

# IDENTIFICATION OF FRICTIONAL COEFFICIENTS OF STAINLESS STEEL SLIDING BASE ISOLATORS

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## SUMMARY

Sliding types of base isolator are used widely for lightweight structures instead of laminated rubber isolators. Many experimental studies were carried out regarding the frictional coefficient of sliding base isolators. In those experiments, the frictional coefficients are assigned as  $\mu = 0.05 \sim 0.10$ . However, even if  $\mu$  is in the range 0.10~0.20, it may be effective in the case of a big earthquake. And those values of  $0.10 \sim 0.20$  are obtained by using simple devices made of stainless steel. The purpose of this study is to identify those frictional coefficients by experimental and analytical methods. A stainless steel cart is placed on the sliding stainless steel surface. The responses of the cart are measured by laser sensor. Two different loading cases are adopted. Case-1: Constant horizontal force and vertical sinusoidal vibration are given simultaneously. Case-2: Horizontal earthquake excitation (JMA KOBE-NS and JMA KOBE-EW) only. In this research, the newly developed slowdown-Newton method is introduced to identify the frictional coefficients. The frictional coefficient is assumed to be a function of the static frictional coefficient  $\mu_s$  , the dynamic frictional coefficient  $\mu_d$  and the exponential damping coefficient c , expressed as  $\mu = \mu_d + (\mu_s - \mu_d)e^{-c|\nu|}$ . These three parameters  $\mu_s$ ,  $\mu_d$  and c are identified by minimizing a target function  $f = \sum (x_i^* - x_i)^2$ , where  $x_i^*$  is the measured displacement and  $x_i$  are the calculated displacement using trial values of  $\mu_s$ ,  $\mu_d$  and c. The validity of identified values is assured by comparing the time displacement curves of the cart. It shows good agreement with those of experimental curves. It is also proved that the maximum acceleration of the cart decreases even when the frictional coefficient  $\mu$  equals to 0.2. The maximum acceleration response is about 200Gal in the case of horizontal earthquake excitation, both JMA KOBE-NS (max. 818Gal) and JMA KOBE-EW (max. 617Gal).

## **INTRODUCTION**

In recent years, the sliding bearing as a seismic base isolator for lightweight structures has been increasingly used. Many experimental studies [1-3] regarding the frictional coefficient of sliding base

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isolators have been carried out. It is known that the frictional coefficient depends on the pressure at the sliding surface, the sliding speed and the amplitude of horizontal displacement. But the vertical component of the earthquake excitation was not considered precisely. Moreover, in most of those studies the frictional coefficient is assumed to be between 0.05 and 0.1 in order to get sufficient response reduction even for medium scale of earthquake by using Teflon as a slide material. However, it has not been fully investigated about the response reduction effect in the case of a frictional coefficient greater than 0.1. Now Teflon is often used as a slide material. But there is fear of creep in Teflon and creep is related to durability. In addition, Teflon is expensive.

The response reduction effect in the wide range of frictional coefficients in the range 0.001~0.5 was investigated by a three-dimensional finite element method and it turned out that the response reduction effect was acquired even when a frictional coefficient is about 0.2 [4]. The dependency of frictional coefficient on the vertical motion (maximum acceleration is 400 Gal) was investigated experimentally and it turned out that the dependency in the case of using stainless steel is larger than in the case of using Teflon as a slide material [5, 6]. And lubrication is not used because it deteriorates with the time. In this study, for the purpose of development of cheap and durable slide isolator, a couple of different types of stainless steel are adopted as the slide surface. And the frictional coefficients are identified experimentally. Furthermore, influences of lubrication and of vertical vibration on the frictional coefficients are examined.

## CASE-1 HORIZONTAL CONSTANT LOAD AND VERTICAL VIBRATION

## **Experimental equipments**

The guide table equipped with the sliding surface (made of stainless-steel) is fixed on the top plate of the vertical vibrator. A movable cart is placed on the sliding surface. The experimental equipments are shown in Fig. 2-1. Horizontal forces are given by a suspended weight via a pulley. Lead balls and water are used as weight. The details of the cart are shown in Fig. 2-2. There are three projections at the bottom of the cart and their total contact surface area is 1cm<sup>2</sup>. The cart can move only in the horizontal direction. In order to minimize the resistant force caused by contact with the sidewalls of the guide table, bearings are attached to the four sides of the cart.



## **Condition of experiments**

The experiments are performed on the two types of conditions shown in Table 2-1.

## **Table 2-1 Condition of experiments**

	Sliding surface	Horizontal force	Vertical vibration	Measurement
	No-lubrication	Adding water drops is		Horizontal
Type-1	Organic molybdenum	continued until the cart	No	force
	Solid molybdenum	begins to move.		Displacement
	No-lubrication			
Type-2	Organic molybdenum	Maximum of Type-1	Sinusoidal wave	Displacement
	Solid molybdenum			

The effects of lubrication are examined in three cases in both Types: no-lubrication, organic molybdenum, and solid molybdenum. And the following two types of experiments were performed.

Type-1: Addition of water drops into a water vessel until a cart begins to slide. Laser sensors measure the displacements of the cart. The weights of the water vessel ( $W_{water}$ ) when the cart begins to slide are measured. The maximum static frictional coefficients are calculated from dividing  $W_{water}$  by the total weight of the cart ( $W_{cart}$ ). The above is repeated five times for each three sliding surfaces.

Type-2: The maximum weights in each set of the five experiments for the three sliding surfaces in Type-1 are adopted as the horizontal force. The cart is allowed to slide at the same time a vertical vibration is imposed. Laser sensors measure the displacements of the cart. The above is repeated five times for each three sliding surfaces.

## Parameters of vertical vibration

Experimental parameters of vertical vibrations are shown in Table 2-2. Those maximum vertical accelerations are assigned near the value of gravity. When maximum acceleration exceeds 1G(=980Gal), continuous jumping may arise. In order to investigate the influence of that behavior on  $\mu$ , 900Gal, 1000Gal and 1100Gal are adopted as maximum vertical acceleration.

Froqueney	Maximum
Frequency	acceleration
	900Gal
10Hz	1000Gal
	1100Gal
	900Gal
20Hz	1000Gal
	1100Gal

#### Table 2-2 Input sinusoidal waves

#### **Experimental results (Case-1)**

Type-1

The time history responses of the displacement of the cart are shown in Fig. 2-1. These graphs show that at the case of lubrication the cart moves faster than in the case of no-lubrication. This means that the frictional coefficient decreases by lubrication.



Table 2-3 shows the results of measured weights ( $W_{water}$ ). The average value of five times is calculated, but in the case of no lubrication, the third experiment, and in the case of solid molybdenum, the first experiment are omitted because they are differ greatly from the other correspondent experiments.

Tablez-0 The results of measured weights wwater							
W <sub>water</sub>	1	2	3	4	5	Average	
No lubrication [gf]	1431.4	1427.6	1881.9	1486.8	1361.2	1426.8	
Organic molybdenum [gf]	1598.5	1373.0	1399.8	1477.7	1553.8	1480.6	
Solid molybdenum [gf]	1869.2	1516.8	1617.7	1776.4	1728.2	1659.8	

## Table2-3 The results of measured weights W<sub>water</sub>

By dividing the average weights by the total weight of the cart, the maximum static frictional coefficients are estimated. In the case of no lubrication and organic molybdenum the value of the static frictional coefficient  $\mu_s$  is about 0.17, and in the case of solid molybdenum it is about 0.20.

Table2-4	Maximum	static	frictional	coefficients

$\mu_s$ is evaluated from $W_{water}/W_{total}$ .								
	No lubrication	Solid molybdenum						
W <sub>water</sub> [gf]	1426.8	1480.6	1659.8					
W <sub>total</sub> [gf]	8500.7	8500.7	8500.7					
$\mu_{s}$	0.168	0.174	0.195					

 $\mu_{\rm s}$  is evaluated from W<sub>water</sub>/W<sub>tota</sub>

## Type-2

The time history responses of the displacement of the cart are shown in Fig. 2-2. These graphs show that the cart moves more slowly in the case of vertical vibration than the case of non-vibration. The difference between the cases in which the acceleration is more than 1G (1100Gal) and the cases in which it is less than 1G (900Gal) is not large. Response curves vary widely in the case of solid molybdenum.

#### **IDENTIFICATION OF FRICTIONAL COEFFICIENTS FOR CASE-1**

With these results the static and dynamic frictional coefficients and exponential damping coefficient are estimated by introducing the system identification method.



#### Method of identification

The equation of motion is expressed as follows

$$(m + m^*)\ddot{x} + \operatorname{sgn}(\dot{x})\mu mg = m^*g$$
 (3-1)  
where  $m$ : mass of the cart  
 $m^*$ : mass of the weight  
 $\mu$ : frictional coefficient  
 $g$ : gravity

In general, the frictional coefficient may depend on surface pressure, velocity and temperature. In this paper, considering the effect of velocity, the frictional coefficient is assumed to be expressed by the following relation.

$$\mu = \mu_d + (\mu_s - \mu_d)e^{-c|v|}$$
(3-2)  
where  $\mu_s$ : static frictional coefficient  
 $\mu_d$ : dynamic frictional coefficient  
 $c$ : exponential damping coefficient  
 $v$ : relative velocity

The values of  $\mu_s$ ,  $\mu_d$  and c are presumed by using the results of the displacement responses in Type-1 and Type-2. The observed responses are denoted as  $x_i^*$ . Giving the initial values of velocity and acceleration to Equation (3-1), the displacement responses  $x_i$  can be calculated by the Newmark  $\beta$ method. The following target function f is adopted.

$$f = \sum (x_{i}^{*} - x_{i})^{2}$$
(3-3)

Since  $x_i$  is determined by  $\mu_s$ ,  $\mu_d$  and c uniquely, this target function will be close to zero when  $\mu_s$ ,  $\mu_d$  and c approach appropriate values. The slowdown Newton method described below is used as the method for minimization.

#### **Slowdown Newton Method**

The minimizing procedure of the target function f can be summarized as follows.

(1) Let:  $\mathbf{p}^T = \{ p(1), p(2), p(3) \}$ 

where the components of **p** are  $p(1) = \mu_s$ ,  $p(2) = \mu_d$  and p(3) = c.

(2) Put:  $p^k, k = 1$ 

(3) Calculate:  

$$\mathbf{g}^{T}(\mathbf{p}^{k}) = \{\frac{\partial}{\partial p_{1}}, \frac{\partial}{\partial p_{2}}, \dots, \frac{\partial}{\partial p_{n}}\}$$
(4) Calculate:  

$$\mathbf{H}(\mathbf{p}^{k}) = \begin{bmatrix} \frac{\partial^{2}f}{\partial p_{1}^{2}} & \frac{\partial^{2}f}{\partial p_{1}\partial p_{2}} & \dots & \frac{\partial^{2}f}{\partial p_{1}\partial p_{n}} \\ \frac{\partial^{2}f}{\partial p_{2}\partial p_{1}} & \frac{\partial^{2}f}{\partial p_{2}^{2}} & \dots & \frac{\partial^{2}f}{\partial p_{2}\partial p_{n}} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{\partial^{2}f}{\partial p_{n}\partial p_{1}} & \frac{\partial^{2}f}{\partial p_{n}\partial p_{2}} & \dots & \frac{\partial^{2}f}{\partial p_{n}^{2}} \end{bmatrix}$$
(5) Calculate:  

$$\Delta \mathbf{p}^{k} = -\mathbf{H}^{-1}(\mathbf{p}^{k})\mathbf{g}(\mathbf{p}^{k})$$
(6) Put:  

$$\mathbf{p}^{k+1} = \mathbf{p}^{k} + \alpha\Delta \mathbf{p}^{k}$$
(7) If:  

$$\|\mathbf{p}^{k} - \mathbf{p}^{k+1}\|_{\infty} < \varepsilon \cdot \max\left\{\mathbf{l}, \|\mathbf{p}^{k}\|_{\infty}\right\} \text{ then go to Step (9)} \\ \text{where } \|\mathbf{p}^{k}\|_{\infty} = \max\left\{p_{1}^{k}, \dots, p_{n}^{k}\right\}$$
(8) Replace:  

$$\mathbf{p}^{k} = \mathbf{p}^{k+1}, \ k = k+1 \text{ and return to Step(3)} \\ (9) \text{ Solution:} \qquad \mathbf{p} = \mathbf{p}^{k+1} \text{ and stop}$$

In step (6), in this research  $\alpha = 0.1$  is adopted. It is true that  $\alpha = 1$  is valid if the target function *f* is convex. But generally we could not expect convexity. In that case, over shooting occurred and the solution will diverge. In order to avoid those overshootings and divergence, the value of  $\alpha$  needs to be smaller than 1. We call this simple method as the slowdown Newton method.

## Identified values of $\mu_s$ , $\mu_d$ and c

#### Type-1

Table 3-1 shows identified values of  $\mu_s$ ,  $\mu_d$  and *c* using the results of Type-1. Comparing the  $\mu_s$  of Table 3-1 with those of Table 2-4, identified values seem to be reasonable. The static frictional coefficients  $\mu_s$  are about 0.17~0.18. The dynamic frictional coefficient  $\mu_d$  in the case of no lubrication is about 0.16, in the case of organic molybdenum is about 0.04, and in the case of solid molybdenum is about 0.07.  $\mu_d$  in the case of lubrication are much smaller than  $\mu_d$  in the case of no lubrication.

Table 5-1 The results of identification (Type-1)							
	No lubrication	Organic molybdenum	Solid molybdenum				
$\mu_{s}$	0.166	0.173	0.178				
$\mu_{d}$	0.164	0.037	0.069				
С	0.030	0.352	0.149				

Table 3-1 The results of identification (Type-1)

## Type-2

Table 3-2 shows identified values of  $\mu_s$ ,  $\mu_d$  and *c* using the results of Type2. The static frictional coefficients  $\mu_s$  are in the range 0.16~0.19. The dynamic frictional coefficients  $\mu_d$  in the case of no lubrication and solid molybdenum are in the range 0.14~0.20, while in the case of organic molybdenum they are in the range 0.08~0.10. These tables show that the frictional coefficients are a little larger in the case of vertical vibration than in the case of no vibration. Especially in the case of solid molybdenum the

values of  $\mu_d$  are almost same as the values of  $\mu_s$ . The values of  $\mu_d$  vary widely in the case of solid molybdenum. Therefore, organic molybdenum is a suitable lubricant. The difference between acceleration greater than 1G (1100Gal) and less than 1G (900Gal) is not large.

Table 3-2 The results of Identification (Type-2)	Table 3-2 The	eresults of	identification	(Type-2)
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a) The static frictional coefficients  $\mu_s$ 

	No lubrication		Organic m	olybdenum	Solid molybdenum	
	10Hz	20Hz	10Hz	20Hz	10Hz	20Hz
900Gal	0.181	0.176	0.174	0.170	0.175	0.177
1000Gal	0.138	0.191	0.168	0.186	0.189	0.190
1100Gal	0.159	0.181	0.162	0.186	0.190	0.181

b) The dynamic frictional coefficients  $\mu_{d}$ 

	No lubrication		Organic m	olybdenum	Solid molybdenum	
	10Hz	20Hz	10Hz	20Hz	10Hz	20Hz
900Gal	0.170	0.209	0.101	0.079	0.184	0.165
1000Gal	0.172	0.170	0.100	0.077	0.192	0.154
1100Gal	0.177	0.168	0.076	0.089	0.164	0.142

c) The exponential damping coefficient *c* 

	No lubrication		Organic m	olybdenum	Solid molybdenum	
	10Hz	20Hz	10Hz	20Hz	10Hz	20Hz
900Gal	0.074	0.276	0.209	0.237	0.126	0.283
1000Gal	0.343	0.242	0.285	0.263	0.404	0.531
1100Gal	0.207	0.216	0.225	0.300	0.508	0.394

## **CASE-2 HORIZONTAL EARTHQUAKE EXCITATION**

## **Experimental equipment**

Experimental equipment is shown in Fig.4-1. The details of the cart are shown in Fig. 4-2. This system is a modified version of that shown in Fig.2-2 except the size of the guide table and the cart.

## **Condition of experiments**

The experiments are performed with the two types of conditions shown in Table 4-1.

Table 4-1 Condition of experiments						
Sliding surface	Earthquake excitation	Measurement				
No lubrication						
Lubrication	JIVIA KUDE-INS	Displacement and				
No lubrication		and the cart				
Lubrication						

# Table 4-1 Condition of experiments



To investigate the effect of lubrication, the cases of no lubrication or lubrication are used. Organic molybdenum is adopted as the lubricant. JMA KOBE-NS (maximum acceleration =818Gal) and JMA KOBE-EW (maximum acceleration =617Gal) are introduced as input earthquake excitations. The time scale of these earthquake excitations is condensed to half of its original size due to restriction of displacement of shaking table. Laser sensors measure the horizontal displacement and acceleration of the cart and the shaking table as well. The above test is repeated three times for each two sliding surfaces.

#### **Experimental results (Case-2)**

The time history curves of acceleration are shown in Fig.4-1. For observing near the peak point, the time scale of Fig.4-1 is expanded and shown in Fig.4-2. Although the maximum acceleration of the shaking table is 818Gal or 617Gal, the maximum acceleration of the cart decreases about 200Gal. The rate of RMS value [(RMS value of the cart)/(RMS value of the shaking table)] is 58.4~59.2% in the case of JMA KOBE-NS, and 69.9~71.0% in the case of JMA KOBE-EW. When lubrication is used, response acceleration was a little smaller than when lubrication is not used.





The time history of displacement response is shown in Fig.4-3. The dominant sliding occurred at around 2.5 seconds. Relative displacement is about 15.1~15.3cm in the case of JMA KOBE-NS, and 10.1~11.4cm in the case of JMA KOBE-EW. When lubrication is used, the cart slides a little better than when lubrication is not used.



#### **IDENTIFICATION OF FRICTIONAL COEFFICIENTS FOR CASE-2**

Frictional coefficients are identified by using the result of Case-2. After comparison with the frictional coefficients of Case-1, the validity of the values is checked.

## Method of identification

Method of identification is same as the first half. The equation of motion is different from (3-1) and expressed as follows:

$$m\ddot{x} + \mathrm{sgn}(\dot{x})\mu mg = m\ddot{x}_{g} \tag{5-1}$$

where m: mass of the cart

 $\mu$ : frictional coefficient

g: gravity

 $\ddot{x}_{o}$ : acceleration of shaking table

Definition of the frictional coefficient is different from (3-2) and is expressed by (5-2) and (5-3).

$$\mu = \mu_d + (\mu_s - \mu_d) e^{-c|v|}$$
(5-2)  

$$\mu = \begin{cases} \mu & v \le -v_{\lim}, v_{\lim} < v \\ \frac{|v|}{v_{\lim}} \mu & -v_{\lim} < v \le v_{\lim} \end{cases}$$
(5-3)  
where  $v_{\lim}$ : limit velocity  

$$(5-2) \qquad \mu_s \\ \frac{|\mu_s|}{v_{\lim}} \\ \frac{|\nu_s|}{v_{\lim}} \\ \frac{|\nu$$

Fig.5-1 frictional coefficient

## Identified values of $\mu_s$ , $\mu_d$ and c in Case-2

The results of identified values are shown in Table 5-1. These are average values of every three tests. The static frictional coefficients  $\mu_s$  are 0.17~0.22. The dynamic frictional coefficients  $\mu_d$  are 0.07~0.09. These tables show that the frictional coefficients are a little bigger in the case of lubrication than in the case of no lubrication. Identified values are different from those identified for Case-1. It seems that the values shown in Table 5-1 are more reliable, because they are based on a sampling frequency larger than that of Case-1 and the two lines show good agreement.

(a) JMA KOBE-NS				(b) JMA KOBE-EW		
	No lubrication	Organic molybdenum	_		No lubrication	Organic molybdenum
$\mu_s$	0.218	0.223		$\mu_{s}$	0.165	0.183
$\mu_{d}$	0.072	0.086	_	$\mu_{_d}$	0.074	0.083
С	0.044	0.045	_	С	0.038	0.113

#### Table 5-1 The results of identification (Case-2)

#### Accuracy of identified values

In order to verify the accuracy of the identified values, the time-displacement curves are compared with experimental results. Those are shown in Fig.5-2. The light line shows simulated displacements that are calculated by identified values and the dark line shows experimental results. Here the simulated displacements are based on the identified value corresponding experimental result. As mentioned before experiments are carried out three times for each condition, and identifications have been done three times as well. The following figures show only one case in three. As can be seen from Fig.5-2, the two lines agree with each other. The dominant sliding occurred at around 2.5 seconds.





Fig.5-2 Comparison of simulated displacement (light line) and experimental displacement (dark line)

#### CONCLUSIONS

Aiming at the development of a cheap and simple slide isolation for lightweight structures, a couple of different types of stainless steel are proposed as sliding bearings. Frictional coefficients are identified from compact experimental systems composed of a stainless steel cart and a stainless steel sliding surface. The expression for the frictional coefficient is assumed as  $\mu = \mu_d + (\mu_s - \mu_d)e^{-c|v|}$ . The values for  $\mu_s$ ,  $\mu_d$  and *c* are identified by using the slowdown Newton method and experimental results. It appeared that the static frictional coefficients  $\mu_s$  are 0.17~0.22 and the dynamic frictional coefficients  $\mu_d$  are 0.07~0.09. It is notable that the maximum acceleration of the cart decreases at 200Gal although that of the shaking table is 818Gal or 617Gal. This means that stainless steel can be applicable as a sliding isolator in the case of big earthquakes. The validity of the identified values is assured by comparing the time displacement curves. It shows good agreement with those of experimental curves.

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