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STUDY ON ELASTIC RIGIDITY OF COMPOSITE GIRDER IN STEEL STRUCTURE

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SUMMARY

In this paper, when the framed structures are subjected to lateral load and sustained load, the elastic rigidity of composite girder varies with the change of moment and of rotation angle. In this case, an approximate equation on the elastic rigidity of composite girder is described by the constant value of elastic rigidity. Analytical assumption is that the both ends rotation angle of composite girder are equal. The principal parameters are sustained load, section of steel girder, thickness of slab, column width, shape of column section (H and box), and depth of deck plate. The elastic rigidity of composite girder varies with these. Therefore, this dispersion is described by method of least squares.
The constant value on the elastic rigidity of composite girder is calculated by using steel girder rigidity and composite effect, which has nothing to do with lateral load and sustained load. This analysis was calculated by 1512 cases with the principal parameters. We propose that the constant value on the elastic rigidity of composite girder shows rigidity ratio and composite effect relation. Where, when equivalent moment of inertia in composite girder under positive bending divided by the moment of inertia in steel girder, composite effect. And when an approximate elastic rigidity of composite girder divided by elastic rigidity of steel girder, rigidity ratio.

INTRODUCTION

Figure 1 shows the Moment-Rotation angle relation. The elastic rigidity of composite girder under the positive bending is very different from that under negative bending. The elastic rigidity of composite girder under the positive bending is higher than steel girder, but under negative bending is similar to steel girder. In case of Figure 1, cMy is the yield moment of composite girder. $c\,\theta y$ is the yield rotation angle of composite girder. sMp is the yield moment of steel girder. $s\,\theta y$ is the yield rotation angle of steel girder.
Figure 2 shows Load relation. As the occurrence of sustained load and horizontal load and the both ends rotation angle of composite girder is equal, the superposition principles of elastic theory cannot be used for the elastic rigidity of composite girder. The design of composite girder is difficult to be used in the elastic theory. As shown in Figure 2, hMa is the A end moment under horizontal load. hMb is the B end moment under horizontal load.
For example, the average value of the positive bending rigidity to negative one was used expediently as elastic rigidity of composite girder. It is not exact and unfounded. Therefore, We propose that the constant value on the elastic rigidity of composite girder make use of method of least squares. When the both ends rotation angle of composite girder is equal.

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CASE STUDY ON ELASTIC RIGIDITY OF STEEL COMPOSITE GIRDER

Analytical assumption

Sectional capacity of composite girder is calculated by Design Recommendations for Composite Constructions (AIJ, 1985)[1] and Recommendation for Limit State Design of Steel Structures (AIJ, 1998) [2].
Table 1 is numerical incremental analysis by the principal parameters of 1512 cases, when the sustained load and horizontal load on the fixed end span. sWy(=8sMy/L^2) is the sustained load in elastic limit of steel girder on a simply supported span.
Composite girder is full composite action between steel girder and concrete slab. When the both ends rotation angle of composite girder is equal. The Moment-Rotation angle relation is calculated. Neglect tensile strength of concrete slab under the negative bending.
Sectional shape of composite girder under the positive bending is shown in Figure 3. In case of Figure 3, Be is effective width of the end in composite girder under the positive bending. D is column section.

① Effective width of the end in composite girder under the positive bending is different from column section. When square hollow section column is 1.3 times and H shape section column is 2.0 times.
② A change in effective width from the end of composite girder is 45°.
③ Compressive strength in concrete slab is 0.85 times of compressive strength in concrete slab[3].

Table 1. The principal parameters

steel girder	girder depth (H)	400、500、594、600、612、700、800、900 (mm)
	flange width	≒H/2、≒H/3
span (L)		≒20×H (m)
deck plate depth (hr)		0、75 (mm)
slab thickness (tc)		100、150、200 (mm)
sustained load (W/sWy)		0、0.1、0.2、0.3、0.4、0.5、0.6
stress ratio (σy/Fc)		240/21、330/21
column	section shape	Square hollow section、H-shape section
	size (D)	Square hollow section：300、400、500(mm)
		H-shape section： 400、500、600、700、800、900(mm)

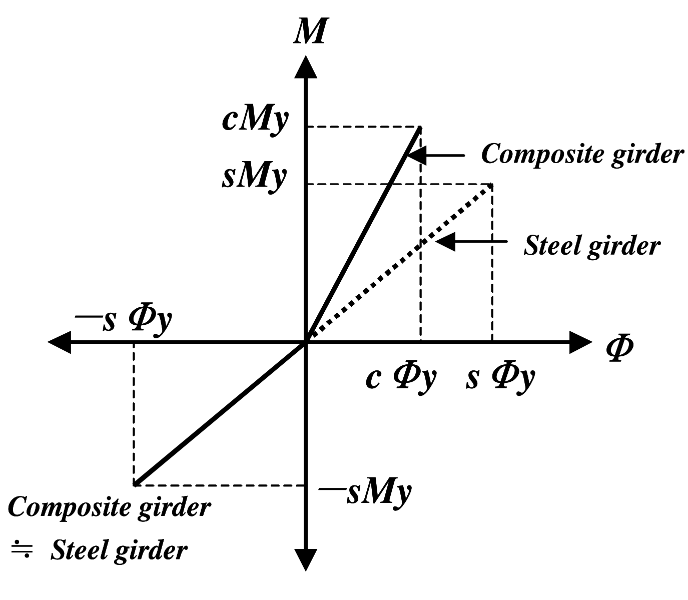

Figure 1. The Moment-Curvature relation

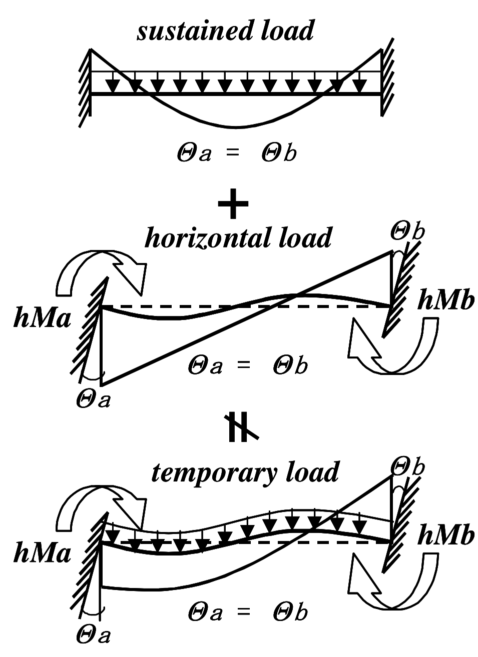

Figure 2. Load relation

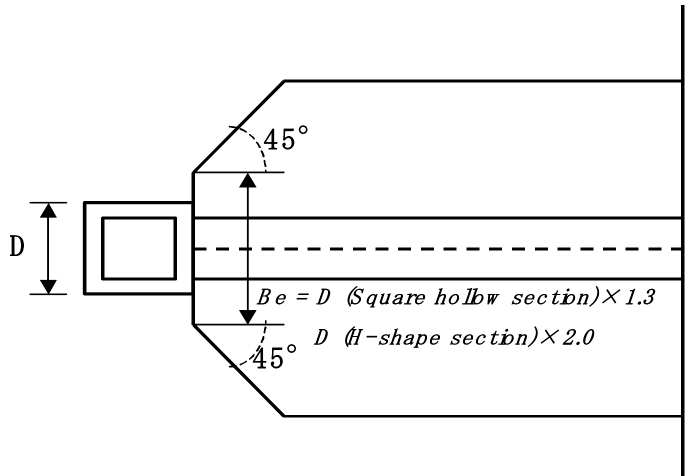

Figure 3. Sectional shape of composite girder under the positive bending

A change of elastic rigidity of composite girder by sustained load

The moment-rotation angle relation and The rigidity-rotation angle relation of composite girder by sustained load are shown in Figure 4 and Figure 5, when consisting of H shape section column of H-600×300×12×20, 75mm deck plate depth and 150mm concrete slab thickness. In Figure 4, the moment shows the sum total of positive bending moment and negative one.

In case of horizontal load (W=0) is a straight line. However, In case of temporary load is a curved line at the start by sustained load and a straight line has come, because negative bending regions of composite girder under sustained load change positive bending regions by horizontal load. Therefore, Initial rigidity is low. According to increase of horizontal load, Elastic rigidity of composite girder under temporary load is similar to elastic rigidity under horizontal load.

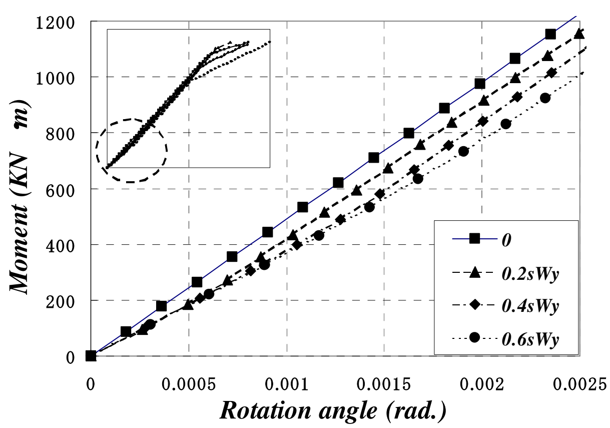

Figure 4. Moment-Rotation angle relation by sustained load

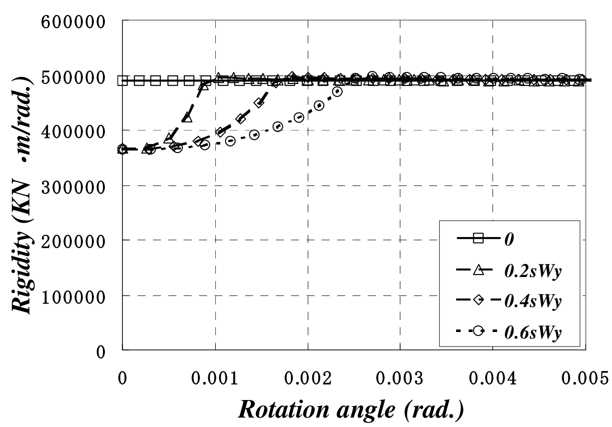

Figure 5. Rigidity-Rotation angle relation by sustained load

PROPOSITION OF ELASTIC RIGIDITY OF COMPOSITE GIRDER

An approximate elastic rigidity of composite girder

We propose an approximate elastic rigidity of composite girder. Figure 6 shows the elastic rigidity model of composite girder. A definition of $\theta 1$ rotation angle as shown in Figure 7, when negative bending regions of composite girder under sustained load change positive bending regions by horizontal load, Moment is M1, Rotation angle is $\theta 1$. When arrive in first yield moment between the both ends of composite girder under temporary load, Moment is My, Rotation angle is θy. When arrive in first yield moment between the both ends of composite girder under only horizontal load, Moment is hMy, Rotation angle is $h\theta y$.

Ksc1 of approximate elastic rigidity connected between the starting point (0, 0) and point (M1, $\theta 1$). Ksc2 of approximate elastic rigidity connected between point (M1, $\theta 1$) and point (My, θy). Ksc3 of approximate elastic rigidity connected between the starting point (0, 0) and point (My, θy). Ksco of approximate elastic rigidity connected between the starting point (0, 0) and point (hMy, $h\theta y$).

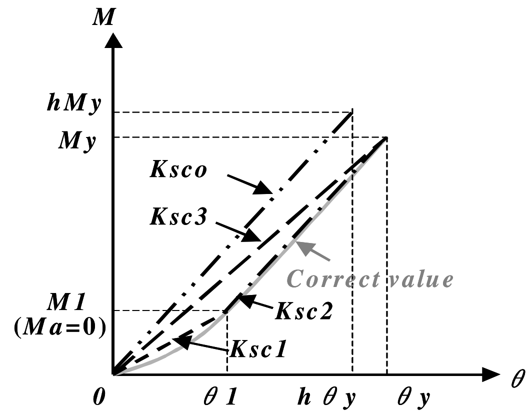

Figure 6. The elastic rigidity model of composite girder

An approximate rigidity and sustained load relation are shown in Figure 8, when consisting of H shape section column of H-600×300×12×20, 75mm deck plate depth and 150mm concrete slab thickness. A horizontal axis is sustained load divided by sWy($=8sMy/L^2$) that it is the sustained load in elastic limit of steel girder on a simply supported span. A vertical axis is an approximate elastic rigidity divided by elastic rigidity of steel girder.

Ksc1 is smaller than Ksc2 and Ksc3 by affecting on sustained load. According to increase of sustained load, Ksc2 become smaller, and Ksc3 is smaller than Ksc2.

Figure 9 shows an approximate rigidity and composite effect relation in all 1512 cases. A horizontal axis is composite effect, when equivalent moment of inertia in composite girder under positive bending divided by the moment of inertia in steel girder. A vertical axis is an approximate elastic rigidity divided by elastic rigidity of steel girder. An approximate elastic rigidity of composite girder is described by method of least squares.

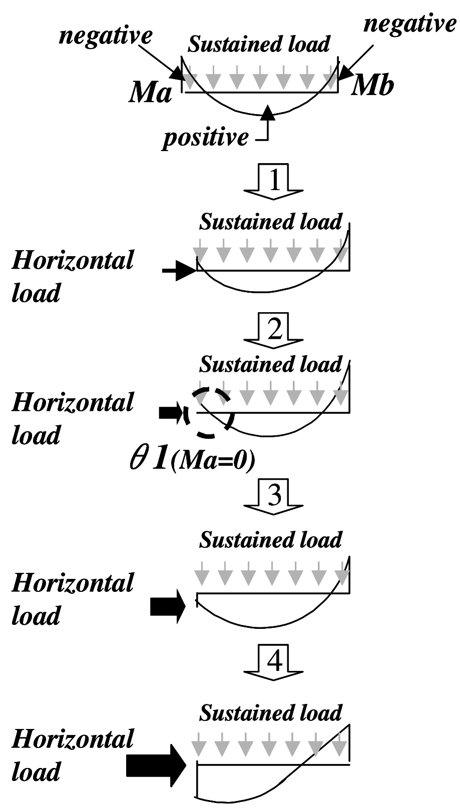

Figure 7. A definition of $\theta 1$ Rotation angle

Ksc1 = Ks { 0.16 (cI/sI) ＋ 0.9 } -------------- (1)
Ksc2 = Ks { 0.29 (cI/sI) ＋ 0.7 } -------------- (2)
Ksc3 = Ks { 0.27 (cI/sI) ＋ 0.73 } -------------- (3)
Ksco = Ks { 0.29 (cI/sI) ＋ 0.7 } -------------- (4)

Where, Ks = (12・E・sI)/L (E: elastic modulus, sI: The moment of inertia in steel girder, L: span)

Proposition of rotation angle θ y and θ 1

When negative bending regions of composite girder under sustained load change positive bending regions by horizontal load, rotation angle is θ 1. When arrive in first yield moment between the both ends of composite girder under temporary load, rotation angle is θ y. Rotation angle (θ 1 , θ y) change by stress alteration of member. However, if Rotation angle (θ 1 , θ y) divided by s θ y yield rotation angle of steel girder, regardless of stress alteration of member, and rotation angle ratio (θ 1/ s θ y , θ y/ s θ y) have the constant value.

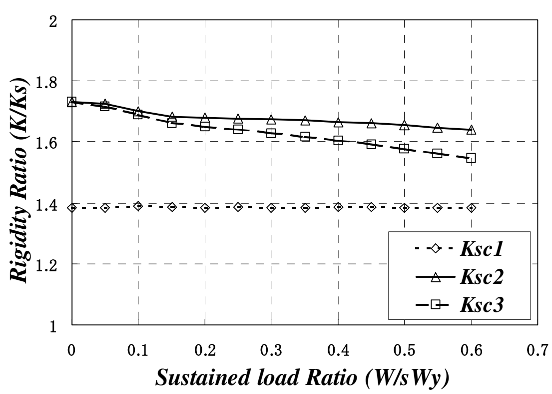

Figure 8. Approximate rigidity-Sustained load

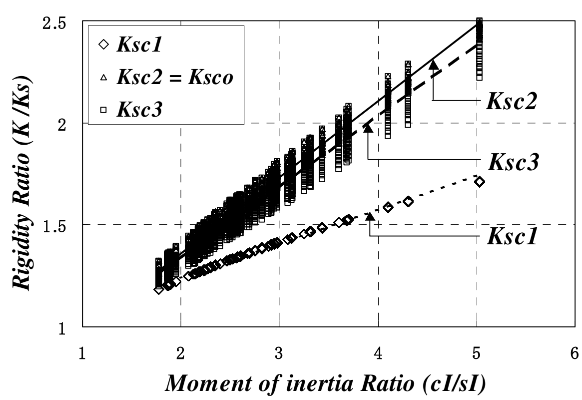

Figure 9. Approximate rigidity-Composite effect

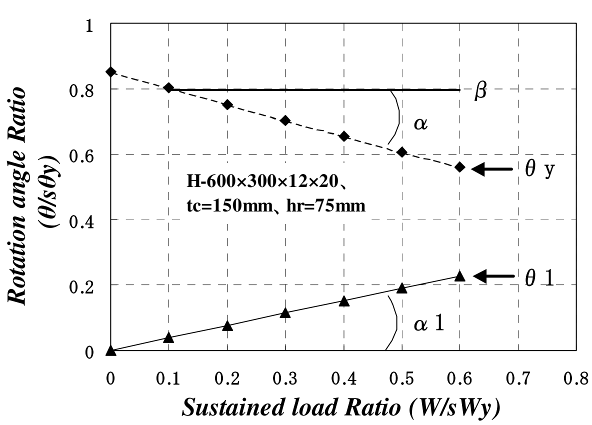

Figure 10. Rotation angle (θ 1, θ y)-Sustained load

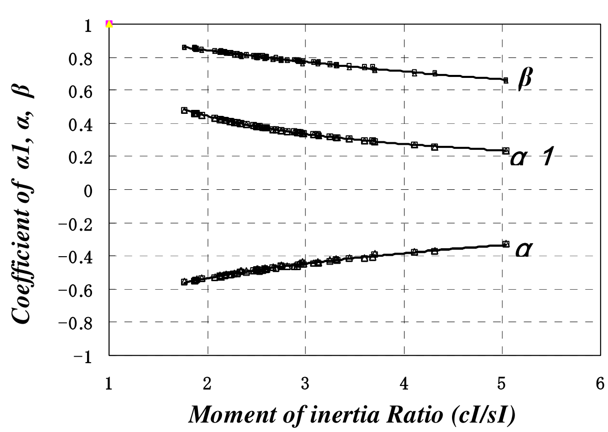

Figure 11. Coefficient (α 1, α , β)-Sustained load

Rotation angle ($\theta 1$, θy) and sustained load relation are shown in Figure 10, when consisting of H shape section column of H-600×300×12×20, 75mm deck plate depth and 150mm concrete slab thickness. Rotation angle ($\theta 1$, θy) is proportional to sustained load ratio(W/sWy) and linear relationships. $\alpha 1$ shows rotation angle ratio $\theta 1/ s\theta y$ and sustained load ratio(W/sWy) relation. When β is 0.1 sustained load ratio(W/sWy), α shows rotation angle ratio $\theta y/ s\theta y$ and β relation.

$$\theta 1 = \alpha 1 \times (W/sWy) \times s\theta y \qquad \text{-------------- (5)}$$
$$\theta y = \{ \alpha \times (W-0.1)/sWy + \beta \} \times s\theta y \qquad \text{-------------- (6)}$$

Figure 11 shows a coefficient of $\alpha 1$, α, β and (cI/sI) composite effect relation by all 1512 cases. A coefficient of $\alpha 1$, α, β described by method of least squares.

$$\alpha 1 = 0.713 \times (cI/sI)^{-0.7} \qquad \text{------------- (7)}$$
$$\alpha = 0.22 \times \ln(cI/sI) - 0.69 \qquad \text{-------------- (8)}$$
$$\beta = 0.006 \times (cI/sI)^2 - 0.1 \times (cI/sI) + 1.02 \qquad \text{------------ (9)}$$

ERROR BETWEEN APPROXIMATE VALUE AND CORRECT VALUE

We checked absolute and relative errors in all 1512 cases. A definition of absolute error and relative error are shown as in Figure 12 suggest, Absolute error means difference between correct value and approximate value divided by yield moment of composite girder, and Relative error means difference between correct value and approximate value divided by at that time moment of composite girder.

Figure 13 shows the relative error of an approximate rigidity. Figure 14 shows the absolute error of an approximate rigidity. A horizontal axis is rotation angle divided by θy rotation angle when arrive in first yield moment between the both ends of composite girder under temporary load. Ksc1,2 shows Ksc1 and Ksc2. Ks+c shows the average value of the positive bending rigidity to negative one was used expediently as elastic rigidity of composite girder. However, it is not exactly and unfounded.

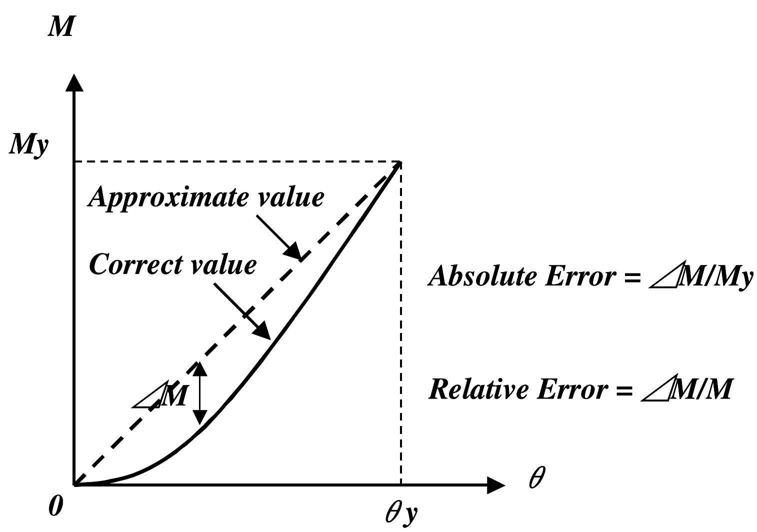

Figure 12. Definition of Absolute error and Relative Error

According to the increase of rotation angle ratio (θ / θ y), Relative error is low, but Absolute error is high. Relative error and Absolute error of Ksc1,2 are smaller than Ksc3, Ksco and Ks+c. Relative and Absolute errors of Ksco are similar to Ksc3. And Relative error and Absolute error of Ks+c is higher than the other approximate rigidity.

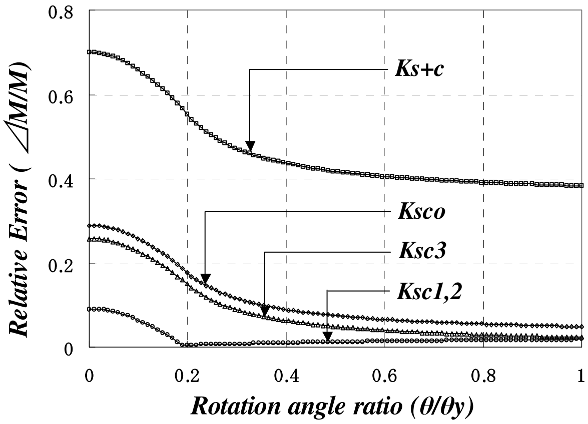

Figure 13. Relative Error of Approximate rigidity

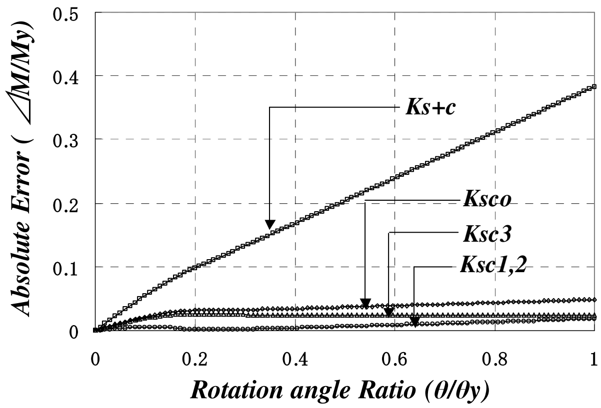

Figure 14. Absolute Error of Approximate rigidity

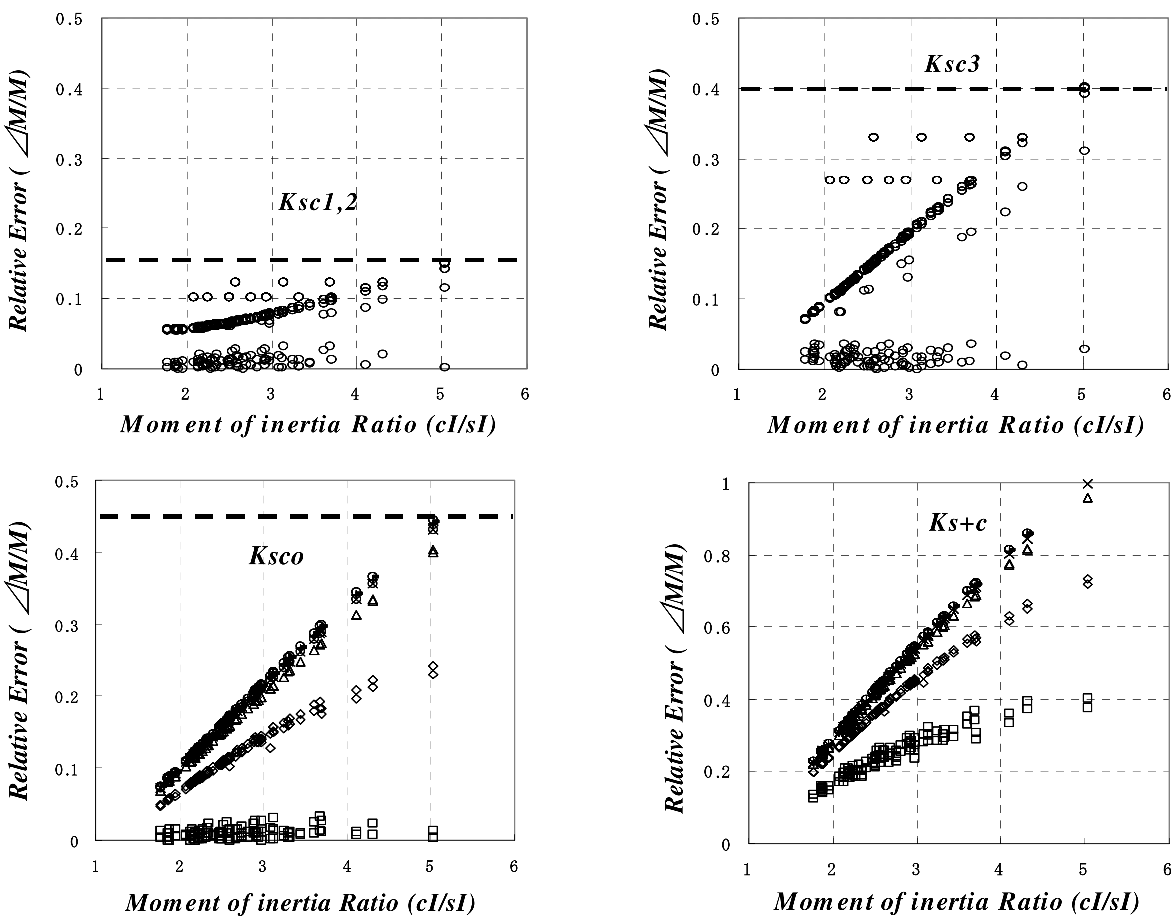

Figure 15. The maximum Relative Error of Approximate rigidity

The maximum relative error of approximate rigidity is shown in Figure 15, a horizontal axis is composite effect, and a vertical axis is relative error. Ksc1,2 shows below 15% of the maximum relative error. Ksc3 shows below 40%. Ksco shows below 45%. According to increase composite effect (cI/sI), Ks+c of the maximum relative error is higher.

The maximum absolute error of approximate rigidity is shown in Figure 16, a horizontal axis is composite effect, and a vertical axis is absolute error. Ksc1,2 shows below 4% of the maximum absolute error. Ksc3 shows below 6%. Ksco shows below 6%. Ks+c shows below 50%.

Therefore, are shown in Figure 15 and Figure 16, Ksc3 and Ksco are near equal, and an error of Ks+c is higher than the other approximate rigidity.

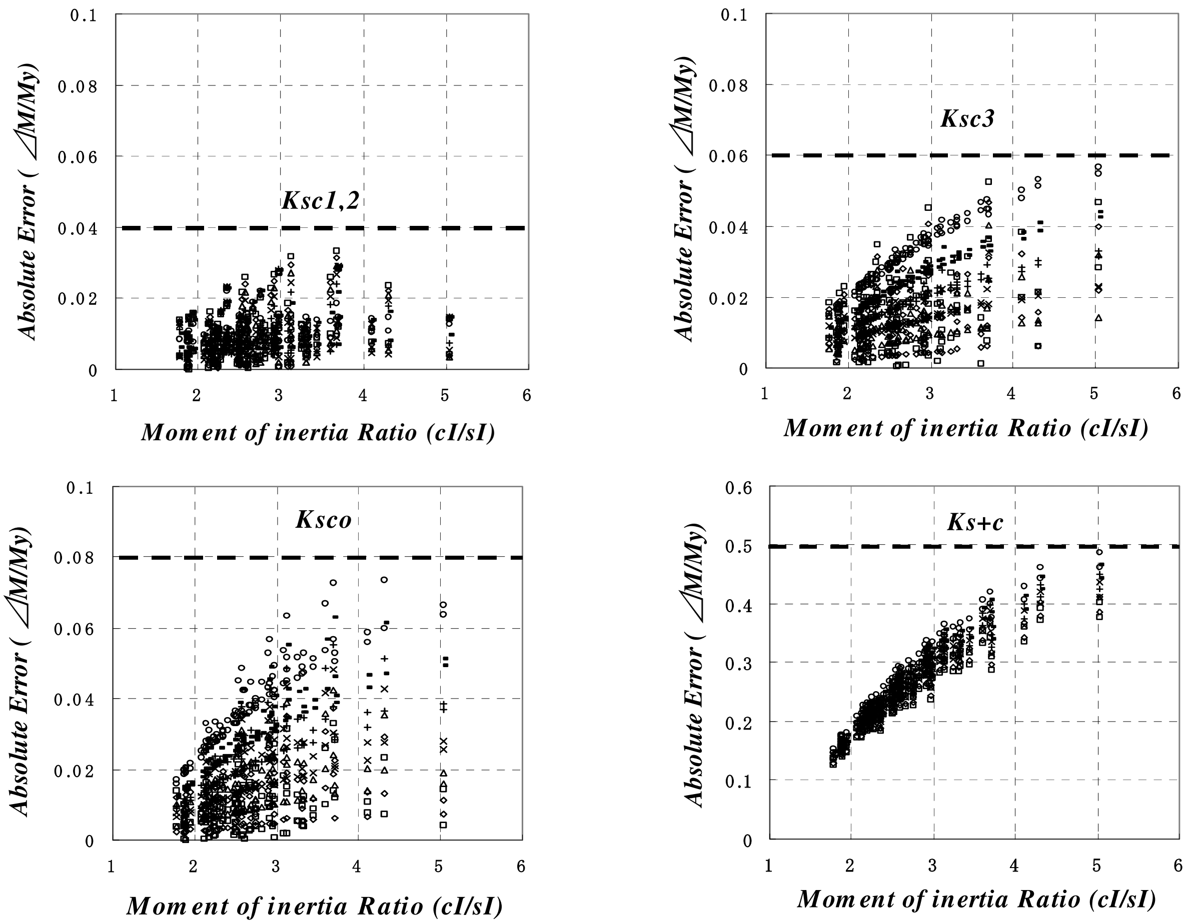

Figure 16. The maximum Absolute Error of Approximate rigidity

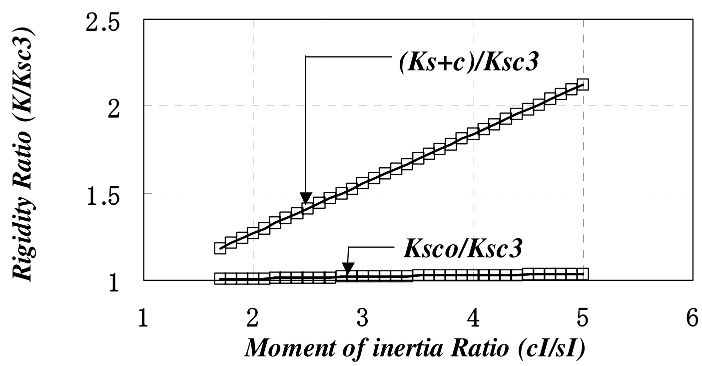

Figure 1 7 . Compare of Approximate rigidity by Composite effect

Compare of approximate rigidity by composite effect are shown in Figure 17, Ksco of approximate rigidity is high 1.03 times of Ksc3. Ks+c of approximate rigidity is high 1.5 times of Ksc3.

CONCLUSION

1. One elastic approximate rigidity of composite girder is described as follow,

   $K = Ks \{ 0.27 (cI/sI) + 0.73 \}$

   One elastic approximate rigidity shows below 6% of the maximum absolute error in 1512 cases.

   Where,
   Ks =elastic rigidity of steel girder,
   cI = The equivalent moment of inertia in composite girder under positive bending,
   sI = The moment of inertia in steel girder.

2. Two elastic approximate rigidity of composite girder are described as follow,

   For $\theta < \theta 1$,

   $$K = Ks \{ 0.16 (cI/sI) + 0.9 \}$$

   For $\theta >= \theta 1$,

   $$K = Ks \{ 0.29 (cI/sI) + 0.7 \}$$

   Where, $\theta 1 = 0.713 \times (cI/sI)^{-0.7} \times (W/sWy) \times s\theta y$
   $\theta 1$ = rotation angle when negative bending regions of composite girder under sustained load change positive bending regions by horizontal load,
   θ = rotation angle of composite girder under temporary load,
   $s\theta y$ = yield rotation angle of steel girder,
   Ks = elastic rigidity of steel girder,
   cI = The equivalent moment of inertia in composite girder under positive bending,
   sI = The moment of inertia in steel girder,
   (W/sWy) = sustained load ratio,
   sWy = the sustained load in elastic limit of steel girder on a simply supported span.

3. Ks+c of elastic approximate rigidity is high 1.5 times of elastic approximate rigidity formula in conclusion 1.

   Where,
   Ks+c shows the average value of the positive bending rigidity to negative one was used expediently as elastic rigidity of composite girder.

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