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DYNAMIC SOIL STRUCTURE INTERACTION FOR LAYERED AND INHOMOGENEOUS GROUND: A COMPARITIVE STUDY

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SUMMARY

A series of centrifuge tests have been performed on a scaled containment structure resting on a rigid foundation by using the 10m-beam centrifuge at Cambridge. This scaled structure has been tested using different foundation soil stratifications and layering. It has been subjected to low, medium and high frequency earthquakes generated by using the Stored Angular Momentum earthquake actuator. The experimental results have been compared with the numerical predictions obtained by using the state of the art numerical code called SWANDYNE. The comparisons have indicated that numerical predictions are accurate when layering and stratification is present in the subsoil if the parameters are chosen judiciously. The various numerical modelling aspects are discussed and it is shown that layering introduces a mismatch in impedances of the soil medium and changes the overall nature of dynamic soil structure interaction.

INTRODUCTION

The overall performance of a structure during seismic shaking is influenced by the interaction between three inter linked systems: the structure, the foundation, and the complex geological media underlying and surrounding the foundation. A seismic soil structure interaction analysis evaluates the collective response of the three systems to the specified free field ground motion. The complete solution of the Soil Structure Interaction (SSI) would include consideration of issues like handling the boundary conditions for infinite lateral extent of soil, behaviour at the interface of the structure and soil etc. Remarkable progress has been made in the study of SSI problems, particularly in evaluating the dynamic characteristics of embedded structures in the last two decades and is summarised by Trifunac et al. [1]. The application of the SSI analysis has been developed to compute the seismic response of important structures such as nuclear power plants resting on rigid raft foundations, bridge abutments, and high-rise buildings on pile foundations. The practical analysis aims to predict the overall response of the concerned structure during an earthquake. This response involves the determination of the following important characteristics:

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- Specifying the design input motion to the structure.
- Modelling the dynamic soil properties specifically soil damping and stiffness on the basis of experimental data and past experience.
- Idealising the analytical model of the soil structure system either by direct method or sub structure method.
- Treating the non-linear soil model by a suitable constitutive model.
- Modelling the boundary between the bounded structure and the unbounded soil.
- Coping with the possible separation of the soil and the structure during strong shaking.
- Computing the response of the entire model by suitable numerical techniques (frequency domain/ time domain).

These complicated interdependent factors make the complete solution of the Soil Structure Interaction problem difficult. Each of the bulleted points highlights a specific area of the SSI problem. In this paper some of the above mentioned aspects are discussed by idealizing the analytical model of the soil structure system by direct method. In recent years observations from physical model tests (Centrifuge, 1-g shaking table, and field testing) have shed light on our understanding of the various mechanisms guiding soil responses during earthquake liquefaction. Results from these experiments have generated a large database for calibration and verification of computational models. But most of these tests have been carried out on homogenous loose soil layers where the natural variability of the ground has not been taken into account. Very few calibration studies apart from NSF sponsored VELACS [2] project exists for such inhomogeneous ground.

The present paper looks into the SSI effects for layered and inhomogeneous grounds through physical modelling. The present work intends to investigate the seismic response of heavy foundations resting on layered and inhomogeneous saturated soils. As the database of well characterised and instrumented case studies on the seismic performance of heavy foundations on liquefiable soil is very limited, physical modelling is used to generate the required database. These results are also used to calibrate the finite element code SWANDYNE [3]. A numerical model is advantageous because different numerical scenarios can be analyzed and various design parameters can be easily adjusted to determine their influence. The major problem in applying a numerical model for the analysis of the SSI is its numerical uncertainty. This concern is limited in this study as a series of validation studies have been performed. It is important to remember that SSI is always present for all types of subgrade even if the engineer chooses to neglect it either explicitly or implicitly.

CENTRIFUGE MODELLING

The need for dynamic earthquake modelling is closely associated with the nature of infrequent earthquakes in the field. It is extremely expensive preposition to instrument a potentially seismic site in preparation for an earthquake to occur. Thus foundations rarely have instrumentations to measure the dynamic responses incorporating interaction effects. Very little quantitative field data exists, which quantify and qualify the nature of SSI effects during strong shaking. Dynamic centrifuge modelling is useful in identifying SSI effects at reduced scale, and providing data on an idealized prototype for the development of analytical techniques. Similar to other experimental techniques it suffers from certain inherent inaccuracies arising from a number of factors like variation of g level in both tangential and radial directions, performance of shake tables, boundary conditions imposed by the container system and limitation of equipment transducers and data acquisition system. A well controlled centrifuge test can justifiably validate ideas, analytical methods and technological developments in a general sense.

The centrifuge tests described in this paper were conducted by using the 10m beam centrifuge at Cambridge. Specific discussion of the design and operation of the CUED 10m beam centrifuge can be found in Schofield [4]. Briefly the centrifuge has a 10m long beam, mounted on a central spindle. This spindle contains power, data and hydraulic slip-rings, giving great versatility in in-flight operations. At each end of the beam there is a swinging platform on which the model and counterweight are mounted, with the surface of the swinging platforms being at 4.125m radius in their swung-up position. The SAM (Stored Angular Momentum) earthquake actuator has been developed in-house to apply strong lateral motions simulating earthquakes to centrifuge packages. Detailed description of this shaker is available (Madabhushi [5]). The actuator is capable of operating at different g levels, generating earthquakes of maximum peak ground acceleration up to 0.4g, and duration varying in the range of 2.5 to 100s. The frequency of the motion can be altered between 1-5 Hz in prototype scale. Generally the motion is single frequency sinusoidal input.

Figure 1 shows the general arrangement for the centrifuge tests performed at Cambridge University. The soil is finely graded laboratory Fraction E sand whose properties are reproduced on Table 1, enclosed in an ESB (Equivalent Shear Beam) container, which matches the stiffness of the end wall with the stiffness of soil column (Schofield and Zeng [6]) during shaking. The internal dimensions of this box are 560mm x 235mm x 220mm. This is equivalent to a soil bed 28m x 11.75m in plan and 11m deep in a 50g test. The soil was saturated using silicon oil having a viscosity of 50cS to correct the dynamic scaling laws. The overburden consists of a rigid containment structure similar to the pre-stressed containments for nuclear power plants slightly embedded in the soil and applying a bearing pressure of 150 kPa at 50g. Test BG-04 consisted of a loose layer having a thickness of 2.5m deposited (R_D 45%) uniformly between dense layers having a R_D of 85%. Dense sand is generally considered non susceptible to liquefaction. It is however necessary to evaluate the response and the deformation of the ground against extremely strong motion for vital structures. The instrumentation in this tests consisted of accelerometers, pore pressure transducers and LVDT's.

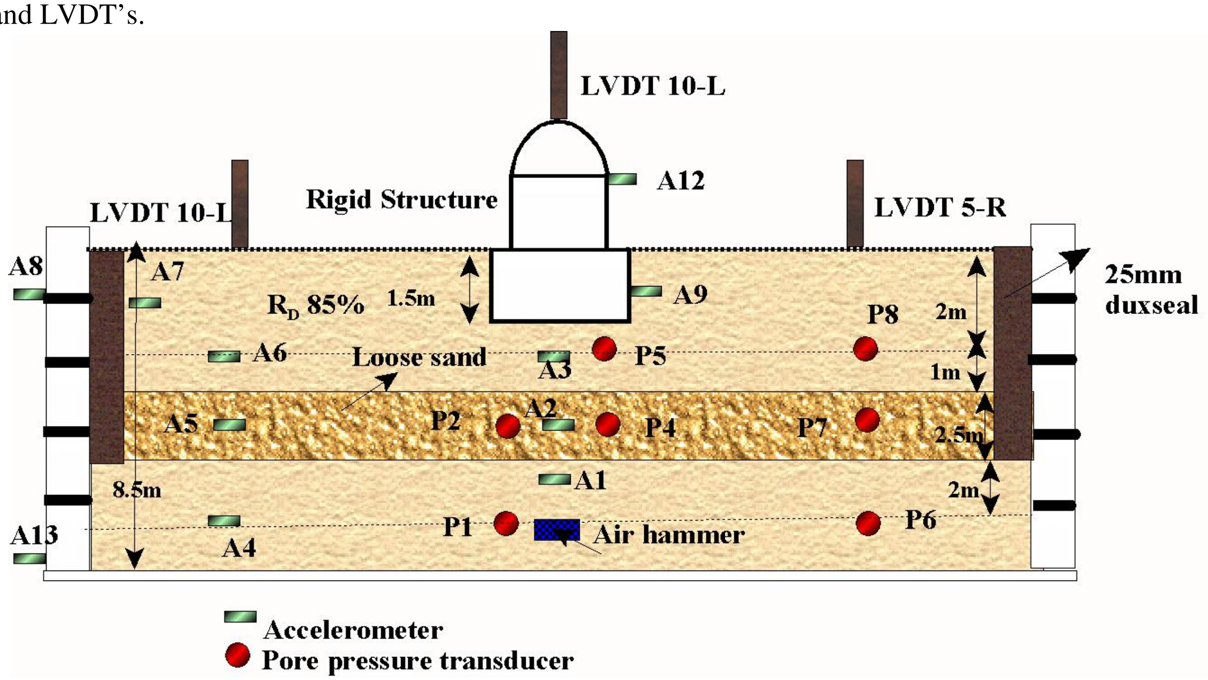

Figure 1: Instrumentation and test layout BG-04

The model was prepared by air pluviation of Fraction E silica sand whose properties are shown in Table 1. Different densities were achieved by varying the rate of pouring. The total depth of the prototype was 8.5m. The sand was poured up to a depth of 30mm and then the air hammer (Ghosh et al.[7]) was placed carefully in the model. The air hammer is a small actuator, which is used as a source to generate waves within the soil model. The propagation of shear waves through a model soil profile was measured in flight using an array of vertical accelerometers at different centrifugal accelerations in liquefiable soil. The values of the shear wave velocity measured were used in characterizing the soil layers for their evaluation of one dimensional property. These values were used in modelling the ground response in the free field.

Table 1. Properties of Fraction E silica sand

Property	ϕ_{crit}	D_{10}	D_{50}	D_{60}	e_{min}	e_{max}	k with water at e = 0.72
Value	32^0	0.095 mm	0.14 mm	0.15 mm	0.613	1.014	0.98×10^{-4} m/s

Post Test observations and test results

A series of earthquakes were fired and only a few relevant results are discussed here. More detailed results can be seen in the next section of the paper where the numerical comparisons are made. The tilt and rotation of the foundation after seismic shaking is considered as a performance criteria for raft foundations. It was seen that the tilt and rotation of the foundation was reduced significantly as the soil was densified. The angle of tilt was 4° in dense soil. The final settlement of the superstructure was 150mm. As the strength of the earthquake is increased and a strong earthquake is fired (15% g) acceleration traces show that the entire loose sandwiched layer had nearly liquefied in test BG-04 and the transmission of the shear waves is significantly reduced as seen in Figure 2.

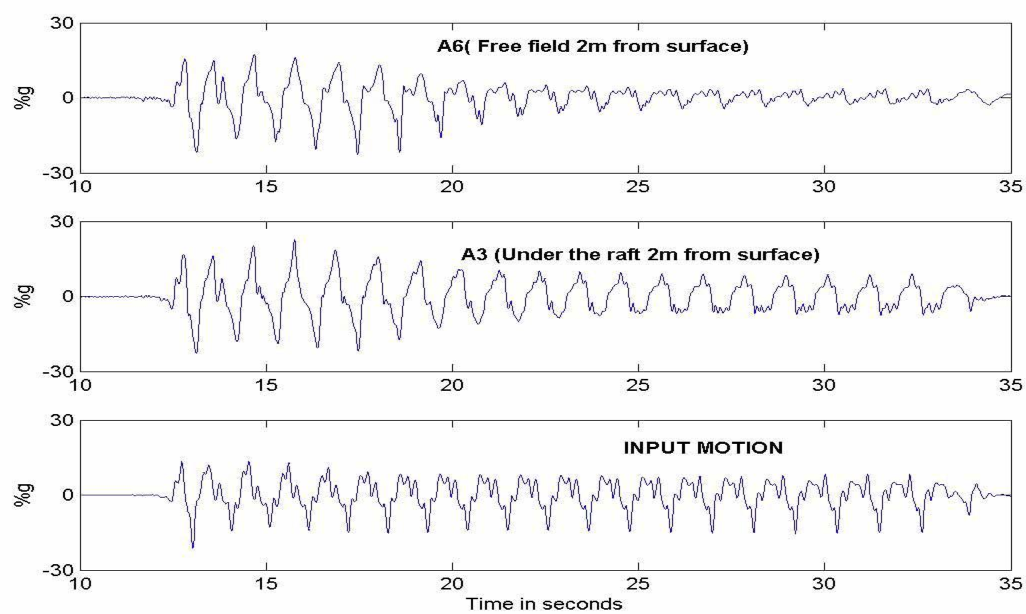

Figure 2: Acceleration transmission through dense and loose soil in BG-04 with a sandwiched loose layer being liquefied after strong shaking.

Accelerations in the loose sand layer show progressive and dramatic overall de-amplification of the earthquake motion. In the beginning of shaking cycle motion is amplified as it transmitted through the dense soil in the first few cycles. The high frequency components are filtered out as the motion is propagated to the surface. It is seen in Figure 2 that the loose layer under the high overburden has softened considerably but was still transmitting accelerations whereas the free field had liquefied and dense sand was slumping into the loose zone. This behaviour is similar to the tests on homogeneous loose soil where the free field had softened more than underneath the raft foundation. The phenomena can be compared with the mass spring system transmitting motion from the base. In the initial stages the spring is stiff enough to transmit all the frequencies. The progressive degradation of the soil stiffness due to excess pore pressure generation and the cyclic shear strain amplitude can soften the spring. The system may undergo resonance at lower harmonics while the higher harmonics is being attenuated. At the final stage the spring can become so flexible that fundamental earthquake frequency band cannot be transmitted as this corresponds to the full liquefaction of the soil. This phenomenon is similar to the isolation mechanism observed in the Kobe earthquake [8]. The benefits of the isolation mechanism is however available only during strong shaking, whereas for small strength earthquake considerable amount of shear waves is still being transmitted to the surface.

The large strength earthquakes induce large dynamic shear stresses and the magnitude of the excess pore pressure build up is related to the cyclic shear strains induced. Figure 3 presents the pore pressure measured for model for BG-04 for a high strength earthquake (15%g). The acceleration trace for this earthquake has been presented in Figure 2. The excess pore pressure ratio reached for P7, which is located in the middle of the loose layer at the free field is very close to 1 indicating very low effective stresses by the end of the shaking period and conditions close to initial liquefaction. Under the structure none of the pore pressure measurements reached their initial vertical effective stress. The presence of the structure created a sustained static shear stress in the soil and thus has a significant effect in the pore pressure build up. By the end of the shaking period this loose sandwiched layer had liquefied and was transferring much less shear as seen in Figure 2.

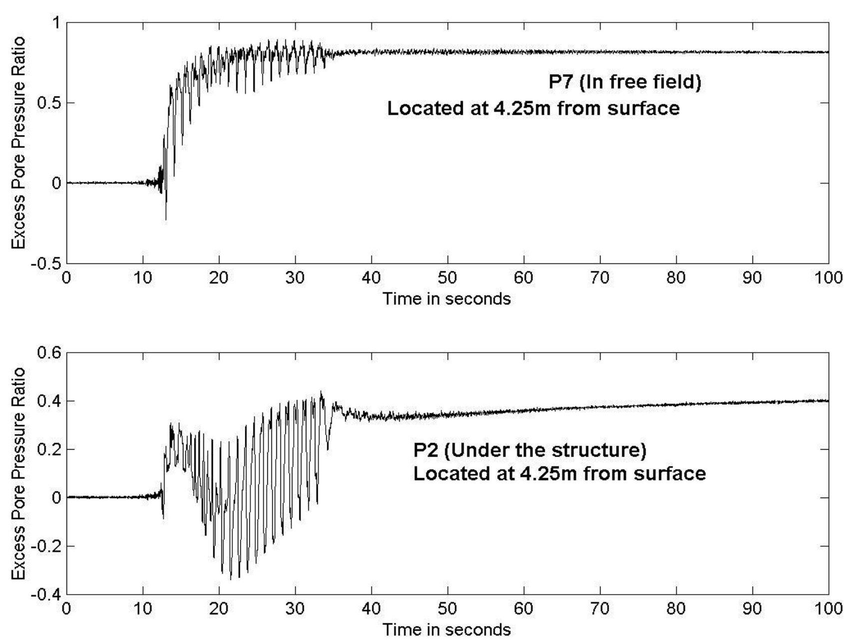

Figure 3: Pore pressure measurements in the model with uniform loose layer for BG-04.

Pore pressure transducer P2, which is under the structure in the sandwiched loose layer, shows a significant transient decrease in pore pressure and corresponding soil strength gain due to suppressed dilatancy as the event is undrained. Large shear displacements can occur at low shear stresses but at some critical point dilatancy triggers negative pore pressures that momentarily increase the effective stresses and stiffens the soil. This would potentially lead to an increase in the bearing capacity of the soil. However, permanent shear strains accumulate in the dense soil even after the stress path has stabilized at the strength envelope. This behaviour has been reported in cyclic undrained triaxial tests on dense sands by numerous researchers (Hyodo et al. [9]).

Thus the centrifuge test results indicate that the loose layer sandwiched in between the dense layers is capable of changing the transmission characteristics of the input waves as they travel through the medium. The numerical code has to be used to predict these behaviour patterns to develop confidence in its usage. The numerical code is first discussed briefly and then the constitutive model used is briefly presented. This is followed by some simulation results obtained by using the code.

NUMERICAL MODELLING

Numerical codes have been developed to analyze soil liquefaction though they have often lacked field data for validation. Dynamic centrifuge modeling has often been used for checking the accuracy of these numerical models. This was the primary objective of the NSF sponsored VELACS [2] project which brought numerical and physical modellers together. The numerical analysis methods that were used were broadly categorised by Arulanandan et al. [10] as decoupled methods, partially coupled methods and fully coupled methods. None of the above methods were able to model accurately the bench mark liquefaction problems, but the codes which came closest to capturing the essentials of the complicated behavior of saturated sands under cyclic loading were the fully coupled codes. In these codes, which included DYSAC2 [11], DYNAFLOW [12], and SWANDYNE [3], the differential equations governing the motion of the solid and fluid phases are coupled with the mass balance equation resulting in the fully coupled differential equation. These are then approximated by a weighted residual method. These equations are solved using the finite element techniques. The type of constitutive model used must predict the pore pressure generation and dissipation, which are fully coupled and linked with the deformation of the soil skeleton according to the Biot’s formulation [13]. The numerical code SWANDYNE will be briefly described along with the constitutive model adopted to capture the essentials of dynamic soil behavior.

SWANDYNE: P-Z Mark III model

SWANDYNE is a unified, general-purpose, 2-D, finite element program, which uses Biot’s fully coupled dynamic equation with the assumption that the fluid acceleration in relation to the solid skeleton is negligible. It uses the ‘u-p’ formulation, with displacement u and pore pressure p being chosen as the field variables. The time integration scheme used is the GNpj (Generalised Newmark pth order interpolation scheme for jth order differential equation) developed by Katona et al. [14]. This scheme uses a staggered approach and a single field variable is obtained using the extrapolated value of the other. For example displacement ‘u’ and pore pressure ‘p’ can be chosen as the field variables and the displacement can be obtained using the extrapolated values of the value of the pore pressure. It can be used to deal with static, consolidating and dynamic conditions under drained or undrained conditions. There are different soil models incorporated into the program such as linear elastic, elastic with variable moduli and Coulomb cut off surface, original Cam clay, Modified Cam clay and Pastor Zienkiewicz (P-Z) Mark III models. Of all these constitutive models in SWANDYNE, the last one is best suited to model cyclic behaviour of soils.

The P-Z model developed by Pastor et al. [15] is a generalised plasticity bounding surface model with a non-associative flow rule. It models the effects of dilation, permanent deformations and the hysteric properties of saturated sands under dynamic loads. A detailed description of the model is available elsewhere [15]. The parameters needed to define the model completely can be obtained from routine triaxial tests. The capabilities of this program in handling dynamic problems have been tested time and again. Madabhushi [16] used it for the numerical analysis of tower-soil interaction problems and compared the results with centrifuge test data. Chan et al. [17], and Zienkiewicz et al. [18], also used this in the VELACS project for the Class A predictions. These results have been compiled and compared with the post test results [2]. All these simulations have been on level homogeneous loose soil and thus the code has not been sufficiently tested for layered soils. The present simulation is an attempt to validate the code for such inhomogeneous ground conditions.

FE mesh and solution sequence

Centrifuge experiments were modelled using the prototype scale dimensions. The dynamic response of the model foundation embedded in layered soil was generally obtained by the following steps, as suggested by Zienkiewicz et al. [19].

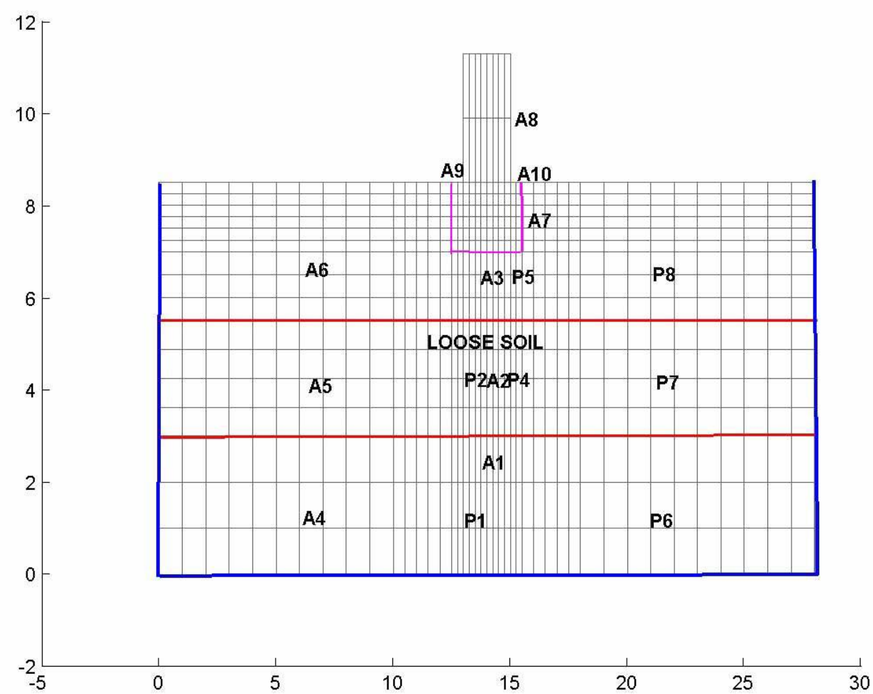

Figure 4: Finite element mesh for BG-04 with loose layer sandwiched between dense sand layers. Actual test set up shown in Figure 1.

- A finite element mesh was initially created using an in-house pre-processor written in MATLAB. The mesh (Figure 4) was discretised using 8 noded quadrilateral elements having 4 fluid nodes. This was done to reduce the numerical difficulties associated with nearly incompressible fluid phase. The ESB container comprises of rigid aluminium rings separated by thin rubber rings. Whilst each ring is stiff, maintaining a K_0 condition at the end walls, the flexible rubber layers enable the rings to move relative to each other during dynamic loading and mimic the shear stiffness of a semi-infinite soil layer. In order to model these boundary conditions in a 2D analysis, the mass of each ring was included in the wall elements at the box ends and the overall stiffness of the shear-stack was matched by that of the columns of wall elements.

- Each solid node is associated with two degrees freedom for the lateral and vertical displacements, and each fluid node is associated with 1 degree of freedom for pore pressure. The appropriate boundary conditions were then applied at the boundary of the model. Tied nodes are used to model the 'laminar box' (Hushmand et al. [20] type behaviour. This implies that the horizontal and vertical nodal displacements at the two ends of the soil are restrained to have the same value. This creates some problems as our ESB box is not really a laminar box. The boundary conditions are best simulated by rigid conditions for our ESB box.
- The saturated condition of the soil medium was prescribed by specifying the initial hydrostatic pressure at the fluid nodes. The fluid nodes are located at the same co-ordinates as the displacement or solid nodes. The structure load is simulated by applying the bearing pressure on the appropriate solid nodes.
- Static analysis was then performed to determine the initial stress state of the model. Different types of soil models can be used for the static run. It was seen that the initial stress state was the one of the important parameters that affects subsequent dynamic analysis. This was due to the fact that this stress state was the 'initial stress' state for consequent dynamic analysis. Figure 5 shows the stress distributions obtained after using the slip elements at the interface of the soil and the structure. The stress distribution shows symmetric pattern of the pressure and a load dispersion which varies between the active and the passive states. The area of the soil underneath the building is subjected to higher static shear stresses due to higher effective overburden pressure. Figure 5(b) shows the magnitude of these static shear stresses as 20kPa. Negative and positive shear stresses are essentially due to the sign convention adopted.

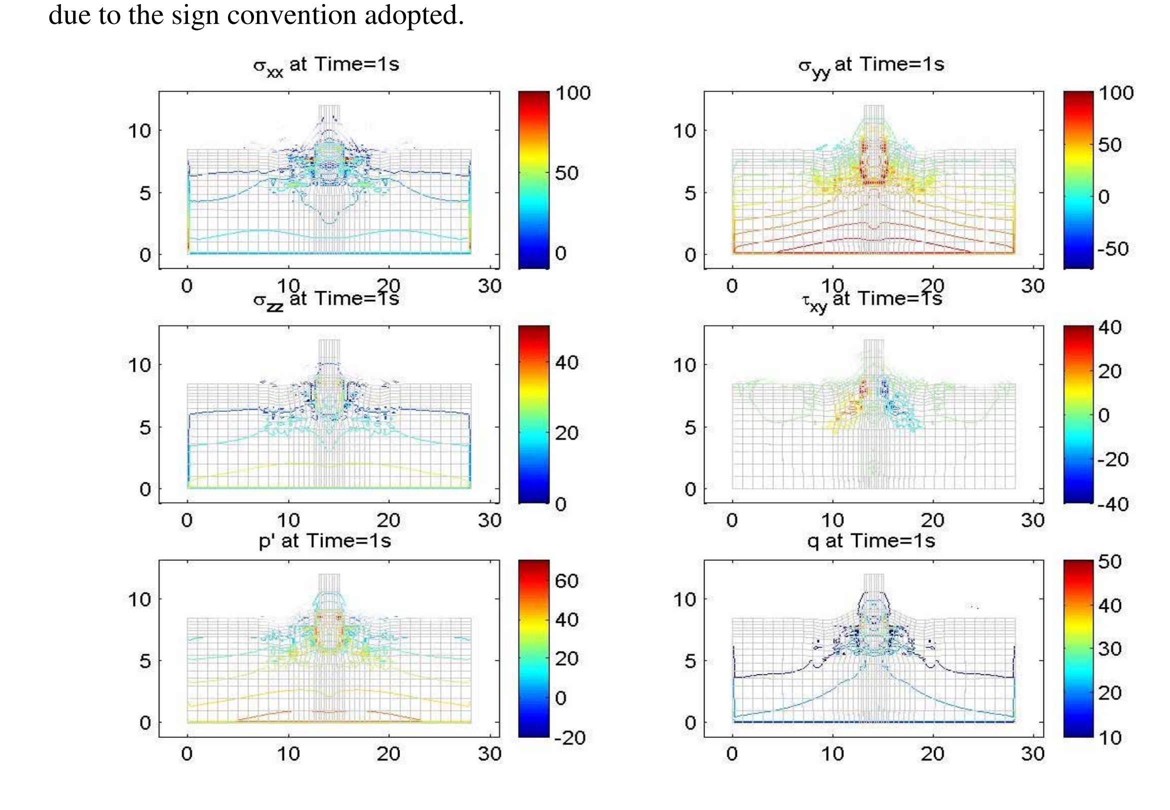

Figure 5: Static stress distribution with the superstructure; (b) Static shear.

- Following this, a non-linear analysis was performed for the earthquake stage with the assumed cyclic loading similar to the earthquake applied in the centrifuge at the base of the model. In the present case the cyclic loading was the input acceleration applied during the testing. The analysis was performed using a Generalised Newmark scheme with non-linear iterations.
- Various plots are then created for the final comparisons. A post processor program was written in MATLAB for this purpose. A similar post processor was also used to post process the centrifuge test data. As the post processors use consistent output formats direct comparisons are straight forward.

Parameter identification

The parameters needed to define the PZ model completely are obtained from routine triaxial tests. Jeyatharan [21] has identified the numerical values of these parameters from the triaxial tests performed at Cambridge. These parameters are defined and obtained by standard triaxial cyclic tests. Figure 6(a) presents the variation of the deviatoric stress against axial strain as reported by Jeyatharan [21] from a cyclic undrained test. Figure 6(b) shows some of the experimental test results with the P-Z Mark III model predictions using the element-testing program for comparison purposes. As the numerical simulation agrees very well with the experimental result thus Mark III model was selected for the present study. Thus the PZ model requires

- 2 parameters for initial conditions (D_R and e)
- 2 Elastic constants (K, G)
- Critical state (M_g)
- Shape (M_f/M_g, α_g and α_f)
- Deviatoric strain hardening (β_0 and β_1)
- Unloading (H_u and γ_{Hu})
- Loading (H_l and γ_{dm})

Thus there are 12 variables needed to define this model completely. This is a fairly reasonable number for defining standard dynamic constitutive models. The values used in the simulation are shown in Appendix.

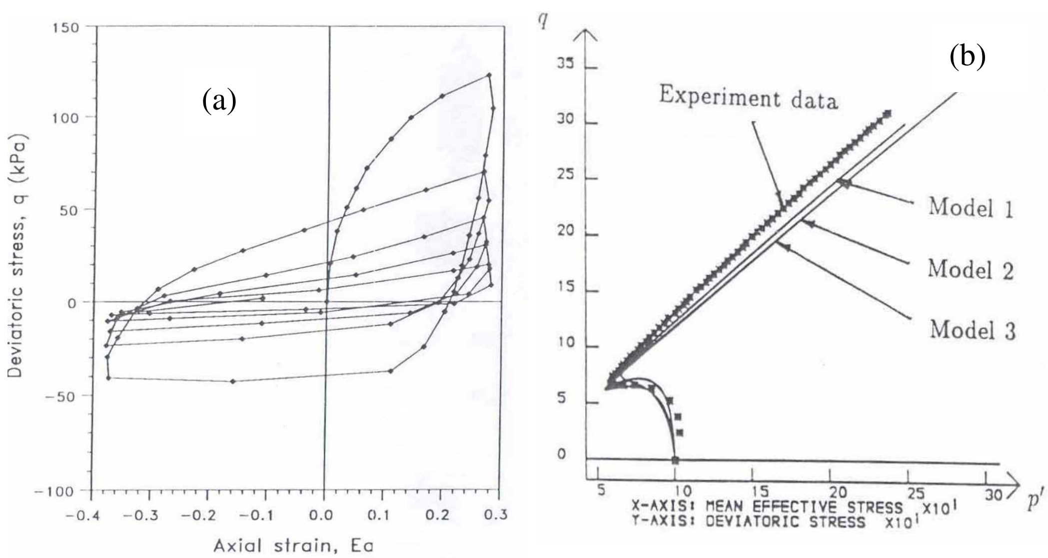

Figure 6: (a) Variation of the deviatoric stress against axial strain for cyclic undrained triaxial test ;(b) P-Z Mark III model predictions for undrained compression test (Jeyatharan 1991).

VALIDATION OF CENTRIFUGE TEST RESULT

The outline of the method used for the numerical simulation of the centrifuge test results were discussed in the previous sections. In this section a few of the validation results are presented. In this paper the free field accelerations and pore pressures are presented. Detailed simulation results can be seen at Ghosh [22] where the comparisons underneath the raft foundation have been presented along with the settlement and building deformations.

Accelerations in the free field

Figure 7 presents the earthquake accelerations predicted by the numerical model in the free field for test BG-04. The instrument identification is depicted in Figure 1. The input motion is not symmetric and at a depth of 7.5m from the surface, the predictions match the measured accelerations closely as seen in Figure 7 although the peak cycle acceleration is higher in the centrifuge test result. This accelerometer is located in dense soil.

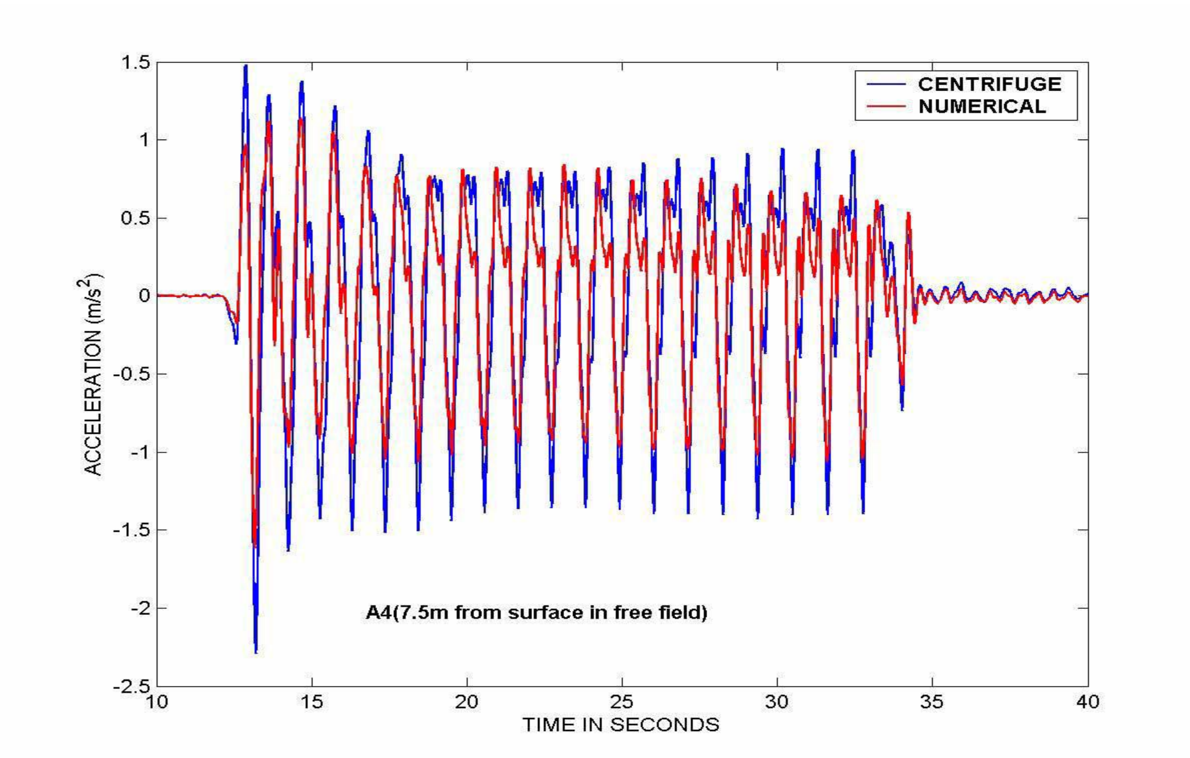

Figure 7: Comparison of acceleration time history in the free field for test BG-04 for A4.

At the middle of the loose layer as seen in Figure 8, centrifuge test results show attenuation due to considerable softening of the loose layer after a few cycles of shearing, although predictions are still reasonably accurate. The accelerometers located at shallow depth match the measured and predicted accelerations well as seen in Figure 9 in the first few cycles of shaking. It is seen that the numerical values show more attenuation at the beginning. As the soil weakens the value of accelerations depart from the experimental value. This is possibly due to the amount of the shear wave energy being transmitted from the bottom to the top. As the soil weakens the magnitude of shear waves transmitted should diminish. However, due to the oscillatory nature of the pore water pressure, the mean effective stress is not reduced as much as in the physical experiment. Therefore the shear modulus is not reduced adequately leading to excessive transfer of shear wave energy.

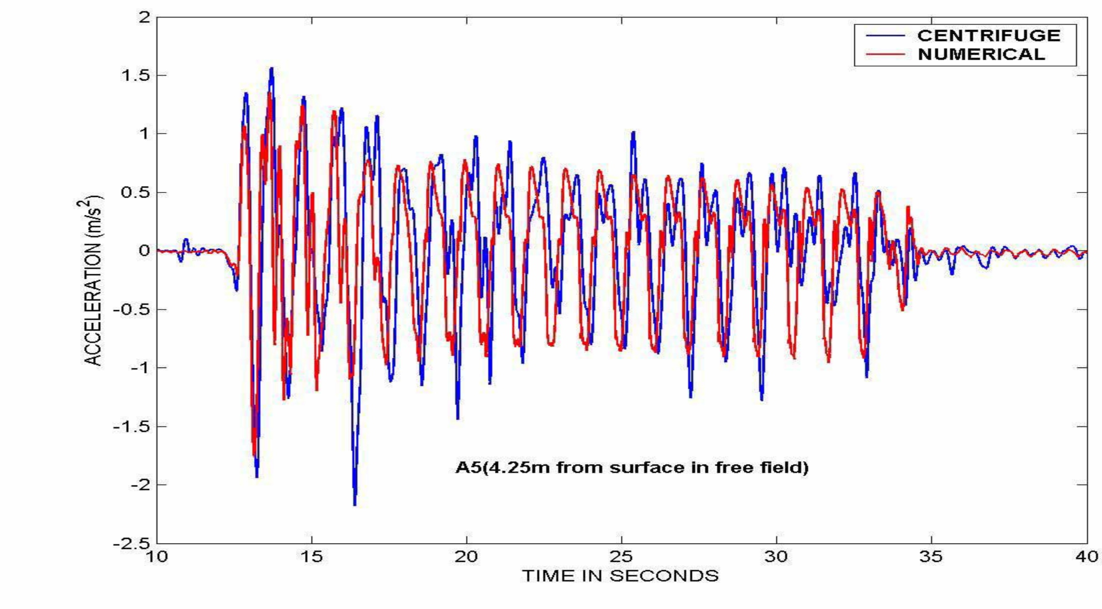

Figure 8: Comparison of acceleration time history in the free field for test BG-04 for A5.

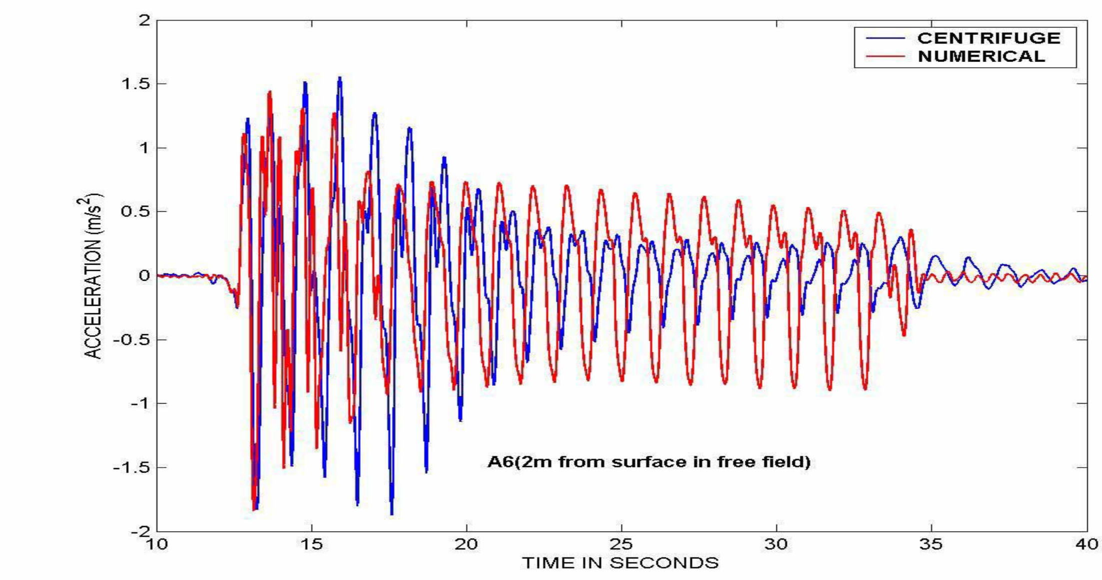

Figure 9: Comparison of acceleration time history in the free field for test BG-04 for A6.

In summary the computed accelerations were in reasonable agreement with the recorded counterparts for most of the simulations presented in this paper. The reduction in the transmission of shear waves following the onset of liquefaction is captured adequately in loose homogeneous soil. In layered soil this effect is not adequately modelled. This is true in the free field. The accelerations predicted in dense soil are usually under predicted.

Pore pressures in the free field

Figure 10 compares the excess pore pressures predicted in the free field in the dense sand layer for BG-04 by the pore pressure transducer P6, and it is seen that the average values of pore pressure compare well although the initial rate of generation is different. The maximum pore pressure is attained in the first few cycles**.**

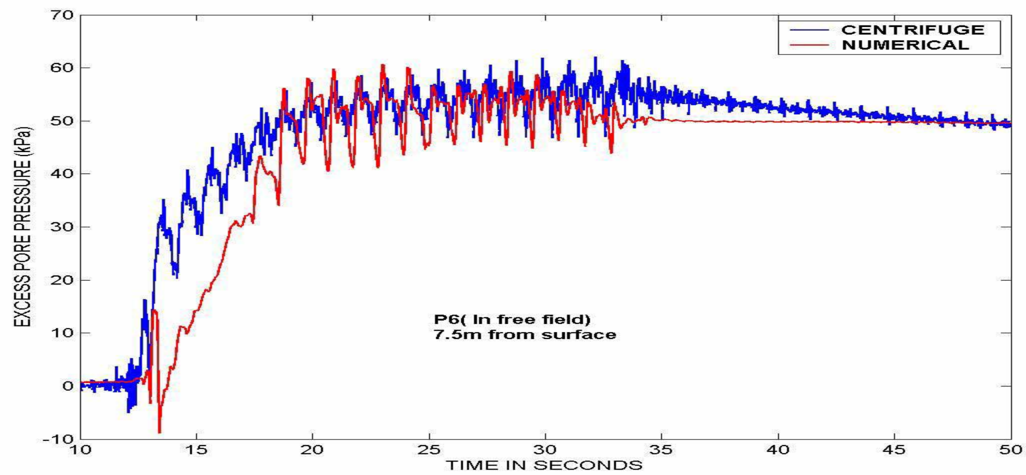

Figure 9: Comparison of excess pore pressures in free field for test BG-04 in dense soil.

At a shallower depth within the loose sand layer the predictions match well in an average sense as seen in Figure 11 although the dilation induced suction spikes are more in the centrifuge test results. The interesting fact to notice is that the parameters used to separate the loose and dense layers seem to capture the essential difference in the behaviour between these layers. Higher pore pressures are generated within the loose layer and the numerical model predicts this well in the free field.

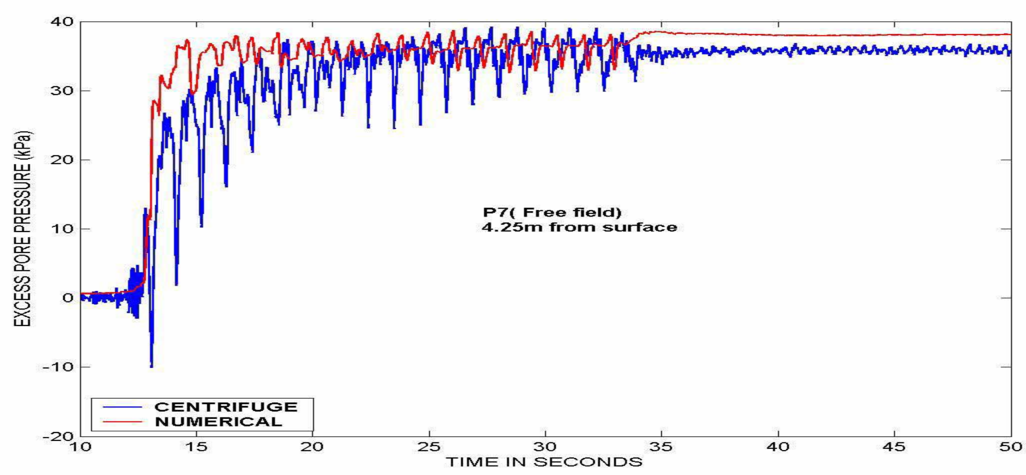

Figure 11: Comparison of excess pore pressures in free field for test BG-04 in loose soil.

At shallower depths (Figure 12) the predictions do not match well at the beginning of the shaking cycle. The centrifuge test results show much more aggressive dilating behaviour of dense sand especially in the initial period of shaking where suction spikes are seen and transient reductions of the pore pressures are in the range of 10-15 kPa. The transducer P8 is located in the dense sand as seen in Figure 1.

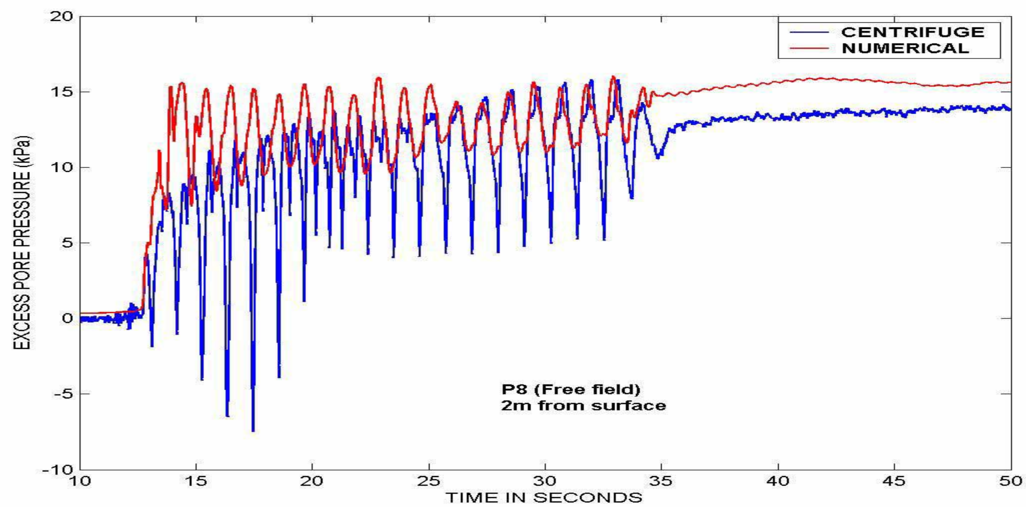

Figure 12: Comparison of excess pore pressures in free field for test BG-04 in dense soil.

Thus it can be seen that the overall response of the predicted excess pore pressure at several locations is in agreement with the experimental observations as seen in the comparisons in the free field. But the PZ model is unable to model the first few cycles of the excitation and it is seen in all the pore pressure traces that the rise in the excess pressure is much higher in the first few cycles.

CONCLUSIONS

The dynamic response of a saturated soil bed consisting of alternate layers of dense – loose –dense sand layer was investigated. The superstructure consisted of a containment structure supported on a rigid raft foundation. Test results indicated the complex mechanisms changing the transmission characteristics of the input waves as they traveled through the medium. This centrifuge test was also simulated by using the FE code SWANDYNE.

In this paper SWANDYNE was successsfully used to validate the centrifuge test results on layered soil in the free field. The test simulated had complicated geometry and layering which made their numerical simulations difficult. In any FE model the elements provide the spatial approximation to the model geometry whereas the mathematical model for material properties provides the physical representation. In this paper emphasis was placed on of these factors and it was shown that SSI problems have to be modelled carefully in order to obtain better results. Interface or slip elements have been positioned between the underside of the foundation and the soil so that the correct interface behaviour can be modelled. The importance of modelling the box was demonstrated and introduction of a layer of slip elements next to the box improved the performance.

It was also shown that intricate details in the dynamic behaviour can be predicted in a global sense. Some of the numerical discrepancies in the results can be traced back to the way the constitutive model has been formulated and some parametric studies are suggested. Thus the results from the validation exercise suggest that the code can be used in layered soil provided that the boundary conditions are modelled accurately.

The computed accelerations were in reasonable agreement with the recorded counterparts for most of the simulations in test BG-04. The reduction in the transmission of shear waves following the onset of liquefaction is captured adequately in loose homogeneous soil. In layered soil this effect is not adequately modelled. This is because the simulated stationary acceleration responses were different in different layers and was mainly due to the overestimation of the shear wave propagation through liquefied soil. This overestimation of the acceleration is due to using a constant Rayleigh damping proportional to the initial stiffness of the soil. In case a damping parameter is used which updates with the current tangent stiffness the accelerations underneath the containment will be simulated properly. The transferred accelerations predicted in dense soil are usually under predicted.

APPENDIX

Material parameters used in the simulation

Model : DEP08Q	Loose soil	Dense soil
M_g	1.15	1.32
$`M_f$	0.75	1.1
α_f, α_g	0.45	0.45
K	38MPa	50MPa
G	21MPa	25MPa
β_0	4.2	4.2
β_1	0.2	0.2
Ref p´	25kPa	25kPa
H_o	200	40
H_{uo}	400MPa	4MPa
γ_{Hu}	2	2
γ_{DM}	0	4

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