

# SIMPLIFIED PREDICTION METHOD FOR SEISMIC RESPONSE OF ROCKING STRUCTURAL SYSTEMS WITH YIELDING BASE PLATES

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# SUMMARY

The rocking structural systems with yielding base plates, which cause rocking vibration during strong earthquakes to reduce seismic responses of steel buildings, are now under development. In this paper, simplified methods are proposed to predict the seismic responses of these systems based on the energy conservation law and so on. These methods are verified by shaking table tests using a half scale steel frame with three stories and one bay. The prediction values for uplift displacement, base share, lateral roof displacement and vertical compressive force at column bases of the rocking structural systems are in good agreement with the corresponding test results.

# INTRODUCTION

To reduce seismic responses of steel building structures, we are now developing structural systems which can cause rocking vibration under appropriate control during sever earthquakes [1]-[7]. One of these systems has base plates yielding in uplift motion at the column bases as shown in Fig.1. The base plates yield before yielding of the super structure. In this paper, we propose simplified prediction methods for seismic responses of this rocking structural system [BPY system], and verify the adequacy of these prediction methods comparing with the results of shaking table tests on a half scale steel frame with three stories and one bay[5][6].

# SIMPLIFIED PREDICTION METHOD FOR SEISMIC RESPONSES OF BPY SYSTEMS

### Uplift, base shear and lateral roof displacement

### Basic concept

To predict uplift at the column bases, base shear of the super structure and lateral roof displacement of the whole structure, the energy conservation law [8] which is generally used for practical seismic design of building structures is applied. This method evaluates the energy which is dissipated by the system until

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Fig.1 Basic conception of rocking structural systems with yielding base plates



relationship of base plates

plates

the uplift at the column bases reaches the maximum from 0, and predicts seismic responses such as the maximum uplift considering the balance between the dissipated energy and the input earthquake energy. It is assumed that the total energy dissipation is composed of the energy dissipation by elastic deformation of the super structure  $E_s$ , that by up-lifting of the super structure's gravity center  $E_G$  and that by plastic deformation of the yielding base plates  $E_B$ . We ignore the energy dissipation by compression stress of columns brought by impact force here. From the following section, the super structure is replaced to the equivalent one mass model as shown in Fig.2. According to this figure, uplift is expressed by the product of the building width B and the rigid rotational angle  $\theta$ . And the restoring force characteristic of base plates is modeled into the uplift-force relationship shown in Fig.3 based on tensile static tests [7].

#### Energy dissipation by elastic deformation of the super structure

model

Using the equivalent one mass model, the base shear of the super structure can be evaluated by Eq.(1).

$$Q(H + \delta_{\mu\nu}) = M \cdot g(0.5B - \delta_h) + n \cdot N \cdot B \tag{1}$$

Where, *Q*: the base shear of the super structure, *H*: the first effective height of the super structure,  $\delta_{up}$ : uplift displacement of the gravity center of the super structure, *M*: mass of the super structure,  $\delta_n$ : horizontal displacement of the super structure, *n*: the number of columns existing in the uplift side, *N*: tensile force at the column base.

Because the uplift displacement  $\delta_{up}$  of the gravity center and the horizontal displacement  $\delta_h$  of the super structure are relatively slight to the height and the width of the super structure respectively, we ignore them. The tensile force N at the column base can be evaluated by Eq.(2) based on the restoring force characteristic of the base plate shown in Fig. 2.

$$N = N_{y} + K_{2} \left( B \cdot \theta - \delta_{y} \right) \tag{2}$$

Where,  $N_y$ : tensile yield strength of the base plate,  $K_2$ : secondary stiffness of the base plate,  $\delta_y$ : yield uplift deformation of the base plate.

Substituting Eq.(2) into Eq.(1), we can derive the following equation.

$$Q = \frac{n \cdot B^2 \cdot K_2}{H} \cdot \theta + \frac{B}{H} \left( 0.5 \cdot M \cdot g + n \cdot N_y - n \cdot K_2 \cdot \delta_y \right)$$
(3)

The energy dissipation by elastic deformation of the super structure is evaluated by Eq.(4). Substituting Eq.(3) into this equation, the energy dissipation can be expressed as the function of the rigid rotational angle  $\theta$ .

$$E_U = 0.5 \cdot Q^2 / K = 0.5 \cdot Q^2 / \left( \omega^2 \cdot M_U \right)$$
<sup>(4)</sup>

Where,  $\omega$  the first fundamental frequency,  $M_u$ : the first effective mass.

#### Energy dissipation by uplift of the super structure's gravity center

Energy dissipation by uplift of the super structure's gravity center is evaluated by Eq.(5) based on the geometric relationship shown in Fig. 2.

$$E_{G} = M \cdot g \cdot \int (0.5B - H \cdot \theta) d\theta$$
  
= 0.5 \cdot M \cdot g \cdot \theta (B - H \cdot \theta) (5)

Because the member  $H\theta$ , which means horizontal displacement is relatively slight to the width of the structure, we ignore it as shown by Eq.(6).

$$E_G = 0.5 \cdot M \cdot g \cdot B \cdot \theta \tag{6}$$

#### Energy dissipation by plastic deformation of the base plates

The bold line in Fig. 4 illustrates the uplift response of the base plate. In this figure, the tensile stiffness of the base plate is estimated to be degraded with its yielding to several waves until the uplift reaches the maximum. However, it is difficult to evaluate the degraded tensile stiffness precisely because the time history of the uplift response with yielding depends on the earthquake characteristic. Thus we evaluate energy dissipation by plastic deformation of the base plates using the force - deformation relationship shown by the dotted line in this figure. This idea provides the most conservative evaluation. This energy dissipation is evaluated by Eq.(7).

$$E_{B} = 0.5 \cdot n \cdot B \cdot \theta \left( B \cdot \theta - \delta_{y} \right) \cdot K_{2}$$
<sup>(7)</sup>

#### *Prediction using the energy conservative law*

The energy conservative law of the BPY system is expressed by Eq.(8).

$$E_I = E_U + E_G + E_B \tag{8}$$

Suppose the member  $E_I$  is nearly equal to the elastic potential energy of the super structure whose bases are fixed, it is evaluated by Eq.(9).

$$E_I = 0.5 \cdot Q_E / \left( \omega^2 \cdot M_U \right) \tag{9}$$

Where,  $Q_E$ : Base shear of the super structure whose bases are fixed

Finally, by substituting Eq.(4), (6), (7) and (9) into Eq.(8), we can predict the rigid rotational angle  $\theta$ . Furthermore, we can predict the uplift by multiplying the  $\theta$  by the building width *B*, the base shear *Q* of the super structure by substituting the  $\theta$  into Eq.(3) and the horizontal roof displacement of the whole structure by adding the elastic deformation of the super structure which corresponds to the base shear *Q* to the rigid deformation which is the product of  $\theta$  and the building height *h*.

### Vertical compressive force at column bases

The past shaking table tests cleared that impact effect on the structure's landing amplifies vertical compressive force at the column bases [5][6]. The model shown in Fig. 5 which is composed of concentrated masses and vertical springs is used to evaluate this effect. This mass-spring model expresses vertical structural characteristics of the landing side of the super structure. Each mass is equal to the half of mass at the corresponding floor level. Rigidity of each vertical spring is equal to vertical rigidity of the corresponding column. It is supposed that the mass-spring model starts oscillating vertically with the impact velocity  $v_0$  of column bases as the initial velocity from the moment when the column bases land. The variable and vertical compressive force  $N_C^i$  at the column base on each floor is predicted by the following equation. This equation regards that  $IMP^i$ , which is the force caused by the impact effect on the i-th floor, is equal to vertical force on each spring of the mass-spring model.

$$N_C^i = -N_T^i + IMP^i \tag{10}$$

Where,  $N_T^i$ : Variable and vertical tensile force at the column base on the i-the floor

Suppose that there is no impact effect, the absolute value of the variable and vertical force in the compressive side is equal to the one in the tensile side. Then it is considered that  $N_C^i$  is equal to  $-N_T^i$  in Eq.(10). The initial velocity  $v_0$  in Fig. 5 used to predict *IMP*<sup>*i*</sup> is calculated by Eq.(11).

$$v_0 = \frac{2\pi}{T_{uplift}} \max \delta_{uplift}$$
(11)

Where,  $T_{uplifi}$ : response vibration period of uplift and  $\max \delta_{uplifi}$ : the maximum uplift

The response vibration period of uplift is obviously equal to that of the whole system. This period can be calculated using the equivalent linear stiffness of the whole system.



Fig.5 Mass-spring model for evaluation of impact effect on column bases

In order to estimate structural safety of columns, we need the maximum value of the  $N_C^i$  by Eq.(10). The maximum value of  $N_T^i$  is estimated at the summation of tensile strength of base plates corresponding to their uplift deformation and the self weight supported by the column. On the other hand, the maximum value of  $IMP^{i}$  in Eq.(10) can be evaluated using the maximum vertical deformation of the equivalent onemass system of the multi-mass-spring model shown in Fig. 5 by Eq.(12).

$$\Delta_{\max} = \frac{T_v}{2\pi} v_0 \tag{12}$$

Where,  $T_{\nu}$ : the first natural period of the mass-spring model shown in Fig.5

After calculating  $\Delta_{max}$  and distributing it to each spring of the original multi-mass-spring model, the maximum compressive force of each story can be predicted by multiplying the deformation by the rigidity of each vertical spring. In Eq.(12), it is assumed that all kinetic energy which the multi-mass-spring model possesses on the landing is replaced to the potential energy of columns with no energy dissipated by viscous damping. Finally, the maximum value of  $N_C^i$  is evaluated by substituting the maximum  $N_T^i$  and the maximum  $IMP^i$  in Eq.(10).

# **VERIFICATION OF THE PROPOSED METHOD**

The adequacy of the seismic prediction methods proposed in the previous chapter is verified using results of the past shaking table tests [5][6].

Photo.1 shows a steel frame with three stories and one bay used for the shaking table tests. Yielding base plates shown in photo.2 and Fig.6 are installed at each column base on the ground floor. The shaking table is oscillated only in one lateral direction.



Photo.1 Test steel frame



Photo.2 Base plate Table 1 Cross section Column H-148x100x6x9 Beam \$11 (PC steel bar) Brace



Fig.6 Plan of base plate

Table 2 Mass at floor level			
Floor	RF	3F	2F
Mass[t]	4.60	5.20	5.20

Table 3 Characteristics values of base plates of BP6 model				
Qy(kN)	$\delta y (mm)$	<i>K1</i> (kN/mm)	<i>K2</i> (kN/mm)	K2/K1
23.75	1.83	12.98	4.33	0.33

$2y(\mathbf{K}(\mathbf{Y}))$	<i>by</i> (IIIII)	$\mathbf{M}$ ( $\mathbf{M}$ ( $\mathbf{M}$ ) $\mathbf{M}$	$K_{2}$ (KI (/IIIII))	$M_2/M_1$
23.75	1.83	12.98	4.33	0.33

Table 4 Characteristics values of base plates of BP9-2 model

Qy(kN)	$\delta y (\mathrm{mm})$	Kl (kN/mm)	<i>K2</i> (kN/mm)	K2/K1
23.66	1.84	12.85	2.52	0.20

First-mode effective height H	4.13 m	
First-mode effective mass Mu	12.82 t	
First-mode natural circular frequency	33.07  rad/s	
of the super-structure $\omega$	55.07 Tad/s	
Relation between base shear and the		
maximum input acceleration $a_{max}$ (from test	$Q_E = 55.25 \times a_{\max}$	
results)		
Relation between frame roof deformation and	$\delta_{n} = 0.0715.0$	
base shear (from test results)	$OS = 0.0/15 Q_E$	

Table 5 System parameters used for seismic response prediction

The height of the first story is 1.7 m and that of the other stories is 1.8 m. The total height of the frame is 5.3 m. The frame width in the oscillation direction is 3.0 m. Braces made of PC steel bar are installed in each story. Each brace is stressed previously so that the tensile strain is  $1000 \mu$ .

Mass at each floor level and cross section of structural members are listed in Table 1 and Table 2 respectively. Every column is arranged so that its weak axis direction coincides with the oscillation direction. The first natural period and viscous damping ratio are 0.18 s and 0.5% respectively. They are derived by white noise oscillation tests.

Table 3 and 4 show yield strength Qy, yield deformation  $\delta y$ , elastic stiffness *K1* and secondary stiffness *K2* of the base plates in the uplift motion. The BP6 model of Table 3 has the base plates of which thickness is 6 mm. The BP9-2 model of Table 4 has the ones of which thickness is 9 mm. Every base plate has four wings as shown in photo.2 and Fig.6. But in the case of BP9-2 model, the bolts fixing wings to base beams which are arranged in the vertical direction to the oscillation direction are removed. Thus the base plates of the BP9-2 model have only two wings. The yield strength of the base plates is calculated regarding their wings as beam models. The other values in Table 3 and 4 are derived using the curves enveloping the uplift force-deformation relationships which are obtained by static tensile tests of the column bases with the base plates [7].

The input ground motion for the shaking table tests is 1940 El Centro NS of which time axis is shrunk to  $(1/2)^{0.5}$ . Each test model is oscillated several times amplifying input level gradually without changing yielding base plates.

To predict uplift, base shear and roof displacement of each model by the proposed method, system parameters listed in Table 5 and restoring force characteristics of base plates listed in Table 3 and 4 are used. The first natural mode shape, which is used to calculate the first effective mass and height shown in Table 5, is estimated to be reversed triangle shape. Elastic base shear and elastic roof displacement of the super structure listed in Table 5 are derived based on the test results on the fixed base models [5][6].

Fig.7 shows test results of uplift, base shear and roof displacement comparing with the corresponding predicted values. The predicted values are approximately in good agreement with the corresponding test results. But the predicted values for uplift and roof displacement of the BP9-2 model overestimate test results a little when PGA becomes larger. The reason of these errors is probably that energy dissipation by base plates is evaluated too conservatively by Eq.(7) and Fig.4. As the another reason, it is pointed out



(c) Kool displacement

Fig.7 Comparison of between predicted values and test results: uplift, base shear and roof displacement

that the difference between the predicted EI by Eq.(9), which is based on the elastic first period, and the real one might become large considerably after the frame uplifts.

Fig.8 shows test results of vertical compressive force at the column base on the first floor comparing with the corresponding predicted values, which are the summation of the calculation result by Eq.(10) and the self weight supported by the column. Predicted values are in good agreement with test results or estimate test results conservatively a little.



Fig.8 Comparison of between predicted values and test results: maximum vertical compressive force

Using the proposed prediction methods, preliminary structural design of the BPY system can be performed according to the following procedure. But prior to this procedure, the BPY system should be ensured not to cause rocking vibration under moderate earthquakes.

Step 1: Set the criteria,  $\delta_0$  and  $Q_0$ , for uplift and base shear of the super structure.

Step 2: Calculate the criterion  $N_0$  for tensile force of base plates by Eq.(13).

$$N_0 = Q_0 H/B - 0.5Mg \tag{13}$$

If the criterion  $N_0$  is 0, the structure needs no base plate. If it is smaller than 0, the BPY system can not be applied to this structure.

*Step 3*: Choose the specification of base plates so that tensile force of base plates becomes lower than or equal to their criterion when the uplift becomes equal to its criterion.

Step 4: Confirm that the uplift predicted by the proposed method does not surpass its criterion.

*Step 5*: Confirm that the vertical compressive force at the column bases does not surpass the corresponding yield force.

On the Step 4, if it is judged that uplift surpasses its criterion, the criterion of uplift or base shear must be changed into larger one on the Step 1. If the criterion of base shear is changed, the criterion of tensile force of base plates is also changed into larger one on the Step 2. Then harder base plates can be chosen. Using base plates of which secondary stiffness is smaller is also effective to decrease uplift, because they have superior energy dissipation capacity.

On the Step 5, if it is judged that vertical compressive force at the column bases surpass the corresponding yield force, the column must be changed into one which possesses higher strength. Decreasing the

criterion of base shear might be also effective to decrease the vertical compressive force at the column bases. However uplift and the impact effect due to uplift become larger in this case.

# CONCLUSION

- [1] To predict uplift, base shear and roof displacement of the BPY systems, the simplified method was proposed based on the energy conservation law.
- [2] To predict vertical compressive force at the column bases on the first floor, which is affected by impact force on the landing, the simplified method was proposed. A mass-spring model which expresses structural properties of the landing side of the structure is used in this method.
- [3] The adequacy of these prediction methods was verified comparing with the results of shaking table tests using a half scale steel frame with three stories and one bay.

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