory data (i.e. numbers, location and characteristics of the exposed system of elements of a system)
- Integration and visualization capabilities (i.e. data management framework, integration or seismic risk and graphical projection of the results)

Among the above ingredients, factors influencing the vulnerability curves, such as ground motion and material, are closely investigated using a multistory RC structure, as presented in subsequent sections.

BENCHMARK STRUCTURE

Description of selected structure

A structure, which was tested under earthquake loading, is selected for a realistic and reliable benchmark study of the derivation of vulnerability curves. The prototype structure was originally designed for the purpose of an experimental study [Bracci et al, 1992]. It is a three story ordinary moment resisting concrete frame designed for gravity loads and is non-seismically detailed. The provisions of ACI 318-89 code, with Grade 40 steel [$f_y = 276$ MPa (40 ksi)] and ordinary Portland cement [$f_c' = 24$ MPa (3.5 ksi)], was employed. The plan and elevation of the structure are given in Figure 1. For detailed design information, reference is made to Bracci et al. [1992].



Analysis model and verification

Selection of analysis method

In literature, several analysis methods have been proposed to determine the seismic demand on structures subjected to earthquake loading. Static analysis methods, such as conventional or adaptive pushover analysis, are computationally cost effective but lack accuracy when a structure is irregular or higher mode effects are predominant. Since structures not designed to resist seismic loads usually fail in localized modes, their response is not likely to be well estimated by these static methods. Inelastic dynamic time history analyses give more realistic results than do static analysis. But most dynamic analyses tools are using constant $M - \phi$ relationship and point-hinge model which are quite different from reality.

It was decided to deploy the most accurate method available for seismic demand and supply evaluation, inelastic dynamic response history analysis, in order to focus attention of other approximation in the vulnerability function derivation. Use is made of the Mid-America Center analysis environment ZEUS-NL [2002], in which the sectional response is calculated at each loading step from inelastic material models which account for stiffness and strength degradation. Consequently, there is no need for sweeping assumptions on the moment-curvature relationships required by other analysis approaches. The verification of the analytical environment as well as the analysis model is introduced in the following section.

Verification of analytical model

Damping, stiffness, and mass properties are key parameters for the time history analysis of a frame. Hence, the analytical model is verified through comparison with the experimental result in the view of structural period and displacement time history.

In order to represent structural modeling, columns and beams are divided into six and seven elements respectively. The subdivision is of varying lengths. Lumped masses are applied at the beam and column connection. Material properties are taken from the material test result of experimental model. From eigen value analysis, the elastic structural periods were 0.898 sec, 0.305 sec, and 0.200 sec for the 1st, 2nd, and 3rd mode, respectively. The periods from 1/3 scale experiment [Bracci et al. 1992], converted to full scale, were 0.932 sec, 0.307 sec, and 0.206 sec. Due to minor cracking in the tests, the fundamental periods of

the structure from the experiment under small amplitude testing are $3\sim4\%$ longer than the analytical values. These values give credence to the analytical model.

Figure 2 shows the comparison of the 3rd story displacements of 1/3 scale experiment and the analytical model for moderate and severe ground motions, 0.20g and 0.30g in PGA, respectively. Damping other than hysteretic is not included for these analyses since hysteretic damping is the main source of energy dissipation in these levels of ground motion. Figure 2(a) and 2(b) show congruency between the experimental and analytical results. However, for the analysis at 0.05g PGA level Rayleigh damping measured from the experiment was necessary to dissipate energy. In conclusion, assuming the same level of damping for a small amplitude of ground motion to the collapse level ground motion could result in non-conservative collapse limit state vulnerability curves as the extra damping reduces response quantities. For the purposes of this study, it is assumed that there is no source of damping other than hysteretic. Consequently, for low limit states, the vulnerability curves tend to the conservative side.

From the comparison of structural periods and displacement time history, it is verified that the analysis environment and the analytical model can replicate the experimental result very well. In addition to this, it is found that the effect of damping is negligible for moderate-to-large earthquake motions which bring inelastic behavior on the structure.



(a) 3rd story displacement - Taft 0.20g



Time, sec

(b) 3rd story displacement – Taft 0.30g Figure 2. Comparison of dynamic analysis

LIMIT STATES DEFINITION

There has been ample research and publications with proposed limit states, which are physically meaningful damage states for the purpose of damage evaluation and intervening action. In ATC 40 [1997] and FEMA-273 [1997], four limit states are defined based on global behavior (interstorey drift) as well as element deformation (plastic hinge rotation). Rossetto and Elnashai [2003] used five limit states for derivation of vulnerability curves based on observational data while Chryssanthopoulos et al. [2000] used only two limit states. In the latter studies, the global limit states are independent of the specific response of the structure. For example, the FEMA-273 [1997] 'life safety level' limit state of interstorey drift (ISD) for non-ductile moment resisting frame is 1.00% regardless of gravity force levels or the details of structural configuration within the sub-class of structure.

It is necessary, for rigorous analysis, to define limit states for each individual structure, since the deformation capacity could be a function of many other factors. These could include factors such as gravity force level, irregularity, anticipated plastic hinging mechanism, etc. Three limit states are defined for the prototype structure based on the first yielding of steel, attainment of maximum element strength, and maximum confined concrete strain during the adaptive pushover analysis. These are termed, 'serviceability', 'damage control', and 'collapse prevention', respectively. The 1st story drift which corresponds to each limit state, for the prototype structure, are 0.57%, 1.2% and 2.3% for the selected limit states.

RANDOM VARIABLES

Vulnerability curves are expressed in probabilistic terms because the seismic demand as well as structural capacity is not a deterministic variable. The uncertain variables need to be defined in terms of probabilistic parameters such as mean, standard deviation, and probability distribution function, or there should be a set of data with which simulation can be run. In this study, the parameters of material properties are adopted from previous researches. Ground motion uncertainties are considered by using 9 sets of ground motion data.

Ground motion uncertainty

Selection of ground motion set

Parameters that define seismic hazard of a site include magnitude, source mechanism, attenuation characteristic, local site response, etc [Wen et al., 2003]. It is difficult to consider all those factors in the derivation of vulnerability curves due to lack of information. Also vulnerability curves based on those specific information lacks applicability in general situation since the derived curves depends on the selected ground motion set. Nine sets of ground motions are used for the purpose of understanding the effect of input ground motion.

The first three sets of motions, set low a/v, normal a/v, and high a/v, are categorized based on the ratio of peak ground acceleration to peak ground velocity (a/v) as defined by Zhu et al. [1988]. The a/v ratio is a single and simple parameter. It, however, implicitly accounts for many seismo-tectonic and site characteristics of earthquake ground motion records. Sawada et al. [1992] performed a statistical study and concluded that low a/v ratios signify earthquakes with low predominant frequencies, broader response spectra, longer durations and medium-to-high magnitudes, long epicentral distances and site periods. Conversely, high a/v ratios represent high predominant frequencies, narrow band spectra, short duration, and small-medium magnitudes, short epicentral distance and site periods. Based on this categorization, three sets of ground motions are selected (Table 1). The average response spectrum of selected ground motion sets (Figure 3) show distinctive differences among each ground motion set.

A/V Ratio	Earthquake event / Location	ML	Date	Soil Type	D [km]	A max [m/s2]	V max [cm/s]	a/v (g/ms ⁻¹)
low a/v	Bucharest / Romania	6.40	3/4/1977	rock	4	-1.91	-70.55	0.275
	Erzincan / Turkey	unknown	3/13/1992	stiff soil	13	-3.82	101.83	0.382
	aftershock of Montenegro / Yugoslavia	6.20	5/24/1979	alluvium	8	-1.17	18.87	0.634
	Kalamata / Greece	5.50	9/13/1986	stiff soil	9	-2.11	32.73	0.657
	Kocaeli / Turkey	unknown	8/17/1999	unknown	101	-3.04	41.33	0.750
normal a/v	aftershock of Friuli / Italy	6.10	9/15/1976	soft soil	12	-0.81	7.95	1.040
	Athens / Greece	unknown	9/7/1999	unknown	24	-1.09	-10.17	1.090
	Umbro-Marchigiano / Italy	5.80	9/26/1997	stiff soil	27	-0.99	9.12	1.108
	Lazio Abruzzo / Italy	5.70	5/7/1984	rock	31	-0.63	-5.64	1.136
	Basso Tirreno / Italy	5.60	4/15/1978	soft soil	18	0.72	-6.19	1.183
high a/v	Gulf of Corinth / Greece	4.70	11/4/1993	stiff soil	10	-0.67	4.79	1.432
	aftershock of Montenegro / Yugoslavia	6.20	5/24/1979	rock	32	-0.67	4.46	1.526
	aftershock of Montenegro / Yugoslavia	6.20	5/24/1979	alluvium	16	-1.71	-11.14	1.564
	aftershock of Umbro-Marchigiana / Italy	5.00	11/9/1997	rock	2	0.41	-2.21	1.902
	Friuli / Italy	6.30	5/6/1976	rock	27	3.50	-20.62	1.730

Table 1. Selected ground motions based on a/v ratio

Table 2. Property of artificial ground motions for Memphis, TN

City	Perc- entile	Peak Ground Motion Parameter	Set #	Scenario 1 7.5 @ Blytheville, AR	Set #	Scenario 2 6.5 @ Marked Tree, AR	Set #	Scenario 3 5.5 @ Memphis, TN
Memphis, TN (Lowlands)		PGA (g)	Set L-1	0.1427	Set L-2	0.0632	Set L-3	0.0958
	84th	PGV (m/s)		0.152		0.0576		0.0665
		PGD (m)		0.0606		0.0202		0.0138
Memphis, TN (Uplands)		PGA (g)	Set U-1	0.1407	Set U-2	0.0676	Set U-3	0.103
	84th	PGV (m/s)		0.129		0.0516		0.0609
		PGD (m)		0.0537		0.0178		0.0118



Figure 3. Average response spectrum of selected ground motion sets

The other six sets of motions are artificial ground motions that are generated for Memphis, Tennessee by Drosos [2003]. Three of the sets, set L-1, L-2, and L-3, are generated based on Lowlands soil profile in Memphis area. The other three sets, set U-1, U-2, and U-3, are generated based on Uplands soil profile in the same region. Each of the three sets of ground motions are based on three scenario earthquakes. Each set contains ten ground motions. Table 2 shows the peak ground motion quantities for Memphis, TN. The percentile corresponds to the amplification factors used for the generation of the ground motions.

Selection of duration and scaling factors

When a structure undergoes inelastic response, the duration of the significant part of strong-motion affects the response of the structure. There are many previous studies aimed at defining the duration of strong ground motion. Trifunac and Brady [1975] used significant duration concepts based on the integrals of the squares of acceleration, velocity and displacement where the duration is defined as the interval between the times at which 5% and 95% of the total integral is attained. This range of duration is meaningful in characterizing ground motions. This duration may not be very practical when viewed from the point of structural analyses. For instance, if this interval, between 5% and 95% of total integral is used, the ground motion acceleration could start at very large value, which may apply an unrealistic pulse to a structure. Also, since the majority of ground motions' energy is skewed to the early part of the motion, use of identical margins for start and end of the duration is not a reasonable approach. For the study described in this paper, the interval between 0.5% and 95% of the integrals are used. There were minor adjustments on the start and end time of strong motion based on judgment to exclude insignificant motions.

The capacity spectrum method was used (Figure 4) to determine the range of a reasonable PGA scale. The capacity curves were obtained from adaptive pushover analysis, and the demand curves were converted from the elastic displacement and acceleration spectra of each ground motion set. For accurate estimation of maximum PGA scale at which the structure collapse, elastic demand needs to be decreased to consider inelasticity using effective damping [ATC 40, 1997, Borzi et. al, 2001] or ductility ratio [Chopra, 1999]. In this analysis, however, rough estimation of PGA scales were undertaken using elastic 5% damped demand spectra and inelastic capacity spectra.

Material properties

Concrete strength

Bartlett and Macgregor [1996] investigated the relationship between the strength of cast-in place concrete and specified concrete strength. When the concrete is one year old, the ratios of the average inplace strength to the specified strength were 1.33 and 1.44 for short and long elements, respectively, with a coefficient of variation of 18.6%. The variation of strength throughout the structure for a given mean inplace strength depends on the number of members, number of batches, and type of construction. The coefficient of variation is 13.0% for multiple batches of concrete and for multiple members. It is assumed that there is no variability of concrete strength within the considered structure as the structure is a low-rise building of limited volume that would have been constructed in a relatively short period of time. Hence, a coefficient of variation of 18.6% is used. The specified concrete strength (or design strength) of the considered structure was 24 MPa. The in-place concrete strength is assumed to be 1.40 times larger than the specified strength (33.6 MPa).

Steel strength

Mirza and MacGregor [1979] reported results of about 4000 tests on Grade 40 and 60 bars. The mean values and coefficient of variation of the yield strength were 337 MPa (48.8 ksi) and 10.7 %, respectively for Grade 40 bars. The probability distribution of modulus of elasticity of Grade 40 reinforcing steel followed a normal distribution with a mean value 201,327 MPa (29,200 ksi) and a coefficient of variation of 3.3 %. Due to the low level of variability observed, the modulus of elasticity is assumed to be deterministic (201,327 MPa). The structure was designed with grade 40 steel, thus the mean steel strength is assumed to be 337 MPa.



Figure 4. PGA scale selection using Capacity spectrum method

Figure 5. Collapse state and lognormal distribution assumption (normal a/v ratio)

SIMULATION

In order to investigate the effect of ground motion set, material strength, and effects of sample size, combinations of material strengths were chosen. For ground motion set low $a/v \sim high a/v$, ten ultimate concrete strengths, f_c , and ten steel yield strengths, F_y , were generated and a full combination of material strengths are used resulting total 100 frames. The analysis results are used to study the effect of material properties on the structural responses. For ground motion set L-1, L-2, and L-3, which are artificial ground motions based on Lowlands profile, 50 concrete and steel strengths are arbitrarily generated resulting 50 different frames. For ground motion set U-1, U-2, and U-3 based on Uplands profile, 100 concrete and steel strengths are arbitrarily generated. From the analysis result of these frames, the effect of sample size is investigated.

The total simulation of the frame using ground motion set low, normal, and high a/v ratio required 456 hours using a Pentium 4-2.65 GHz system for a total of 23,000 response history analyses. For the analysis of frames using ground motion set L-1 through U-3, mass-simulation environment, which can handle 32 processes simultaneously, were developed for super computer, IBM p690, in NCSA. Interstorey drifts are retrieved from each analysis and used for statistical analysis.

DERIVATION OF VULNERABILITY CURVES

If geometric nonlinearity as well as material inelasticity is considered in the dynamic time history analysis, the structure becomes unstable when it is subjected to gravity forces and large lateral displacement due to earthquake loading. The story drift of that state cannot be included in the statistics of structural response since the unstable structural behavior more rely on convergence criteria used in the analysis rather than on seismic demand. In this study, it is determined that the structure is in the collapse state if the maximum interstorey drift (ISD_{max}) is larger than the defined 2.3% of interstorey drift. This value, 2.3%, is applicable only to the studied structure, and possibly the sub-class of non-seismically designed medium-rise RC frames. Lognormal distribution of interstorey drift was assumed for structures with ISD_{max} < 2.3% as shown in Figure 5. The statistics of the two states are combined using total probability theorem from which probability of exceeding limit states at each PGA level are calculated.

EFFECT OF GROUND MOTION AND MATERIAL VARIABILITY

Effects of ground motion set

Figure 6 compares vulnerability curves for each limit state derived from nine ground motion sets. For general application of the vulnerability curve, it is necessary to perform regression analysis to obtain functional forms of vulnerability curves. But for the purpose of comparison of vulnerability curves from different ground motion sets, actual data with linear interpolation are plotted in order that differences are not masked by regression. From Figure 6 the following is observed.

- a) Each ground motion set shows significantly different vulnerability curves
- b) The difference in the vulnerability curves increases with limit state levels since the material variability takes effect at large PGA level, which will be explained later.
- c) The vulnerability curves are not monotonically increasing due to instability of structure and statistical manipulation at large PGA level.

Based on this observation, we can conclude that the ground motion sets should be selected with great care since the derived vulnerability curves are dependent on the selected ground motion.

Effects of material properties

The effects of material properties are investigated using analysis results from a/v ratio ground motion sets for which 10 concrete ultimate strengths and 10 steel yield strengths are combined. It is assumed that ISD_{max} is a function of ground motion sets, concrete ultimate strength, and steel yield strength.

$$USD_{\max} = g(X_1, X_2, X_3)$$
 (1)

where X_1 : ground motion

 X_2 : concrete strength

 X_3 : steel strength

Mean of ISD_{max} can be calculated from the result of full simulation, thus using 100 frames, or from the simulation of single frame using mean material properties. Figure 7 compares the means of IDS_{max} from the two methods. Up to 0.25g, the means from the two methods are almost identical. The difference of mean values between two methods becomes large as ground motion level increases. This is due to the fact that ISD_{max} becomes much more sensitive to material properties at a large PGA level than at a small PGA level.

Assuming all the uncertain variables, i.e. ground motion, concrete, and steel properties, are statistically independent, variance of ISD can be calculated from Eq.(2)

$$Var(ISD_{max}) \quad c_1^2 Var(X_1) + c_2^2 Var(X_2) + c_3^2 Var(X_3)$$
(2)

where $c_i = \partial g / \partial X_i$ evaluated at $x_i = \mu_{X_i}$. The c_1 of the first term, which is $\partial (ISD_{max}) / \partial X_1$ by definition, is not quantifiable as differentiation of ISD_{max} with ground acceleration itself is not possible. The c_2 and c_3 values can be calculated numerically in the vicinity of mean values of concrete and steel properties. Variances, $Var(X_2)$ and $Var(X_3)$, can be calculated from coefficient of variation, COV=0.186, and COV = 0.107, for concrete and steel respectively.

Figure 8 (a), (b), (d), and (e) shows ISD_{max} against f_c and F_y at small and large PGA level. Figure 8 (c) and (f) shows the contribution of material variability on the variance of ISD_{max} , i.e. the second and third term of Eq.(2). From Figure 8(a), each ground motion has different slope, $d(ISD_{max})/df_c$, since each ground motion has different displacement response spectrum. And in each ground motion, $d(ISD_{max})/df_c$ varies with f_c . It's because the elastic modulus of concrete is affected by ultimate strength, f_c , and, hence, structural period is affected by f_c . As a result, the relationship between ISD_{max} and f_c cannot be represent-



(c) Probability of exceeding ISD 2.3% Figure 6. Vulnerability curves for each ground motion set



Figure 7. Mean ISD_{max} from mean material properties and full simulation





Figure 9. Comparison of variance with mean frame and multiple frames

-ed as general trend, such as generally increasing or decreasing ISD_{max} with f_c , since ISD_{max} and f_c is correlated with spectral displacement, which is random in nature. On the contrary, the ISD_{max} is rarely affected by the yield strength of steel at low PGA level from Figure 8 (d), as the elastic modulus of steel is constant regardless of yield strength, and the ground motion level is not large enough to cause yielding of the steel. Comparison of Figure 8.(a) and (b) shows that the effect of concrete strength to ISD_{max} increases with increasing PGA level. From comparison of Figure 8 (d) and (e), it is speculated that the variability of yield strength of steel affects the ISD_{max} when ground motion is large enough to cause yielding of steel. Figure 8 (c) and (f) show contribution of concrete and steel strengths, which are second and third terms of Eq (2), on the variance of ISD_{max} . As it is shown, for small PGA levels, the contribution of material properties on the variance of ISD_{max} is small. Also from the increasing variances with PGA level, it is expected that the confidence interval of derived vulnerability curves can be increased, resulting less reliable curves.

Effect of sample size

The analysis result of ground motion set U-1 is used where 100 concrete and steel properties are arbitrarily combined to see the effect of the sample size on the variance of ISD_{max} . From the discussion in the previous section, we can reasonably assume that the material has very little effect on the variability of ISD_{max} at low PGA level. Based on this assumption, the variances of ISD_{max} from mean frame, 10 frames, 50 frames, and 100 frames were compared. Variance can be calculated as Eq. (3).

$$\operatorname{Var}_{1} = \sum_{i=1}^{n} \frac{(x_{i} - \mu_{X_{i}})^{2}}{n - 1}$$
(3)

For mean frame we have 10 samples since we have 10 ground motions. Thus, n = 10. If we use 100 frames, number of samples become n' = jn = 1000.

$$\operatorname{Var}_{2} = \sum_{i=1}^{jn} \frac{(x_{i} - \mu_{x_{i}})^{2}}{jn - 1}$$
(4)

where jn = 1000, j = 100, and n = 10. Since we did not increase the number of ground motions, and from the assumptions that the material variability has little effect on the outcome of result, we can rewrite Eq. (4) as below.

$$\operatorname{Var}_{2} = \sum_{i=1}^{jn} \frac{(x_{i} - \mu_{Xi})^{2}}{jn - 1} = \sum_{i=1}^{n} j \frac{(x_{i} - \mu_{Xi})^{2}}{jn - 1}$$

$$= \frac{n - 1}{n - 1/j} \sum_{i=1}^{n} \frac{(x_{i} - \mu_{Xi})^{2}}{n - 1} = \frac{n - 1}{n - 1/j} = \frac{10 - 1}{10 - 1/100} \operatorname{Var}_{1} = 0.90 \operatorname{Var}_{1}$$
(5)

In other words, the increase in sample size considering material variability, which does not increase variability of ISD_{max} , reduces the variance of ISD_{max} considering only ground motion uncertainty. Figure 9 (a) compares the variances from mean frame with 10, 50, and 100 frames. The variance of mean frame is larger than that of frames with variable material properties. If you take into account the assumption that material properties have little effect on ISD_{max} , which is true for small PGA levels, the correction factor in Eq. (5) is applied and the resulting variances are compared in Figure 9 (b). As it was expected, the variances of four curves are very similar in small PGA levels, up to PGA of 0.15g, and discrepancies increase with increasing PGA, since material variability takes effect at large PGA levels.

CONCLUSIONS

Vulnerability curves were derived for a three story structure with as few simplifying assumptions as possible. Several aspects of the process of vulnerability curves derivation were discussed: effects of damping, selection of limit states, selection of ground motion set, its duration and PGA level and simulation methods. The effects of ground motion sets and material variability on the derived vulnerability curves are thoroughly examined. The main observations from the reported studies are reiterated below:

- Different ground motion sets lead to widely varying vulnerability curves. Therefore, vulnerability curves are in general hazard-specific.
- The difference in vulnerability curves from different ground motion sets increase with PGA level. The hazard-sensitivity is therefore more pronounced for high damage and collapse limit states.
- Concrete strength can affect the ISD_{max} at low PGA level, but such effect is much lower than that from ground motion variation. The effect of steel yield strength is negligible at low PGA levels.
- At high PGA levels, the variability of material properties affects the response of structure significantly.
- An increase in the sample size of frames without increasing the number of ground motions reduces the variability of ISD_{max} low PGA level.
- The reliability of derived vulnerability curves at large PGA level is lower than that from small PGA level due to the increasing variability in the structural response.

The investigation reported above is continuing, with the objectives of quantifying the reliability of vulnerability functions derived using various models, methods and input assumptions.

ACKNOWLEDGEMENT

This paper is a product of the Mid-America Earthquake (MAE) Center Project DS-3 'Response analysis tools'. The MAE Center is a National Science Foundation Engineering Research Center (ERC), funded through contract reference NSF Award No. EEC-9701785. Thanks are due to Drs. T. Prudhomme, C. Beldica and J. Towns who facilitated the use of the parallel processors at the National Center for Supercomputing Applications

REFERENCES

- 1. ATC-13 (1985) Earthquake damage evaluation data for California, Applied Technology Council, Redwood City, Palo Alto, California
- 2. ATC-40 (1997) Seismic Evaluation and Retrofit of Concrete Buildings, Applied Technology Council, Redwood City, Palo Alto, California
- 3. Ang, A. H. and Tang, W. H., (1975), "Probability Concepts in Engineering Planning and Design," John Wiley & Sons, Inc.
- 4. Barlett, F.M, and MacGregor, J.G. (1996), "Statistical analysis of the compressive strength of concrete in structures," *ACI Material Journal*, 93(2), 158-168
- 5. Borzi, B., Calvi, G. M., Elnashai, A. S., Faccioli, E., and Bommer, J. J. (2001), "Inelastic spectra for displacement-based seismic design," *Soil Dynamics and Earthquake Engineering*, 21(1), 47-61

- 6. Bracci, J.M., Reinhorn, A.M.,and Mander, J.B. (1992), "Seismic Resistance of Reinforced Concrete Frame Structures Designed only for Gravity Loads: Part I-Design and Properties of a One-Third Scale Model Structure.", Technical Report NCEER-92-0027
- 7. Chopra, A.K., and Goel, R.K. (1999), "Capacity-Demand-Diagram Methods for Estimating Seismic Deformation of Inelastic Structures: SDF Systems," Report No. PEER-1999/02, Pacific Earthquake Engineering Research Center, University of California, Berkeley.
- 8. Chryssanthopoulos, M.K., Dymiotis, C., and Kappos, A.J. (2000), "Probabilistic evaluation of behaviour factors in EC8-designed R/C frames," *Engineering Structures*, 22(8), 1028-1041
- 9. Drosos, V.A. (2003). "Synthesis of Earthquake Ground Motions for the New Madrid Seismic Zone," MS Thesis, Georgia Institute of Technology, Atlanta, GA.
- 10. Elnashai, A.S. (2002), "Do we really need inelastic dynamic analysis?," *Journal of Earthquake Engineering*, Vol. 6, Special Issue 1(2002) 123-130
- Elnashai, A.S., Papanikolaou, V. and Lee, D. (2002). Zeus NL A System for Inelastic Analysis of Structures, Mid-America Earthquake Center, University of Illinois at Urbana-Champaign, Program Release Sept. 2002
- 12. FEMA 273 (1997) NEHRP guidelines for seismic rehabilitation of buildings, Federal Emergency Management Agency, Washington D.C.
- 13. Mosalam, K.M., Ayala, G., White, R.N., and Roth, C. (1997), "Seismic fragility of LRC frames with and without masonry infill walls," *Journal of Earthquake Engr.*, 1 (4), 693-720
- 14. Mirza, S.A., and MacGregor, J.G. (1979), "Variability of Mechanical Properties of Reinforcing Bars," *Journal of structural division*, Vol.105, No.ST5
- 15. NIBS (1999), "HAZUS, Earthquake Loss Estimation Technology," Technical Manual prepared by the National Institute of Buildings Sciences (NIBS) for the Federal Emergency Management Agency (FEMA)
- 16. Orsini G (1999), "A model for buildings' vulnerability assessment using the parameterless scale of seismic intensity (PSI)," *Earthquake Spectra*, Vol 15(3), 463-483
- 17. Reinhorn, A.M., Barron-Corvera, R., and Ayala, A.G. (2001), "Spectral evaluation of seismic fragility of structures Structural safety and reliability," (ICOSSAR 2001)
- 18. Rossetto, T. and Elnashai, A. (2003), "Derivation of vulnerability functions for European-type RC structures based on observational data," *Engineering Structures*, 25(10), 1241-1263
- 19. Sawada, T., Hirao, K., Yamamoto, H. and Tsujihara, O. (1992), "Relation between maximum amplitude ratio and spectral parameters of earthquake ground motion," *Proc. 10th World Conf. on Earthquake Engineering*, Madrid, Spain, Vol. 2, 617-622
- 20. Trifunac, M.D. and Brady, A.G. (1975), "A study on the duration of strong earthquake ground motion," *Bulletin of the Seismological Society of America*, 65(3), 581-626
- 21. Wen, Y.K., Ellingwood, B.R., and Veneziano, D., Bracci, J. (2003), "Uncertainty Modeling in Earthquake Engineering," Mid-America Earthquake Center Project FD-2 Report, January.
- 22. Zhu, T.J., Heidebrecht, A.C. and Tso. W.K. (1988), "Effect of peak ground acceleration to velocity ratio on ductility demand of inelastic systems," *Earthquake Engineering and Structural Dynamics*, 16, 63-79