

COINCIDENT VERTICAL RESPONSE ANALYSIS OF STRUCTURE AT MAXIMUM HORIZONTAL RESPONSE

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SUMMARY

A spectrum-basis response analysis for evaluating the vertical response with which coincides maximum horizontal responses of structures subjected to horizontal and vertical ground motions is developed. The ratio of response coincidence, the vertical response to its absolute maximum at the maximum horizontal response, is introduced to express the intensity and direction of the coincident vertical response. The ratio forms a certain probabilistic distribution regardless of types of earthquake, soil, structure and linearity/non-linearity of structural restoring system properties. The combination of the maximum expected value of the ratio based on the allowable probability of exceedance and the modal responses enables to compute the maximum expected coincident vertical response at the instance of the maximum horizontal response.

INTRODUCTION

To accomplish competent structures with minimizing seismic hazards, it is prudent to design for the worst case, which takes into account the simultaneous response on each structural axis. Unlike numerical integration techniques, no method is available to calculate exact quantities of structural responses at a specific time of interest.

The translational ground motion is usually resolved into three components; two in the horizontal plane and one in the vertical direction, and rotational ground motions are neglected. The peak response of structures due to each component of the ground motion is commonly evaluated by means of the response spectrum method. Here, the problem arises if such a separate evaluation can appropriately provide a severe response to be verified in designing the seismic resistance system of structures, although the simple combination of these separate evaluations gives surely the severest case. However, the senior author pointed out the rareness of the response coincidence between maximum horizontal and maximum vertical components viewing from the probability of occurrence [1, 2, 3]. Moreover, applying the severest case for

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all structure needs careful consideration, because the recent knowledge shows that the structures should possess the reasonable strength according to their importance. However, the major seismic design codes do not have any solution in terms of combination of structural responses [4, 5, 6].

As the beginning of series research and owing to the large number of parameters involved in this problem, the scope of this investigation is limited to the structures subjected to the horizontal and vertical ground motions. The recent study of seismic damages implies importance of consideration for the vertical response and corresponding response analysis methods were proposed [7, 8, 9].

The objective of this paper is to develop the analysis method for evaluating the vertical response of structure, with which coincides the maximum horizontal response of structure. The proposed method is based on the response spectrum method with assistance of the probability of response coincidence, which expresses the intensity and direction of the coincident vertical response, since the response spectrum has no information about the time space such as when the horizontal response reaches its maximum. This paper implicitly assumes that the critical response occurs at the maximum horizontal response of structures.

COINCIDENT VERTICAL RESPONSE ACCELERATION ON SINGLE SPRING-MASS SYSTEM

To find fundamental properties of the vertical response of the structure at the instance of its maximum horizontal response, the characteristics of response intensity and direction are examined with a simple spring-mass system. This is because the properties can be immediately applied to designing a simple structure and may be done in prediction of the vertical response of multi-story structures.

Consider the single spring-mass system (SSM system), which possesses independent freedom in both horizontal and vertical directions with 1% structural damping. The mass and spring are specified to have predefined natural period whose range is set from 0.01 to 1.0 second with 0.01-second intervals in both directions, respectively. The horizontal restoring system has three types of spring properties; linear, bilinear and slip type, which are adopted according to the analysis purpose. This is because most inelastic structural behavior can be explained by the combination of these spring types [10]. The inelastic region considered is up to 20 in the ductility factor, which may be enough according to recent study on seismic damages [10]. In contrast, the vertical restoring system maintains its linearity despite the extent of horizontal response. This study uses 104 accelerograms recorded around Japan that are disseminated by National Information Center for Earthquakes and Disasters [11]. According to the classification of soil types [6], 34 of 104 are recorded on the hard soil (the natural period of the soil is less than 0.2 second); 29 of 104 are on the soft soil (over 0.6 second); and 41 of 104 are on the medium soil (between 0.2 second and 0.6 second).

This investigation compiles statistics on the ratio, λ , of the coincident vertical response acceleration $\ddot{z}_v + \ddot{y}$ to the absolute maximum vertical response acceleration $|\ddot{z}_v + \ddot{y}|_{max}$ at the instance when the horizontal response acceleration reaches its absolute maximum acceleration $|\ddot{z}_H + \ddot{x}|_{max}$. Here, x and y are the horizontal and vertical displacements of the SSM system, respectively. \ddot{z}_H and \ddot{z}_v are the horizontal and vertical ground accelerations, respectively (See Figure 1(a)). From the combination of the natural period of the SSM system, the number of experiments for a set of accelerograms is 10,000.

$$\lambda = \frac{\ddot{z}_V + \ddot{y}}{\left| \ddot{z}_V + \ddot{y} \right|_{\max}} \text{ at the instance of } \left| \ddot{z}_H + \ddot{x} \right|_{\max}$$
(1)





(a) Single spring-mass model

(b) Multi-story structure model

Model	n	m	р	q
5-story	5	20,000	2	12,000
15-story	15	60,000	3	18,000
30-story	30	120,000	4	24,000
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(c) Dimension of actual sections

5	5-story	1:	5-story	30-story		
story	section	story	section	story	section	
1-3	550X550	1-3	700X700	1-2	800X800	
4-5	500X500	4-11	600X600	3-10	750X750	
		12-15	550X550	11-18	650X650	
				19-26	550X550	
				27-30	500X500	

(d) Dimension of unrealistic sections

4	5-story	1:	5-story	30-story		
story	section	story	section	story	section	
1-5	600X600	1-15	650X650	1-30	700X700	

FIGURE 1. ANALYTICAL MODELS AND THEIR STRUCTURAL DETAILS

From this definition, λ is positive while the SSM system responses upward and λ is negative while the SSM system responses downward, and its range is from -1.0 to 1.0.

Figures 2(a) to 2(c) show the probability density of λ along the specific rank of λ , while the horizontal restoring system is the linear type. The probability density λ is classified according to the soil type. Figures 3(a) to 3(c) show the probability density of λ for all ductility factors on each soil type, while the horizontal restoring system is the bi-linear type. Figures 4(a) to 4(c) show the probability density of λ for all ductility factors on each soil type, while the horizontal restoring system is the bi-linear type. Figures 4(a) to 4(c) show the probability density of λ for all ductility factors on each soil type, while the horizontal restoring system is the slip type. In computation of inelastic response, a set of recorded accelerograms is amplified until the horizontal response reaches the specific ductility. From these figures, the following remarks are drawn.

1) The ratio may be independent of the natural period of SSM system.

2) The ratio may be independent of the scale of earthquake.

3) The ratio may be independent of the type of earthquake.

4) The ratio may be independent of the type of soil.

5) The ratio may be independent of the extent of inelastic response.

6) The ratio may be independent of the spring type of restoring system.

7) The ratio may have probabilistic properties regardless of parameters concerned herein.



FIGURE 4. PROBABILITY DENSITY OF λ (SLIP TYPE)

The probability densities can be modeled by a beta distribution. Table 1 shows the variance of each probability density according to analytical conditions. Generally, the variance becomes smaller when the natural period of soil and/or the ductility factors become larger. However, since we can not find a particular trend in them, we take the mean value 0.142 as their representative value. In addition, the solid line on each figure is the beta distribution, which represents the probability densities of all case, whose

variance is 0.142 while the mean is zero. Based on this probabilistic investigation, the following remarks are also yielded.

- 1) The maximum vertical response acceleration may rarely coincide with the maximum horizontal response acceleration.
- 2) Most of vertical response accelerations are about zero at the instance when the horizontal response acceleration reaches its maximum acceleration.

Therefore, applying maximum horizontal and vertical responses in verification of the lateral strength of a simple structure independently or simultaneously may yield under- or overestimation of actual seismic event. The senior author reported the same remarks based on the linear analysis [1, 2, 3].

Spring	, typ	e	Hard soil	Medium soil	Soft soil
Liniear type			0.172	0.166	0.148
Bilinear type	1		0.158	0.134	0.120
		3	0.163	0.135	0.120
	ductility	5	0.159	0.136	0.120
		10	0.153	0.134	0.118
		20	0.155	0.131	0.115
Slip type		1	0.167	0.169	0.128
	ctility	3	0.146	0.152	0.120
		5	0.150	0.158	0.119
		10	0.142	0.156	0.117
	qι	20	0.134	0.155	0.117

TABLE 1 VARIANCE OF λ

Moreover, since the ratio of response coincidence maintains its probabilistic properties despite types of horizontal restoring system, the proposed method can calculate the vertical response to be combined with the horizontal response for evaluating the ultimate lateral strength of the member based on such as the energy-conservation-rule.

COINCIDENT AXIAL FORCES ON MULTI-STORY STRUCTURES

Practitioners need to know a pair of member forces to verify the seismic resistance system of structures. In multi-story structures, the practitioner would like to know the column axial force induced by the vertical ground motion, with which coincides the maximum column moment and corresponding axial force induced by the horizontal ground motion. The previous investigators pointed out the necessity of consideration of the coincident column axial force for strictly evaluating the seismic resistance system and its analysis method has been vigorously investigated [7, 8, 9].

This section probabilistically investigates the ratio, μ_r , of the coincident axial force A_r to its maximum axial force of column $|A_r|_{\text{max}}$ when the moment of the column reaches its absolute maximum moment $|M_r|_{\text{max}}$.

$$\mu_r = \frac{A_r}{|A_r|_{\max}} \text{ at the instance of } |M_r|_{\max}$$
(2)

The suffix r distinguishes the column of interest.

This investigation uses three building models with different stories and spans and 1% structural damping illustrated in Figure 1(b) and 104 accelerograms. The section of each column, which is followed actual building design, is also shown in Figure 1(c). Their natural frequencies of the first mode are 0.53 second for 5-story, 1.42 second for 15-story and 2.91 second for 30-story, respectively. To highlight effects of column stiffness on the probability densities, three buildings with the same configurations but uniform stiffness in height, that are rather unrealistic, are also considered (See Figure 1(d)). By the linear analyses with actual and unrealistic sections, the probability densities of the coincident axial forces at top, middle and base of the left column and its adjacent column are examined. In contrast, using three buildings with the actual section, the probability densities of the coincident axial forces when the horizontal restoring system of these buildings' columns are bi-linear and slip types are investigated. The inelastic region considered is up to 20 in the ductility factor. The numerical analyses are carried out by the software TDAP III [12].

Figures 5 to 7 are results of linear analyses and Figures 8 to 13 are results of inelastic analyses. Here, according to results of the SSM system, the soil type does not significantly contribute on the distribution of probability densities. Therefore, the results of the coincident axial forces are processed irrespective of soil types. Figures 5(a), 5(b) and 5(c) show the probability density of μ_r observed at top, middle and base column of five-story building, respectively. Figures 6(a) to 6(c) and 7(a) to 7(c) show those of fifteen-story and thirty-story buildings, respectively. The legends on each figure identify the section used and the column of interest. Employing the bi-linear type in the horizontal restoring system, Figures 8(a), 8(b) and 8(c) show the probability density of μ_r observed at top, middle and base column of five-story building. Figures 9(a) to 9(c) and 10(a) to 10(c) show those of fifteen-story and thirty-story buildings, while Figures 11(a) to 13(c) show those of slip type. The legends in each figure identify the ductility factors considered. From these figures, the following remarks are yielded.

1) The ratio may be independent of the building configurations.

2) The ratio may be independent of the column stiffness.

3) The ratio may be independent of the location of column.

4) The ratio may be independent of the scale of earthquake.

5) The ratio may be independent of the type of earthquake.

6) The ratio may be independent of the type of soil.

7) The ratio may be independent of the extent of inelastic response.

8) The ratio may be independent of the spring type of restoring system.

9) The ratio may have probabilistic properties regardless of parameters concerned herein.



^O: Actual section, x: Unrealistic section, \triangle : Adjacent column, -: approximation FIGURE 5. PROBABILITY DENSITY OF μ_r (5-STORY, LINEAR TYPE)



◇: ductility factor=1.0, +: ductility factor=3.0, □ ductility factor=5.0, x: ductility factor=10.0,
△: ductility factor=20.0, -: approximation





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    ◊: ductility factor=1.0, +: ductility factor=3.0, □ ductility factor=5.0, x: ductility factor=10.0,
    △: ductility factor=20.0, -: approximation
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FIGURE 13. PROBABILITY DENSITY OF μ_r (30-STORY, SLIP TYPE)

The probability density can be modeled by a beta distribution. Table 2(a) to 2(c) show the variance of probability density according to the horizontal restoring system types. As observed on the SSM system, since we can not find a particular trend in them, we take the mean value 0.147 as their representative value. In addition, the solid line on each figure is the beta distribution, which represents the probability density of all cases, whose variance is 0.147 while the mean is zero. Based on this probabilistic investigation, the following remarks are also yielded.

1) The maximum axial force may rarely coincide with the maximum moment.

2) Most of axial forces are about zero at the instance when the moment reaches its maximum.

Therefore, applying the maximum moment induced by horizontal shaking and maximum axial force induced by vertical shaking in verification of the lateral strength of structural member independently or simultaneously may yield under- or overestimation of actual seismic event. Regarding the multi-story structures, the remarks on the literature by the senior author slightly differ from those on above [3]. These arise from the shortage of analytical cases.

	5-story			15-story	-story			30-story		
	Base	Middle	Тор	Base	Middle	Тор	Base	Middle	Тор	
Actual section	0.125	0.113	0.118	0.179	0.166	0.107	0.172	0.175	0.192	
Unrealistic section	0.104	0.096	0.118	0.187	0.168	0.151	0.139	0.199	0.205	
Adjacent column	0.112	0.103	0.108	0.187	0.156	0.146	0.142	0.187	0.171	

TABLE 2(a) VARIANCE OF μ_r (LINEAR TYPE)

		5-story			15-story			30-story		
		Base	Middle	Тор	Base	Middle	Тор	Base	Middle	Тор
ity r	1	0.088	0.106	0.114	0.182	0.199	0.138	0.175	0.189	0.200
	3	0.102	0.087	0.083	0.181	0.197	0.144	0.149	0.204	0.181
ctil: actc	5	0.097	0.119	0.079	0.181	0.195	0.128	0.170	0.168	0.172
du fi	10	0.105	0.140	0.098	0.181	0.172	0.123	0.182	0.167	0.134
	20	0.121	0.106	0.084	0.194	0.139	0.139	0.158	0.152	0.119

TABLE 2(b) VARIANCE OF µr (BI-LINEAR TYPE)

TABLE 2(c) VARIANCE OF µr (SLIP TYPE)

			5-story			15-story			30-story	
		Base	Middle	Тор	Base	Middle	Тор	Base	Middle	Тор
	1	0.090	0.116	0.109	0.181	0.195	0.153	0.176	0.195	0.187
ity r	3	0.115	0.106	0.113	0.154	0.180	0.149	0.172	0.206	0.178
ctil	5	0.103	0.101	0.100	0.173	0.167	0.146	0.163	0.179	0.153
du fi	10	0.119	0.131	0.096	0.176	0.158	0.163	0.154	0.177	0.143
	20	0.108	0.118	0.088	0.175	0.175	0.133	0.159	0.157	0.152

It is worth to note that the probability density of μ_r is almost the same as λ with the reasonable accuracy despite analysis conditions considered herein. It implies that the characteristics of the coincident axial force on multi-story structures have inherited ones of the coincident vertical response acceleration

observed on the SSM system. It suggests the possibility of the application of the superposition of modal response for estimating the coincident axial forces of arbitrary columns that are induced by vertical ground motion.

APPLICATION OF MODAL ANALYSIS TO COINCIDENT AXIAL FORCE ANALYSIS

Since the same probabilistic properties appear in both the coincident vertical response acceleration of SSM system and the coincident axial force of column of multi-story structures, the application of the modal analysis to predict the coincident axial force of the column is examined. This section examines the applicability of the modal analysis to the analysis of the coincident axial force of multi-story structures at the instance of the maximum moment of the column.

The modal analysis is known as computing a maximum axial force at the *r* th floor column $(A_r)_{max}$ of the n-story structure with assistance of response spectrum.

$$(A_{r})_{\max} = \sqrt{\sum_{j=1}^{n} \left\{ \sum_{s=r}^{n} M_{s} \phi_{Vsj} \beta_{Vj} S_{VA}(\omega_{Vj}, h_{Vj}) \right\}^{2}}$$
(3)

Here, M_s = the concentrated mass at *s* th floor, ϕ_{sj} = the modal vector at *s* th floor in *j* th mode, β_j = the participation factor in *j* th mode, ω_j = the natural frequency of *j* th mode, h_j = the damping coefficient of *j* th mode, S_A = the value of acceleration response spectra in corresponding conditions. The suffix V shows quantities of interest are in a vertical direction.

It is noted that the modal analysis holds no particular advantage over the numerical integration technique if an exact answer is desired. Thus it is desirable to use the modal analysis procedure only to compute the maximum modal responses rather than a complete time history of response, and superpose the modal maxima to obtain an upper bound on the true response. Moreover, the modal analysis does not deal with the time space and, hence, provides no answer to the questions such as when the response reaches its maximum. This implies that applying the modal analysis to the coincident axial force analysis shall constitute without referring any information from the time space.

On the contrary, an assumption, which all horizontal modes reach their maxima simultaneously, enables to combine the ratio of the response coincidence with the modal analysis of multi-story structures. The vertical response acceleration of the *j* th mode $\{S_{VA}(\omega_{vj}, h_{vj})\}_{coin}$ at the instance of the maximum horizontal response acceleration of any mode can be calculated as;

$$\left\{S_{VA}\left(\omega_{vj},h_{vj}\right)\right\}_{coin} = S_{VA}\left(\omega_{vj},h_{vj}\right) \cdot \lambda_{j}$$

$$\tag{4}$$

Here, $S_{VA}(\omega_{vj}, h_{vj})$ is a value of the vertical response spectrum, which gives a modal maximum and is regulatory specified. λ_j is the ratio of the coincident vertical response acceleration to the maximum vertical response acceleration in *j* th vertical mode at the instance of the maximum horizontal response acceleration of any mode. The determination procedure of λ_j is discussed later. The substitution of Eq. (4) into Eq. (3) gives a coincident axial force at the *r* th column $(A_r)_{coin}$.

$$(A_{r})_{coin} = \sqrt{\sum_{j=1}^{n} \left\{ \sum_{s=r}^{n} M_{s} \phi_{V_{sj}} \beta_{V_{j}} \left\{ S_{VA} \left(\omega_{V_{j}}, h_{V_{j}} \right) \right\}_{coin} \right\}^{2}}$$
(5)

From the probabilistic properties of λ_j described earlier, the ratio is independent of types of earthquake, soil and structure, Eq. (5) can be rewritten as;

$$\left(A_{r}\right)_{coin} = \lambda \sqrt{\sum_{j=1}^{n} \left\{\sum_{s=r}^{n} M_{s} \phi_{Vsj} \beta_{Vj} S_{VA}\left(\omega_{Vj}, h_{Vj}\right)\right\}^{2}}$$
(6)

The derivation of Eq. (6) also employs another assumption that setting the ratio λ_j to each mode is invalid because the probabilistic occurrence of all modes shall be equally treated. The calculation of the ensemble average of Eq. (6) yields an expected coincident axial force at the *r* th column $\langle (A_r)_{coin} \rangle$.

$$\left\langle \left(A_{r}\right)_{coin}\right\rangle = \sqrt{\left\langle\lambda^{2}\right\rangle} \sqrt{\sum_{j=1}^{n} \left\{\sum_{s=r}^{n} M_{s} \phi_{Vsj} \beta_{Vj} S_{VA}\left(\omega_{Vj}, h_{Vj}\right)\right\}^{2}}$$
(7)

Here, < > denotes the operation of the ensemble average. Since the probability density of λ can be approximated as the beta distribution, the term related to λ gives the standard deviation of λ .

$$\sqrt{\left\langle \lambda^2 \right\rangle} = \sqrt{\int_{-1}^{1} \lambda^2 p(\lambda) d\lambda} = \frac{2}{q+r} \sqrt{\frac{qr}{q+r+1}}$$
(8)

Here, $p(\lambda)$ is given as;

$$p(\lambda) = \frac{1}{B(q,r)} \frac{(\lambda+1)^{q-1} (1-\lambda)^{r-1}}{2^{q+r-1}}, \quad -1 \le \lambda \le 1$$
(9)

Here, q = r = 2.901. Equation (8) means that the mean power of λ is given by the root-mean-square value, which is the same root of the modal analysis. It enables to combine the ratio of response coincidence with the modal analysis. In addition, the theory of extreme value distribution can be applied to this problem, and the maximum expected value of the ratio, λ , should be determined by the probability of passage of a threshold, denoted the allowable probability of exceedance. Computing the coincident axial force through this approach means that the vertical response of structures is probabilistically predicted backed up by the probability of occurrence. The practitioners implement the seismic design with realistic responses and the structural safety is explicitly secured under a certain probable condition [2]. Viewing from this sense, setting the allowable probability of exceedance should be related to the importance of and/or the allowable damage of structures. However, it is out of intent of the paper.

Figure 14 shows the probability of λ and its values corresponding to the allowable probability of exceedance. Using linear structures with the realistic column sections and setting the 5% allowable probability of exceedance, the analytical accuracy of the proposed method is examined. The abscissa of Figure 15(a) to (c) shows the exact coincident axial force of column of interest computed by the numerical integration technique, while their ordinates show that computed by the proposed method. The number of approximated axial force, which exceeds exact axial force, is within the number specified by the allowable

probability of exceedance assumed. Therefore, the proposed method can adequately calculate an upper bound of the coincident axial force of column of multi-story structures.

Moreover, since the probabilistic properties of the ratio of response coincidence are the same despite properties of horizontal response, the vertical response to be combined with the horizontal response for evaluating the ultimate lateral strength of the member based on the energy-conservation-rule can be calculated by the proposed method.



10%	0.515	
	 THO THE T	

0.631

5%







CONCLUSION

- 1. Based on the response spectrum with assistance of the probability of response coincidence, the proposed method enables to compute the coincident vertical response of structure at the instance of maximum horizontal response of structure.
- 2. The response coincidence is quantified as the ratio, the vertical response to its absolute maximum at the maximum horizontal response, which expresses the intensity and direction of the coincident vertical response. Its probabilistic properties are investigated with the single spring-mass systems and multi-story structures. Despite types of earthquake, soil, structure and linearity/ non-linearity of structural response, the probabilistic properties of the ratios of the single spring-mass systems and multi-story structures are the same and form a certain beta distribution.
- 3. The combination of the modal responses and the maximum expected value of the ratio of response coincidence based on the allowable probability of exceedance enables to calculate the maximum expected coincident vertical response. The proposed method is accurate, computationally simple and easy to implement in standard dynamic analysis. However, the appropriate value of the allowable probability of exceedance may be discussed view from the structural importance and/or the allowable damage of structures.
- 4. Since the ratio maintains its probabilistic properties irrespective of linearity/non-linearity of structural restoring system, the proposed method can compute the coincident vertical response to be combined with the horizontal response for evaluating the ultimate lateral strength by such as the energy-conservation-rule.

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