



NONLINEAR STATIC AND DYNAMIC ANALYSES — THE INFLUENCE OF MATERIAL MODELLING IN REINFORCED CONCRETE FRAME STRUCTURES

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SUMMARY

There has been considerable research on modelling inelastic behaviour of reinforced concrete. However, nonlinear material models used for seismic response history analyses and for nonlinear static analysis (NSA) procedures tend to be simple. It can be argued that sophisticated material models for a complex material like reinforced concrete are perhaps not essential for earthquake analysis in view of several other uncertainties associated with the seismic phenomenon. This paper examines the influence of material modelling on RHA responses for a simple reinforced concrete frame structure. Five acceleration time histories compatible to elastic design spectrum of Eurocode 8 are used for RHA. Two material models are considered: a concrete damaged plasticity model that uses the Drucker Prager criterion and in which concrete and reinforcement are modelled separately and a homogenized Drucker Prager model. In both cases the influence of strain hardening and strain rate effects are considered. The results show that the design response from RHA analyses is significantly different for the two models. The paper then compares the NSA and RHA responses for the two material models for reinforced concrete. The NSA procedures considered are the Displacement Coefficient Method (DCM) and the Capacity Spectrum Method (CSM). A comparison of RHA and NSA procedures shows that there can be a significant difference in local response even though the target deformation values at the control node match. Moreover, the difference between the mean peak RHA response and the pushover response is not independent of the material model.

INTRODUCTION

Numerical simulation of the behaviour of plain and reinforced concrete has been a subject of intense research. The past two decades have seen a plethora of diverse mathematical models being developed (Lee and Fenves 1998, Lubliner et al. 1989; Bicanic and Pankaj 1990; Willam et al. 1993; Park and Klingner 1997; Lowes 1999, Hibbit et al. 2001). While there has been research to validate these models in a slow loading environment, there has been relatively little work to examine their efficacy under dynamic or seismic conditions. This paper examines in brief the influence of two similar material models, the Concrete Damaged Plasticity model and the Drucker Prager model, on seismic response. Both these

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models are essentially based on the Drucker Prager yield criterion. The comparison is made for a simple reinforced concrete frame structure using response history analysis.

In the past decade there has also been considerable research to develop non-linear static analysis (NSA) procedures that can provide seismic design values. Displacement-based non-linear static analysis exists in several codes in one form or the other. The existing nonlinear static techniques can be broadly divided into two categories: Displacement Coefficient Method (DCM) and Capacity Spectrum Method (CSM). Theoretically, for a general non-linear multiple degrees of freedom system, the peak seismic response (required for design), can at best be approximated by a static procedure. There has been considerable research directed towards improving pushover procedures so they can reflect various aspects of a non-linear dynamic analysis (Chopra and Goel, 1999, 2001, 2002; Antoniou et al., 2002; Farfaj et al. 1988, 1996, 1999). NSA response is frequently compared with that obtained using RHA, which also uses the same material models, to verify the accuracy of the static procedure. A number of features exhibited by reinforced concrete during dynamic or cyclic loading (e.g. progressive degradation with each cycle of loading, influence of strain rate) cannot be easily included in a static procedure. Therefore it is important to examine whether the difference between RHA and NSA response is influenced by the choice of material models. This paper examines some of the differences for both DCM and CSM procedures. A more detailed comparison can be found elsewhere (Lin, 2003; Pankaj and Lin, 2004).

SEISMIC EXCITATION

In this study the seismic excitation was prescribed using the elastic design spectrum of Eurocode 8 (1998) corresponding to Soil Subclass B (limits of the constant spectral acceleration branch $T_b = 0.15$ sec and $T_c = 0.60$ sec respectively) were taken with 5% critical damping and amplification factor of 2.5. The peak ground acceleration used was 0.3 g. The pushover analysis procedures adopted used this spectrum directly.

For response history analyses, to avoid the peculiarity of a particular time history, five compatible time histories were used as suggested by Eurocode 8. For the generation of time histories, the program developed by Basu (unpublished) and used in Basu et al. (1985) was used. Five acceleration time histories (called V, W, X, Y and Z) were generated. All generated histories were also checked to ensure that they satisfy the requirements of Eurocode 8.

MATERIAL MODELS AND EXAMPLE PROBLEM

As discussed earlier two material models, concrete damaged plasticity (CDP) and Drucker Prager (DP), each with a number of variations were considered.

The CDP Model

ABAQUS (Hibbitt, Karlsson and Sorensen, 2001) supplies the Concrete Damaged Plasticity (CDP) model for monotonic, cyclic and dynamic loading. The yield criterion is pressure sensitive and based on the work by Lubliner et al. (1989) and Lee and Fenves (1998). In biaxial compression, the criterion reduces to the Drucker-Prager criterion. The material model uses isotropic damaged elasticity in association with isotropic tensile and compressive plasticity, to represent the inelastic behaviour of concrete. Both tensile cracking and compressive crushing are included in this model. Beyond the failure stress in tension, the formation of micro-cracks is represented macroscopically with a softening stress-strain response. The post-failure behaviour for direct straining is modelled using tension stiffening, which also allows for the effects of the reinforcement interaction with concrete. Thus, this material model reflects the key characteristics of concrete well. The reinforcement was modelled separately using rebar elements with metal plasticity models. In this study, only longitudinal reinforcement was included.

The elastic parameters used for concrete were: Young's modulus, $E_c = 28.6 \times 10^9 \text{ N/m}^2$ and Poisson's ratio, $\nu = 0.15$. The Young's modulus of reinforcement, $E_s = 20 \times 10^{10} \text{ N/m}^2$ is assumed. The parameters of the CDP model, which remained unchanged in this study, were: dilation angle, $\psi = 15^\circ$; ratio of initial equibiaxial compressive yield stress to initial uniaxial compressive yield stress, $\sigma_{b0}/\sigma_{c0} = 1.16$; ratio of the second stress invariant on the tensile meridian to that on the compressive meridian, $K_c = 2/3$; compressive yield strength, $f_c = 20.86 \times 10^6 \text{ N/m}^2$; ultimate compressive strain, $\epsilon_{cu} = 0.0035$; and initial tensile crack stress, $\sigma_{t1} = 1.78 \times 10^6 \text{ N/m}^2$. The yield stress for reinforcement, $f_y = 460 \times 10^6 \text{ N/m}^2$ was assumed.

Either perfect plasticity or hardening plasticity was assumed in compression. For hardening plasticity the hardening modulus $H_c = 0.05E_c$ was assumed, which is similar to some other studies (e.g. Correnza et al. 1992). No strain softening was assumed in compression. Although it is now well recognised that strain softening is not a material property and the strain softening modulus has mesh (or element) size dependence (e.g. Bicanic and Pankaj, 1990), for simplicity a constant strain softening modulus in tension of $H_T = 0.122E_c$ was assumed for all CDP models.

To examine the influence strain rate on dynamic response the results of Bischoff and Perry (1991) were used. They compiled a range of tests conducted by different authors and plotted the ratio of dynamic compressive strength to static strength against logarithm of the strain rate. In this study this linear variation (on log-linear graph) is assumed to increase from static strength to twice the static strength from a strain rate of 5×10^{-5} to 1×10^1 per sec. This is similar to the upper limit suggested by Bischoff and Perry (1991).

The DP Model

The DP model used is relatively simple. It represents the behaviour of reinforced concrete as homogenized continuum with yielding defined by the Drucker Prager criterion. The Drucker Prager criterion uses the cohesion and friction angle as parameters to define yield. Based on previous studies (e.g. Lowes 1999) the friction angle, β was taken as 15 degrees and the uniaxial compressive strength (from which cohesion can be determined) was assumed to be $20.86 \times 10^6 \text{ N/m}^2$. A Young's modulus of $28.6 \times 10^9 \text{ N/m}^2$ and a Poisson's ratio of 0.15 were also assumed.

Once again for the DP model either hardening plasticity ($H_c = 0.05E$) or perfect plasticity was assumed.

Example Problem

The simple four-storey single-bay reinforced concrete frame structure shown in Fig. 1 was used to examine the variation in response due to different analysis methods and material models. The total mass including live load for the frame was calculated as 97000 kg. A damping ratio of 5% was assumed. The finite element model used 2-node cubic beam elements.

ANALYTICAL METHODS

The response history analysis (RHA) is conducted using an implicit integration approach (Hibbit, Karlsson and Sorenson, 2001).

As discussed, two pushover analysis techniques are used. The DCM approach was based on FEMA 273 (1997). FEMA 273 recommends that two different loading patterns be considered. However, in this study the loading is applied according to the first mode pattern only. FEMA 273 does not provide a clear methodology for the determination of yield displacement and strength from the pushover curve. The bilinear curve determined from the pushover curve is often sensitive to the target displacement. This has

been recognised in FEMA 274 (1997). In this study an iterative process is used to evaluate the yield values.

The CSM procedure adopted is numerical (rather than graphical) based on the studies of Fajfar (1999), Chopra and Goel (1999) and Vidic et al. (1994).

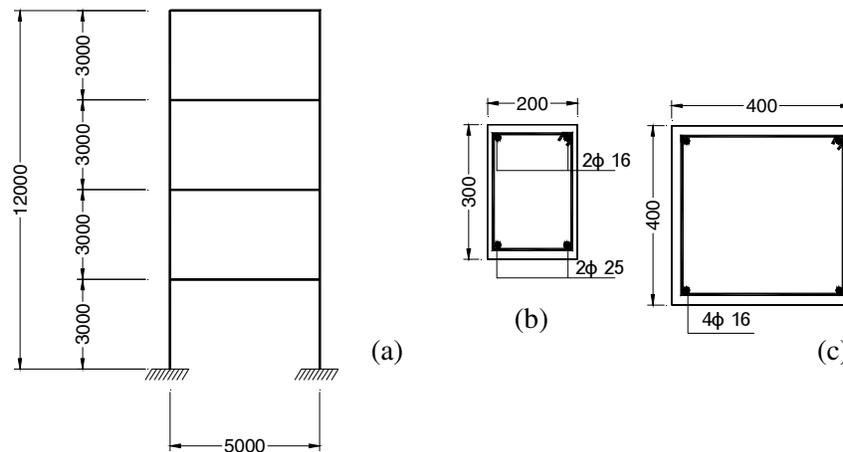


Fig. 1: The four-storey frame used (a) dimension; (b) beam cross section; (c) column cross section

INFLUENCE OF MATERIAL MODELLING ON DYNAMIC RESPONSE

CDP Material Model

Some typical responses of the structure subjected to different excitation histories were examined. A visual inspection of the time histories of response shows that there is little influence of hardening parameter or strain rate on the design parameters.

The values of typical peak responses were examined for all time histories. For example, Table 1 lists the peak top displacements. In this table HP indicates hardening plasticity and PP indicates perfect plasticity. From Table 1 it can be seen that there is little influence of strain rate for any of five earthquakes. Comparing the response between the hardening and perfect plasticity, it can be seen that the differences are again small with maximum for earthquake Y (~5%). In fact, the major difference in the peak response is due to different excitation histories. For example the top deformation of earthquake history X is around 28% higher than the mean peak value. The analysis showed that the peak strain rate during seismic excitation was around 0.004 per sec. However, this does not appear to influence the peak response significantly. Similarly it was found that the influence of strain rate on base shear (not shown) is small and the influence of hardening parameter is even smaller. Interestingly, the base shear values did not vary significantly for different earthquake histories. The maximum variation was found to be around 8% from the mean. This indicates that earthquake excitation histories have larger influence on top deformation than on base shear. This is apparently due to the flat load-displacement response in the inelastic range.

Response of a local parameter *viz.* peak moment at a base node showed a slightly higher variation due to strain rate effect (maximum ~9%), but the influence of hardening parameter was still found to be small (maximum ~4%).

The above results show that the hardening parameter and strain rate effects as used in this study have little influence on the peak response for the CDP model.

Table 1: The peak top deformation (m) in the structure modelled using CDP

Model	Strain rate included	Earthquake history					Mean
		V	W	X	Y	Z	
CDP-HP	No	0.18	0.17	0.27	0.18	0.25	0.21
	Yes	0.18	0.17	0.26	0.17	0.24	0.21
CDP-PP	No	0.18	0.17	0.27	0.17	0.25	0.21
	Yes	0.18	0.17	0.26	0.16	0.25	0.21

DP Material Model

Typical responses of the frame for different excitation histories were again examined. These indicated that the general pattern of responses is similar in all the cases. There appeared to be no difference in the frequency content of the response while the amplitude quantities for different cases appear to have a relatively more significant difference.

Once again the peak values of various response quantities were examined. The peak top deformation (Table 2) shows that strain rate can play a relatively more significant role in this model (this model is isotropic and with no strain softening). The response to excitation Z shows a 23% difference due to strain rate. On the other hand the difference is only about 2% for excitation W. The influence of hardening parameter also varies significantly from one excitation to another. It is interesting to note that in the dynamic environment hardening can cause either an increase or decrease in the peak response.

The peak base shear variations were also examined and once again showed that the variation of base shear for different histories is not as significant as top deformation. Examining the local parameter – moment at a base node again showed a significant influence of strain rate and hardening parameter for some excitation histories.

Table 2: The peak top deformation (m) in the DP structure

Model	Strain Rate included	Earthquake history					Mean
		V	W	X	Y	Z	
DP-HP	No	0.20	0.15	0.26	0.18	0.21	0.20
	Yes	0.17	0.15	0.24	0.17	0.16	0.18
DP-PP	No	0.18	0.14	0.27	0.18	0.21	0.20
	Yes	0.16	0.14	0.24	0.15	0.17	0.17

Comparison of CDP and DP Material Models

In this section, the response of the frame structure when modelled using CDP and DP is compared. It should be noted that both the models are based on the Drucker Prager criterion. Although CDP and DP models come into play only in the post-elastic domain, it is important to realise that the two models are different even in the elastic domain — the CDP model includes reinforcement bars separately whilst the DP model does not. As a result the CDP model has slightly higher natural frequencies.

Figure 2 shows the variation of top deformation for excitation V. The figure shows the response histories can be significantly different when two different material models are used. Similarly relatively large variation was found for other response quantities for the two material quantities. It is also interesting to see that the peaks and troughs for the two models are similarly located. It can be seen that the direction of the peak response can be different for the two models. For example, the maximum top deformation in the DP model is positive whilst the same quantity for the CDP model is negative.

Time history of the typical internal force responses were found to be consistently smaller for the CDP models. This is apparently because of strain softening included in the CDP models. Comparing the mean peak values from the five earthquakes for the two material models, it was found that the mean top deformation is not significantly different; on the other hand, the base shear values are almost half for the CDP models when compared to the DP models. The low internal force peak responses from the CDP model are clearly due to strain softening.

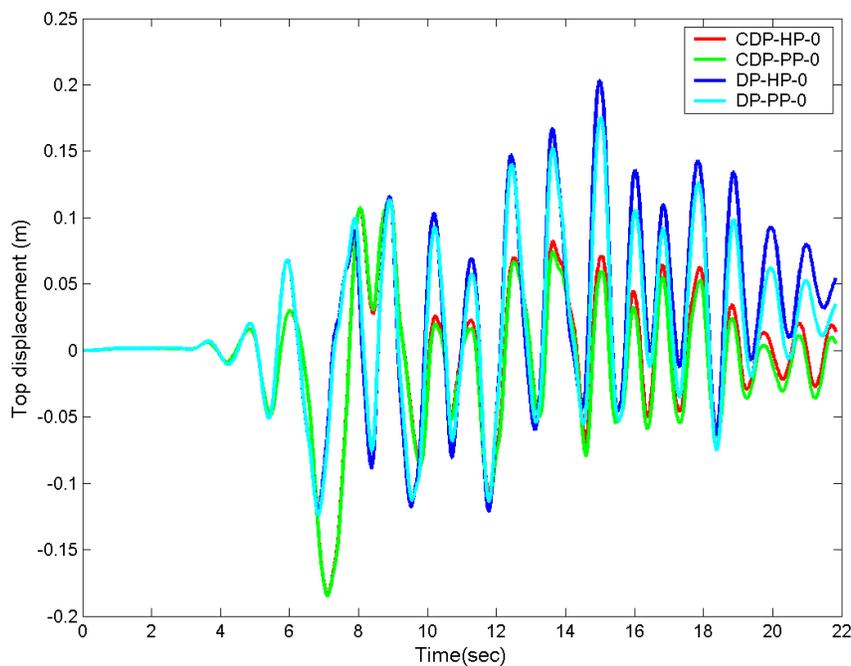


Fig. 2: Top deformation history for different material models for excitation V

PERFORMANCE OF PUSHOVER PROCEDURES FOR DIFFERENT MATERIAL MODELS

In this section the pushover analysis procedures discussed are evaluated with respect to response history analysis for different material models. The motivation is to examine how these nonlinear static procedures perform without the inclusion of cyclic loading presented in a real seismic situation for different material models. Both Concrete Damaged Plasticity (CDP) and Drucker-Prager (DP) material models are

considered. For both models only the hardening plasticity cases are included. Once again the four-storey single-bay frame discussed earlier was used in all analyses.

Using pushover procedures the target displacement was obtained for both DCM and CSM procedures. These are given in Table 3 (deformation floor 4) along with the peak deformation obtained from RHA. The RHA values are the mean of the peak deformation values from the five earthquake motions. It can be seen that the target displacement from pushover procedures match the RHA values very well, more so for the CDP model than for the DP model. In general the pushover values are slightly lower than the RHA values.

In Table 3 typical responses for the pushover procedures are compared with the mean peak RHA values for some typical response quantities. It can be seen that while the top deformation values from DCM and CSM match the RHA values closely, the error increases for deformation in lower floors for both CDP and DP structures. This may be partly attributed to the choice of pushover loading pattern. The base shear values are underestimated by the pushover procedures by around 22% for the CDP structure and by about 30% for the DP structure. The moment for a node at the base of the frame is underestimated by about 47% and 8% respectively. This illustrates that even though the evaluation of the top displacement response may be relatively accurate, some of the design quantities may be predicted inaccurately by pushover procedures. This is consistent with the findings of Chopra and Goel (2001).

Table 3: Peak responses from RHA, DCM and CSM for CDP and DP structures

Response quantity	CDP-HP			DP-HP		
	RHA	DCM	CSM	RHA	DCM	CSM
Deformation (m) floor 4	0.209	0.206	0.201	0.199	0.182	0.179
Deformation (m) floor 3	0.181	0.173	0.168	0.145	0.134	0.131
Deformation (m) floor 2	0.130	0.121	0.116	0.086	0.079	0.077
Deformation (m) floor 1	0.061	0.055	0.051	0.032	0.028	0.027
Base shear (kN)	126	98	97	236	166	166
Moment base node (kN-m)	132	69	70	358	329	328

The variation of inter-storey drifts is shown in Figs. 3 and 4. It may be noted that for RHA, the drifts are not evaluated from the peak deformations, but from the peak of the time-wise variation of drifts. It can be seen that the pushover procedures underestimate the drift of the lowest storey, and overestimate the drifts of other storeys for the CDP model. However, for the DP model the drifts are underestimated for all storeys by the pushover procedures. Thus the difference in results between RHA and pushover response is not independent of the choice of the material model.

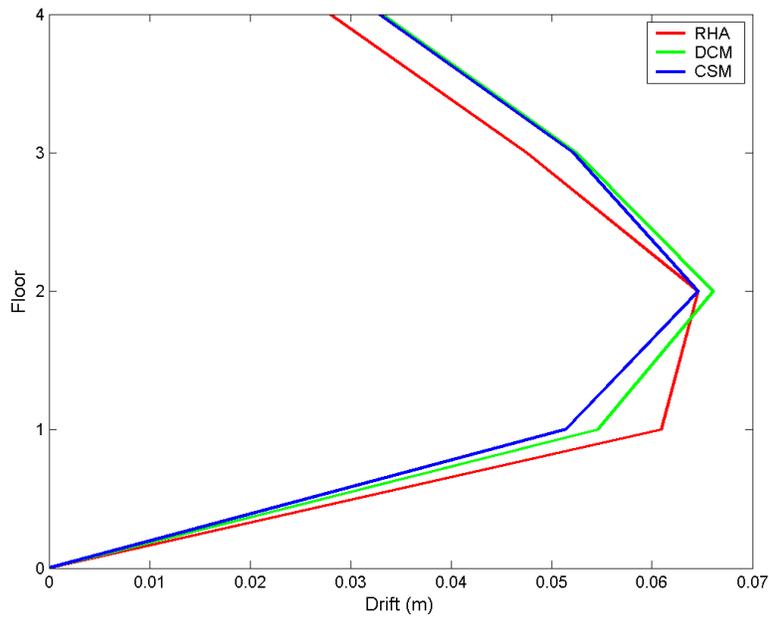


Fig. 3: Heightwise variation of storey drifts for CDP-HP structure

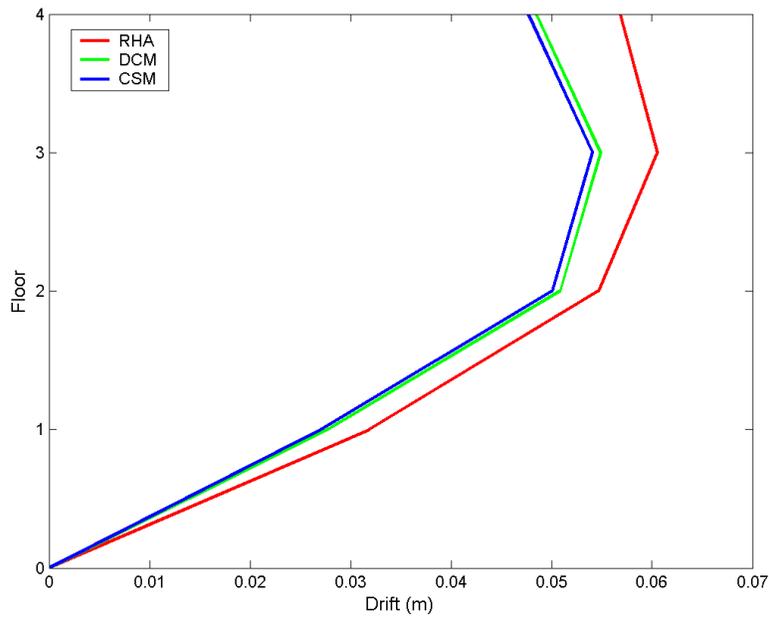


Fig. 4: Heightwise variation of storey drifts for DP-HP structure

CONCLUSIONS

This simple study shows that the influence of strain rate on the seismic analysis of reinforced concrete structures is small. The inclusion of a small value of hardening parameter also has only a small influence. For a given material model the peak RHA response from different excitation histories causes significantly larger variation than does inclusion or exclusion of compression hardening and strain rate parameters.

However, when the RHA response of the two material models is compared a significant difference is observed. The internal force peak response from CDP is significantly lower than from DP.

A comparison of RHA response with that obtained using DCM and CSM procedures shows that there can be a significant difference in the internal force response between dynamic and static procedures even though the target deformation values at the control node match. Moreover, the difference between the mean peak RHA response and the pushover response is not independent of the material model i.e. the static and dynamic procedures can yield similar values for one material model and fairly dissimilar for another.

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