

# 13<sup>th</sup> World Conference on Earthquake Engineering Vancouver, B.C., Canada August 1-6, 2004 Paper No. 3475

# GEOTECHNICAL HYBRID SIMULATION OF LIQUEFIABLE INCLINED GROUND CONSIDERING PORE WATER MIGRATION

Motoki KAZAMA<sup>1</sup>, Noriaki SENTO<sup>2</sup>, Young-Cheul KWON<sup>3</sup> and Akira YAMAGUCHI<sup>4</sup>

#### **SUMMARY**

The authors have developed a new geotechnical hybrid simulation technique for pore water migration. The technique enables a quantitative evaluation of residual deformation such as lateral spreading and settlement caused by liquefaction. In the system, the seismic responses in a one-dimensional slightly inclined multi-layered soil system are taken into consideration. The ground response is governed by both equation of motion and the continuity equation, into which soil behavior is directly introduced instead of using a soil constitutive model. This paper introduces the concept and specifications of the developed system. By applying the system to an example problem, the permeability effect on the seismic response during cyclic shear is studied. The importance of the volume change characteristics of sandy soil during and after cyclic shear is shown in conclusion.

#### INTRODUCTION

# **Background of the research**

In recent years, there has been a need for a more rational technique to evaluate residual deformation caused by liquefaction for practical design purpose. When we consider the residual deformation of ground and soil structure caused by liquefaction, we must consider deformations both during and after an earthquake. The final amount of deformation is the sum of these two deformations. Considering past earthquake disasters, it is clear that post earthquake progressive deformation can not be neglected. Post-earthquake delayed failures have been reported by several large earthquakes. For example, it was reported that Kawagishi-cho apartment continued tilting for as long as half an hour and the Showa bridge collapse occurred after the earthquake motion terminate in the 1964 Niigata earthquake, JSCE report [1]. It is also well known that the lower San Fernando dam failed following the earthquake during the 1971 San Fernando earthquake, Seed [2]. In the 2003 northern Miyagi earthquake in Japan, witnesses claimed that an artificial silty sand slope failed two to three minutes after the earthquake motion ceased, JGS report [3].

<sup>&</sup>lt;sup>1</sup> Professor, Dept. of Civil Eng, Tohoku University, Sendai, Japan, E-mail: m-kazama@civil.tohoku.ac.jp

<sup>&</sup>lt;sup>2</sup> Research Associate, ditto, E-mail: nsentoh@civil.tohoku.ac.jp

<sup>&</sup>lt;sup>3</sup> Graduate Student, ditto, E-mail: kwon@soil1.civil.tohoku.ac.jp

<sup>&</sup>lt;sup>4</sup> Lecturer, Dept. of Civil and Envir. Eng., Tohoku Gakuin University, Miyagi, Japan, E-mail: yamaguti@tjcc.tohoku-gakuin.ac.jp

It is considered that effective stress redistribution due to pore water migration plays an important role on the delayed failure, Sento [4].

## The need to consider pore water migration during and after earthquake

In all the research to date, either fully drained or undrained conditions have been adopted for the cyclic shear test. The reality is, however, that actual site condition are somewhere between these two extremes. There is a practical reason for not carrying out the shearing test under partial drainage conditions in that the drainage condition is decided not only by material properties but also by boundary conditions. In other words, depending on the boundary condition, it is necessary to consider the drainage, even if it is dynamic problem. A typical problem where drainage conditions must be considered is gravel drain columns.

In addition, being able to accurately predict post-earthquake damage as a result of excess pore water migration is another important issue which needs further research with regard to delayed failure. Another key point to take into consideration is the cyclic stress strain behavior of the soil subjected to optional volume change as a result of pore water migration. Furthermore, volumetric change after liquefaction directly related to the settlement of the ground. For these reasons, it is necessary to consider pore water migration both during and after earthquakes.

To solve the problem explained above, the authors have developed an on-line hybrid simulation system for pore water migration. As well as introducing the concept and specifications of the developed system, this paper provides a detailed analysis of its application to an example problem on the permeability effects during cyclic shear

# HYBRID ON-LINE TEST HISTORY IN GEOTECHNICAL EARTHQUAKE ENGINEERING

## Development in earthquake geotechnical engineering

The first application of on-line test to earthquake geotechnical engineering was carried out by Katada [5,6]. They developed a system with a computer controlled cyclic triaxial test apparatus in order to examine the earthquake ground response when subjected to earthquake motion as a single-degree-of-freedom system. Using this, Katada [7,8] showd that liquefied soil exhibited a cyclic softening behavior. After this research, it was shown that liquefaction of the ground differs remarkably depending on the duration of time and the frequency content, even the earthquake motion is equal maximum acceleration. Adachi [9] carried out the on-line test using hollow cylindrical torsion test equipment, and they studied the relationship between the excess pore water pressure and maximum value index of the input earthquake motions. These are typical expansion of the way the single degree of freedom on-line test.

It is crucial to model the multi-layered ground in appropriate degrees of freedom, when this technique is practically applied in the analysis of the earthquake response of the ground. Kusakabe [10] constructed a system consisting of six hollow torsion cyclic shear testing equipment (Photo-1), in which some of the layers are replaced with numerical models. In so doing, they completed a one-dimensional earthquake response hybrid on-line test system of the ground using a cyclic shear test under undrained conditions.

After the 1995 Hyogo-ken Nambu Earthquake, there were new developments of this technique because of demands for more rational liquefaction potential evaluations of the ground for better earthquake proof designs. Fujii [11] constructed an on-line testing system, in order to discuss the stability of embankments, and a stress state of three positions on a circular arc slip plane was reproduced. Yamaguchi [12] examined the seismic response of the Kobe Port Island by using a three-degrees-of-freedom on-line system with actual soil samples.

Until this time, on-line experiment had mainly applied the liquefaction ground earthquake response analysis under undrained conditions. Now, though in Japan, this method is applied to predict the practical liquefaction damage, Sento [13]. Furthermore, Sento [14] conducted an on-line test for post-liquefaction seepage failure and Kwon [15] have developed an on-line system for the consolidation of problems presented by clay soil.

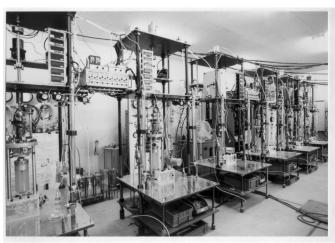


Photo-1 On-line testing system developed by Kusakabe [10]

#### CONCEPT OF THE ON-LINE HYBRID TEST FOR PORE WATER MIGRATION

Understanding and creating models of the cyclic shear behavior of soil has been one of the main research subjects in geotechnical earthquake engineering, because it is the key issue governing the response of the ground and of soil structures as a boundary value problem. Though a lot of constitutive models have been proposed, it is still difficult for a practical engineer to choose a suitable model and to determine the parameters used in it. Soil variation is a primary factor with grain size distribution and secondary factors like its in-situ soil structures making the problem complex. Ideally, the boundary value problem should be solved using the cyclic shear behavior of the actual soil.

Cyclic shear behavior under optional volume change caused by seepage is very complex. None of the constitutive model is available at present provides adequate representation of the fully cyclic behavior of soil under optional volume change conditions. In addition, because incremental volume change depends on not only soil properties but also on the boundary conditions, the problem must be treated as a boundary value problem.

## **Governing equations**

In the system introduced here, the seismic response of one-dimensional slightly inclined multi-layered soil system is taken into consideration. In general, the excess pore water pressure generated due to cyclic shear shows irregular distribution in the depth direction. The generation of the hydraulic gradient of pore water pressure results in pore water migration during and after the cyclic shear. Thus the ground response is governed by both a wave propagation equation (Figure 1) and the volumetric continuity condition (Figure 2) as follows:

Force equilibrium in shear wave propagation;

$$\rho \frac{\partial^2 u_x}{\partial t^2} = -\frac{\partial \tau}{\partial z} \tag{1}$$

where  $\rho$  = density of soil as a whole;  $u_x$  = horizontal displacement;  $\tau$  = shear stress; t = time; and z = the vertical coordinate. On the other hand, the condition of continuity as shown in case of Figure 2, can be expressed as

$$\frac{\partial \mathcal{E}_{v}}{\partial t} = -\frac{\partial Q}{\partial z} \tag{2}$$

where  $\varepsilon_{v}$  = volumetric strain; Q = flux of pore water per unit area in which the plus side shows compression. Introducing Darcy's law, equation (2) can be written as

$$\frac{\partial \mathcal{E}_{v}}{\partial t} = \frac{\partial}{\partial z} \left( k \cdot \frac{\partial}{\partial z} \left( \frac{u_{e}}{\gamma_{w}} \right) \right) \tag{3}$$

where k = coefficient of permeability;  $u_e = \text{excess pore water pressure}$ ; and  $\gamma_w = \text{the unit weight of water}$ .

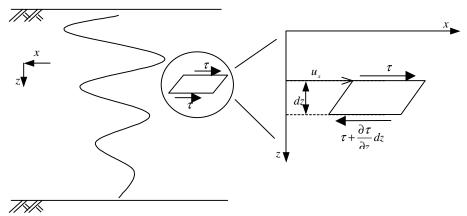


Figure 1. Schematic diagram of one-dimensional wave propagation considered here.

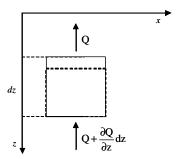


Figure 2. Schematic diagram of the continuity condition.

# **Constitutive relationships**

Regarding the constitutive relationship, the relationship between shear stress and strain, and the effective stress and volumetric strain can be expressed as

$$d\tau = Gd\gamma \tag{4}$$

$$d\varepsilon_{y} = m_{y}d\sigma_{z}' \tag{5}$$

where G = shear modulus;  $\gamma$  = shear strain;  $m_{\nu}$  = the coefficient of volume compressibility;  $\sigma'_{z}$  = the vertical effective stress. When the total stress increment  $d\sigma$  is zero, the relationship between the effective stress and the excess pore water pressure can be written as

$$d\sigma'_{\cdot} = -du_{\cdot} \tag{6}$$

Thus, the equation (5) can be rewritten as

$$d\varepsilon_{x} = -m_{x}du_{x} \tag{7}$$

In this study, the equations (4) and (7) are coupled because of the dilatancy of soil, and these non-linear relationships are obtained from elementary tests directly.

# Spatial discretization of equation of motion

The lumped mass discrete method is adopted for the equation of motion as shown in Figure 3. In the system, both the viscous damping and the energy dissipation into the under ground are considered. The equation of motion of the whole system can be expressed as

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+\{F\}=-[M][I]\alpha$$
 (8)

where [M] = the mass matrix; [C] = the viscous damping matrix;  $\{I\}$  = the unit vector,  $\alpha$  = the input motion;  $\{F\}$  = the restoration force vector;  $\{\ddot{x}\}$  = the relative acceleration vector, and  $\{\dot{x}\}$  = the relative velocity vector.

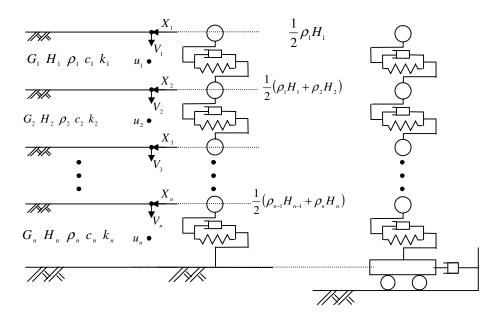


Figure 3. Model of the spring-mass system.

## Spatial discretization of continuity equation

Figure 4 show the discrete model for the condition of continuity. The degree of freedom of excess pore pressure lay at the center of the layer, Sandhu [16]. Since pore water flows through two layers with different permeability, the average permeability  $\bar{k}$  is introduced into equation (9). In the condition of

continuity in *i*-th layer, the equation is one dimensional in a vertical direction, and the volumetric strain is equal to the vertical strain. Therefore, equation (3) can be discretized as

$$\dot{V}_{i} - \dot{V}_{i+1} = \frac{\bar{k}_{i}}{\gamma_{w}} \cdot \frac{u_{i-1} - u_{i}}{\frac{H_{i-1} + H_{i}}{2}} - \frac{\bar{k}_{i+1}}{\gamma_{w}} \cdot \frac{u_{i} - u_{i+1}}{\frac{H_{i} + H_{i+1}}{2}}$$
(9)

where  $\dot{V}$  = the vertical velocity;  $H_i$  = the thickness of *i*-th layer, and  $\bar{k}_i$  = the average permeability between *i*-th and i+1-th layer which is expressed as

$$\bar{k}_{i} = \frac{k_{i-1}k_{i}}{k_{i-1}H_{i} + k_{i}H_{i-1}} \cdot (H_{i-1} + H_{i})$$
(10)

Consequently, the equation representing the condition of continuity in the whole system can be expressed as:

$$[A]\{\dot{V}\} = [\bar{k}]\{u\} \tag{11}$$

where  $\{\dot{V}\}$  = the vertical velocity vector;  $\{u\}$  = the pore pressure vector; [A] = the coefficient matrix; and  $[\bar{k}]$  = the permeability matrix.

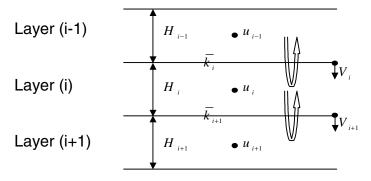


Figure 4. Pore water flow in *i*-th layer.

#### **Numerical integration**

For discretization in the time domain in a hybrid on-line system, an explicit scheme has to be used as a numerical integration using small time intervals, because an implicit scheme can not be adopted for an element test. For an equation of motion, the linear acceleration method is used initially, while the central difference method is used after the second step. For the condition of continuity, the forward difference method is used.

#### TEST PROCEDURE

In the hybrid on-line system proposed here to solve the equations from the period while there are vibrations to the process of excess pore water dissipation. The procedure can be divided into three parts by a controlling method.

## **During the forced vibration**

From a static equilibrium condition, seismic motion is applied to the base of the system. The governing equations are the equation of motion and the continuity condition, as shown previously. For the first step of the equation of motion, a response is obtained from the following equation

$$\{X\}_{i} = -\left(\frac{6}{(\Delta t)^{2}}[M] + \frac{3}{\Delta t}[C] + [G_{0}]\right)^{-1}[M]\{I\}\alpha_{1}$$
 (12)

where  $\Delta t$  = the time increment. After the second step, the response at the next time step can be obtained from the following equation.

$$\{X\}_{j+1} = \left(\frac{[M]}{(\Delta t)^2} + \frac{[C]}{2\Delta t}\right)^{-1} \times \left(-[M]\{I\}\alpha_j + \frac{2[M]}{(\Delta t)^2}\{X\}_j - \left(\frac{[M]}{(\Delta t)^2} - \frac{[C]}{2\Delta t}\right)\{X\}_{j-1} - \{F\}_j\right)$$
(13)

For the continuity condition, the vertical velocity at the next time step can be obtained from

$$\{V\}_{i+1} = \{V\}_i + \Delta t [A]^{-1} [k] \{u\}_i$$
 (14)

From constitutive relationships, in general, the restoration force and the excess pore pressure at the next step can be expressed as the function of the horizontal and vertical displacement (shear and volumetric strain) as follows.

$$\{F\}_{j} + \{F_{initial}\} = f(\{X\}_{j} + \{X_{initial}\}, \{V\}_{j} + \{V_{initial}\})$$
(15)
$$\{u\}_{j} = f(\{X\}_{j} + \{X_{initial}\}, \{V\}_{j} + \{V_{initial}\})$$
(16)

where f = the function representing the soil response;  $\{F_{initial}\}$  = the initial shear stress vector that corresponds to the inclination of the slope;  $\{X_{initial}\}$  and  $\{V_{initial}\}$  = the horizontal and vertical displacement vectors corresponding to the initial shear stress, respectively.

Firstly, from equation (12), the horizontal displacement (the shear strain) in next time step can be calculated using the elastic shear modulus. The vertical displacement (volumetric strain) also can be obtained from the initial distribution of excess pore water pressure, which is zero in ordinary cases, from equation (14). Secondarily, the restoration force and excess pore pressure in next step can be obtained from the constitutive relationships using equations (15) and (16). In case of the element test layer, these values are measured directly from the experiment by applying the shear and volumetric strain calculated in the previous step. These values are then transmitted to the computer by means of the on-line system which performs the response calculation in the succeeding step. On the other hand, in the case of the numerical model layer, these values are obtained from a constitutive model. The seismic ground response analysis is proceeded by repeating this procedure within the time duration of earthquake motion.

#### **During the free vibration process**

The inertia force remains in the free vibration condition even after input earthquake motion ends. The same equations can be used as equations (13) with  $\alpha = 0$  and (14). In this process, whether or not the inertia term is negligible is evaluated.

## **During the dissipation process**

This process is the dissipation of excess pore water pressure when the inertia term is small enough to be negligible. In this process, only the continuity condition is considered, maintaining the initial shear stress. For the experiment, both the volumetric strain and the constant restoration force are applied as input data, and the shear strain and the excess pore water pressure are observed from the specimen as outputs. Therefore, in this process, the stress is controlled by the system.

Finally, the residual displacement at the surface both in horizontal and vertical directions are evaluated as the sum of layers. In practice, since the number of elementary test layers is limited, only the layers that expected to perform with strong nonlinearity are considered experimental. A schematic diagram of the presented system is shown in Figure 5.

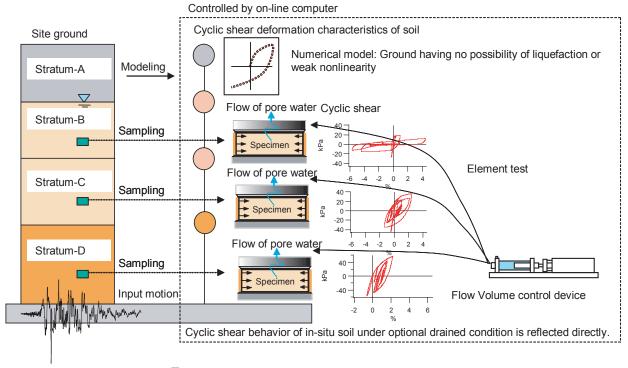


Figure 5. Schematic diagram of a practical application

## **TEST SYSTEM**

# **System configuration**

In this system a personal computer controls everything including the cyclic shearing apparatus, the data acquisition and seismic response analysis. Therefore, all testing devices are automatically controlled by a computer program that was developed by the authors originally. A typical configuration of the hardware of the shearing apparatus for a single layer is shown in Figure 6. It is easy to expand to a multi-layered system to add the necessary number of sets. The developed program can perform the hybrid on-line test on up to 100 layers, including up to eight experimental layers for which the cyclic shear performances and pore pressures can be monitored in real time during the test.

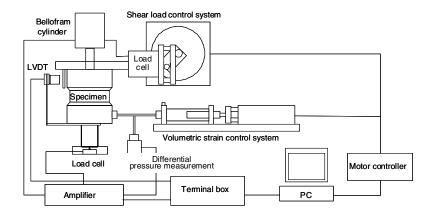


Figure 6. Simple shear test apparatus for element test layer

A simplified simple shearing apparatus (Kusakabe et al., 1999) is used in development work. The strain resolution depends on the size of the specimen, the displacement sensor, the shear load control and the volumetric strain control system. For example, for a specimen with an outer diameter of 60mm and a height of 25mm, the resolution of shear strain and volumetric strain are  $2.0 \times 10^{-5}$  / step and  $5.0 \times 10^{-6}$  / step, respectively. It is small enough for the test.

#### APPLICATION EXAMPLE OF PERMEABILITY EFFECT ON SEISMIC RESPONSE

## Example problem

Conditions of the example problem are shown in Figure 7. A total layer thickness of 6m consisting of homogeneous sandy soil was divided into three experimental test layers for the on-line test. The soil sample used here is Hitachinaka sand with a fine content about 5% and produced with a relative density of about 80%. Three on-line tests were conducted with the assumed permeability of 0.1cm/s, 0.01cm/s and 0.001cm/s.

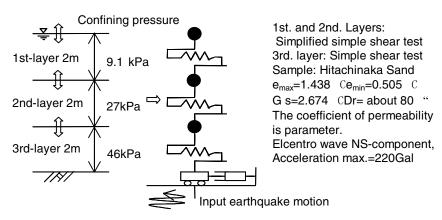


Figure 7. Example problem for the permeability effect

#### **Test results**

Figure 8 shows the distribution of the residual horizontal displacements in the depth direction obtained from the on-line tests. The relative displacement between the adjacent layers represents the shear strain in the sandwiched layer. Therefore, the residual shear strain of the third layer clearly contributes to the residual displacement of the ground surface. It is also found that the smaller the coefficient of permeability is, the larger the residual shear strain.

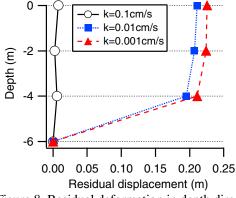


Table 2 Volumetric strain at the end of on-line test The coefficient of 0.001 0.1 0.01 permeability (cm/s) 1st layer 1.0% 0.2% 0.6% (at the end of shaking) (0.4%)(-0.007%)(0.005%)2nd layer 3.7% 1.2% 1.3% (at the end of shaking) (-1.0%)(-0.4%)(-0.039%)3rd layer 1.5% 2.0% 2.1% (at the end of shaking) (1.4%)(0.4%)(0.041%)Corresponding ground 12.4 6.8 7.5 surface settlement (cm)

Figure 8. Residual deformation in depth direction

Table 2 shows the residual vertical displacement at the ground surface and volumetric strain of each layer. Most settlements occurred in the third layer, and the settlement values are larger when there is high permeability. This is in contrast to horizontal displacement. This is a result of the fact that the dilatant behavior during the shaking affects the volumetric compression behavior during the dissipation process.

Figure 9 shows the shear stress-strain relationships of the third layer obtained from the on-line test. In this figure only the relationships for the first four seconds are plotted. The relationships at the initial monotonic shear loading of up to about 0.5% are almost the same. However, after that the relationships gradually begin to differ. As shown in the figure, it is found that the cyclic shear resistance of the condition with a smaller coefficient of permeability had smaller shear resistance because the amount of dissipation of pore water pressure due to drainage is smaller than in the other two cases.

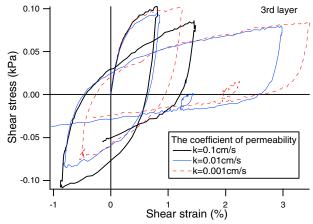


Figure 9. Stress and strain relationships of the third layer from zero to four seconds obtained from the on-line test

Figure 10 shows the volumetric strain during the shaking process in which the positive side represents the compression. Most pore water flow was from 3rd layer to 2nd layer. When the coefficient of permeability is 0.01cm/s, 0.5% volumetric change was generated during the shaking process. This is larger than expected.

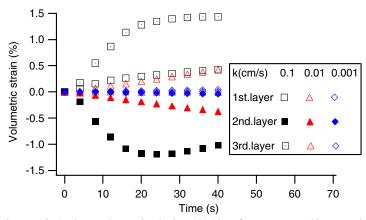


Figure 10. Volumetric strain during shaking from zero to 40 seconds

Figures 11 and 12 show the excess pore water pressure, the volumetric strain, the stress-strain relationships and the effective stress path of the main layers. Regarding the volumetric strain, the final volume change of all layers is the compression side, while the volumetric change in the expansion side

occurred in the 2nd layer in the early stage. It is also found that excess pore water pressure reached close to the initial effective stress of the 3rd layer in all cases. However, in the case where k=0.1cm/s, the decrease of excess pore water pressure started while it was still shaking, and the effective stress recovered to 0.6 at the end of shaking. For highly permeable ground, this is considered reasonable.

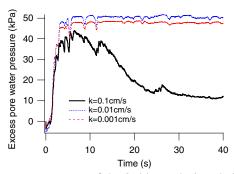


Figure 11. Excess pore water pressure of the 3rd layer during shaking from zero to 40 seconds

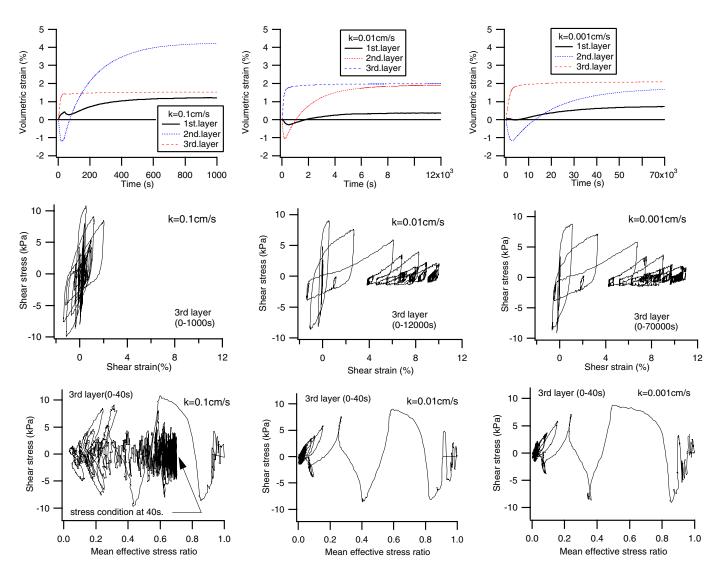


Figure 12. Volumetric strain, stress-strain relationships and effective stress path.

#### CONCLUSION

The concept of new geotechnical hybrid simulation technique for pore water migration has been introduced. The technique enables the quantitative evaluation of residual deformations, such as lateral spreading and settlement caused by liquefaction. The system is based on the seismic response of a one-dimensional slightly inclined multi-layered soil system. Ground response is governed by both equations of motion and the continuity equation, into which soil behavior is directly introduced instead of using a soil constitutive model. This concept is expected to be applied to various cases in future studies in earthquake geotechnical engineering.

The permeability effect on the seismic response during cyclic shear was studied by applying the system to an example problem. From the study, it is clear that the volumetric change during cyclic shear can not be considered as negligible to the seismic response. The authors consider the volume change characteristics of sandy soil during and after cyclic shear to be key issues for understanding liquefaction related phenomena.

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