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STOCHASTIC DYNAMIC ANALYSIS OF STRUCTURE UNDER GENERALIZED FULLY NONSTATIONARY EARTHQUAKE MODEL

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SUMMARY

A fully non stationary earthquake model based on the family of sigma oscillatory process is developed here. The conventional evolutionary PSDF is expressed as function of two different frequencies applying double Fourier transform to the associated auto covariance function. The response of structure to this general non-stationary PSDF is now utilizes to derive the non stationary response quantities. To elucidate the methodology in a focused and specific manner, a building frame subjected to ground motion due to El Centro 1940 earthquake is taken up and the results with proposed model are compared with that of evolutionary spectral density model.

INTRODUCTION

The effects of natural seismicity on structural responses are markedly non-stationary due to non-stationary random excitation process and also for the transition nature of responses. Typically the seismic motions have a sudden beginning and slow decay. Though the motion becomes stationary after some time and can be suitably modeled by Kanai-Tajimi model, sudden rise of the ground acceleration may become critical for satisfactory performance and safety of various structures. To incorporate the non-stationary character of ground motion various models are proposed in literature [1] i.e. Priestley evolutionary power spectrum, the energy spectra, double frequency spectrum, Weigner process, random pulse train model, sigma oscillatory process etc. But the evolutionary earthquake models normally neglects the frequency non-stationary character. Moreover, it is well verged that the maximum response and its likely duration can be determined approximately from the shape of the envelope function without carrying out the nonstationary response analysis [2].

The objective of the work is to present the stochastic dynamic analysis of structure subjected to generalized nonstationary earthquake. The formulation has been developed in double frequency domain.

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In doing so, a fully non stationary model based on the family of sigma oscillatory process as described by Conte and Peng [3] has been taken. The model has been readily extended to derive generalized PSDF in double frequency spectrum. The response of structural system to special class of this general non-stationary PSDF is utilized to derive the non stationary response quantities

THE GENERALIZED NON-STATIONARY GROUND MOTION MODEL

The model views earthquake ground motion as the superposition of component process described by their own arrival time, frequency content, and time intensity function. The ground acceleration is expressed as

$$\ddot{u}_g(t) = \sum_{k=1}^{p} X_k(t) = \sum_{k=1}^{p} A_k(t) S_k(t) \tag{1}$$

$A_k(t)$ is the time modulating function of the k-th component process defined as,

$$A_k(t) = \alpha_k (t-\zeta_k)^{\beta_k} \exp(-\gamma_k (t-\zeta_k) H(t-\zeta)) \tag{2}$$

where, α_k and γ_k are the positive constants, β_k is a positive integer and ζ_k is the arrival time of the k-th sub-process. $S_k(t)$ is the k-th Gaussian stationary process characterized by its autocorrelation function

$$R_{S_k S_k}(\tau) = e^{-\upsilon_k |\tau|} \cos(\eta_k \tau) \tag{3}$$

and corresponding PSD function is

$$\phi_{S_k S_k}(\omega) = \frac{\upsilon_k}{2\pi} \left[\frac{1}{\upsilon_k^2 + (\omega+\eta_k)^2} + \frac{1}{\upsilon_k^2 + (\omega-\eta_k)^2} \right] \tag{4}$$

where, υ_k and $\eta_k(t)$ are two free parameter representing the frequency band width and predominant or central frequency of the sub-process $S_k(t)$. The autocorrelation function of $\ddot{U}_g(t)$ can be expressed as

$$R_{\ddot{u}_g \ddot{u}_g}(t_1, t_2) = \sum_{k=1}^{p} A_k(t_1) A_k(t_2) R_{S_k S_k}(t_1 - t_2) \tag{5}$$

The generalized non-stationary cross-spectral density matrix for ground motion can be represented in double frequency domain as a function of two frequencies by applying double Fourier transform [4,5] to the associated auto covariance function as defined in equation (5) i.e.

$$S_{\ddot{u}_g \ddot{u}_g}(\omega_1, \omega_2) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{\ddot{u}_g \ddot{u}_g}(t_1, t_2) e^{-i(\omega_1 t_1 - \omega_2 t_2)} dt_1 dt_2 \tag{6}$$

Now use of equation (5) in (6) yields

$$\begin{aligned} S_{\ddot{u}_g \ddot{u}_g}(\omega_1, \omega_2) &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\sum_{k=1}^{p} A_k(t_1) R_{S_k S_k}(t_1 - t_2) A_k(t_2) \right) e^{-i(\omega_1 t_1 - \omega_2 t_2)} dt_1 dt_2 \\ &= \frac{1}{4\pi^2} \sum_{k=1}^{p} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_k(t_1) \left(\int_{-\infty}^{\infty} \phi_{S_k S_k}(\omega) e^{i(\omega t_1 - \omega t_2)} d\omega \right) A_k(t_2) e^{-i(\omega_1 t_1 - \omega_2 t_2)} dt_1 dt_2 \end{aligned} \tag{7}$$

and simplifying above, the generalized PSD function of the ground motion is obtained as,

$$\begin{aligned} S_{\ddot{u}_g \ddot{u}_g}(\omega_1, \omega_2) &= \sum_{k=1}^{p} \int_{-\infty}^{\infty} \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} A_k(t_1) e^{i(\omega-\omega_1)t_1} dt_1 \right) \phi_{S_k S_k}(\omega) \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} A_k(t_2) e^{-i(\omega-\omega_2)t_2} dt_2 \right) d\omega \\ &= \sum_{k=1}^{p} \int_{-\infty}^{\infty} M_k(\omega-\omega_1) \phi_{S_k S_k}(\omega) M_k^*(\omega-\omega_2) d\omega \end{aligned} \tag{8}$$

where,

$$M_k(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_k(t)\, e^{i\omega t}\, dt = \frac{\alpha_k e^{i\omega\xi_k}}{(\gamma_k - i\omega)^{(\beta_k+1)}}\,\Gamma(\beta_k + 1) \tag{9}$$

In subsequent numerical study, the six parameters needed for each component process are taken from Conte and Peng [3] corresponding to the El Centro (1940) earthquake record. The parameters and the corresponding PSD function in double frequency domain are furnished in Table 1 and Figure 1, respectively.

Table 1: Parameters of non-stationary ground motion model correspond to El Centro (1940) Earthquake.

k	α_k	β_k	γ_k	ζ_k	ν_k	η_k
1	37.2434	8	2.7283	-0.5918	1.4553	6.7603
2	104.0241	8	2.9549	-0.9857	2.4877	11.0857
3	31.9989	8	2.6272	1.7543	3.3024	7.3688
4	43.8375	9	3.1961	1.686	2.1968	13.5917
5	33.1958	9	3.1763	-0.0781	3.1241	14.3825
6	41.3111	9	3.1214	-7.096	6.7335	25.1532
7	4.2234	10	2.9904	-0.9464	2.6905	48.0617
8	19.9802	6	1.895	1.402	7.2086	37.6163
9	2.4884	10	2.6766	5.3123	6.1101	19.4612
10	24.1474	10	3.3493	8.8564	1.9862	9.04
11	2.5916	2	0.224	3.2558	2.4201	9.3381
12	2.2733	3	0.5285	16.2065	1.5244	14.1067
13	24.2732	3	1.0361	17.5331	1.7141	24.0444
14	41.3111	9	3.1214	-7.096	6.7335	25.1532
15	1.3697	10	2.5936	21.683	1.9362	12.9198
16	15.4646	2	0.7044	27.2979	1.7897	12.0205
17	0.0174	10	1.8451	-2.4168	4.9373	98.628
18	2.9646	10	3.1137	1.5751	1.9726	61.8316
19	0.0007	10	1.3686	2.5173	3.2479	43.90675
20	0.8092	4	0.5969	6.4396	3.6749	26.3365
21	16.7115	2	0.7294	12.493	1.7075	37.1139

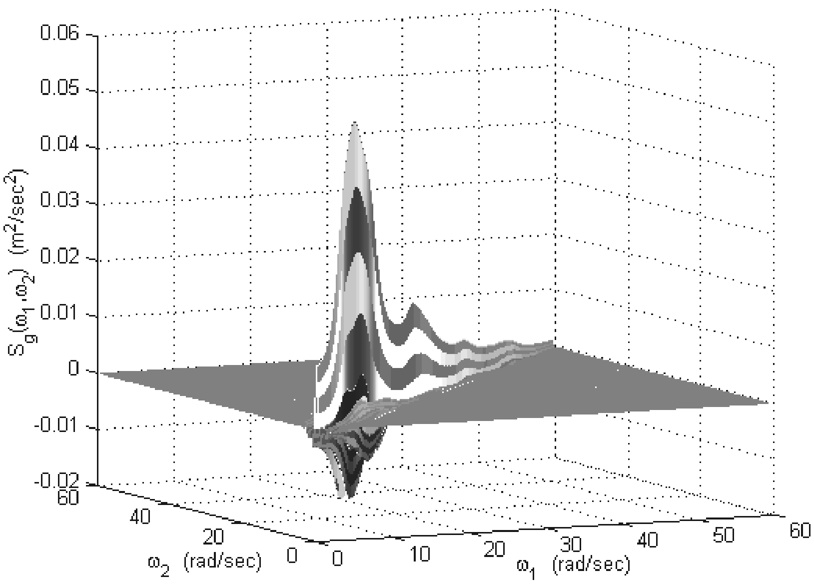

Figure 1 Generalized non-stationary power spectral density correspond to El Centro (1940) Earthquake.

STOCHASTIC DYNAMIC RESPONSE

The equation of motion for a multi degree of freedom (MDoF) system under ground acceleration can be readily expressed as

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = -[M]\{L\}\ddot{u}_g(t) \tag{10}$$

Where, $[M]$ $[C]$ and $[K]$ are the global mass, damping and stiffness matrixes of the structure, respectively. $\{u(t)\}$ is the total displacement vector of the structure and $\ddot{u}_g(t)$ is ground acceleration. $\{L\}$ is the influence coefficient vector which represents the pseudo-elastic response in all degrees of freedom due to unit support motion.

The ground motion that has only one horizontal component is considered in present study. The evolutionary zero mean vector process can be expressed in Fourier-Stieltjes integral form [1]as,

$$\ddot{u}_g(t) = \int_{-\infty}^{\infty} \ddot{U}_g(\omega, t)\exp(i\omega t)dZ(\omega) \tag{11}$$

and for linear structural behaviour, the response $u(t)$ can also be expressed as,

$$\{u(t)\} = \int_{-\infty}^{\infty} \{U(\omega,t)\}\exp(i\omega t)dZ(\omega) \tag{12}$$

For a stable system i.e. damping ratio, $\zeta>0$, the response of the system dies out soon after the excitation dies out. Assuming that the response of the system is independent of time, the dynamic equilibrium in frequency domain can be obtained as

$$\left[-\omega^2[M]+i\omega[C]+[K]\right]\{U(\omega,t)\}=-[M]\{L\}\ddot{U}_g(\omega,t)$$
$$\text{i.e. } [D(\omega)]\{U(\omega,t)\}=-\{F\}\ddot{U}_g(\omega,t) \quad , \tag{13}$$

Where the dynamic stiffness matrix is,

$$[D(\omega)]=-\omega^2[M]+i\omega[C]+[K] \tag{14}$$

To obtain the evolutionary PSD function of response, equation (13) is simplified to the following form,

$$[D(\omega)]\{H_u(\omega)\}=\{F\}$$
$$i.e.\ \{H_u(\omega)\}=[D(\omega)]^{-1}\{F\} \tag{15}$$

Where, $\{U(\omega,t)\}=\{H_u(\omega)\}\ddot{U}_g(\omega,t)$ and $\{F\}=-[M]\{L\}$ .For linear systems with known complex frequency response function, the evolutionary PSD function $[S_u(\omega,t)]$ of displacements at any d.o.f. can be readily written as,

$$[S_{uu}(\omega,t)]=\{H_u(\omega)\}S_{\ddot{u}_g\ddot{u}_g}(\omega,t)\{H_u^*(\omega)\}^T \tag{16}$$

Where, $\{H_u^*(\omega)\}$ is the complex conjugate $\{H_u(\omega)\}$. The PSD function of structural response under evolutionary ground motion $S_{\ddot{u}_g\ddot{u}_g}(\omega,t)$ as presented above is obtained based on the assumption that the amplitude of forcing function is a slowly varying function of time such that it can be treated as being nearly independent of time. Otherwise, computation of nonstationary response involves accurate evaluation of $\{H_u(\omega)\}$ at each instant of time. Thus computation for each frequency is equivalent to performing a time history solution. The response statistics is then computed from the pdf approximation of nonstationary process under the assumptions that a nonstationary process behaves like a stationary process at each instant of time.

However, it can be easily expressed as a function of two different frequency i.e. ω_1 and ω_2 by double Fourier transform as described earlier for ground motion model. When the forcing function on the right hand side of equation (10) are non-stationary, the cross correlation matrix of the responses can be expressed as,

$$[R_{uu}(t_1,t_2)]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{H_u(\omega_1)\}S_{\ddot{u}_g\ddot{u}_g}(\omega_1,\omega_2)\{H_u^*(\omega_2)\}^T\ e^{i(\omega_1 t_1-\omega_2 t_2)}d\omega_1 d\omega_2 \tag{17}$$

Thus, for a linear system with known frequency response function, the PSD function $[S_u(\omega_1,\omega_2)]$ of any response variable $\{u(t)\}$ for known PSD function of ground motion can be readily obtained as

$$[S_{uu}(\omega_1,\omega_2)]=\{H_u(\omega_1)\}S_{\ddot{u}_g\ddot{u}_g}(\omega_1,\omega_2)\{H_u^*(\omega_2)\}^T \tag{18}$$

where $\{H_u(\omega_1)\}$ and $\{H_u(\omega_2)\}$ are the well known transfer functions at two different frequencies obtained from equation (15) considering it as independent of time. Like displacement, force developed at any section can be obtained. The FRF of total base shear force along the direction of ground motion is

$$H_{bs}(\omega)=\{L\}^T[K]\{H_u(\omega)\} \quad . \tag{19}$$

and the psd of base shear can be obtained similarly as in equation (18). In general, if $x(t)$ is any generalized response quantity, the PSD function and autocorrelation function of $x(t)$ can be defined by

$$S_{xx}(\omega_1,\omega_2)=H_x(\omega_1)S_{\ddot{u}_g\ddot{u}_g}(\omega_1,\omega_2)H_x^*(\omega_2) \tag{20}$$

$$R_{xx}(t_1,t_2)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}S_{xx}(\omega_1,\omega_2)e^{i(\omega_1 t_1-\omega_2 t_2)}d\omega_1 d\omega_2=\int_{-\infty}^{\infty}\phi_{xx}(t_1,\omega)e^{-i\omega(t_2-t_1)}d\omega \tag{21}$$

Where $\phi_{xx}(t_1,\omega)$ represents the evolutionary PSD function of the nonstationary process $x(t)$. The time varying spectral moments which are useful in reliability evaluation can be obtained from the evolutionary PSD function as,

$$\begin{aligned}\lambda_j(t_1) &= \int_{-\infty}^{\infty}|\omega|^j\phi_{xx}(t_1,\omega)d\omega \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|\omega_2|^j S_{xx}(\omega_1,\omega_2)e^{i(\omega_1 t_1-\omega_2 t_1)}d\omega_1 d\omega_2\end{aligned} \tag{22}$$

Where 'j' is the order of the spectral moment. The RMS value of the response and its derivatives are

$$\sigma_x(t)=\sqrt{\lambda_o(t)} \text{ and } \sigma_{\dot{x}}(t)=\sqrt{\lambda_2(t)} \tag{23}$$

NUMERICAL EXAMPLES AND RESULTS

An unsymmetrical 3-d building frame idealized as a space frame as shown in figure 2 subjected to EL Centro 1940 earthquake ground motion described by the PSDF as shown in figure 1 is taken up to explain the proposed generalized nonstationary analysis. The size of the columns and beams are 0.3m×0.3m and 0.25m x 0.45m respectively. As most of the masses are concentrated at the roof level, the mass of the beam is taken ten times that of column. Mass density and concrete members are taken as 2400kg/m^3 and 2x10^{7}kN/m^2, respectively.

Figure 3, 4 and 5 compares the time varying root mean square value of displacement at node 16 with evolutionary model for 2%, 5% and 10% damping ratio respectively.

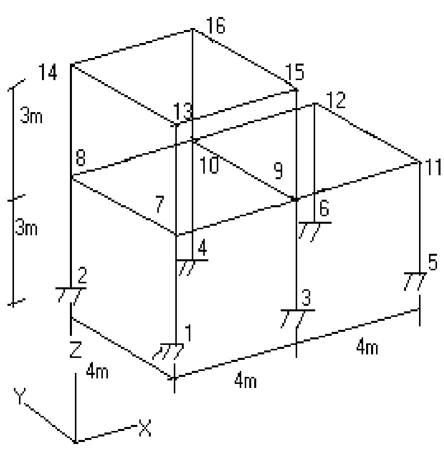

Figure 2 The three dimensional Frame

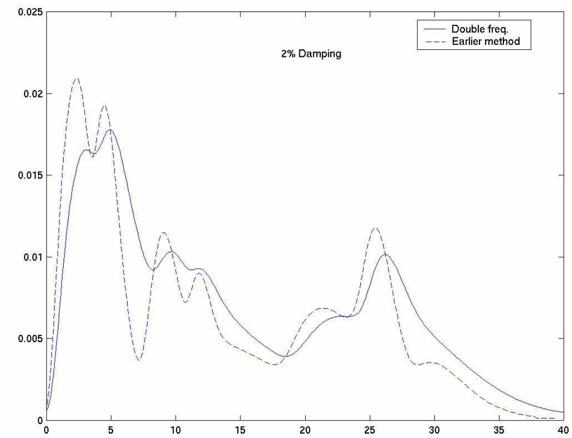

Figure 3 RMS value of displacement at node 16 along X axis 2% damping

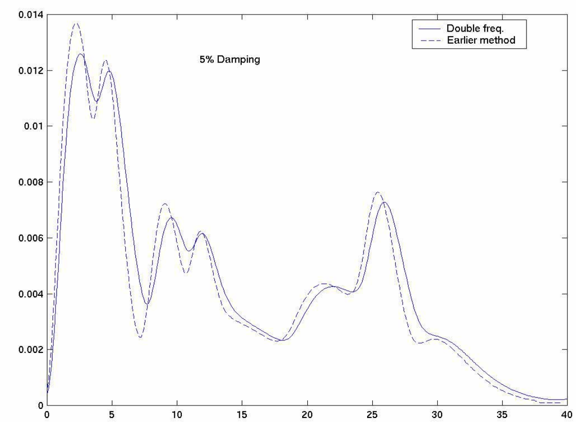

Figure 4 RMS value of displacement at node 16 along X axis for 5% damping

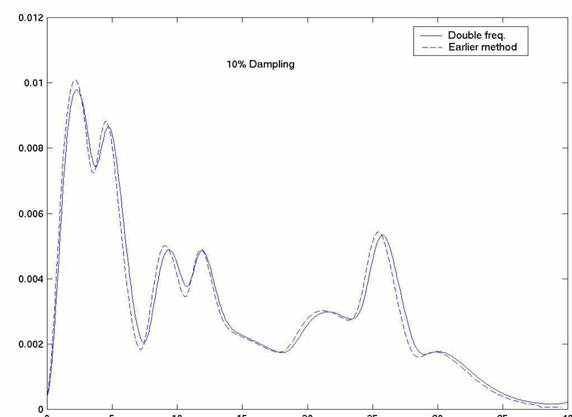

Figure 5 RMS value of displacement at node 16 along X axis for 10% damping

DISCUSSION AND CONCLUSION

An evaluation procedure to compute nonstationary response statistics for MDoF system is developed in double frequency domain. The conventional approach is based on the assumption of slowly varying function of time such that it can be treated as being nearly independent of time. When the nonstationary input is described as a function of time $'t'$ and frequency $'\omega'$ the transient nature of response can only be included through convolution integral. However, the conventional evolutionary PSD function can be easily expressed as function of two frequencies and the analysis can be done in double frequency domain avoiding the double integration. From numerical study, a definite change in generalized nonstationary response by double frequency spectrum with that of evolutionary results are seen, particularly when damping is small.

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