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SLIP DISPLACEMENT ANALYSIS OF FREESTANDING RIGID BODIES SUBJECTED TO EARTHQUAKE MOTIONS

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SUMMARY

The practical analysis method for estimating the slip displacement of freestanding rigid body placed on the undergoing ground or floors of multi-story structures is developed. The proposed procedure uses the prepared sinusoidal wave to compute the basic slip displacement of the body. Its amplitude is identical to the peak horizontal acceleration of the ground or building floors and its period meets the principal period of horizontal ground motion or building structures. The effects of vertical acceleration are monotonously incorporated into it to reduce the friction force. The extent of the reduction is determined by the statistics of the vertical acceleration with which coincides the peak horizontal acceleration. Employing over a hundred of accelerograms and some friction coefficients, the maximum slip displacement of the body relative to the base is computed. The comparison with that induced by the prepared sinusoidal wave yields the adjustment factor, which can be modeled by the logarithmic normal distribution regardless of analytical conditions and suggested probability-basis estimation of the slip displacement. It is shown that the proposed method can compute the maximum expected slip displacement of the body wherever it is placed on if an adjustment factor, as suggested in this study, is applied.

INTRODUCTION

In the recent highly developed and complicated social system, the damage of structures no longer represent total effects of earthquakes on it. The functional loss of the social system that the damage of nonstructural component may concern should take into account. To avoid such the systematical down, the design of nonstructural components receives the same degree of consideration as primary members to remain the critical facilities operationally during and after an earthquake. The former investigators clearly pointed out that there is need to investigate the seismic behavior of nonstructural component in order to assess their vulnerability under the seismic events [1]. In the various kinds of nonstructural components on the social system, this research concerns an important class of nonstructural components, the mechanical/electrical equipment, which can be essentially modeled as a rigid body.

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Effects of base excitation on a freestanding rigid body have been investigated by many researchers [2-8]. Its seismic response showed very complex and highly nonlinear and was classified into five modes (rest, slide, rock slid-rock and jump) [9-12]. While fundamental researches, it was pointed out that not only the peak horizontal acceleration and friction coefficient but also the principal period of base excitation elongated the slip displacement of the body [12].

For the practical convenience, some procedures for estimating the slip displacement of the body have been developed. Shao and Tung [13] prepared a graph, which enabled to determine the mean-plus-standard deviation of the maximum sliding distance of an unanchored body. However, the study does not consider the vertical excitation and its applicability to the slip displacement of the body placed on the building floors. Including the vertical excitation, Garcia and Soong [1] presented the fragility information for sliding related failure modes. They classified the fragility curves according to the friction coefficient and peak horizontal ground acceleration. Although it gives appropriate slip displacement of the body placed on the ground, the prediction accuracy that on the building floors deteriorates.

In contrast, Newmark [14] presented a simple formula to determine the sliding distance of a freestanding body subjected to a single rectangular acceleration pulse of short duration at the base concerning earthquake responses of embankments. Choi and Tung [15] concluded that Newmark's formula could be used if an adjustment factor, which was given as a function of the friction coefficient and peak base acceleration, was applied. However, the study is limited to the slip response under the action of horizontal base excitation. In addition, it assumes the existence of the constant base velocity after a single rectangular acceleration pulse terminates, which may overestimate the slip displacement of the body. Although the procedure gives appropriate slip displacement of the body placed on the ground, the prediction accuracy of that on the building floors whose structural natural period is slightly long deteriorates.

As mentioned above, the previous procedures for predicting the slip displacement of the body does not include effects of the period of the excitation. Consequently, the prediction accuracy of the slip displacement of the body on the building floors may deteriorate. This is because that the filtering process of acceleration records by the vibration system of the building structure may strengthen the advent of the principal period of excitation, which may be close to the natural period of building structure.

The objective of this study is to develop the simple and reasonable procedure that can estimate the slip displacement of a body wherever it is set on. To make up for a deficiency in the previous researches, this research introduces the use of the prepared sinusoidal wave to calculate the basic slip displacement. It intends to use an upper bound of slip displacement induced by a single action of the sinusoidal wave as the first approximation of the slip displacement during an earthquake. The basic slip displacement is modified by multiplying the probability-basis adjustment factor, which also gives the information of the probability of passage of the threshold to the slip displacement. The amplitude of the prepared sinusoidal wave is identical to the peak horizontal acceleration of the ground or building floors and its period meets the principal period of horizontal ground motion or building structures. Generally, the peak horizontal acceleration and principal period are used to characterize the nature of earthquakes. In ground motion, the peak horizontal acceleration is not always induced by the wave component, which possesses the principal period of the earthquake. In the floor response, by the filtering effects of vibration systems, the peak response acceleration may be likely induced by the wave component, which possesses the principal period of the building response. The use of sinusoidal wave may have an advantage in this sense. In contrast, Shao and Tung [13] suggested that effects of vertical ground motion on the mean-plus-standard deviation of the maximum sliding distance was little. However, it is indispensable to compute adequate slip displacement of the body. Viewed in developing the simple calculation procedure, the problem is how to treat the randomness of the vertical excitation in the simple calculation process. This study proposes monotonous reduction of friction force as if the downward vertical acceleration constantly acts. The vertical acceleration is determined according to the probability of occurrence of the vertical acceleration at the instance of the peak horizontal acceleration. The extent of the reduction is determined by the statistics of the coincident vertical acceleration [16, 17].

The first part of the paper examines the applicability of the proposed procedure to the problem that the body is placed on the horizontally and vertically undergoing ground. The ensemble of 104 real earthquakes is then employed, the equations of motion are numerically solved and the maximum slip displacement of the body relative to the base is computed. The computed displacement is then compared with that induced by the prepared sinusoidal wave. The estimation errors by the prepared sinusoidal wave are compiled to examine the adjustment factor based on the probability. This information can then be used to quantify slipping-related risks of this kind of components given its specified earthquake environment.

The second part of the paper extends to the case in which a body is set on the building floors subjected to simultaneous horizontal and vertical excitations. The five, ten and twenty-story shear frame are selected as the building's models. The shear frames meet the latest design specification, which requires having post yielding lateral strength [18]. The 10 ground acceleration time histories are filtered through the shear frames and for each floor the 10 acceleration time histories are computed and used as input to the equation of motion of the slipping body placed on that floor. As in the case in which the body is placed on the ground, the maximum slip displacement of the body placed on a floor is computed and compared with that induced by the corresponding prepared sinusoidal wave. When the body is subjected to the horizontal floor shaking, the estimation errors by the prepared sinusoidal wave shows the same tendency of ones obtained by the examination of the body placed on the ground. It is shown that the proposed procedure can be used wherever the body is placed on. In contrast, the estimation accuracy deteriorates when the body is set on the building floor where horizontally and vertically undergoes.

EQUATION OF MOTION

The equation of motion that governs the slip behavior of the body wherever it is placed on and subjected to simultaneous horizontal and vertical motions can be expressed as follows. Figure 1 presents the mechanical model to be analyzed. Figure 1(a) shows the body is placed on the ground while Fig. 1(b) shows the body is set on the building floor. From Fig.1, it is understood that the subjects are essentially the same by replacing the shaking motions of interest according to the problem.

i) while slip

$$\ddot{x} = -\ddot{z}_h - \nu(g + \ddot{z}_v)\, sign(\dot{x}) \tag{1}$$

ii) while stationary

$$\ddot{x} = 0 \tag{2}$$

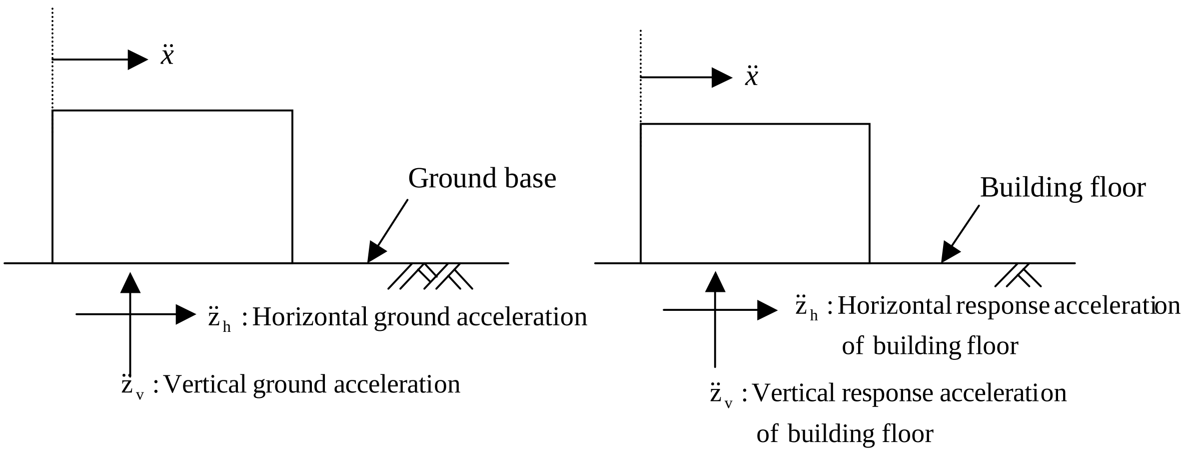
iii) slip commencement condition

$$|\ddot{z}_h| > \mu(g + \ddot{z}_v) \tag{3}$$

iv) slip terminate condition

$$\dot{x} = 0 \tag{4}$$

Here, $\ddot{z}_h$ and $\ddot{z}_v$ are the horizontal and vertical accelerations of ground or building floor, respectively. The g, μ and ν are the gravitational acceleration, static friction coefficient and kinetic friction coefficient, respectively. Here, to simplify the problem, the static and kinetic friction coefficients are assumed as the same and denoted as μ thereafter. The function $sign(\dot{x})$ gives the sign of the variable. Employing the earthquake records in $\ddot{z}_h$ and $\ddot{z}_v$, the slip displacement is numerically computed. The numerical computation is performed by ACSL with 0.001 seconds intervals [19].



(a) The body on the ground (b) The body on the building floor

FIGURE 1. ANALYTICAL MODELS

In contrast, for the analysis of the slip displacement of the body subjected to the prepared sinusoidal wave, Eqs. (1) and (3) are rewritten as follows to incorporate the vertical motion effects monotonously. The details of the prepared sinusoidal wave appear next section.

$$\ddot{x} = -\ddot{z}_h - \mu(g - \sigma \cdot PVGA \cdot \eta)\, sign(\dot{x}) \tag{5}$$

$$|\ddot{z}_h| > \mu(g - \sigma \cdot PVGA \cdot \eta) \tag{6}$$

Here, σ is the standard deviation of the ratio of the coincident vertical acceleration to the peak vertical acceleration at the instance of the peak horizontal acceleration. The product of σ and peak vertical ground acceleration (PVGA) gives the root mean square value of the vertical acceleration with which coincides the peak horizontal ground acceleration (PHGA) [16, 17]. This study uses σ=0.46 and monotonously approximates the vertical acceleration based on the probability of occurrence. In contrast, η is the magnification factor of the structural response of buildings in vertical direction. Employing the prepared sinusoidal wave in $\ddot{z}_h$, the basic slip displacement is calculated. The appendix shows its approximate solution process.

INPUT MOTIONS

Earthquake records

This study uses a hundred and four earthquake records observed around Japan and disseminated by National Information Center for Earthquakes and Disasters [20]. These 104 earthquakes are of different characteristics in terms of energy content in various frequency bands, duration, and variation of intensity with time. The statistical properties of the sliding response of a body obtained using these earthquakes as input must therefore be interpreted accordingly. The time histories of the ground acceleration of these earthquakes are first normalized. That is, for each time history of ground acceleration, the absolute maximum horizontal acceleration, denoted by $A_{gx}\ g$, is scaled to a value of g. When one is interested in the statistics of the sliding response of a body to $A_{gx}\ g$, other than g (say, 0.4 g), all the horizontal acceleration time histories are scaled by multiplying them by the value of 0.4. Maintaining the relationships between the PHGA and PVGA of each earthquake record, the vertical acceleration time history is also normalized and scaled in a same manner.

Prepared sinusoidal waves

The form of a prepared sinusoidal wave used for computing the basic slip displacement is given as follows. It possesses the same peak horizontal ground acceleration of the earthquake of interest, when the body is placed on the ground. It should be replaced by the peak horizontal response acceleration of building floor when the body is set on the building floor.

$$\ddot{z}_h = A_{gx}\ g \sin\left(\frac{2\pi}{T} t\right) \tag{7}$$

Here, T is the principal period of horizontal acceleration (PPHA). In this study, it is identical to the predominant period of the ground when the body is placed on the ground, while it is identical to the natural period of the building when the body is placed on the building floors.

ANALYSES AND DISCUSSIONS

Slip displacement of the body on the ground

Consider the absence of the vertical ground acceleration. The slip displacement of the body under the action of horizontal earthquake motion is numerically computed and compared with that under the action of the prepared sinusoidal wave, whose nature corresponds to the earthquake motion used. Introducing the slip ratio β_h the characteristics of estimation errors are statistically examined.

$$\beta_h = \frac{\left| x_h^{Eq} \right|_{Max}}{x_h^{Sin}} \tag{8}$$

Here, $\left| x_h^{Eq} \right|_{Max}$: the absolute maximum slip displacement of the body induced by the horizontal earthquake, x_h^{Sin} : the basic slip displacement. There are three primary reasons why the slip displacement by the prepared sinusoidal wave differs from that by the earthquake records.

1) The form of the acceleration of the earthquake, which causes the slip, differs from one of the prepared sinusoidal wave.
2) The period of the wave component of the earthquake, which causes the slip, differs from one of the prepared sinusoidal wave.
3) The chronical slip during an earthquake is accumulated in a certain direction.

From these reasons, the basic slip displacement should be adjusted to give appropriate slip displacement. In addition, these are general characteristics of estimation errors and interpreted accordingly. Therefore, the estimation errors are statistically processed.

Figures 2(a) to 2(c) show the probability density of the slip ratio β_h. The slip ratios are classified according to the combination of PHGA and μ. The legend in each figure shows the value of μ. From these figures, the slip ratio β_h possesses almost the same probability density irrespective of the combination of PHGA and μ. Figure 2(d) shows the probability density of the slip ratio β_h that are classified according to the natural period of the ground without identification of PHGA and μ. This study follows the classification of the soil type according to the natural period of the ground [21]. Table 1 shows the range of the natural period of the ground and the number of earthquake records classified. From Fig. 2(d), the probability density of the slip ratio β_h is almost the same irrespective of soil types.

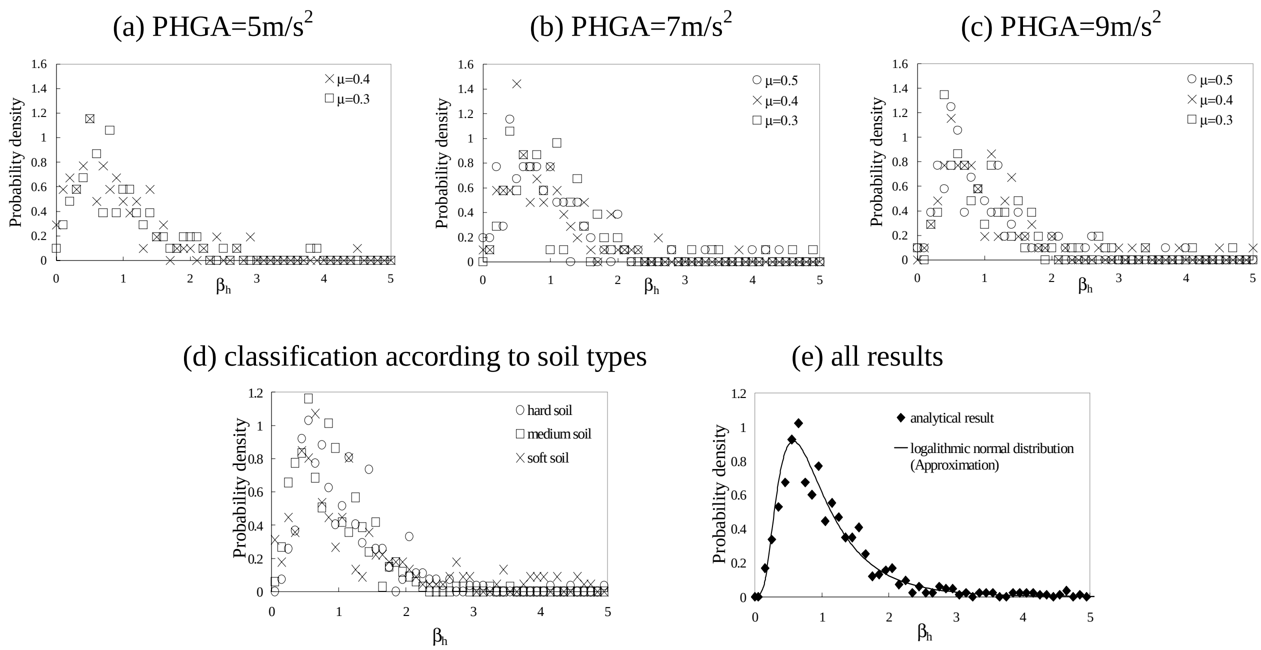


FIGURE 2. PROBABILITY DENSITIES OF SLIP RETIO β_h

TABLE 1. RANGE OF NATURAL PERIOD OF GROUNDS AND NUMBER OF EARTHQUAKE RECORDS CLASSIFIED

The soil type	The natural period of the ground	The number of earthquake records classified into it
Hard soil	T<0.2	34
Medium soil	0.2≤T<0.6	42
Soft soil	0.6≤T	28

As mentioned above, the probability density of the slip ratio β_h shows the same distribution regardless of the analytical conditions. This implies that the proposed procedure appropriately approximate the slip displacement of the body at any conditions within the reasonable errors specified by the distribution β_h.

These considerations suggest that the probability density of the slip ratio β_h can be compiled irrespective of analytical conditions considered herein. Figure 2(e) is the probability density of the slip ratio β_h of all results. The distribution can be modeled by the logarithmic normal distribution function whose mean and standard deviation are 1.03 and 0.71, respectively. The solid line in Figure 2(e) presents the approximation.

This consideration also yields that the slip displacement of the body can be calculated based on the probability of passage of the threshold. Figure 3 shows the probability of the slip ratio β_h converted from the Fig. 2(e). Table 2 shows the probability-basis adjustment factors $\beta_h^{\mathrm{Pr}ob}$ for the selected values of the probability of nonexceedence of specified threshold. By multiplying the slip ratio $\beta_h^{\mathrm{Pr}ob}$ with the basic slip displacement induced by the prepared sinusoidal wave x_h^{Sin}, the maximum expected slip displacement of the body under the action of horizontal earthquake motion based on the probability of nonexceedence x_h^{Exp} can be calculated as follows.

$$x_h^{Exp} = \beta_h^{\mathrm{Pr}ob} \cdot x_h^{Sin} \tag{9}$$

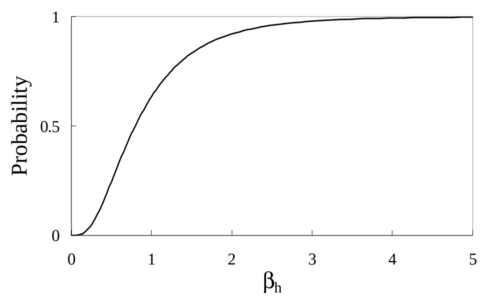


FIGURE 3. PROBABILITY OF SLIP RATIO β_h

TABLE 2. PROBABILITY-BASIS ADJUSTMENT FACTORS $\beta_h^{\mathrm{Pr}ob}$ FOR SELECTED VALUES OF PROBABILITY OF NONEXCEEDENCE

$\beta_h^{\mathrm{Pr}ob}$	Probability of nonexceedence
1.84	90%
2.32	95%
2.70	97%
3.58	99%

Consider the slip displacement of the body under the action of simultaneous horizontal and vertical earthquake motion. Figure 4 examines effects of the vertical ground motion on the slip displacement by comparing slip displacements under the action of horizontal excitation with that under the simultaneous action of vertical and horizontal excitation. It is understood that the vertical ground motion does not always increase the slip displacement. However, it is better for our safe to give the function of calculating increased slip displacement due to the vertical excitation to the proposed procedure. Accordingly, the effects of the vertical motion is monotonously included to reduce the friction force.

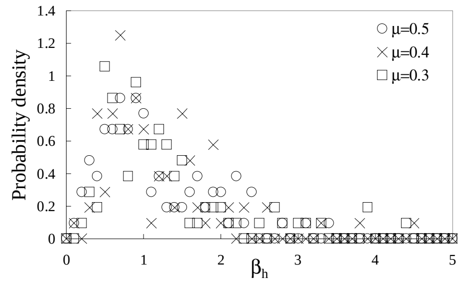


FIGURE 4. EFFECTS OF THE VERTICAL GROUND MOTION ON THE SLIP DISPLACEMENT (EX. PHGA=9m/s^2)

Figures 5(a) to 5(c) show the probability density of the slip ratio $\beta_{h,v}$ classified according to the combination of PHGA and μ. Figure 5(d) also show the slip ratio $\beta_{h,v}$ classified according to the natural period of the ground regardless of the combination of PHGA and μ. Similar to the examination of β_h , the slip ratio $\beta_{h,v}$ forms a certain distribution at any conditions. Figure 5(e) is the probability density of the slip ratio $\beta_{h,v}$ of all results. The distribution can be modeled by the logarithmic normal distribution function whose mean and standard deviation are 0.83 and 0.78, respectively. The solid line in Figure 5(e) presents the approximation.

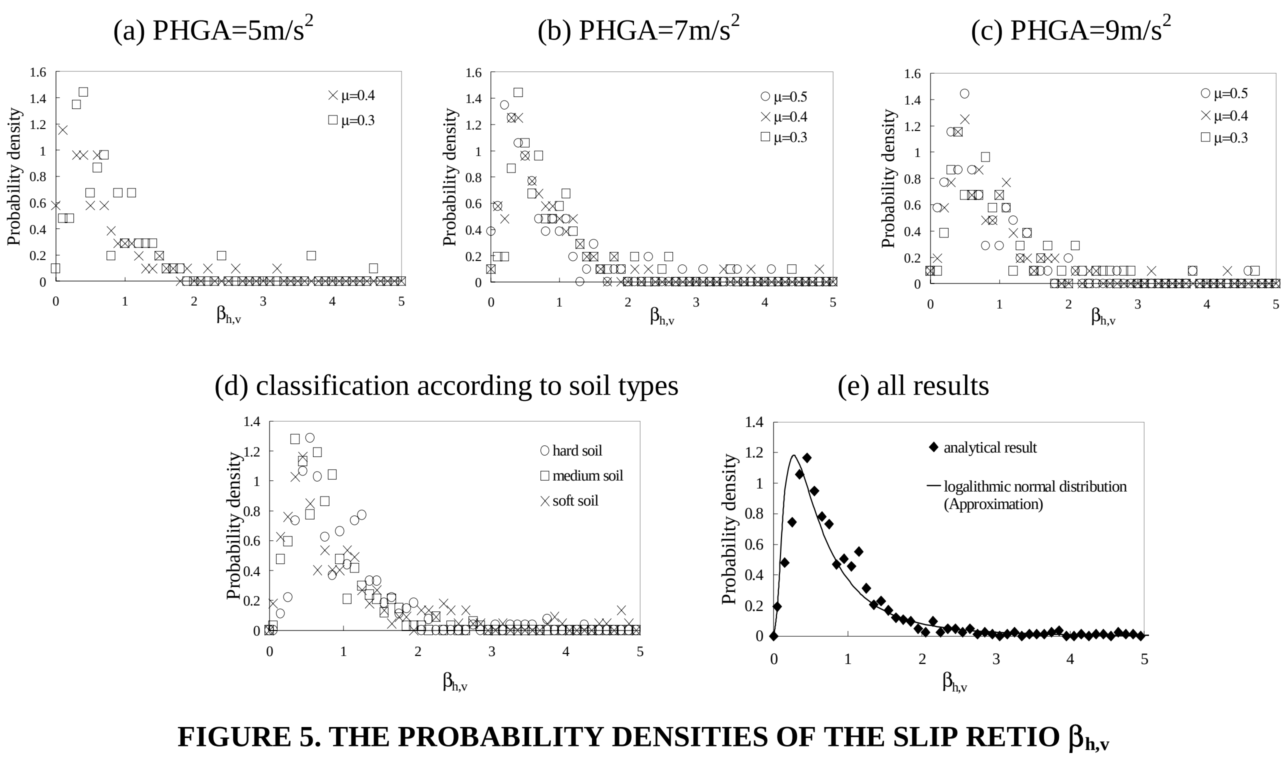


FIGURE 5. THE PROBABILITY DENSITIES OF THE SLIP RETIO $\beta_{h,v}$

Comparison of Figs. 2(e) and 5(e) yields that the estimation accuracy by the proposed procedure does not deteriorate if the action of the vertical excitation is considered, since their standard deviations are almost the same. However, the reduction of the mean value suggests that the proposed procedure overestimates the effects of vertical ground motion on the reduction of the friction force. Similar to Eq. (9), the maximum expected slip displacement of the body $x_{h,v}^{Exp}$ can be calculated as the product of the slip ratio $\beta_{h,v}^{\mathrm{Pr}ob}$ and the basic slip displacement $x_{h,v}^{Sin}$.

$$x_{h,v}^{Exp} = \beta_{h,v}^{\mathrm{Pr}ob} \cdot x_{h,v}^{Sin} \tag{10}$$

Here, Figure 6 shows the probability of the slip ratio $\beta_{h,v}$ converted from Fig. 5(e). Table 3 shows the probability-basis adjustment factors $\beta_{h,v}^{\mathrm{Pr}ob}$ for the selected values of the probability of nonexceedance of specified threshold. These results suggest that the proposed procedure can calculate the slip displacement based on the probability of nonexceedance at any conditions by replacing the slip ratio to be applied.

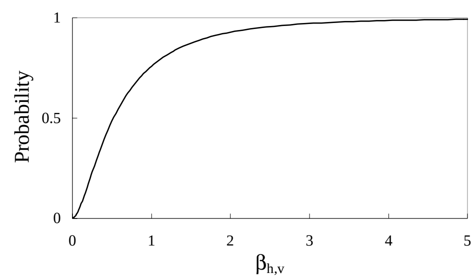


FIGURE 6. PROBABILITY OF SLIP RATIO $\beta_{h,v}$

TABLE 3. PROBABILITY-BASIS ADJUSTMENT FACTORS $\beta_{h,v}^{\mathrm{Pr}ob}$
FOR SELECTED VALUES OF PROBABILITY OF NONEXCEEDENCE

$\beta_{h,v}^{\mathrm{Pr}ob}$	Probability of nonexceedence
1.69	90%
2.35	95%
2.90	97%
4.40	99%

From this discussion, it is understood that the proposed procedure can estimate the maximum expected slip displacement of the body whether we consider the vertical ground motion.

Slip displacement of the body on the building floors

The study is extended to analysis of the slip displacement of the body on the building floors. In the previous section, the maximum acceleration and principal period of the horizontal ground motion were used for the amplitude and period of the prepared sinusoidal wave, respectively. In the case of the body on the building floor, the acceleration time histories to be considered is not the earthquake ground motion but the response acceleration of the building floor. Therefore, the response acceleration time histories of the building models are computed and their physical properties are examined to constitute prepared sinusoidal wave.

The three building models, five, ten and twenty-story shear frame are used. These shear frames meet the latest design specification, which requires having post yielding lateral strength [18]. Employing the earthquake waves whose maximum acceleration is scaled to 1.0 g, the response acceleration time histories of each building model at the top story are numerically computed by the software TDAPIII. [22]. To obtain the reasonable response acceleration time histories at the top story that satisfy the analytical purpose of this study, ten earthquake records whose principal period are close to the natural period of the buildings are used. As reported in reference [23], it is observed that the buildings shake with almost their first natural period regardless of the combination of building type and earthquake ground motion.

Referring to the knowledge obtained in previous section, the constitution of the prepared sinusoidal wave is considered as follows. The amplitude of the prepared sinusoidal wave should meet the peak horizontal response acceleration of the building floors. Moreover, the period of the prepared sinusoidal wave meets the first natural period of each buildings. This is because the filtering effects of the vibration system strengthen the advent of the principle period of structure in the response time histories. Therefore, the first natural period of building model is used for the period of the prepared sinusoidal wave instead of the predominant period of the ground motion.

Firstly, assume the absence of the vertical response of the building floor. The examination process is identical to that described in the previous session "**Slip displacement of the body on the ground.**" Figure 7 shows the probability density of the slip ratio β_h superimposed on Figure 2(e). The symbol of white diagram indicates the results on the building floors. Due to the limitation of analytical conditions considered herein, three building models, two friction coefficients (μ=0.3, 0.5) and ten earthquake waves, the number of data are 60. From Figure 7, although there is some dispersion in the analytical results of building floor, a peak is almost the same and the distribution shows the same tendency comparing to that on the ground. If the number of data increases, it is thought that the dispersion will also become small and the estimation accuracy will be improved. The results suggest that the slip displacement of the body on the building floor subjected to the horizontal response acceleration can be estimated by the same procedure specified by Eq. (9).

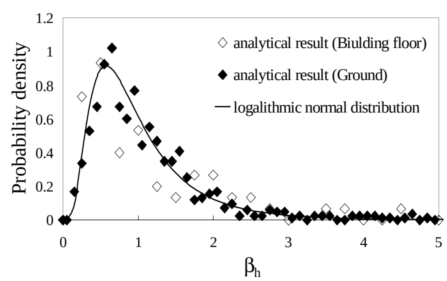


FIGURE 7. PROBABILITY DENSITIES OF SLIP RETIO β_h

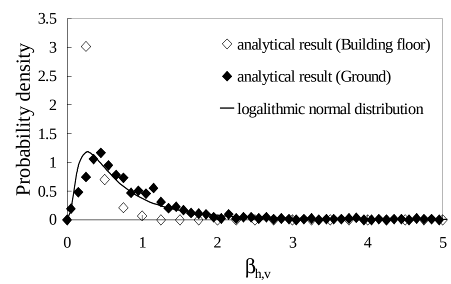


FIGURE 8. PROBABILITY DENSITIES OF SLIP RATIO $\beta_{h,v}$

Secondly, consider the slip displacement of the body subjected to both horizontal and vertical response accelerations. The analytical conditions are the same as the previous investigation in the absence of the vertical response acceleration of building floor. Figure 8 shows the probability density of the slip ratio $\beta_{h,v}$ superimposed on Figure 5(e). Its distribution is completely different. It implies that the effects of vertical acceleration on the slip displacement of the body placed on the building floor differ from that on the ground. Figure 9 presents the changes of the prediction accuracy of slip displacement of the body by the proposed method when we consider the vertical response acceleration of the building floor. It is understand that the vertical response acceleration works to increase the slip displacement of the body. Moreover, its influence is greater than the knowledge obtained in the investigation into the body on the ground. In some cases, it is observed that the downward vertical response acceleration sometimes exceeds the gravitational acceleration. The frictional resistance system completely disappear and the motion of the body and building floor has no relationship. Such discontinuous mechanism induces the large slip displacement. Therefore, in predicting the slip displacement of the body of placed on the building floor, the vertical response acceleration has to be considered. However, it is thought that the different approach other than the proposed method is necessary, which can appropriately deal with the vertical floor motion.

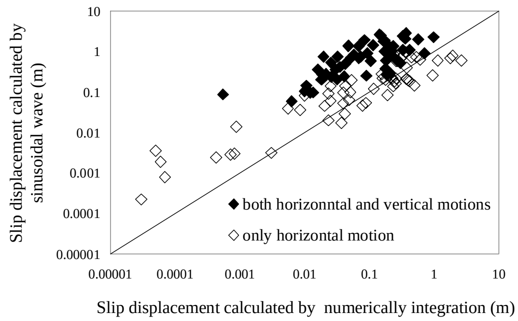


FIGURE 9. PREDICTION ACCURACIES OF SLIP DISPLACEMENT

CONCLUSION

This paper proposed the use of the prepared sinusoidal wave to predict the slip displacement of the freestanding rigid bodies with the assistance of the adjustment factor. The prepared sinusoidal wave is used for calculating the first approximation of the slip displacement, then it is revised with the adjustment factor which possesses the probabilistic properties. The proposed method enables to calculate the maximum expected slip displacement based on the probability of nonexceedance of the threshold if practitioners know the peak horizontal and vertical acceleration, friction coefficient and the principal period of the excitation. The proposed method well estimate the slip displacement of the body subjected to the horizontal and vertical accelerative action, wherever it is set on. However, the estimation accuracy deteriorates when the body is on the building floor and subjected to horizontal and vertical motion.

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APPENDIX

Approximate solution of slip displacement under action of sinusoidal wave

Assume the absence of the vertical acceleration to simplify the problem. According to the observations of the slip behavior of the body subjected the sinusoidal base motion from the absolute rest frame, the examples of tracks of acceleration and velocity of the body and base are graphically depicted in Figure A1.

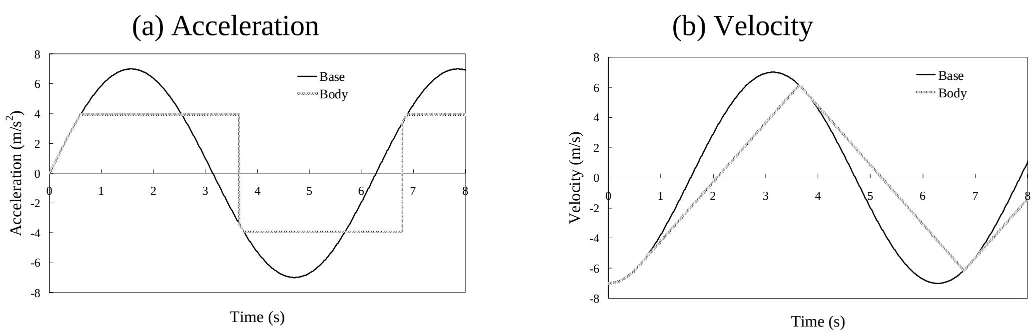

FIGURE A1. ACCELERATION AND VELOCITY TIME HISTORIES OF THE BASE AND BODY

The analytical conditions are $A_{gx}\, g$ =7m/s^2 and T =6.28s μ =0.4. The body begins to slip when the horizontal inertia force overcomes the friction force between the body and the base. This condition is simply given by the base acceleration μg . Since this study does not distinguish types of friction coefficients, the horizontal acceleration of the body during the slip is also given as μg . The slip behavior continues until the velocities of the body and base become the same.

From Eqs. (6) and (7), the time t_0 when the body begins to slip is calculated as:

$$t_0 = \frac{T}{2\pi} Sin^{-1} \frac{\mu}{A_{gx}} \tag{A1}$$

From the horizontal acceleration of the slipping body, its velocity at arbitrary time is calculated as:

$$\dot{x} = \mu g t + C_1 \tag{A2}$$

Similarly, the velocity of the base is calculated as:

$$\dot{z}_h = -A_{gx}\ g \frac{T}{2\pi} \cos\frac{2\pi}{T} t \tag{A3}$$

Here, the initial velocity of the base motion ($-A_{gx}\ gT/2\pi$) is assumed to simplify the problem. In contrast, C_1 in Eq. (A2) is integral constant and should be determined to have the body the same velocity of the base when the body commences the slip, since the observation from the absolute rest frame is assumed.

$$C_1 = -\frac{T}{2\pi}\left(\mu g Sin^{-1}\frac{\mu}{A_{gx}} + A_{gx} g \cos\left(Sin^{-1}\frac{\mu}{A_{gx}} \right)\right) \tag{A4}$$

Since the slip terminates when the velocity of the body reaches the same velocity of the base, the time can be calculated by Eqs. (A2), (A3) and (A4)

$$\mu g t - \frac{T}{2\pi}\left(\mu g Sin^{-1}\frac{\mu}{A} + A_{gx} g \cos\left(Sin^{-1}\frac{\mu}{A_{gx}} \right)\right) = -A_{gx} g \frac{T}{2\pi} \cos\frac{2\pi}{T} t \tag{A5}$$

Solve Eq. (A5) for t to find the slip termination time t_1. Since Eq. (A5) is, however, the transcendental function in terms of the cosine function, it is difficult to obtain the solution directly. This study tries to find its approximate solution employing the Taylor's series. According to the Figure A1, the slip terminates when t is around $T/2$, it implies that the Taylor's series around t=$T/2$ may give the appropriate approximation. The Taylor's series of the cosine function around t=$T/2$ is given as:

$$\cos\frac{2\pi}{T} t \cong -1 + \frac{2\pi^2}{T^2}\left(t - \frac{T}{2} \right)^2 \cdots \tag{A6}$$

Substitution of Eq. (A6) into Eq. (A5) yields the slip termination time t_1 as follows.

$$t_1 = \frac{T}{2\pi A_{gx}}\left(-\varphi + \sqrt{\varphi^2 + 2A_{gx}\phi} \right), \tag{A7}$$

$$\text{where, } \varphi = \mu - \pi A_{gx},\ \phi = \mu g Sin^{-1}\frac{\mu}{A_{gx}} + A_{gx} g \cos\left(Sin^{-1}\frac{\mu}{A_{gx}} \right)$$

Therefore, the slip displacement of the body, i.e. the basic slip displacement x_h^{Sin}, can be calculated by integrating the slip acceleration over the duration of the slip.

$$x_h^{Sin} = \int_{t_0}^{t_1}\int_{t_0}^{t_1}\left(\mu g - A_{gx} g \sin\frac{2\pi}{T}t\right)dtdt = \frac{1}{2}\mu g t_1^2 + \frac{A_{gx} g T^2}{2\pi^2}\sin\frac{2\pi}{T}t_1 + D_1 t_1 + D_2 \tag{A8}$$

where, $D_1 = -\mu g t_0 - \frac{A_{gx} g T}{2\pi}\cos\frac{2\pi}{T}t_0$,

$$D_2 = -\frac{1}{2}\mu g t_0^2 - \frac{A_{gx} g T^2}{4\pi^2}\sin\frac{2\pi}{T}t_0 + \mu g t_0^2 + \frac{A_{gx} g T t_0}{2\pi}\cos\frac{2\pi}{T}t_0$$

Here, D_1 and D_2 are integral constant. It is worth nothing that t_0 and t_1 includes the effects of dominant parameter on the slip motion of the body. In addition, it gives an upper bound of slip displacement in a single action of the sinusoidal wave. Figure A2 examines the prediction accuracy of Eq. (A8) by comparing to the numerically obtained slip displacement under the same conditions. From Fig. A2, it is understood that Eq. (A8) calculate the slip displacement with reasonable accuracy at any condition.

Finally, consider the slip displacement of the body subjected to simultaneous action of the horizontal and vertical motions. Replacing the friction coefficient μ of all equations in this Aappendix by the nominal friction coefficent $\mu' = \mu(1 - PVGA \cdot \eta \cdot \sigma / g)$, the slip displacement of interst $x_{h,v}^{Sin}$ can be calculated in a same manner. The nominal friction coefficient includes the reduction of the friction force based on the probability of occurrence of the coincident vertical ground acceleration at the peak horizontal acceleration.

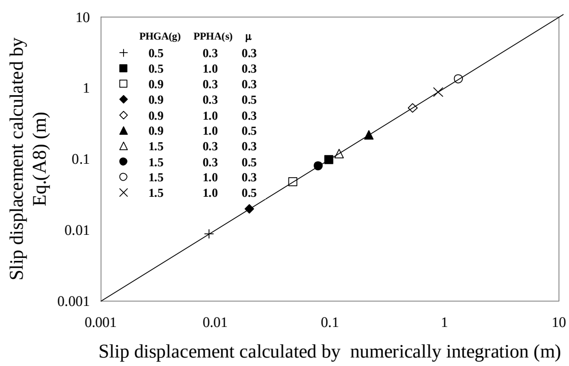

FIGURE A2. PREDICTION ACCURACY OF PROPOSED METHOD