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ANALYSIS OF SCATTERING WAVE BY TOPOGRAPHIC IRREGULARITY USING RAYLEIGH WAVE AND LEAKING MODE

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SUMMARY

We investigate the behavior of scattering waves generated in a ground with topographic irregularity relating with excitation of Rayleigh wave and leaking mode. Analyzed model is simplified as an elastic half space with an edge. Surface motion, when a plane SV wave is incident to the model, is calculated by boundary element method in frequency domain. A new method is presented to calculate amplitude, phase velocity and attenuation of main components forming the surface motion. Using this method, it is found that the scattering wave generated at the edge is mainly composed of Rayleigh wave and leaking mode that is defined as another eigenmode of the same characteristic equation in P-SV wave field as the Rayleigh wave derived. The Rayleigh wave propagates in the far distant. On the other hand, the leaking mode influences the surface motion around the edge, though it is not seen in the distance.

INTRODUCTION

It is well known that the damage of the infrastructure occurs frequently in a region with topographic irregularity during a strong earthquake, since the behavior of the ground motion in the region changes rapidly and amplifies greatly. Therefore, it is important to clarify the behavior of the ground motion in detail for mitigation of earthquake damage.

So far much studies have been done to investigate the seismic wave propagation in such a ground theoretically. For example, Mal and Knopoff [1], Momoi [2] and Fujii et al. [3] examine the reflection and transmission due to the incidence of Rayleigh wave at an elastic edge by solving the equation of motion analytically, though the problem to be solved analytically is restricted. On the other hand, Zama [4], Ohtsuki et al. [5] and Bard and Bouchon [6] begin to calculate the seismic wave propagation through a basin or a cliff using some numerical methods. One of important results in their studies is to clarify that Rayleigh wave is generated at the edge of the ground.

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Rayleigh wave is derived from a characteristic equation in P-SV wave field. Here, leaking mode can also be defined as another eigenmode of the same characteristic equation. Oliver and Major [7] indicate that a long period wave motion observed following P wave is interpreted as leaking mode. Phinney [8], Gilbert [9] and Chapman [10] investigate theoretically how leaking mode affects the ground motion. As suggested above, it is expected that the ground motion in the topographic irregularity contain not only Rayleigh wave but also leaking mode.

In this study, we investigate physical properties of scattering wave generated on the ground with topographic irregularity in P-SV wave field to relate with the excitation of Rayleigh wave and leaking mode. Analyzed ground model is simplified as an elastic half space with an edge. The surface motion, when a plane SV wave incident to the model, is calculated by boundary element method (BEM). For investigating the surface motion quantitatively, we have already proposed a method, which is called the modal expansion method to expand the ground motion in a series of the main component whose amplitude is predominantly large in the P-SV wave field (Akiyama [11]). Applying this method to the ground motion obtained above, it is found that the scattering wave in the ground is composed of Rayleigh wave and leaking mode.

OUTLINE OF THE MODAL EXPANSION METHOD

We present an outline of the modal expansion method. This method is used to identify the pole of the main components of the surface motion in the complex wave-number plane and to determine amplitude, phase velocity and attenuation of these components.

We suppose that displacement vector $\mathbf{u}(x)$ in two-dimensional P-SV wave field is expressed with,

$$\mathbf{u}(x) = \sum_{j=1}^{n} \mathbf{A}_j \exp(-i\xi_j x), \quad \cdots\cdots (1)$$

where $\mathbf{A}_j$ is amplitude vector and ξ_j is complex wave number that is $\xi_j = k_j(1-h_j)$, k_j is real part of the wave number ξ_j and h_j is attenuation factor. In equation (1), the displacement is represented as a superposition of main n wave components.

Equation (1) is a non linear equation for both $\mathbf{A}_j$ and ξ_j, which are unknown values. In order to determine $\mathbf{A}_j$ and ξ_j by use of least-squares method, we convert equation (1) into a linear equation. Defining $\mathbf{A}_{oj}$ and ξ_{oj} as approximate values for $\mathbf{A}_j$ and ξ_j, and expanding equation (1) by these approximate values, the liner equation is obtained as follows,

$$\mathbf{u}(x) = \sum_{j=1}^{n} \{\mathbf{A}_j - i(\xi_j - \xi_{oj}) x \mathbf{A}_{oj}\} \exp(-i\xi_{oj} x). \quad \cdots\cdots (2)$$

Applying least-squares method to equation (2), $\mathbf{A}_j$ and ξ_j can be determined.

When we determine $\mathbf{A}_j$ and ξ_j from equation (2), it is very important to set up the approximate values $\mathbf{A}_{oj}$ and ξ_{oj} appropriately. In the modal expansion method, these approximate values are obtained from a wave number spectrum of the displacement. Using Fourier transform, the wave number spectrum $\tilde{\mathbf{u}}(\xi)$ of the displacement $\mathbf{u}(x)$ in equation (1) is given as follows,

$$\tilde{\mathbf{u}}(\xi) = i \sum_{j=1}^{n} \frac{\mathbf{A}_j}{\xi - \xi_j}, \quad \cdots\cdots (3)$$

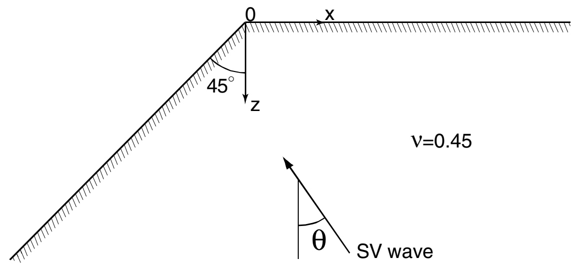

Fig.1 Analyzed model

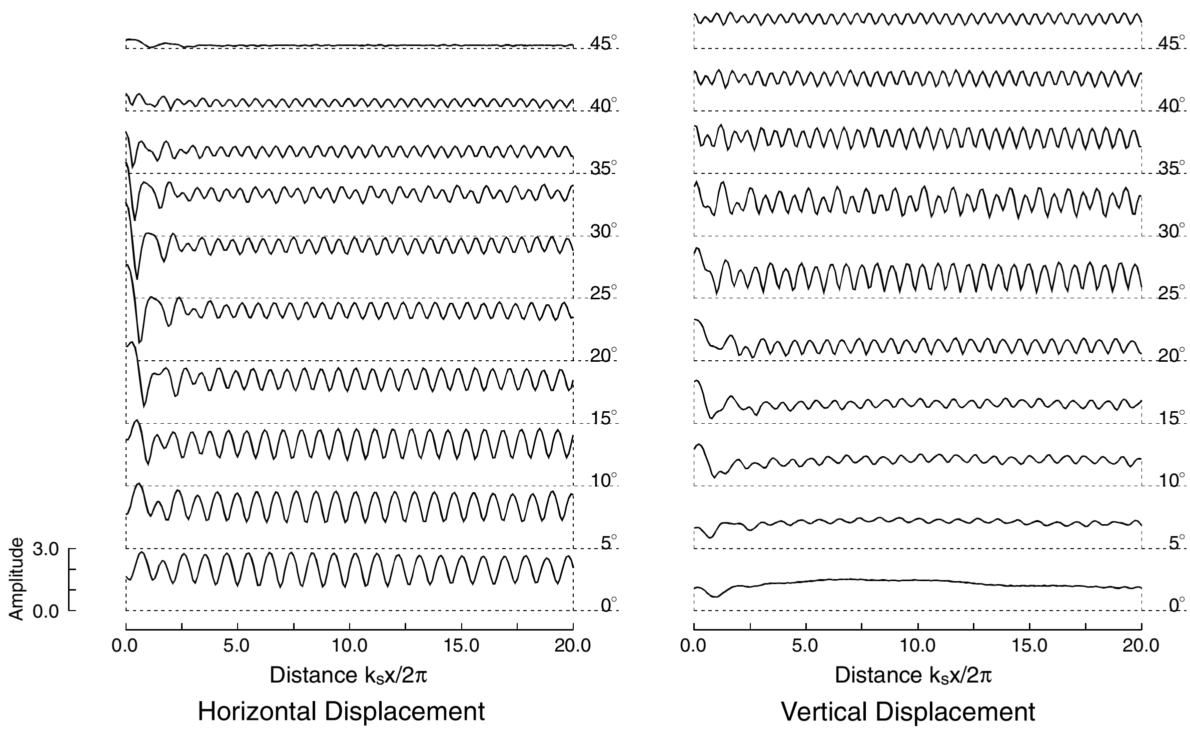

Fig.2 Surface displacement amplitude by wave propagation analysis (BEM)

where ξ is a wave number. Now, we note that ξ_j means a pole of first order on the complex wave number plane. In equation (3), we find that the wave number spectrum $\tilde{\mathbf{u}}(\xi)$ is amplified, when ξ is close to ξ_j. Then, assuming that $\tilde{\mathbf{u}}(\xi)$ is influenced only of the pole ξ_j, when ξ is near ξ_j, we obtain

$$(\xi - \xi_j)\tilde{\mathbf{u}}(\xi) - i\mathbf{A}_j = \mathbf{0}. \qquad \cdots\cdots(4)$$

Applying the least-square method to equation (4), the approximate values $\mathbf{A}_{oj}$ and ξ_{oj} can be set up appropriately.

WAVE PROPAGATION ANALYSIS BY BOUNDARY ELEMENT METHOD

Analyzed model

A ground model of the wave propagation analysis is shown in Fig.1. This model is simplified as an elastic half space containing an edge in which an angle of inclination is 45°. Poisson's ratio ν is 0.45.

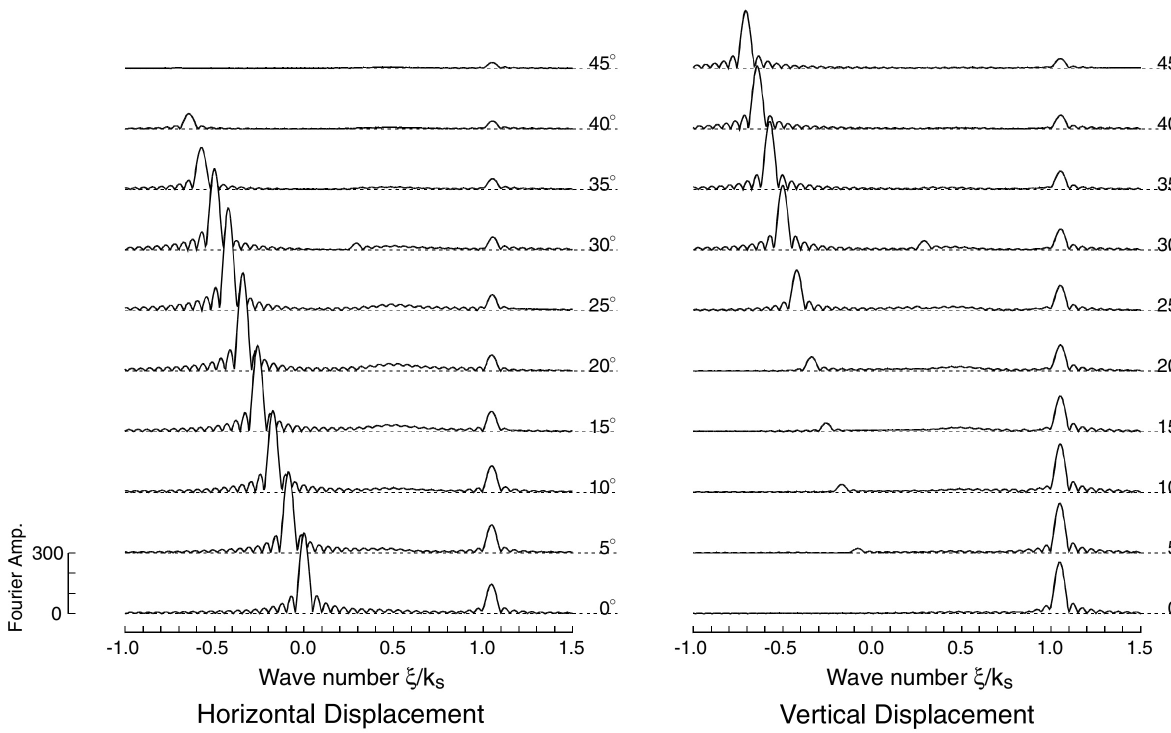

Fig.3 Wave number spectra of displacement by wave propagation analysis (BEM)

Surface displacement of the model, when a plane SV wave is incident from the angle θ on the model, is calculated by boundary element method (BEM) that is formulated in the frequency domain. The angle of incidence is between θ=0° and 45°. In this paper, both the distance from the edge (x=0) and the length of the boundary element are normalized by the incident SV wavelength. The surface of the model is divided into the boundary elements from the edge to the point 20 times of the SV wavelength away. The length of the boundary element is equivalent to 1/10 of the SV wavelength.

Results of wave propagation analysis

Surface displacement amplitudes obtained from the wave propagation analysis by BEM are shown in Fig.2. The horizontal axis of the figure expresses the dimensionless distance ($k_s x/2\pi$). k_s is a wave number of SV wave. Distribution of these amplitudes changes with angle of incidence, because the reflection and the scattering in the ground change with direction of incident SV wave.

Fig. 3 shows the wave number spectra, which are the Fourier transform of the surface displacements in Fig.2. Two main peaks appear in both horizontal and vertical components of the spectra. One of left hand side of these two peaks represents the contribution due to the incident SV wave, since the dimensionless wave number ξ/k_s of the peak migrates from 0.0 to -0.7 for the angle of incidence θ increasing from 0° to 45°. On the other hand, the wave number of the right hand side peak is stationary regardless of the angle of incidence θ, and corresponds the wave number of Rayleigh wave. Therefore, it can be expected that the right hand side peak express the contribution of Rayleigh wave.

In the wave number spectra, some excitations appear between the above two main peaks. In the case of θ=30°, the spectrum forms the peak slightly at ξ/k_s=0.29. This wave number corresponds with that of reflected P wave generated on the slope due to the incident SV wave. Therefore, it is confirmed that this peak indicates the contribution of the reflected P wave. Furthermore, gently sloping excitations which appear notably in the cases between θ=15° and 25° are seen around ξ/k_s=0.5. We expect that these excitations are the contribution of leaking mode, which is derived from the characteristic equation of the

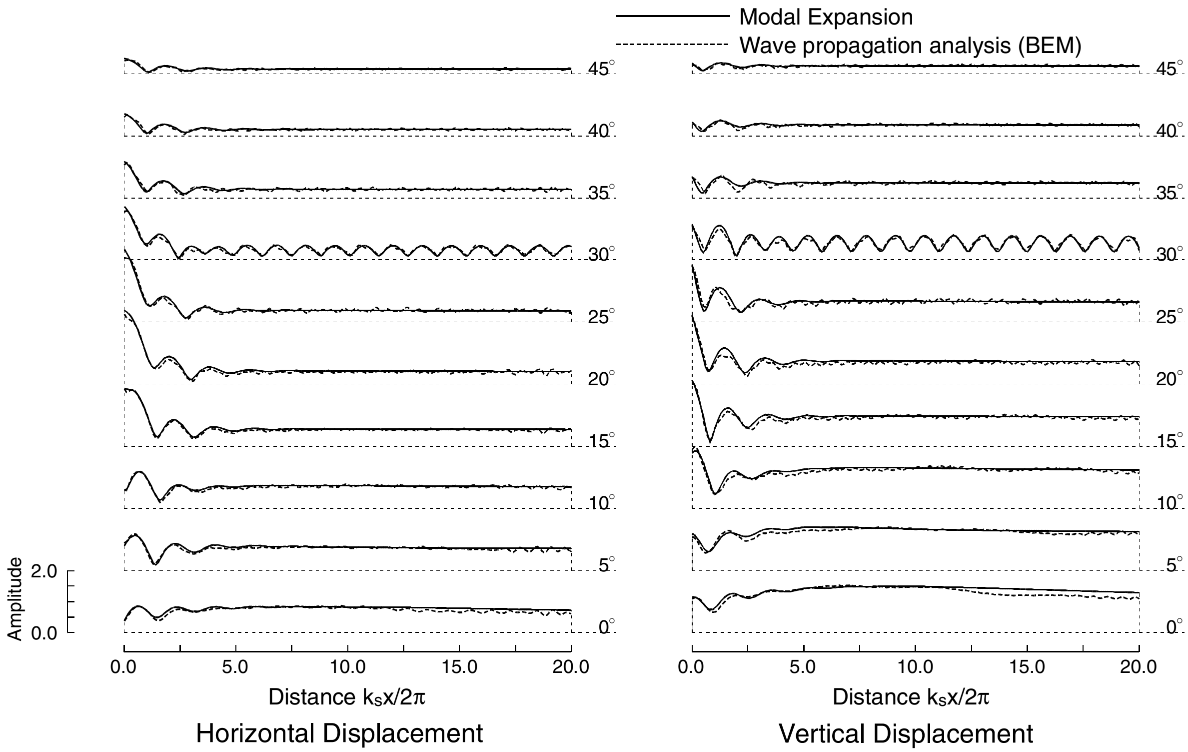

Fig.4 Comparison between the modal expansion method and the wave propagation analysis Displacement amplitude of the scattering wave

P-SV wave field, since ξ/k_s=0.5 is equivalent to the wave number of the leaking mode and the gently sloping form of the excitation shows the effect of some wave component with strong attenuation, such as the leaking mode.

ANALYSIS OF THE SCATTERING WAVES BY MODAL EXPANSION METHOD

Modal expansion for the scattering waves

The surface displacement shown in Fig. 2 is composed of the contribution of the incident SV wave and of the scattering wave generated in the slope or the edge. The contribution of the scattering wave cannot be evaluated without difficulty, however the contribution of the incident SV wave is red simply from considering the relationship between incidence and reflection on a surface of an elastic half space. Therefore, we use the modal expansion method to investigate the behavior of the scattering wave.

Results by the modal expansion method are shown in Fig.4 and 5. In these figures, dashed line indicates the displacement of the scattering wave, which is obtained by removing the contribution of incident SV wave calculated theoretically from the response of the wave propagation analysis shown in Fig.2 and 3.

As shown in Fig.4, the amplitude of the scattering wave fluctuates greatly near the edge, and settles down stationary as away from the edge. The fluctuation of the amplitude distribution is especially large in the cases between θ=10° and 25°. In Fig.4 and 5, displacements and wave number spectra obtained by the modal expansion method correspond to those of the wave propagation analysis correctly. From these results, it is considered that the scattering wave is analyzed by the modal expansion method with sufficient accuracy.

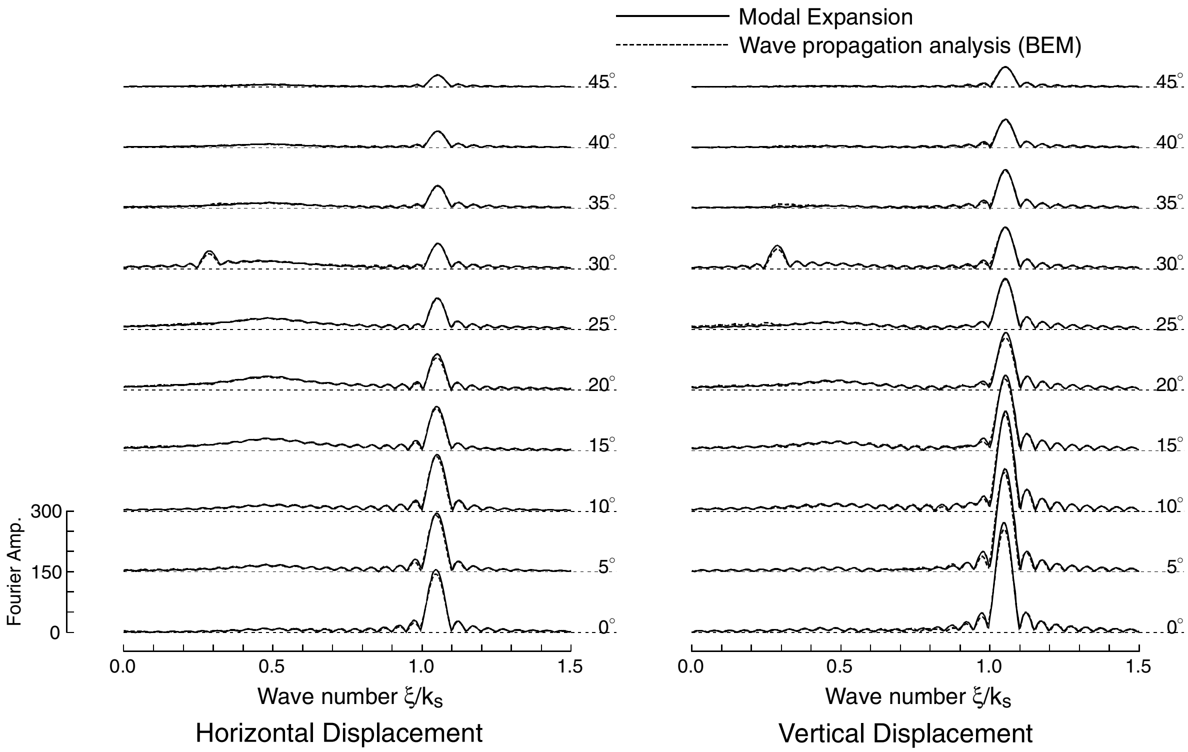

Fig.5 Comparison between the modal expansion method and the wave propagation analysis Wave number spectrum of the scattering wave

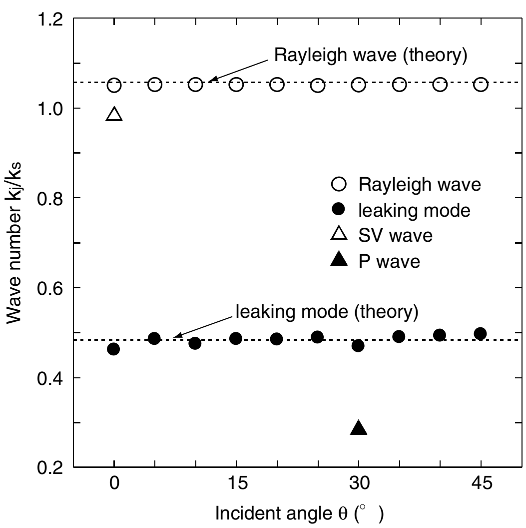

Fig.6 Wave number

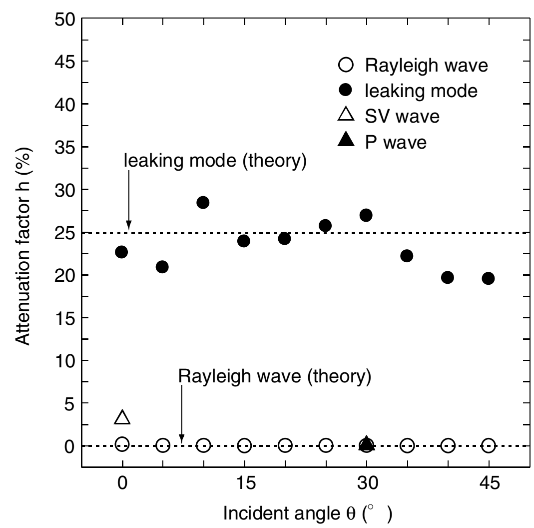

Fig.7 Attenuation factor

Behavior of scattering waves expanded by the modal expansion method

Wave numbers of the components, which are detected from the scattering wave by the modal expansion method, are shown in Fig.6. The wave numbers of these components correspond to P wave, SV wave, Rayleigh wave and leaking mode, respectively. Both the wave numbers corresponding to Rayleigh wave and those corresponding to leaking mode always exist in all cases. These two kinds of wave numbers are

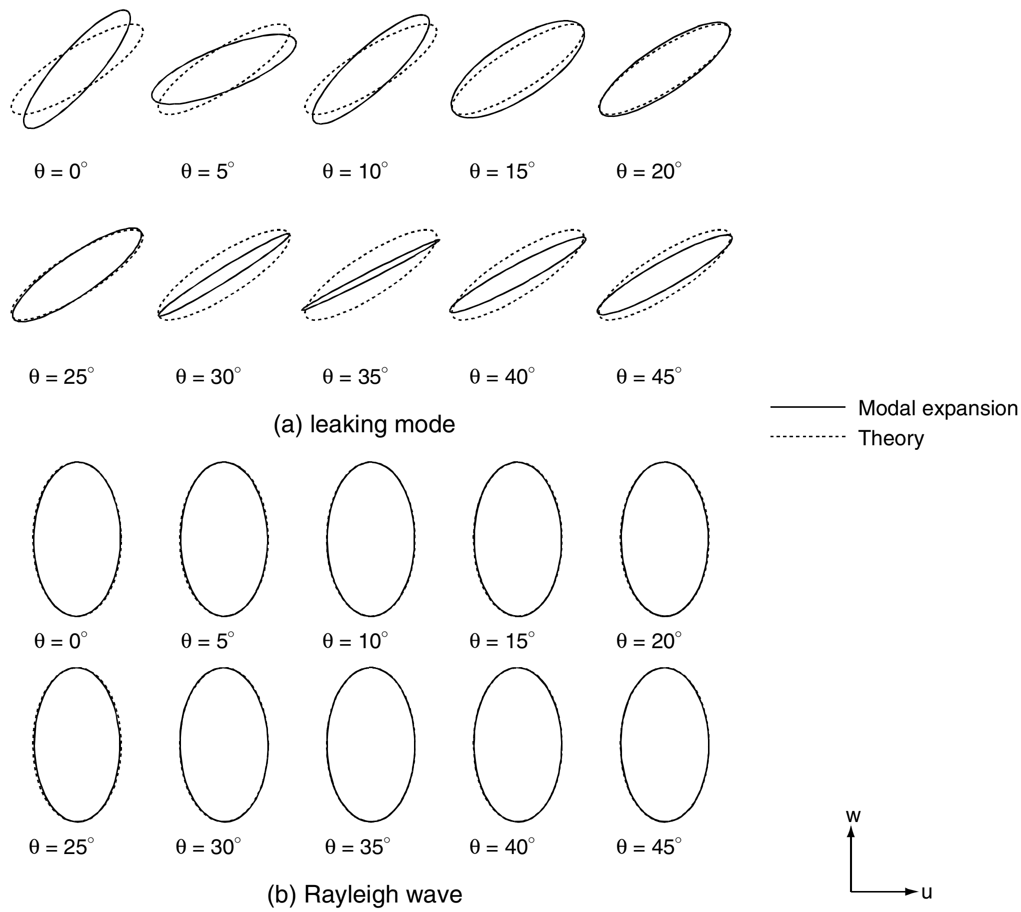

Fig.8 Particle orbit of Rayleigh wave and leaking mode

well in agreement with theoretical values for Rayleigh wave and leaking mode. On the other hand, there is the wave number of SV wave only in the case of θ=0°, since the SV wave generates on the slope by reflection and propagates parallel to the horizontal surface. The wave number of P wave also exists only in the case of θ=30°, because P wave is observed as a homogeneous wave only in this case. Judging from the above results, both Rayleigh wave and leaking mode are main components of the scattering wave.

In Fig.7, attenuation factors of the components are shown. The attenuation factors corresponding to Rayleigh wave are well agreement with the theoretical value that is zero. On the other hand, the attenuation factors corresponding to leaking mode are in general close to a theoretical value, which is about 24.9%, though some of them have comparatively large variation to the theoretical value. It is appropriate theoretically that the attenuation factor, which is close to zero in the case θ=30°, corresponds to that of the reflected P wave, because it is a plane homogeneous wave. Since the attenuation factor of the SV wave is about 3%, it is found that this component disappears at the distant place. From Fig.6 and 7, it is expected that the main components, which form the scattering wave, are Rayleigh wave and leaking mode.

In order to verify the above, the results by the modal expansion method are compared with theory about the particle orbit of the displacement amplitude of the components in Fig.8. The results calculated by the modal expansion method are very well in agreement with the theoretical results about Rayleigh wave. Thus, it is found that Rayleigh waves are detected very accurately from the scattering wave by the modal expansion method. Moreover, the results by the modal expansion method about leaking mode correspond to the theoretical results in general, though those are less in agreement than those about Rayleigh wave.

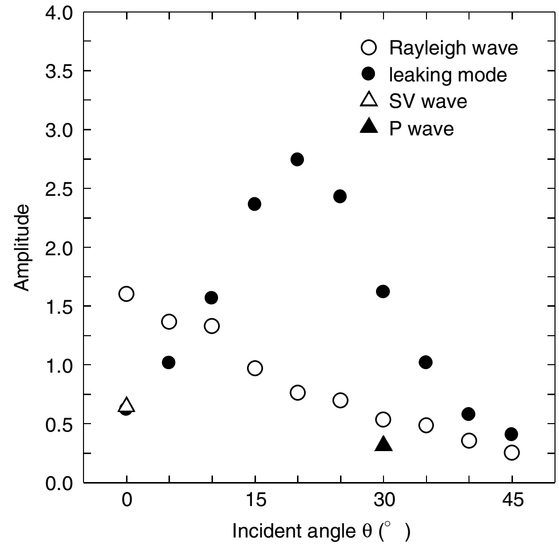

Fig.9 Displacement amplitude

From these results, it is confirmed that the components detected by the modal expansion method are Rayleigh wave and leaking mode.

Displacement amplitudes of the components that are defined as 1/2 of the length for long side of the elliptical orbits in Fig.8 are shown in Fig.9. Amplitude of Rayleigh wave is 1.6, which is the largest, in the case of θ= 0°, and then it decreases monotonously with increase of the angle of incidence. On the other hand, amplitude of leaking mode is 0.6 in the case of θ= 0°, which is nearly half of the amplitude of Rayleigh wave. However, it increases with the shift of the angle of incidence and reaches 2.7 in the case of θ= 20°, that is the largest amplitude on leaking mode.

Summarizing the above results, the scattering wave generated on the slope or the edge is mainly constituted by Rayleigh wave and leaking mode. Since Rayleigh wave does not have attenuation, its influence is extended to an infinite distant place. On the other hand, since leaking mode has strong attenuation, its influence is not extended to a distant place. However, its amplitude may reaches about 3 times of incident SV wave in some case of the angle of incidence. In these cases, leaking mode affects the surface motion greatly, because amplifying the displacement around the edge strongly.

DISCUSSION

We try to consider with the reason that the amplitudes of Rayleigh wave and leaking mode change with the shift of the angle of incidence. Gilbert and Laster [12] indicate that Rayleigh wave is excited by S wave and the leaking mode excited by P wave. Paying our attention to their indication, we have a discussion about the excitation of Rayleigh wave and leaking mode around the edge, focusing on the reflective P and SV waves generated on the slope.

The relationship between the incidence and reflection for plane P and SV waves on the slope is shown in Fig.10. Reflection coefficients of P and SV waves are plotted against angle of incidence in Fig. 11. In this figure, θ_{cr} expresses the critical angle. The reflection coefficient for P wave increases with increase of the angle of incidence from θ= 0° and shows maximum value around θ= 20°. The change of the coefficient of P wave is well similar to that of the amplitude of leaking mode shown in Fig.9. Comparing these two results, it is expected the behavior of reflected P wave generated in the slop relates to excitation of leaking mode.

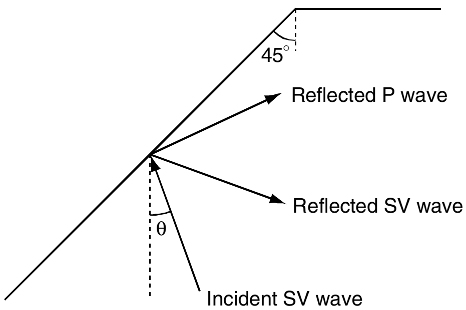

Fig.10 Incidence and Reflection in the slop

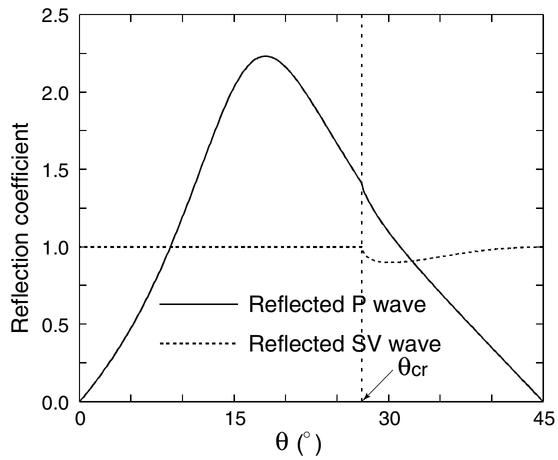

Fig.11 Reflection coefficient due to incident SV wave

When the angle of incidence θ is less than θ_{cr}, total reflection occurs on the slop. In this situation, the reflected P wave is transmitted as an inhogeneous wave along the slope and its energy is concentrated to the edge of the ground surface. Furthermore, as shown in Fig.11, the amplitude of the reflected P wave between θ =15° and 25° grows larger. Therefore, leaking mode is excited by such a behavior of the reflected P wave.

On the other hand, reflection coefficient for SV wave, which is shown in Fig.11, is almost constant regardless of the angle of incidence. As shown in Fig.9, the amplitude of Rayleigh wave becomes the maximum at θ= 0° and it decreases with increase of the angle of incidence. Therefore, the excitation of Rayleigh wave may not be related to the amplitude of the reflected SV wave.

Then, we pay attention to the direction of the reflected SV wave. In Fig.10, when θ= 0°, the reflected SV wave propagates horizontally parallel to the ground surface. According to the increase of the angle of incidence θ, the propagation direction of the reflected SV wave becomes downward and the energy goes away from the horizontal surface. Judging from these results, when the propagation direction of the reflected SV wave is parallel to the ground surface, Rayleigh wave is excited most greatly and it decays as the propagation direction becomes downward.

CONCLUSIONS

The surface motion, when a plane SV wave is incident from arbitrary angle on an elastic half space having a slop in which an angle of inclination is 45° and Poisson's ratio is 0.45, is calculated by the wave propagation analysis using boundary element method. The behavior of the scattering wave generated at the edge is investigated by the modal expansion method. These results are summarized as follows.

(1) The scattering wave generated at the edge is constituted by Rayleigh wave and leaking mode. These components always exist regardless of the angle of incidence of SV wave.
(2) Distribution of amplitude of the scattering wave changes largely at the edge of the surface, and it is converged at the distant from the edge. The large amplitude around the edge expresses the influence of leaking mode. On the other hand, stationary distribution of the amplitude in the place distant from the edge expresses the influence of Rayleigh wave.
(3) The amplitude of Rayleigh wave and leaking mode both which are contained in the scattering wave changes with the angle of incidence of SV wave. Leaking mode has great influence on the surface

motion near the edge, because it's amplitude amounts to about 3 times of the incident SV wave between θ =15° and 25°, especially.

(4) The amplitude of leaking mode changes in relation with the behavior of the reflected P wave generated on the slope. That is to say, leaking mode is excited greatly when the energy of the large reflected P wave may concentrate on the edge in the case that SV wave is incident to the slop between θ=15° and 25°.

REFERENCES

1. Mal A.K, Knopoff L. "Transmission of Rayleigh wave at a corner." Bull. Seism. Soc. Am. 1966; Vol.56, No.2: 455-466.
2. Momoi T. "Scattering of Rayleigh waves in an elastic quarter space." J. Phys. Earth 1980; 28: 385-413.
3. Fujii K, Takeuchi S, Okano Y, Nakano M. "Rayleigh wave scattering at various corners." Bull. Seism. Soc. Am. 1984; Vol.74, No.1: 41-60.
4. Zama S. "Behavior of the elastic waves propagating through the irregular structures, 1. Effect on cliff by earthquake ground motions." Bull. Earthq. Res. Inst. 1981; Vol.56: 741-752 (in Japanese).
5. Ohtsuki A, Yamahara H, Harumi K. "Surface motion of layered medium having an irregular interface due to Rayleigh waves." J. Strct. Mech. Earthquake Eng., JSCE, 1981; No.337: 27-36 (in Japanese).
6. Bard P.Y, Bouchon M. "The seismic response of sediment-filled valleys, Part 2 The case of incident P and SV waves." Bull. Seism. Soc. Am. 1980; Vol.70: 1921-1941.
7. Oliver J, Major M. "Leaking modes and the PL phase." Bull. Seism. Soc. Am. 1960; Vol.50, No.2: 165-180.
8. Phinney R.A. "Leaking modes in the crustal wave-guide, 1, the oceanic PL wave." Jour. Geophys. Res. 1961; Vol.66: 1445-1461.
9. Gilbert F. "Propagation of transient leaking modes in a stratified waveguide." Reviews of Geophysics 1964; Vol.2, No.1: 123-153.
10. Chapman C.H. "Lamb's problem and comments on the paper 'On leaking modes' by Usha Gupta." Pure and Applied Geophysics, 1972; 94: 233-247.
11. Akiyama S. "Analysis of scattering wave by topographic irregularity in P-SV wave field using eigenmodes of Rayleigh wave characteristic equation." J. Strct. Mech. Earthquake Eng., JSCE, 2003; No.731/I-63: 267-282 (in Japanese).
12. Gilbert F, Laster S.J. "Excitation and propagation of pulses on an interfaces." Bull. Seism. Soc. Am. 1962; Vol.52: 294-319.