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STUDY ON CHARACTERIZATION OF NON LINEAR RESPONSE OF SDOF MODEL CONSIDERING LARGE DEFORMATION BY P-DELTA EFFECT

Norihiko YAMASHITA[1] and Takanori HARADA[2]

SUMMARY

Over the past few decades a considerable number of studies have been made on the issues of P-Δ effect on structural performance of lumped mass model. The influence of P-Δ effect is caused by the weight and lateral deformation of the structure. In this study, in order to establish the simple seismic design method considering the large deformation such as P-Δ effect, we investigate the non linear response characteristics using two types of non linear model; the horizontal SDOF and rocking SDOF models. We performed non linear response analysis and calculated ratio of absolute acceleration response spectrum, and we attempted to separate the P-Δ effect.

INTRODUCTION

The decreasing of restoring force of a structure by the gravity is called as the P-Δ effect[1],[2], and it cannot be disregarded in case of the top-heavy structure. Especially, the restoring force characteristics of the negative gradient occurs by the effect of the gravity, when the lateral restoring force responses like the elasto-plastic behaviors, and it seems to be one of the causes of a collapse of the structure by strong earthquake. In U.S.A. and New Zealand, the seismic design beyond the elastic range considers the effects of the ductility of the structure and the effect of the P-Δ effect. On the other hand, in Japan, the seismic design regulation for bridges does not take into consideration the P-Δ effect, because it restricts the residual displacements and the lower limit of the restoring forces of the structures in non linear responses. However, it is still necessary to examine the effect of the P-Δ effect by considering the case exceeding the seismic design loadings. And also, a number of investigations on the P-Δ effect are based on the models assuming the small lateral deformations by ignoring the large deformations[3],[4].

In this study, in order to establish the simple seismic design method considering the P-Δ effect under the large deformations, we investigate the non linear response characteristics using two types of non linear model; the horizontal SDOF (single degree of freedom) and rocking SDOF models. To evaluate and separate the P-Δ effects from the other non linearity, we propose three factors such as the factors of the effect of non linearity, of the P-Δ effect, and of the non linear P-Δ effect by the ratios of the absolute

[1] Assistant Professor, Kobe City College of Technology, Kobe, Japan. Email:yamasita@kobe-kosen.ac.jp
[2] Professor, Miyazaki University, Miyazaki, Japan. Email:harada@civil.miyazaki-u.ac.jp

maximum response acceleration of the non linear SDOF models, considering the non linearity and P-Δ effects, to the absolute maximum response acceleration of linear SDOF model (so-called response spectrum) as a function of natural period of linear SDOF model. The similar 3 factors for the relative maximum response displacements are also introduced. By the numerical investigations using the strong ground motions from the 1995 Hyogoken-Nanbu earthquake, the 2000 Tottoriken Seibu earthquake and the 1940 Imperial Valley earthquake, we shows that the P-Δ effect appears even if the structure has short natural period, depending on the input earthquake ground motions.

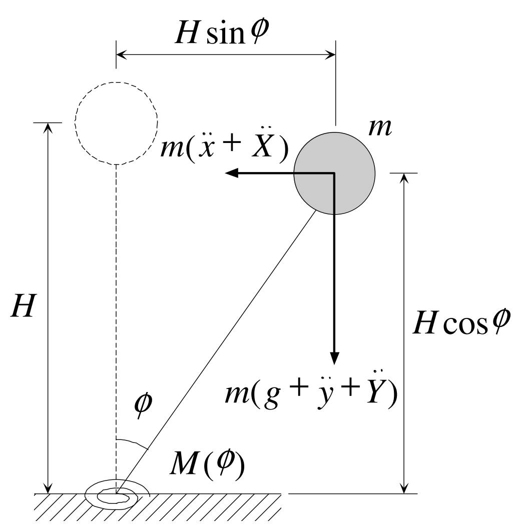
EQUATION OF MOTION OF ROCKING SDOF MODEL

A rocking SDOF model considering the P-Δ effect in this study is shown in Fig. 1. Equation of motion of the rocking SDOF taking into rigorously consideration the large deformation of the model is given by Eq.(1), and its approximation is expressed as Eq.(2) by assuming the small angle of rotation in Eq.(1)($\cos\phi = 1, \sin\phi = \phi$).

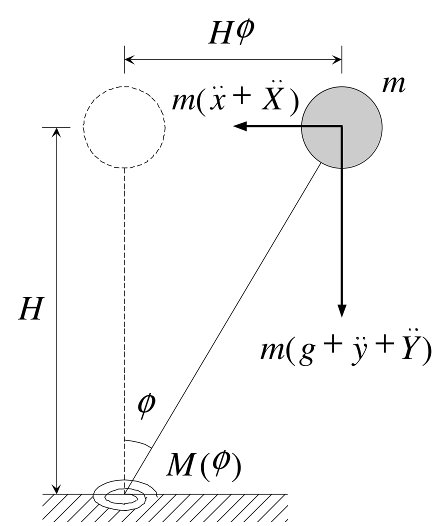
$$\ddot{\phi} + 2\xi\frac{2\pi}{T}\dot{\phi} + \frac{M(\phi)}{mH^2} = -\frac{\ddot{X}}{H}\cos\phi + \frac{g+\ddot{Y}}{H}\sin\phi \tag{1}$$

$$\ddot{\phi} + 2\xi\frac{2\pi}{T}\dot{\phi} + \frac{M(\phi)}{mH^2} = -\frac{\ddot{X}}{H} + \frac{g+\ddot{Y}}{H}\phi \tag{2}$$

where m is the mass of lumped mass, ξ is the damping factor, T is the natural period, $M(\phi)$ is the restoring moment, ϕ is the rotational angle (The pier is assumed to be rigid, and it is equal to the non linear plastic hinge at the bottom of the pier.), g is the gravity acceleration, $\ddot{x}$ and $\ddot{y}$ are the acceleration which are respectively vertical and lateral direction of SDOF model. In addition, $\ddot{X}$ and $\ddot{Y}$ are the horizontal and vertical input motions. In this paper, the non linear plastic hinge of the bottom of the pier column is modeled by a complete elasto-plastic model.



(a) Rigorous rocking SDOF model



(b) Approximate rocking SDOF model

Figure 1. Coordinate system and notations of rocking SDOF model considering the P-Δ effect

P-Δ EFFECT AND RESPONSE SPECTRUM OF ROCKING SDOF MODEL

We note here the linear spring constant of the lateral SDOF model usually used to evaluate the response spectrum (see Fig. 2(a)) as k_H , and also the linear rotational spring constant of the rocking SDOF model (see Fig. 2(b)) as k_R. By comparing the forces acting on the lateral and SDOF models, we obtain the following relation between the two models,

$$k_H = \frac{1}{H^2} k_R \tag{3}$$

By ignoring the damping force in the equation of motion of the linear SDOF model of Fig. 2(a) such as,

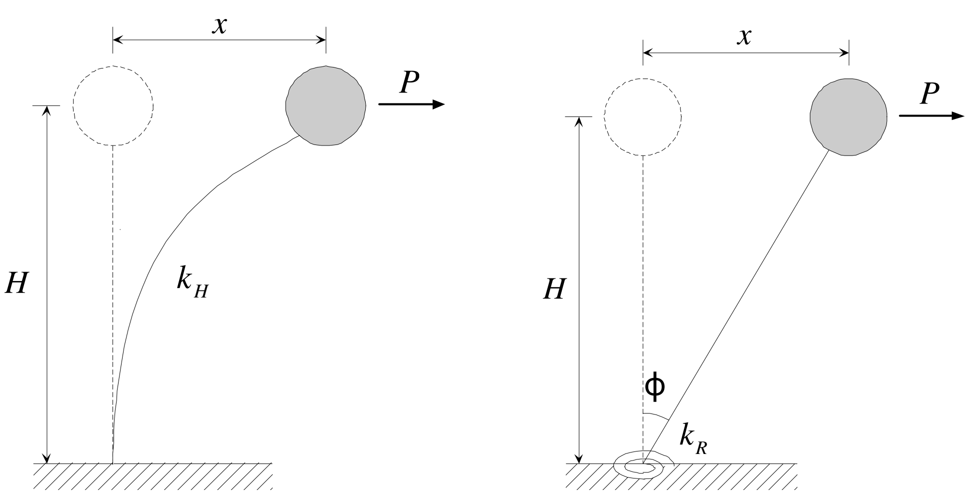
$$\ddot{x} + \frac{k_H}{m} x = -\ddot{X} \tag{4}$$

and then considering $x = \phi H$ and Eq.(3) in Eq.(4), we obtain the following the linear equation of motion of rocking SDOF model.

$$\ddot{\phi} + \frac{k_R}{mH^2} \phi = -\frac{\ddot{X}}{H} \tag{5}$$

Therefore, it can be said that the second term of the right-hand side of Eq.(2) expresses the P-Δ effect in comparison with Eq.(5) and Eq.(2). Inversely, Eq.(2) is rewritten using the relation $x = \phi H$ and Eq.(3) such as,

$$\ddot{x} + \frac{k_H}{m} x = -\ddot{X} + \frac{g}{H} x \tag{6}$$



(a) Lateral SDOF model (b) Approximate rocking SDOF model

Figure 2. Relationship between lateral SDOF model and approximate rocking SDOF model

In Eq.(6),the restoring force is expressed by $(k_H - mg/H)x$. The restoring force is decreased by the term $(mg/H)x$. And, the restoring force given by Eq.(6) can be also expressed by using the natural period of the linear SDOF model such as,

$$\left(k_H - \frac{mg}{H}\right)x = \left(\frac{2\pi}{T}\right)^2 \left(m - \frac{mg}{(2\pi)^2}\frac{T^2}{H}\right)x \tag{7}$$

By using this expression of Eq.(7), the decreasing of the restoring force is proportional to T^2 and inversely proportional to H of the height of the bridge pier. Since the natural period of the buildings is approximately proportional to the height of buildings, the P-Δ effect in buildings increases in proportion to the natural period. However, there is not a clear correlation in the bridges between the natural period and the height of the pier. Therefore, in this investigation of the P-Δ effect on the structural responses, the two parameters of the natural period and the height of the structures are treated as the independent parameters.

Now introducing the ratios of the response spectra defined and schematically shown in Fig. 3,the effect of non linearity without the P-Δ effect (α), the effect of only the P-Δ effect (β), and the composite effect

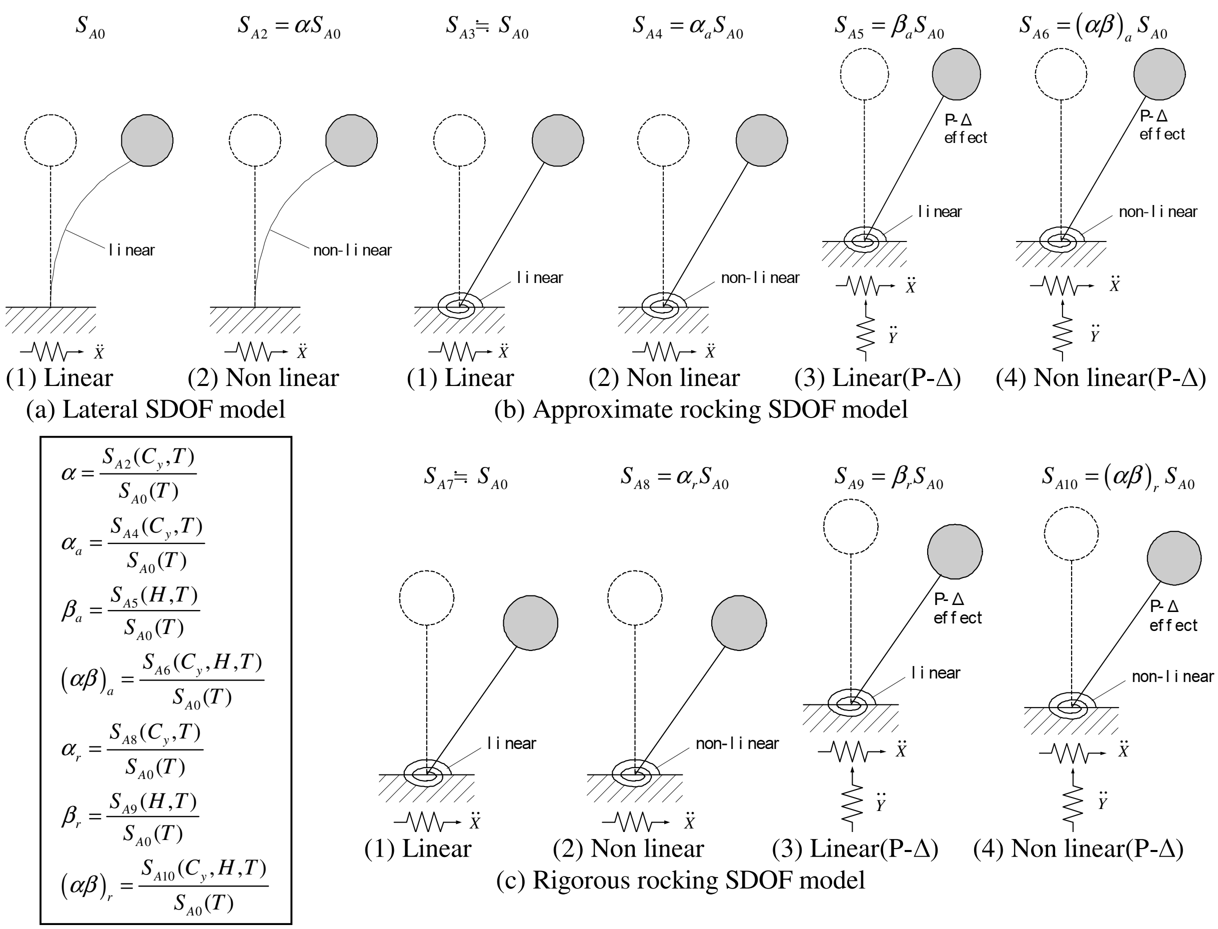


Figure 3. Relationships between absolute acceleration response spectra and the spectrum ratios α, β and $\alpha\beta$

of the non linearity and the P-Δ effect ($\alpha\beta$) are examined. The sub suffixes a and r of α, β and $\alpha\beta$ defined in Fig. 3 indicate that they are obtained from the approximate and rigorous models. In Fig. 3, S_{A0} is the linear absolute acceleration response spectrum of the lateral SDOF model, and S_{An} ($n = 2-10$) is the absolute acceleration response spectrum of each rocking SDOF model. The rotational motion from the rocking SDOF model is converted into the lateral movement of the mass, when both are compared. The peak lateral absolute accelerations of the response from the rigorous and approximate rocking SDOF models are estimated by,

$$\ddot{x}_{\max} = \left[ABS.\left\{ \ddot{X} + \left(-H \sin\phi\left(\dot{\phi}\right)^2 + H\cos\phi\left(\ddot{\phi}\right)\right)\right\}\right]_{\max} \tag{8}$$

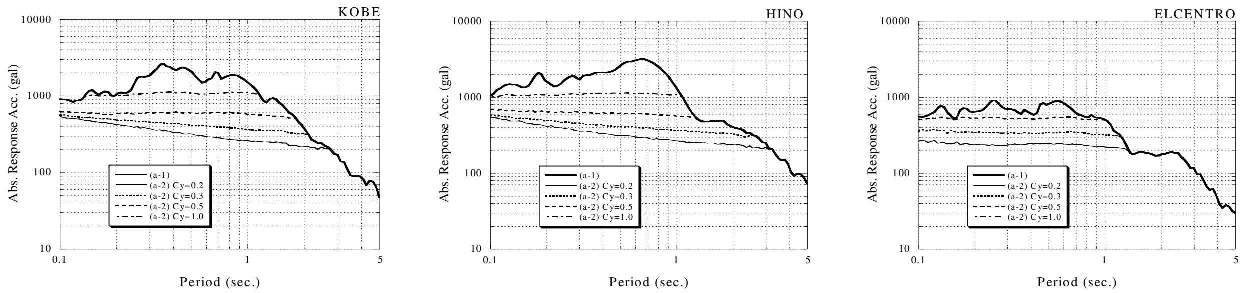
$$\ddot{x}_{\max} = \left[ABS.\left\{ \ddot{X} + H\ddot{\phi} \right\}\right]_{\max} \tag{9}$$

Although for the linear system, the displacement response spectrum can be estimated from the absolute acceleration response spectrum, the non linear effect and P-Δ effect on the displacement response spectrum are also examined numerically by computing the maximum response displacements.

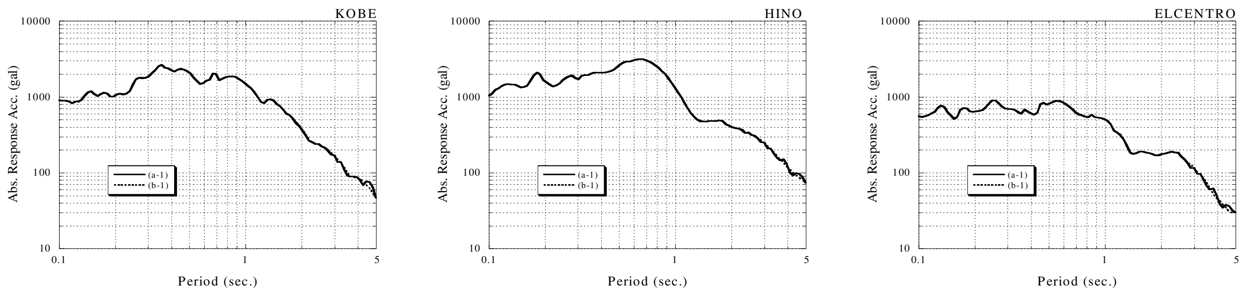
NUMERICAL EXAMPLES

Non linear response analysis was carried out by using the linear acceleration method with the time increment of 0.001 sec. As the structural parameters, we assume 5% of critical damping in linear structure, and the 5 cases of the height H of the structures (10, 20, 30, 50, and 100m), and also the 4 cases of the yielding level of the structures expressed by the seismic coefficient C_y (0.2, 0.3, 0.5, and 1.0). As the input ground motions, we use the ground motions of NS component from JMA Kobe(in the 1995 Hyogoken-Nanbu earthquake), Hino(in the 2000 Tottori Seibu earthquake) and Elcentro(in the 1940 Imperial Valley earthquake). In the present analysis, the vertical component of ground motions is not used because the effect of the vertical ground motions has been negligible from our previous study [5]. We assume that the non linear restoring force is modeled by a complete elasto-plastic model.

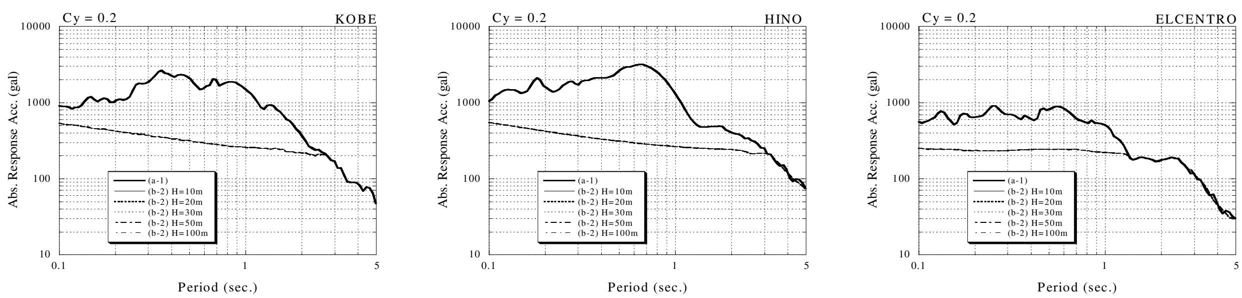
Figure 4 shows the absolute acceleration response spectra from the lateral and rocking SDOF models (The rotational angle was converted into the horizontal displacement (see Eqs. (8) and (9))). In the response spectrum from the non linear lateral model of Fig. 4 (a), the bold curve indicates the linear response spectrum of the lateral SDOF model which is usually used in seismic design, and the other curves show the non linear response spectra of lateral SDOF model by changing the yielding level of the SDOF, C_y =0.2, 0.3, 0.5, and 1.0. The response spectrum decreases, because the non linearity increases as the seismic coefficient of yield C_y is smaller, and the response spectrum from the linear and non linear models agrees in the long period structure. The response spectrum of the linear lateral SDOF model is perfectly coincident to that of the approximate linear rocking SDOF model, regardless of the height H of the pier as shown in Fig. 4 (b), because they are obtained from the linear model. In Fig. 4 (c), the response spectra of the non linear approximate rocking model with C_y =0.2 are shown by changing the height of pier H =10, 20, 30, 50, and 100m indicating that the height H does not affect the non linear response spectra for the case of C_y =0.2. The response spectra obtained from the linear approximate rocking model where the P-Δ effect only is taken into consideration are shown in Fig. 4 (d) by changing the height H of the pier. The difference of the response spectra due to the height of the pier occur for the structure of long period over 2 seconds. And, this difference increases as the height H of bridge pier is lower. Fig. 4 (e)



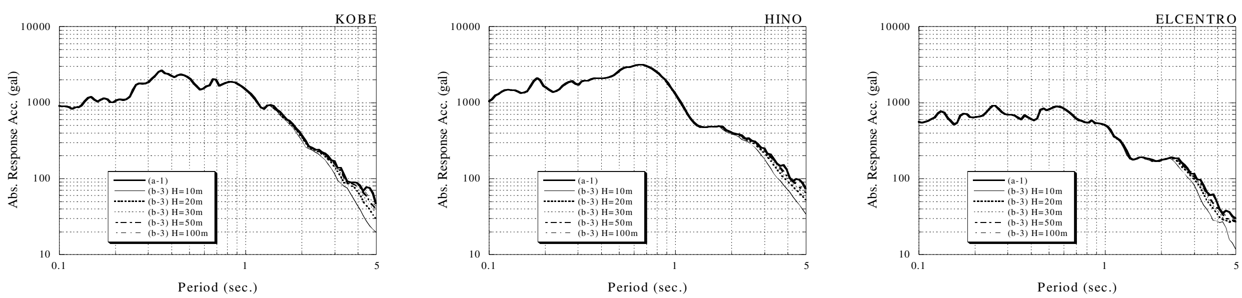
(a) Non linear model (Lateral SDOF model)



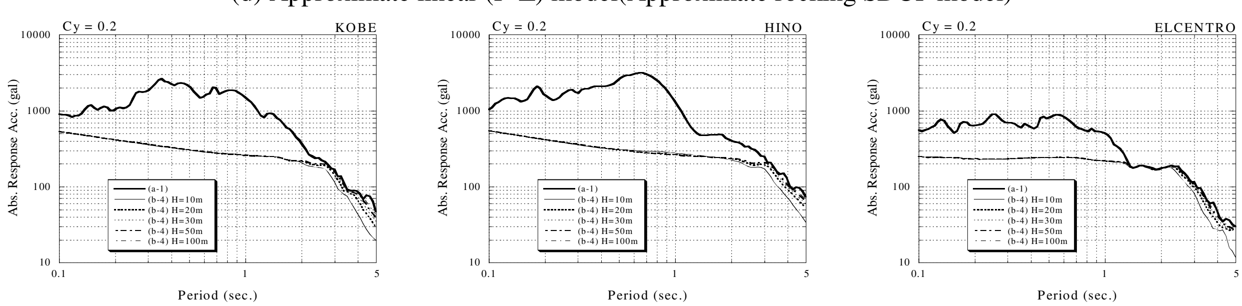
(b) Approximate linear model(Approximate rocking SDOF model)



(c) Approximate non linear model(Approximate rocking SDOF model)



(d) Approximate linear (P-Δ) model(Approximate rocking SDOF model)



(e) Approximate non linear (P-Δ) model(Approximate rocking SDOF model)

Figure 4. Absolute acceleration response spectra (JMA Kobe, Hino and Elcentro)

shows the response spectra obtained from the non linear approximate rocking SDOF model where the both effects of non linearity and decreasing the restoring force due to the gravity (P-Δ effect) are taken into consideration. In Fig. 4 (e), C_y =0.2 is fixed and the height H of the pier are changing as H =10, 20, 30, 50, and 100m.

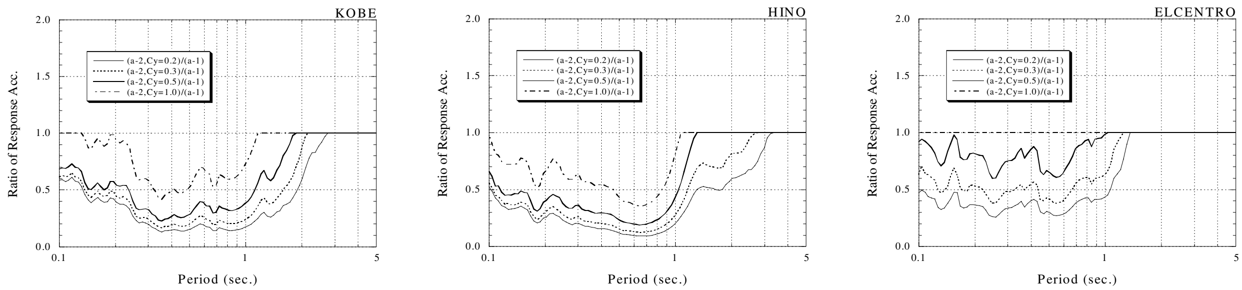
As the summaries of Fig. 4, the P-Δ effect on the absolute acceleration response spectrum appears for the structure of long natural period and short height of the pier as expected from Eq.(7). It should be noted here that the effect of non linearity on the acceleration response is significant and the P-Δ effect on acceleration responses is restricted only for the structure of long natural period and short height of pier when the natural period of structure and the height of the pier are treated as the independent parameters.

Figure 5 shows the ratios of the non linear acceleration response spectra to the linear acceleration response spectrum as defined in Fig. 3. These ratios represent the only non linear effect (α), the only P-Δ effect (β), and the composition of the non linear effect and the P-Δ effect ($\alpha\beta$). Fig. 5(a) shows the ratio α which is the ratio of the absolute acceleration spectrum from the lateral non linear model to that from the lateral linear model. It is observed from Fig. 5(a) that the non linear effect on the structure with the lower level of yielding C_y is larger. In the case of C_y =1.0 and the input motion of Elcentro (the right column of Fig. 5(a)), the response is restricted in the elastic range and the value of α is 1.0. For the structures of long period the response is limited in the elastic range, the value of α takes 1.0 independent of the input motions. The ratios shown in Fig. 5 (b) and (c) show the similar nature of Fig. 5(a), indicating the non linear approximate rocking model and the non linear rigorous rocking model give almost similar responses of those from the non linear lateral model.

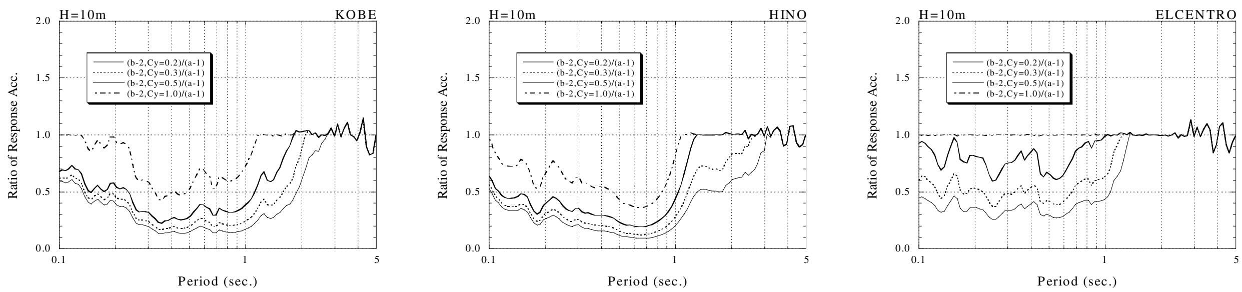
Figure 5(d) shows the ratio β which is the index of the P-Δ effect only. It is observed from Fig. 5(d) that the P-Δ effect is bigger as the height of bridge pier is lower in the long period range. It should be noted here that the structure having the shorter pier becomes the so called top heavy structure relatively under the same natural period of structure, because the structural mass for the short height of pier is bigger than that for the longer height of pier.

Figure 5(e) show the ratio $\alpha\beta$ which is the composite index of both the non linear effect and the P-Δ effect by assuming the yielding level of the structure C_y =0.2. Of course this index shows the composite trends of Figs. 5(c) and (d).

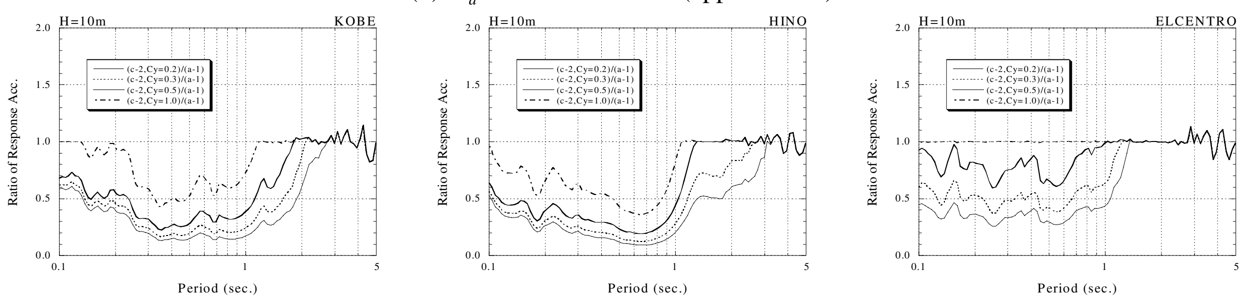
Figure 6 shows the displacement response spectra from the lateral and rocking SDOF models. The displacement response spectra of the lateral and approximate rocking SDOF models are almost perfectly coincident as shown in Fig. 6(a), because they are obtained from the linear models. And also, the displacement response spectra from the approximate non linear rocking model with the height of pier H =10, 20, 30, 50, and 100m and C_y =0.2 are shown in Fig. 6(b). In this case the displacement non linear response spectral values are all similar regardless of the height of pier. The displacement response spectra from the approximate linear rocking model, where the P-Δ effect only takes into consideration (see Fig. 3(4)), are shown in Fig. 6(c) by changing the height of pier. As was observed from Fig. 5(d), Fig. 6(c) indicates that the P-Δ effect appears in the displacement response spectra of the structure of long period over 2 seconds, and also that the P-Δ effect is larger as the height of pier is shorter (see Eq.(7)).
Finally, the displacement response spectra obtained from the approximate non linear rocking model with the height of pier H =10, 20, 30, 50, and 100m and C_y =0.2, where the P-Δ effect and the effect of non



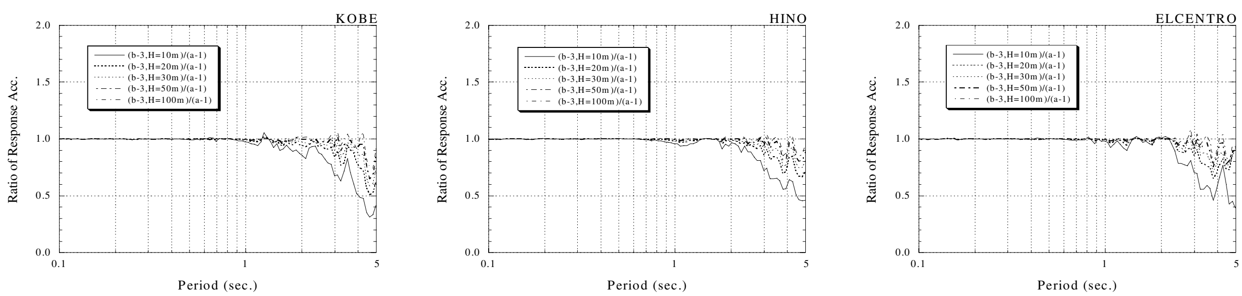
(a) α :Non-linear factor



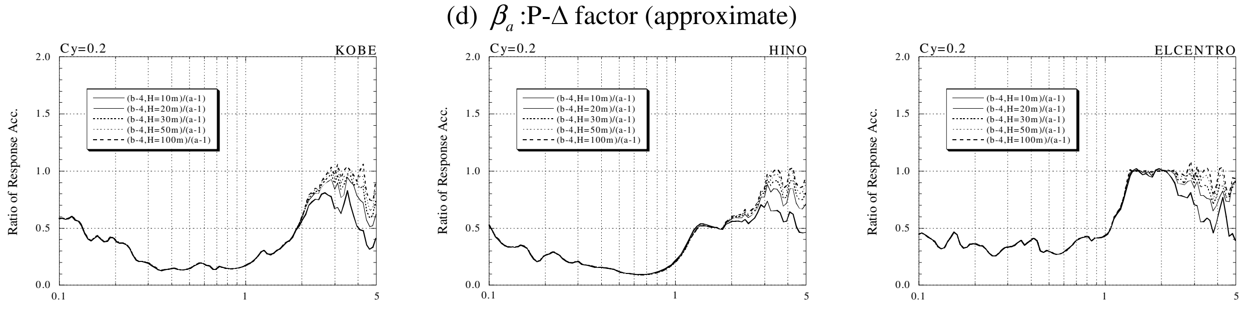
(b) α_a :Non-linear factor (approximate)



(c) α_r :Non-linear factor (rigorous)

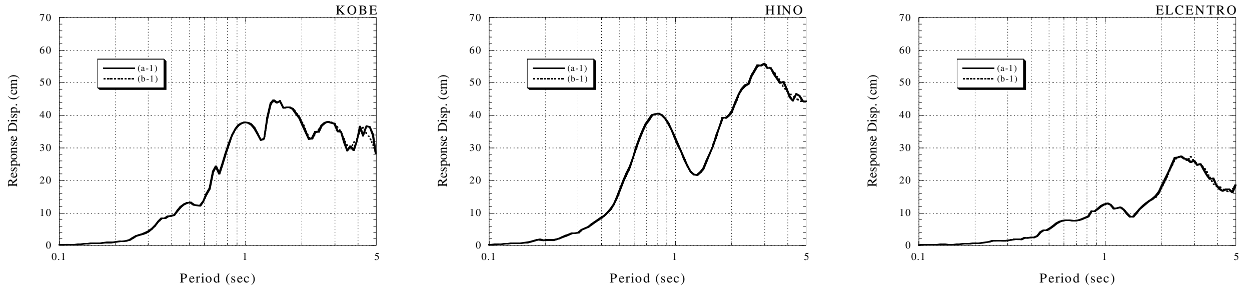


(d) β_a :P-Δ factor (approximate)

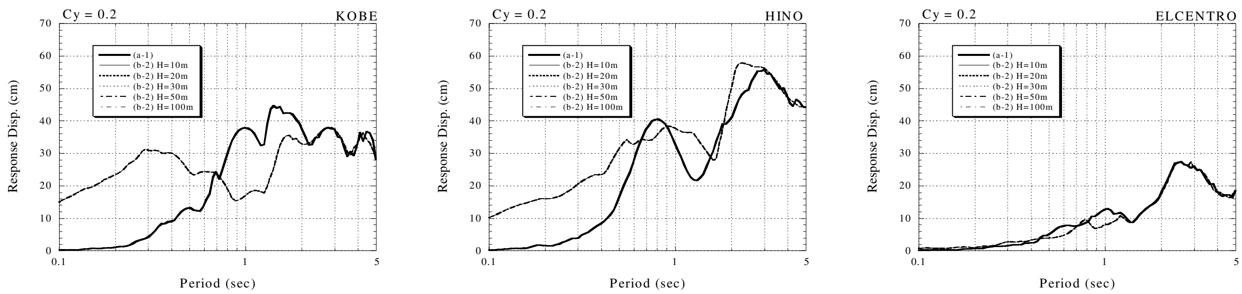


(e) $(\alpha\beta)_a$:Non-linear+P-Δ factor (approximate)

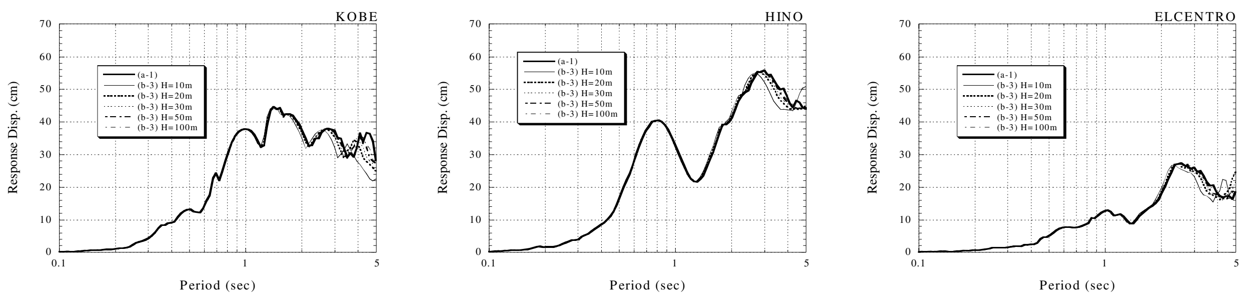
Figure 5. Spectrum ratios α , β , $\alpha\beta$ of absolute acceleration response spectra (JMA Kobe, Hino and Elcentro)



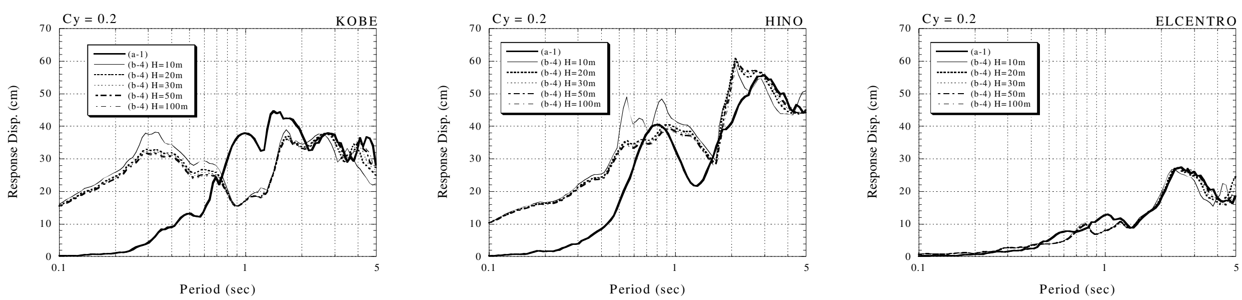
(a) Approximate linear model



(b) Approximate non linear model



(c) Approximate linear (P-Δ) model

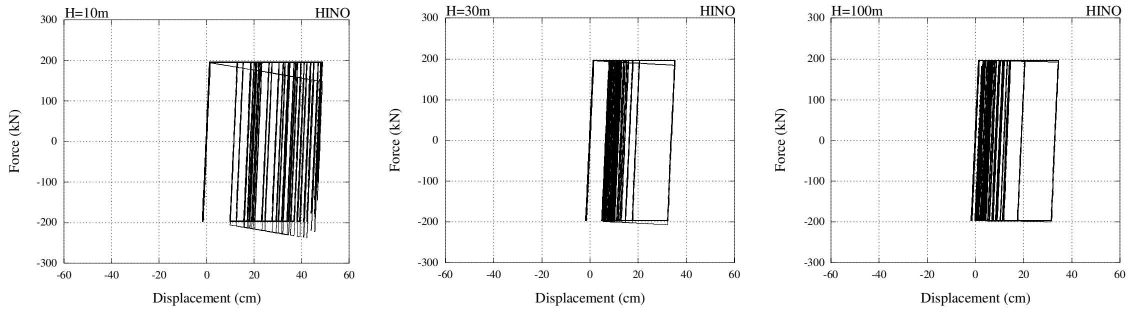


(d) Approximate non linear (P-Δ) model

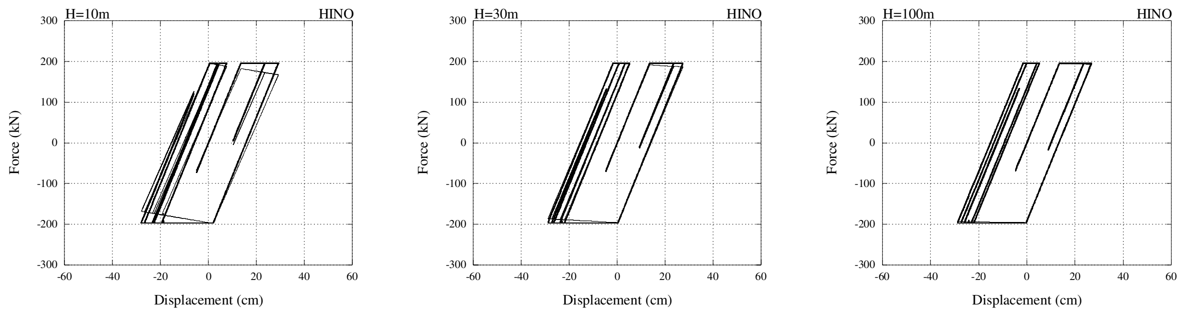
Figure 6. Displacement response spectra of approximate rocking SDOF model (JMA Kobe, Hino and Elcentro)

linearity are taken account for (see Fig. 3(4)), are shown in Fig. 6(d). In this case, it is observed from Fig. 6(d) that the P-Δ effect appears in the structures of not only long period but also short period, although the degree of this observation, of course, depends on the input ground motions. This observation is important as the P-Δ effect in modeling the non linear restoring force of the top heavy structure of short natural period.

To make the above observation from Fig. 6(d) more clear, the restoring forces induced in the structures of natural period of 0.55 sec and 1.65 sec, for the case of C_y =0.2 and the Hino ground motion as the input



(a) Natural period 0.55sec



(b) Natural period 1.65sec

Figure 7. Hysteresis loop of restoring forces induced in the structure of T =0.55 and 1.65 sec for three height of pier (H =10, 30 and 100m)

ground motion, are shown in Fig. 7. The bold lines show the restoring force, and the fine lines are the restoring forces subtracted the effect of the gravity from the restoring forces. It is observed from Fig. 7 that the residual displacement occurs at the structure of natural period of 0.55 sec due to the P-Δ effect. Especially, the decreasing of the restoring force by the gravity is big in the case of the height of bridge pier of 10m. By considering that the bridges, having the short natural period and height of pier such as 0.55sec and 10m, are the typical highway bridges, it seems to be an important issue to reexamine the non linear response behaviors of the highway bridges with the short natural period and the short height of the pier by taking into consideration the P-Δ effect, the non linear restoring characteristics, and the input ground motions.

CONCLUSIONS

The extraction of the P-Δ effect in non linear response behaviors of the SDOF models has been tried in terms of the response spectra. The P-Δ effect on the non linear restoring forces induced in the structure has been examined by indicating the hysteresis loop of the restoring forces. The results of this paper are summarized as followings.

1) In this study, the responses obtained from the rigorous rocking SDOF model were coincident to those from the approximate rocking SDOF model.
2) For the structure of the short natural period, the absolute acceleration response spectrum were primarily controlled by the non linear effect in disregard of the P-Δ effect.
3) In the displacement response spectra, the P-Δ effect appeared in the structures of not only long period but also short period, although the degree of this observation, of course, depended on the input ground motions. This observation is important as the P-Δ effect in modeling the non linear restoring force of the top heavy structure of short natural period.

4) In the hysteresis loops of the restoring forces induced in the structure, the residual displacements occurred when the P-Δ effect was big. Especially, the P-Δ effect was bigger as the height of bridge pier is shorter under the structure of the same natural period. The residual displacement occurred at the structure of short natural period due to the P-Δ effect. Especially, the decreasing of the restoring force by the gravity is big in the case of the height of bridge pier of 10m.
5) By considering that the bridges, having the short natural period and height of pier such as 0.55sec and 10m, are the typical highway bridges, it seems to be an important issue to reexamine the non linear response behaviors of the highway bridges with the short natural period and the short height of the pier by taking into consideration the P-Δ effect, the non linear restoring characteristics, and the input ground motions.

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