

EVALUATION OF SECOND ORDER EFFECTS ON THE SEISMIC PERFORMANCE OF RC FRAMED STRUCTURES: A FRAGILITY ANALYSIS

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SUMMARY

This paper is aimed at probabilistic evaluation of second order effects (P- Δ effects) on the inelastic response of RC framed structures subjected to seismic excitations. To this purpose, the study focuses on the seismic performance of a double span, 8-story RC frame, designed according to the last version of Eurocode 8 (EC8) for the high ductility class. Seismic response of the structure has been obtained by performing a non linear dynamic analysis taking into account the most important degradation factors affecting cyclic behaviour of RC structures. Randomness in seismic input has been introduced by considering an ensemble of twenty time-histories whose mean elastic spectrum fits the assumed EC8 elastic spectrum. Top displacement and interstorey drift have been assumed as response parameters to check the seismic response of the structure and compared with the limit values characterizing the assumed limit states, as provided by Structural Engineers Association of California (SEAOC) for RC framed structures. The probability of exceeding the assumed limit states for the sample structure subjected to seismic excitation having increasing intensity has been represented through fragility curves. Comparison among fragility curves, evaluated with and without second order effects, has evidenced a remarkable influence of such effects in defining structural performance and, then, safety levels related to the assumed limit states.

INTRODUCTION

This work deals with evaluation of second order effects (P- Δ effects) on response of RC framed structures subjected to seismic actions. Previous investigations developed by the authors on RC frames designed according to specifications provided by Eurocode 8 [1] for low and high ductility (De Stefano [2][3][4][5]) proved that the amount of second order effects strongly depends on the level of inelasticity experienced by the structure and, as a consequence, on the accuracy of the adopted models in describing mechanical response in the post-elastic phase. In particular, frames designed according to the high ductility specifications proved to be very sensitive to the second order effects, since they suffer large excursions in the inelastic range under the design seismic actions. In fact, while low ductility structures, which experience smaller inelastic deformations, proved to be not sensitive to second order effects, high ductility frames showed a significant increase in ductility demands. In particular, such increase proved to be much larger than that provided by the simplified procedure proposed by the EC8, consisting in an

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amplification of the effects (both displacements and internal forces) through the following amplification factor α :

$$\alpha = \frac{1}{1 - \theta} \tag{1}$$

where θ is the stability coefficient of each storey. In the EC8 approach to the second order effects, the stability coefficient is the most important quantity governing the analysis. It is defined through the following expression:

$$\theta = \frac{P_{TOT} d_r}{V_{TOT} H}$$
(2)

where P_{TOT} is the storey vertical load, V_{TOT} the storey shear, d_r the design interstorey drift and H the interstorey height. According to EC8 provisions, second order effects can be neglected when θ is below 0.10, while the amplification factor α has to be applied if θ ranges from 0.10 and 0.20. Structures having a stability coefficient exceeding 0.30 are considered unacceptable anyway.

While the stability coefficient proved to be a consistent parameters to predict the influence of second order effects in RC framed structures, the classification proposed by EC8 for θ seems to be less satisfactory. In fact, in a frame designed according to EC8 provisions, θ hardly achieves the minimum limit of 0.10, so that second order effects should be always neglected. However, from the numerical predictions, it turns out that P- Δ effects cannot be neglect even if $\theta < 0.10$.

In this paper the evaluation of second order effects on the seismic response of RC framed structures has been conducted by a probabilistic approach, based on fragility curves capable to relate the amount of such effects to the probability of exceeding assumed limit states of the structure, according to performance based design procedures. The seismic input has been represented by an ensemble of twenty ground motions whose spectra fit with a good approximation the elastic spectrum provided by EC8 for soil class C. The investigation has been developed with reference to a double span, 8-story RC frame, designed according to EC8 for the high ductility class. The inelastic response of the structure has been performed by nonlinear dynamic analyses, which led to an accurate representation of the cyclic behaviour of structural elements, taking into account the most important mechanical phenomena affecting their response beyond the elastic range, such as degradation in strength and stiffness and pinching. The deformation levels obtained from the numerical analyses have been compared with the limit values characterizing the assumed limit states, as provided by SEAOC [6] for RC framed structures. The probability of exceeding the assumed limit states for the sample structure subjected to seismic excitation having increasing intensity has been represented through fragility curves. Comparison among fragility curves, evaluated accounting for or not second order effects, has revealed a remarkable influence of such effects in defining structural performance. As a consequence, P- Δ effects have resulted in a reduction of safety levels.

THE NUMERICAL ANALYSIS

Analyzed sample structure

The assumed sample structure is a double span and 8 storey RC frame, whose geometry is shown in Figure 1, together with the dimensions of each element. The frame has been designed according to specifications of EC8, with seismic input represented through the design spectrum provided by the same code for PGA equal to 0.35 g and for a soil type C. A concrete C25, having a cubic strength of 30 MPa, and a steel Grade 450 A, having a yield stress of 450 MPa have been assumed for determining cross section

capacities. Reinforcement has been dimensioned according to the specifications provided by EC8 for high ductility framed structures. In Figure 1 the values obtained for the stability coefficient θ at each level of the structure are also shown. Although the sample structure has been designed in order to be as much slender as possible according to EC8 provisions, the stability coefficient does not reach the limit value of 0.10. As a consequence, based on EC8 approach, second order effects could be neglected.

F		cross sections: $B \times H$ (cm)				
	storey	external columns	internal columns	beams*	- 0	
3.20	8	30×30	40×40	30×45	0.062	
3.20	7	30×30	40×40	30×50	0.074	
3.20	6	30×35	40×45	30×55	0.078	
3.20	5	30×40	40×45	30×60	0.078	
3.20	4	30×45	40×50	T $(30/70) \times 60$	0.076	
3.20	3	30×50	40×50	T $(30/70) \times 60$	0.073	
3.20	2	30×55	40×55	T (30/70) × 65	0.068	
3.20	1	30×55	40×55	T (30/70) × 65	0.046	

* T shaped beams: $(b/B) \times H$

Figure 1. The sample structure.

Seismic input

6.00

An ensemble of twenty ground motions has been assumed in the analysis in order to account for randomness in the seismic input. In Table 2 the main characteristics of the assumed set of ground motions are listed. Their elastic spectra are shown in Figure 2 with the mean spectrum which fits pretty well the elastic spectrum provided by EC8 for the assumed class of soil (type C) and periods larger than 0.78 sec. Since the sample structure has a fundamental period of 1.2 sec, it can be considered that the selected ensemble represents well the EC8 seismic action for this case study.



Figure 2. Elastic spectra of the assumed ground motions and EC8 spectrum.

EQ code	Description	Magnitude	Distance (km)	Scale Factor	Duration (sec)	PGA (g)
La01	fn Imperial Valley, 1940, El Centro	6.9	10.0	1.675	53.48	0.383
La02	fp Imperial Valley, 1940, El Centro	6.9	10.0	1.675	53.48	0.567
La03	fn Imperial Valley, 1979, Array #05	6.5	4.1	0.842	39.39	0.325
La04	fp Imperial Valley, 1979, Array #05	6.5	4.1	0.842	39.39	0.408
La05	fn Imperial Valley, 1979, Array #06	6.5	1.2	0.700	39.39	0.250
La06	fp Imperial Valley, 1979, Array #06	6.5	1.2	0.700	39.39	0.192
La07	fn Landers, 1992, Barstow	7.3	36.0	2.667	80.00	0.350
La08	fp Landers, 1992, Barstow	7.3	36.0	2.667	80.00	0.358
La09	fn Landers, 1992, Yermo	7.3	25.0	1.808	80.00	0.433
La10	fp Landers, 1992, Yermo	7.3	25.0	1.808	80.00	0.300
La11	fn Loma Prieta, 1989, Gilroy	7.0	12.0	1.492	40.00	0.558
La12	fp Loma Prieta, 1989, Gilroy	7.0	12.0	1.492	40.00	0.808
La13	fn Northridge, 1994, Newhall	6.7	6.7	0.858	60.00	0.567
La14	fp Northridge, 1994, Newhall	6.7	6.7	0.858	60.00	0.550
La15	fn Northridge, 1994, Rinaldi RS	6.7	7.5	0.658	15.95	0.442
La16	fp Northridge, 1994, Rinaldi RS	6.7	7.5	0.658	15.95	0.483
La17	fn Northridge, 1994, Sylmar	6.7	6.4	0.825	60.00	0.475
La18	fp Northridge, 1994, Sylmar	6.7	6.4	0.825	60.00	0.683
La19	fn North Palm Springs, 1986	6.0	6.7	2.475	60.00	0.850
La20	fp North Palm Springs, 1986	6.0	6.7	2.475	60.00	0.825

Table 1. Set of ground motions assumed in the dynamic analysis.

Dynamic analysis

Non linear dynamic analysis, despite of its computational effort, is the most effective tool to investigate the inelastic response of structure under seismic excitation. Of course, effectiveness of the analysis depends primarily on the quality of the representation of each phenomenon describing the inelastic behavior of each element of the structure.

Dynamic analysis has been performed in this study by the IDARC2D program (Valles [7]). The behaviour of each element has been described through bi-linear (beams) or three-linear (columns) moment-curvature relationships, taking into account axial loads due to gravity loads and assuming the Kent and Park model [8] for the confined concrete and the elastic perfectly plastic relationship for the reinforcement steel. The assumed interaction domain for bending moments and axial loads is shown in Figure 3.

A linear distribution of the inelastic deformation has been assumed at the critical regions of the elements, and two different evolutive-degrading hysteretic models (Sivalsen [9]) have been assumed for beams and columns. The hysteretic model, shown in Figure 4 takes into account strength and stiffness degradation and pinching. In order to perform a fragility analysis, a seismic excitation of increasing intensity has been applied to the sample structure. The considered values of PGA range from a minimum of 0.20g to a maximum of 0.55g with a step of 0.05g. The dynamic analysis has been performed with and without P- Δ effects.





Figure 3. Axial load-bending moment domain.

Figure 4. Evolutive-degrading hysteretic model.

RESULTS

Interstorey drifts

Tables 2 and 3 show results, in terms of interstorey drifts, obtained for the sample frame with and without second order effects. In the tables the results found for each ground motion have been listed, together with their mean, standard deviation (*s.d.*) and coefficient of variation (*cov*). As it can be seen, second order effects not only amplify the maximum interstorey drift along the frame height, but also increase the *cov* of the obtained results.

Ground	Interstorey drifts (mm)							
motions	0.2g	0.25g	0.3g	0.35g	0.4g	0.45g	0.5g	0.55g
la01	19.89	29.87	36.45	40.56	38.81	47.56	65.47	76.07
la02	33.87	39.62	47.93	60.85	57.07	61.53	65.14	73.22
la03	19.09	24.50	34.75	43.60	52.67	65.11	80.76	101.68
la04	18.67	20.55	21.41	26.01	27.30	35.52	40.26	50.04
la05	17.46	20.61	26.17	32.76	43.19	58.43	79.14	104.53
1a06	12.57	14.66	16.96	19.44	21.19	25.17	29.86	36.51
la07	20.66	26.58	31.32	35.68	38.62	46.03	52.26	58.59
la08	18.01	23.66	29.80	35.21	40.94	46.09	50.94	59.62
1a09	38.84	53.30	65.35	71.57	85.75	94.56	99.92	107.39
la10	26.72	30.81	33.3	37.56	43.10	47.38	52.32	57.29
la11	31.93	45.63	62.99	80.11	98.01	113.24	128.14	141.25
la12	30.71	43.00	53.01	64.00	65.89	59.51	63.58	65.96
la13	36.77	49.08	53.86	58.33	61.38	67.01	73.62	89.18
la14	32.27	33.61	44.68	50.67	57.08	63.07	82.87	103.38
la15	41.31	47.67	48.97	53.99	57.16	59.38	60.67	71.55
la16	47.49	57.59	67.8	76.75	84.74	92.81	100.24	104.85
la17	25.82	34.32	42.78	49.33	61.40	73.56	83.33	88.47
la18	44.66	60.21	82.41	95.35	104.17	118.99	137.14	138.76
la19	31.18	33.87	34.54	52.44	56.05	41.04	50.26	61.35
la20	37.63	41.84	46.48	59.14	72.56	78.28	91.74	106.55
mean	27.89	34.82	41.96	49.70	55.59	61.65	70.86	80.80
s.d.	11.66	14.96	18.94	21.92	25.02	27.89	31.29	33.32
cov	0.418	0.430	0.451	0.441	0.450	0.452	0.442	0.41

Table 2. Interstorey drifts not accounting for P- Δ effects.

Ground			Ir	nterstorey di	rifts (mm)			
motions	0.2g	0.25g	0.3g	0.35g	0.4g	0.45g	0.5g	0.55g
la01	26.45	35.80	40.25	39.61	43.76	55.07	68.24	91.97
la02	32.32	38.53	54.75	56.80	57.90	62.22	62.49	63.89
la03	19.21	25.57	35.77	43.93	53.87	68.52	88.30	118.91
la04	18.36	20.13	21.40	24.11	27.30	35.52	42.51	57.04
la05	17.46	20.54	27.11	34.18	45.91	65.27	105.15	243.35
1a06	12.09	14.19	16.97	19.89	21.39	25.46	30.69	38.71
la07	21.59	27.12	31.21	35.75	38.68	46.09	48.91	65.82
la08	17.67	24.00	30.13	35.60	41.81	46.16	54.46	61.35
1a09	43.52	53.13	66.68	73.47	88.22	101.23	95.45	110.88
la10	27.73	31.21	33.50	37.00	41.24	46.97	52.79	59.91
la11	33.40	48.21	66.36	85.31	103.67	121.76	138.81	154.28
la12	31.61	42.58	52.80	61.36	60.14	68.56	57.73	59.57
la13	39.87	48.28	53.67	57.15	62.59	72.80	72.21	79.26
la14	30.21	34.40	44.94	50.47	59.04	84.33	113.29	152.43
la15	42.43	47.83	49.77	52.88	54.48	54.11	64.64	76.12
la16	48.86	60.59	70.82	79.01	84.97	89.34	91.03	90.15
la17	26.73	34.97	43.49	51.29	64.58	75.79	83.91	89.11
la18	45.80	63.03	78.80	91.43	111.96	126.60	137.11	139.22
la19	28.57	34.66	40.20	55.70	59.30	51.54	52.55	60.57
la20	35.80	40.92	51.79	64.42	78.20	94.99	104.81	127.50
mean	28.57	35.52	43.37	49.99	57.11	66.32	74.55	92.41
s.d.	12.09	15.40	19.13	22.19	26.45	30.33	34.21	51.64
cov	0.423	0.434	0.441	0.444	0.463	0.457	0.459	0.559

Table 3. Interstorey drifts accounting for P- Δ effects.

The obtained mean results are compared in Figure 5. As it can be seen, the increase in interstorey drift due to P- Δ effects is negligible for PGA below 0.35 g, while it becomes more important for larger values of PGA. In the same figure, the values obtained for the coefficient of variation (*cov*) are reported. The *cov* is almost the same for the two models for PGA under 0.45g, while it increases for larger PGA.



Figure 5. Interstorey drifts from the dynamic analysis: mean values and coefficient of variation.

In order to quantify second order effects on the local response of the sample structure, the obtained mean interstorey drifts have been compared for each value of PGA. The obtained percent difference, shown in Figure 6, increases with the PGA, assuming, for strong ground motions, values over 10%.



Figure 6. Percent difference computed for interstorey drifts.

Assumed limit states

In order to perform a fragility analysis, the obtained interstorey drifts have been compared with assumed limit states. The probability of the structure to exceed each limit state for a given intensity of seismic excitation provides information about its performance level, and therefore about its safety.

The assumed limit states are those provided by SEAOC, shown in Table 4. In the table both the nondimensional values of the interstorey drift and their effective dimensions referred to the sample structure have been listed.

The probability of exceeding the assumed limit states have been also discussed by considering the goals stated in FEMA 356 [10], which indicates detailed objective to achieve as a function of the earthquake hazard and the target performance level. Table 5 shows the matrix provided by FEMA 356 for existing building. The standard objective is the "Base Safety Objective" (BSO), which assume the verification of conditions k and p, evidenced in Table 5.

The goal k refers to a Life Safety (LS) performance level and a 10% in 50 years hazard level (return period equal to 476 years), corresponding to the design PGA equal to 0.35 g. The goal p refers to a Collapse Prevention (CP) limit state and a more severe (2% in 50 years, that is an occurrence period of 2475 years) hazard level. In this study only the goal k can be verified, since the occurrence period of 2475 years corresponds to a PGA value larger than that achieved in the analysis (PGA= 0.55 g).

Limit states	Interstory drift			
Limit states	% H	mm		
Fully Operational (FO)	0.5	16		
Operational (OP)	1.5	48		
Life Safety (LS)	2.5	80		
Collapse Prevention (CP)	5	160		

Table 4. Assumed limit states (SEAOC [9]).

Table 5. Rehabilitation Objectives (FEMA 356, [10]).

		Target Building Performance			
		Levels			
		Operational Performance Level (1-A)	Immediate Occupancy Performance Level (1-B)	Life Safety Performance Level (3-C)	Collapse Prevention Performance Level
ke	50% in 50 years	а	b	С	D
urthqual Hazard Level	20% in 50 years	е	f	g	h
	10% in 50 years	i	j	k	l
Ĕ	2% in 50 years	т	n	0	р

Fragility domains

Fragility curves of the sample structure with and without $P-\Delta$ effects have been determined by assuming a Gaussian statistical distribution over the domain of the response parameter, by using the mean value and the standard deviation found from the sample of the twenty results for every PGA values. Each distribution has been compared with the assumed limit values, so obtaining the corresponding probability of exceed them. For each performance level, therefore, a probability of exceeding, and therefore a point of the fragility curve, has been calculated for every PGAs. Each point of the fragility curve represents the probability of the response parameter (r) of the frame to exceed the assumed limit state (l.s.) under a given intensity of the ground motion (I), according to the following expression:

$$Fragility = P[r>l.s. | I]$$
(3)

The most adopted function to represent the fragility curves is the two-parameters lognormal distribution (Barron Corvera [11]), that can be determined when at least three points are known. In order to fit the curve the points have to belong to the intermediate part of the curve, that is for values not too close to 0 and 1. In this case, the available eight points are fully suitable to fit curves representative of Fully Operational (FO), Operational (OP), Life Safety (LS) limit states, while they seem to be not completely satisfactory to set those corresponding to the Collapse Prevention (CP) limit state. The obtained families of fragility curves for the two studied models (with and without second order effects) are shown in Figure 7.

Figure 8 shows the comparison between the two families of fragility curves referred to each limit state. The domains bounded by the two curves referred to the same limit state represent, for such limit state, the fragility domain due to second order effects. As it can be seen, fragility domains of the FO and OP limit states have a negligible extension, while the LS and CP domains are much larger. It has to be noted that the fragility domain referring to the CP limit state cannot be considered as precise as the others, since the fragility curves involved in such domain are described by only few points. In fact, a better description of such fragility curves would need the achievement of higher values of PGA, which could not be reached because of the onset of model collapse.



Figure 7. Fragility curves with and without second order effects.

The influence of P- Δ effects on the probability of exceeding the assumed limit states has been numerically quantified by comparing the curves referred to the same limit state. The functions expressing the Percent Difference (PD) as a function of PGA for each limit state are shown in Figure 9. As it can be seen, the sensitivity to the second order effects, for each limit state, strongly depends on PGA value. In fact, while the limit states FO and OP are significant for low values of PGA, where P- Δ effects are negligible, the peak of PD for the LS limit state if for PGA=0.45 g, and the one for CP is for PGA=0.65g.



Figure 8. Fragility domains expressing the second order effects.

The increase in probability of exceeding the LS limit state under a ground motion of PGA=0.35g (Base Safety Objective) due to P- Δ effect is of 9.5%. When LS and CP limit states are considered, second order effects are more important for larger values of PGA.



Figure 9. Sensitivity of each limit state to $P-\Delta$ effects.

CONCLUSIONS

A non linear dynamic analysis has been performed on a sample structure in order to evaluate the influence of second order effects on its response under seismic excitation. The slenderness of the assumed sample structure, a double span 8-storey RC frame, measured in terms of stability coefficient, was well below the limit values provided by EC8 to distinguish the second order effects in negligible and significant.

The assumed seismic input consistent of an ensemble of twenty ground motions whose mean elastic spectrum fits very well the elastic spectrum provided by EC8 for the sample structure. The analysis has been repeated for all ground motions and for increasing values of PGA, ranging from 0.20g to 0.55g with steps of 0.05g.

The results obtained from this study show an increase of deformations due to P- Δ effects not negligible at all, since the maximum interstorey drifts along the frame height increases until 13.5% (for PGA = 0.55g). This increase is not only not negligible, but also much larger than that provided by EC8, based on simplified amplification formula.

The obtained second order effects have been analyzed according to the performance based design criteria, by comparing the maximum values of the interstorey drifts with the limit values provided by SEAOC for these response parameters. Four different limit states have been considered: Fully Operational, Operational, Life Safe and Near Collapse. For each value of the PGA, the response domain of the structure, assumed to be normal and therefore characterized by the mean value and the standard deviation calculated over the twenty ground motions, has been compared with the four limit values. The obtained probability of exceeding the assumed limit states has been expressed in terms of fragility curves, in the space PGA-probability of exceeding.

The comparison between fragility curves of the sample frame with and without P- Δ effects evidenced the role played by such effects on the safety level referred to each limit state.

In particular, it has been observed that the probability to exceed the Life Safety limit state increases, due to P- Δ effects, up to 15% even for seismic intensity close to the design one (PGA = 0.4g), while for the Collapse Prevention limit state the increase exceeds 50% for large PGAs.

The obtained results also proved the importance of taking into account P- Δ effects in the evaluation of the response parameters expressed in terms of displacements for RC structures which experience large inelastic deformation. RC building structures designed according to the provisions for high ductility class can present a large sensitivity to P- Δ effects for stability coefficients well below the limit values provided by EC8. Therefore, the classification based on the stability coefficient, as proposed by EC8, seems to need some modification, in order not to underestimate second order effects, and the simplified procedures to predict the increase of response parameters due to such effects should be differentiated for forces and deformation parameters.

REFERENCES

1. CEN. "Eurocode 8: Design of structures for earthquake resistance". Part 1: General rules, seismic actions and rules for buildings. Draft N. 6, Brussels, November 2002.

2. De Stefano, M., Nudo, R., Viti, S. "The influence of P- Δ effects on seismic response of RC framed structures". Proc. of the 3rd International Symposium on Earthquake Resistant Engineering Structures, Malaga, September 2001: 347-356.

3. De Stefano, M., Nudo, R., Viti, S. "Influenza degli effetti del secondo ordine sulla risposta sismica di telai in c.a.". Atti del X Congresso Nazionale "L'Ingegneria Sismica in Italia", Potenza, Settembre 2001 (CD Rom).

4. De Stefano, M., Nudo, R., Viti, S. "Seismic performance of RC multistory frames including P- Δ effects". Proc. of Seventh U.S. National Conference on Earthquake Engineering, Boston (MA), July 2002 (CD Rom)

5. De Stefano, M., Rutenberg, A. "Seismic stability and the force reduction factor of code-designed onestorey asymmetric structures". Earthquake Engineering and Structural Dynamics, 28: 785-803.

6. Kent, D.C., Park, R. "Flexural members with confined concrete". Journal of Structural Division, ASCE, 1971, 97(7): 1969-1990.

7. Sivalsen, M.V., Reinhorn, A.M. "Hysteretic models for deteriorating inelastic structures". Journal of Engineering Mechanics, 2000, 126(6): 633-640.

8. Valles, R.E., Reinhorn, A.M., Kunnath, S.K., Li, C., Madan, A. "IDARC2D: A computer program for the inelastic analysis of buildings". Technical Report NCEER-96-0010, State University of New York at Buffalo, 1996, Buffalo, NY.

9. Structural Engineers Association of California (SEAOC) "Blue Book, Part 2, Preliminary guidelines for performance based seismic engineering", Sacramento, December 1998.

10.Federal Emergency Management Agency. "FEMA-356: Prestandard and Commentary for the Seismic Rehabilitation of Buildings", 2001.

11.Barron Corvera, R. "Spectral evaluation of seismic fragility of structures". PhD Dissertation, Department of Civil, Structural & Environmental Engineering, State University of New York at Buffalo, 2000 Buffalo, N.Y.