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DYNAMIC ANALYSIS OF CONCRETE GRAVITY DAMS BY DECOUPLED MODAL APPROACH IN TIME DOMAIN

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SUMMARY

In this paper, a new approach is proposed for dynamic analysis of general concrete gravity dam-reservoir systems based on the decoupled modal technique in time domain. The method is described initially, and the analysis of Pine Flat Dam is considered as a verification example. The proposed approach is proved to be a very effective technique. The main advantage being that it relies on eigen-vectors of decoupled system, which can be easily obtained by standard eigen-solution routines.

INTRODUCTION

There are different approaches available for seismic analysis of concrete gravity dams by applying finite element method. However, the most natural technique is based on the Lagrangian-Eulerian formulation, which employs nodal displacements and pressure degrees of freedom for the dam and reservoir region, respectively. Meanwhile, it is well known that in this formulation, the induced total mass and stiffness matrices of the coupled system are unsymmetric due to interaction terms [1]. In direct method of analysis in time domain, it is possible to efficiently transform the direct integration algorithm in such a manner that allows one to work with symmetric matrices [1, 2]. However, in modal analysis, the symmetrization process requires introduction of additional variables in eigen-solution routines, which is not very efficient and creates complications in computer programming [3, 4].
In the present study, a modal analysis method is proposed which is dependent on mode shapes evaluated from the symmetric part of the original eigen-problem of the system. The formulation of this method is presented initially and the procedure is implemented in a special computer program “MAP-76” [5]. Subsequently, the analysis of Pine Flat Dam is considered as a numerical example. The proposed technique is applied for this dam and the results are compared against corresponding results related to direct method of analysis. Finally, the accuracy of the method is evaluated and its convergence is controlled.

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METHOD OF ANALYSIS

Let us consider a general concrete gravity dam-reservoir system. In this study, the dam is discretized by plane solid finite elements, while plane fluid elements are utilized for the reservoir region. It can be easily shown that in this case, the coupled equations of the system may be written as [6]:

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{B} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{r}} \\ \ddot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{K} & -\mathbf{B}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} -\mathbf{M}\,\mathbf{J}\,\mathbf{a}_{\mathrm{g}} \\ -\mathbf{B}\,\mathbf{J}\,\mathbf{a}_{\mathrm{g}} \end{bmatrix} \tag{1}$$

M, **C**, **K** in this relation represent the mass, damping and stiffness matrices of the dam body, while **G**, **L**, **H** are assembled matrices of fluid domain. The unknown vector is composed of **r,** which is the vector of nodal relative displacements and the vector **p** that includes nodal pressures. Meanwhile, **J** is a matrix with each two rows equal to a 2×2 identity matrix (its columns correspond to unit rigid body motion in horizontal and vertical directions) and $\mathbf{a}_{\mathrm{g}}$ denotes the vector of ground accelerations. Furthermore, **B** in the above relation is often referred to as interaction matrix.

The relation (1) can also be written alternatively in a more compact form as:

$$\overline{\mathbf{M}}\,\ddot{\overline{\mathbf{r}}} + \overline{\mathbf{C}}\,\dot{\overline{\mathbf{r}}} + \overline{\mathbf{K}}\,\overline{\mathbf{r}} = -\overline{\mathbf{M}}\,\overline{\mathbf{J}}\,\mathbf{a}_{\mathrm{g}} \tag{2}$$

where $\overline{\mathbf{r}}$ and $\overline{\mathbf{J}}$ are defined as follows:

$$\overline{\mathbf{r}} = \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} \tag{3}$$

$$\overline{\mathbf{J}} = \begin{bmatrix} \mathbf{J} \\ \mathbf{0} \end{bmatrix} \tag{4}$$

Meanwhile, the exact forms of $\overline{\mathbf{M}}, \overline{\mathbf{C}}$ and $\overline{\mathbf{K}}$, are well apparent by matching relations (1) and (2) together. Obviously, these matrices can also be written as sum of the symmetric and unsymmetric parts as below:

$$\overline{\mathbf{M}} = \mathbf{M}_{\mathrm{S}} + \mathbf{M}_{\mathrm{U}} \tag{5a}$$

$$\overline{\mathbf{C}} = \mathbf{C}_{\mathrm{S}} \tag{5b}$$

$$\overline{\mathbf{K}} = \mathbf{K}_{\mathrm{S}} + \mathbf{K}_{\mathrm{U}} \tag{5c}$$

It is noted from equation (1) that the damping matrix is totally symmetric, and the following relation also holds:

$$\mathbf{K}_{\mathrm{U}} = -\mathbf{M}_{\mathrm{U}}^{\mathrm{T}} \tag{6}$$

The coupled equation (1) can be integrated and solutions can be obtained through time by direct method as well as modal approach. The direct integration process is usually carried out by applying Newmark's algorithm. In the normal procedure, this is encountered with a non-symmetric system of equations to be solved at each time step that is not going to be efficient. However, this could be avoided by a Pseudo-Symmetric technique, which is discussed elsewhere utterly [2]. In modal approach, which is the basis of the present study, the method relies on obtaining the natural frequencies and mode shapes of the system. Thereafter, the solution can be estimated based on the combination of these modes at different time steps.

Decoupled Modal Technique

The eigenvalue problem corresponding to relation (2) can be written as follows:

$$\overline{\mathbf{K}}\ \overline{\mathbf{X}}_j = \bar{\lambda}_j \overline{\mathbf{M}}\ \overline{\mathbf{X}}_j \tag{7}$$

Physically, it is clear that the eigenvalues of the this system are real and free vibration modes exist. However, it is noted from the form of matrices $\overline{\mathbf{K}}, \overline{\mathbf{M}}$ (relation (5)) that the system is not symmetric and standard eigenvalue computation methods are not directly applicable. Although, there are techniques available to arrive at a symmetric form and reduce the problem to a standard eigenvalue one, it is computationaly costly and additional variables are required to be introduced. Therefore, this path is not pursued in the present study. As a substitute, it was preferred to work with the eigenvalues and vectors extracted from the following eigen-problem:

$$\mathbf{K}_S\ \mathbf{X}_j = \lambda_j \mathbf{M}_S \mathbf{X}_j \tag{8}$$

Where $\mathbf{K}_S, \mathbf{M}_S$ are the symmetric parts of the $\overline{\mathbf{K}}, \overline{\mathbf{M}}$ matrices, as mentioned previously (relation (5)). Of course, the eigenvectors obtained through the above relation, are not the true mode shapes of the coupled system. However, these can be presumed as Ritz' vectors which can be similarly combined to estimate the true solution. The solution of this substitute eigen-problem are easily obtained by standard methods, since the involving matrices are symmetric and positive definite. Having the orthogonality condition and normalizing the modal matrix with respect to mass matrix, one would have:

$$\mathbf{X}^T \mathbf{M}_S \mathbf{X} = \mathbf{I} \tag{9a}$$

$$\mathbf{X}^T \mathbf{K}_S \mathbf{X} = \mathbf{\Lambda} \tag{9b}$$

Where $\mathbf{I}$ is the identity matrix and $\mathbf{\Lambda}$ is a diagonal matrix containing the eigenvalues of the symmetric substitute system. The following relations are also derived easily based on relations (5), (6) and (9):

$$\mathbf{X}^T \overline{\mathbf{M}}\ \mathbf{X} = \mathbf{I} + \mathbf{X}^T \mathbf{M}_U \mathbf{X} \tag{10a}$$

$$\mathbf{X}^T \overline{\mathbf{K}}\ \mathbf{X} = \mathbf{\Lambda} - \mathbf{X}^T \mathbf{M}_U^T \mathbf{X} \tag{10b}$$

As usual in modal techniques, the solution is written as a combination of different modes:

$$\bar{\mathbf{r}} = \mathbf{X}\ \mathbf{Y} \tag{11}$$

The vector $\mathbf{Y}$ contains the participation factors of the modes. Substituting this relation into (2) and multiplying both sides of that equation by $\mathbf{X}^T$, it yields:

$$(\mathbf{X}^T \overline{\mathbf{M}}\ \mathbf{X})\ddot{\mathbf{Y}} + \mathbf{C}^* \dot{\mathbf{Y}} + (\mathbf{X}^T \overline{\mathbf{K}}\,\mathbf{X})\mathbf{Y} = \mathbf{F}^*(t) \tag{12}$$

In this relation, additional matrix definitions are utilized as below:

$$\mathbf{C}^* = \mathbf{X}^T \overline{\mathbf{C}}\,\mathbf{X} \tag{13a}$$

$$\mathbf{F}^*(t) = -\mathbf{X}^T \overline{\mathbf{M}} \, \overline{\mathbf{J}} \, \mathbf{a}_g(t) \tag{13b}$$

Or alternatively, the following relation is obtained by employing (10):

$$(\mathbf{I} + \mathbf{X}^T \mathbf{M}_U \mathbf{X}) \ddot{\mathbf{Y}} + \mathbf{C}^* \dot{\mathbf{Y}} + (\mathbf{\Lambda} - \mathbf{X}^T \mathbf{M}_U^T \mathbf{X}) \mathbf{Y} = \mathbf{F}^*(t) \tag{14}$$

Applying the Newmark's technique for integration of this equation, would yield the following equation at each new time step:

$$\hat{\mathbf{K}} \; \mathbf{Y}_{n+1} = \hat{\mathbf{F}}_{n+1} \tag{15}$$

$\hat{\mathbf{K}}$ and $\hat{\mathbf{F}}_{n+1}$ are denoted as the generalized effective stiffness matrix and the generalized effective force vector of the system at time step n+1, respectively. They are defined as below:

$$\hat{\mathbf{K}} = a_0 (\mathbf{I} + \mathbf{X}^T \mathbf{M}_U \mathbf{X}) + a_1 \mathbf{C}^* + (\mathbf{\Lambda} - \mathbf{X}^T \mathbf{M}_U^T \mathbf{X}) \tag{16}$$

$$\hat{\mathbf{F}}_{n+1} = \mathbf{F}^*_{n+1} + (\mathbf{I} + \mathbf{X}^T \mathbf{M}_U \mathbf{X})(a_0 \mathbf{Y}_n + a_2 \dot{\mathbf{Y}}_n + a_3 \ddot{\mathbf{Y}}_n) + \mathbf{C}^* (a_1 \mathbf{Y}_n + a_4 \dot{\mathbf{Y}}_n + a_5 \ddot{\mathbf{Y}}_n) \tag{17}$$

In general, the vector of participation factors can be solved through relation (15). Thereafter, the unknown vector is obtained by equation (11) as usual in the modal procedure. It must be also mentioned that the generalized effective stiffness matrix employed in relation (15) is inherently unsymmetric. However, it may be easily transformed to a symmetric matrix by multiplying certain rows of the matrix relation (15) by an appropriate factor. This is discussed elsewhere in detail [7].

NUMERICAL EXAMPLE

In this section, the analysis of Pine Flat Dam is considered as a verification example. The dam is 121.92 m high, with the crest length of 560.83 m and it is located on the King's River near Fresno, California.
A special computer program "MAP-76" [5] is used as the basis of this study. The program was already capable of analyzing a general dam-reservoir system by direct approach in the time domain [2]. In this study, the modal analysis option is also included in the program based on the formulation presented in the previous section.

Modeling and Basic Data

The Pine Flat dam-reservoir system is considered over a rigid foundation. The two-dimensional finite element model is displayed in Figure 1. The dam section relates to the tallest monolith (121.92 m), and it is assumed in a state of plane stress. The water level is considered at the height of 116.19 m above the base, similar to the previous study [2]. Meanwhile, a length of 200 m is included in the model for the reservoir domain.
The dam body is discretized with 8-node plane solid elements, while 8-node fluid elements are used for the reservoir region. The model consists of a total of 439 nodes and 568 degrees of freedom and it includes 40, 90 plane solid and fluid elements, respectively.

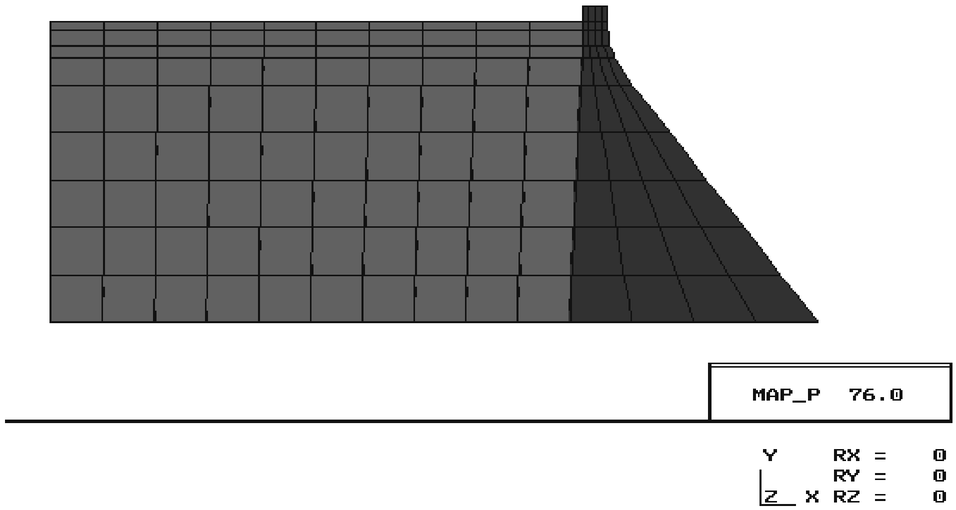


Figure 1. Dam-reservoir system discretization

Basic Parameters

The concrete is assumed to be homogeneous and isotropic with the following basic properties:

- Elastic modulus E_c = 22.75 GPa
- Poisson's ratio ν_c = 0.20
- Unit weight γ_c = 24.8 kN/m^3

The water is taken as compressible, inviscid fluid, with weight density of 9.81 kN/m^3 and pressure wave velocity of 1440.0 m/s. Meanwhile, the sommerfeld boundary condition is imposed at the upstream boundary of the impounded water domain, and the reservoir bottom condition is assumed completely reflective.

The main analysis carried out, is based on modal analysis, and viscous damping is assumed for this case. The viscous damping coefficients are considered constant for all the modes ($\beta_d = 0.05$). However, a second case is also analyzed based on the direct approach, where the Rayleigh damping matrix is applied and the corresponding coefficients are determined such that equivalent damping for frequencies close to the first and third modes of vibration would be 5% of critical damping.

Loading

It should be mentioned that static loads (weight, hydrostatic pressures) are each visualized as being applied in one separate increment of time. Therefore, the same time step of 0.01 second, which is chosen in dynamic analysis, is also considered as time increment of static loads application. It is noted that time for static analysis is just a convenient tool for applying the load sequentially, but it is obvious that inertia and damping effects are disregarded in the process. In this respect, the dead load is applied in one increment and hydrostatic pressures thereafter in another increment at negative range of time. At time zero, the actual dynamic analysis begins with the static displacements and stresses being applied as initial conditions.

The dynamic excitation considered, is the S69E component of Taft earthquake records, which is applied in the horizontal x-direction. The time duration utilized, is 13 seconds in each case.

Analysis Results

As mentioned, the main analysis is performed by modal approach. As a first step of this case, the eigen-problem is solved based on the symmetric parts of the total mass and stiffness matrices. This is actually a decoupled system, and the natural frequencies obtained correspond to either the dam or the reservoir (finite region considered). The first five natural frequencies of each domain are listed in Table 1.

Table 1: Natural frequencies of the dam and reservoir.

Mode number (i)	Natural frequencies f_i (Hz)	
	Dam	Reservoir
1	3.146408	3.115126
2	6.475173	4.749112
3	8.738600	7.795491
4	11.248678	9.300412
5	16.989656	9.958278

It is noticed that the first natural frequency of the reservoir is actually slightly lower than the one corresponding to the dam. Meanwhile, the natural frequencies of the dam are wider spread in comparison to the ones related to the reservoir domain. This means that a much higher number of modes are required for the reservoir in comparison with the dam for an accurate solution.

The first mode shape of the dam is displayed in Fig. 2. Meanwhile, the first two mode shapes of the reservoir region are depicted in Fig. 3. It is noticed that the first mode of the fluid domain corresponds to a nearly symmetric case which pressures are approximately constant in the horizontal direction, while the second mode is very close to a perfect anti-symmetric case.

In the next step, the modal analysis is carried out by utilizing 25, 75 modes for the dam and the reservoir domain, respectively.

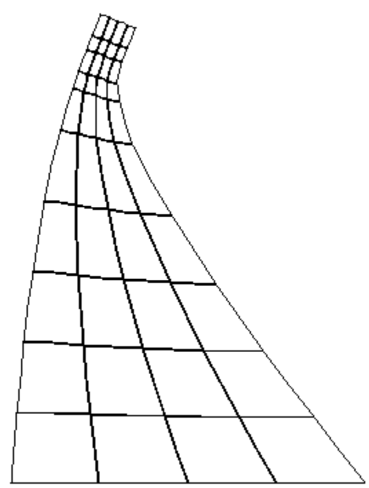
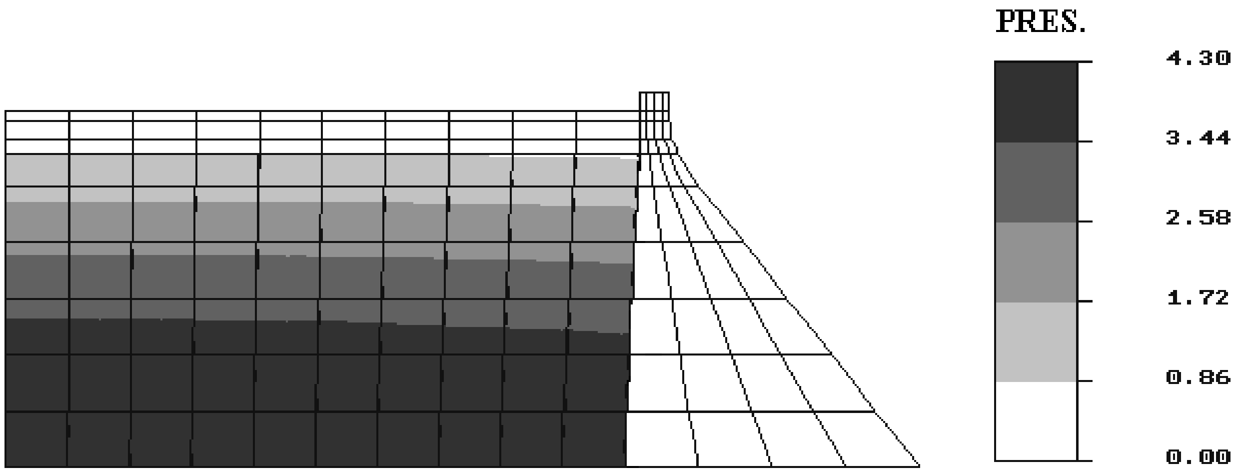
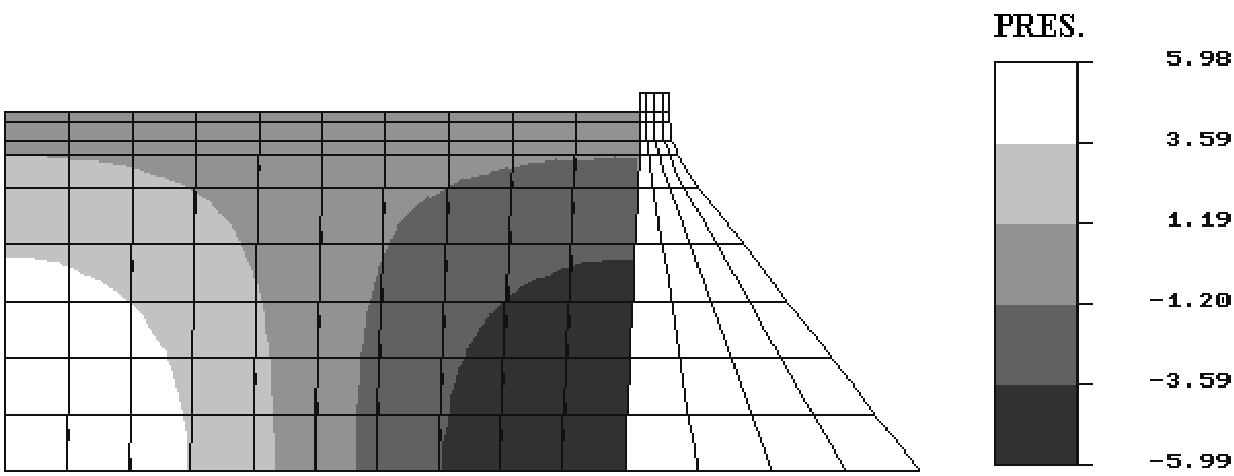


Figure 2. The first mode shape of the dam



(a)



(b)

Figure 3. The first two modes of the reservoir; (a) Mode 1, (b) Mode 2

For comparison purposes, the same model is also analyzed based on the direct approach. All the basic data in this case, are similar to the original case, except for the type of damping. That is Rayleigh damping in comparison with viscous damping. Although, this could cause slight differences in result, it should not be very significant. Neglecting this minor source of difference, it is well known that the direct method results can be considered as exact for the discretization employed, since it is actually equivalent to considering all of the modes of both discrete domains.

Both cases are analyzed, and the result corresponding to envelope of maximum tensile stress is illustrated in Figure 4. It is observed that the distribution is very similar for the modal and direct approaches. However, it is noticed that maximum value of tensile stress, is about 7.9 percent lower for the modal approach in comparison to the direct method.

For a better evaluation of the results, time histories of some important quantities are depicted in Figures 5 and 6. These are the horizontal component of displacement at dam crest and the maximum tensile principal stress at dam heel. In each graph, the result corresponding to direct method is also shown for comparison purposes.

It is noticed that trends of all quantities monitored, are very similar for the modal and direct approaches. However, the modal technique predicts slightly lower maximum values.

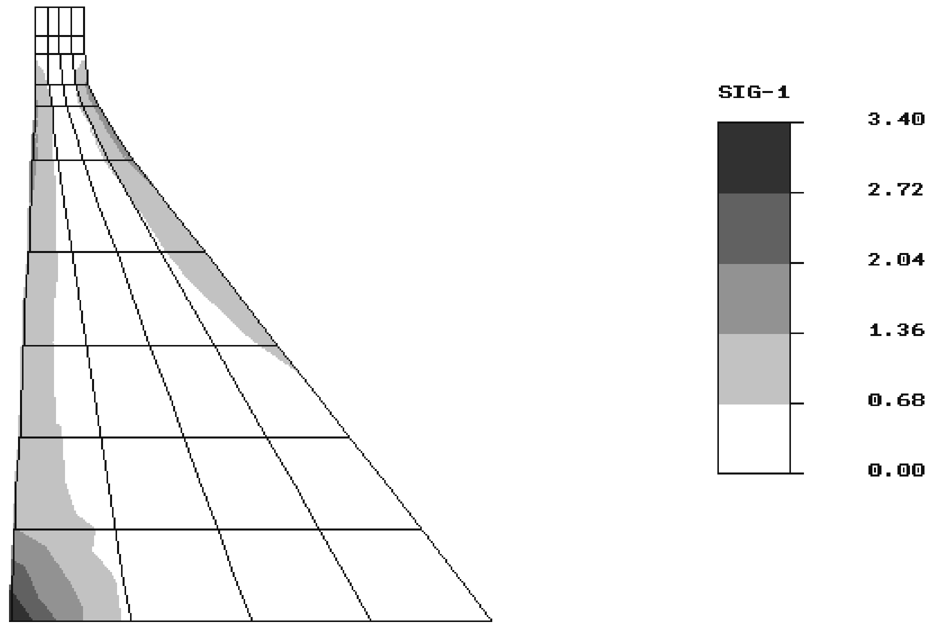
Finally, to evaluate the modal technique more closely, it was decided to double the number of modes utilized over the original case. The envelope of maximum tensile stress for this case is shown in Figure 7. This Figures could be compared with corresponding result of direct method (Figures 4b).

It is noticed that distribution becomes almost precisely the same. Meanwhile, the absolute maximum tensile stress is now merely 2.2 percent lower than the direct approach result. Although, this degree of accuracy is seldom required for practical cases, this illustrates vividly the convergence of the proposed technique.

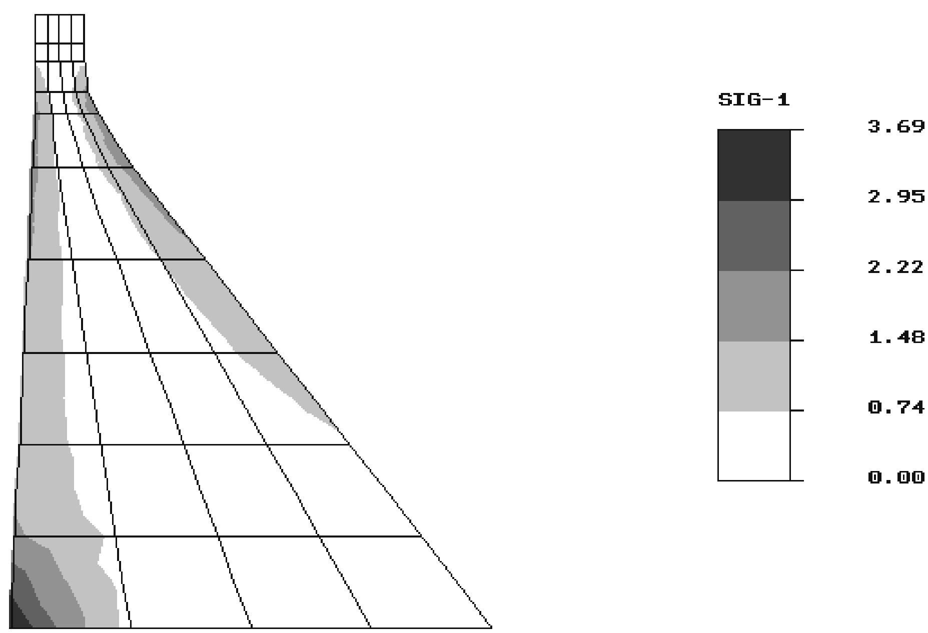
CONCLUSIONS

In this paper, a new technique is proposed for earthquake analysis of concrete gravity dams, which is referred to as decoupled modal approach. The method is explained initially, and the procedure is implemented in a special computer program "MAP-76". Meanwhile, the analysis of Pine Flat Dam is considered as a numerical example and for verification purposes. The original case analyzed, is based on the proposed technique, and it is carried out by utilizing 25, 75 modes for the dam and the reservoir domain, respectively. Meanwhile, the direct method of analysis is used for comparison purposes. Overall, the main conclusions obtained can be listed as follows:

- The results obtained based on the decoupled modal approach, compare very well with the corresponding results of direct method. More specifically, by comparing the envelope of maximum tensile stresses, it is shown that its distribution is very similar for both approaches. Meanwhile, the maximum value of tensile stress is only about 7.9 percent lower for the modal approach in comparison to the direct method. Furthermore, trends of all quantities monitored, are very similar and in good agreement for both methods throughout the execution time.
- By doubling the number of modes utilized over the original case, the convergence of the technique is also controlled. For this case, it is noticed that absolute maximum tensile stress is merely 2.2 percent lower than the corresponding direct method result.
- The proposed decoupled modal approach is proved to be an effective technique for seismic analysis of concrete gravity dams. The main advantage of this modal technique is that it employs eigen-vectors of the decoupled system, which can be easily obtained by standard eigen-solution routines.



(a)



(b)

Figure 4. Envelopes of maximum tensile principal stresses (MPa); (a) Modal, (b) Direct

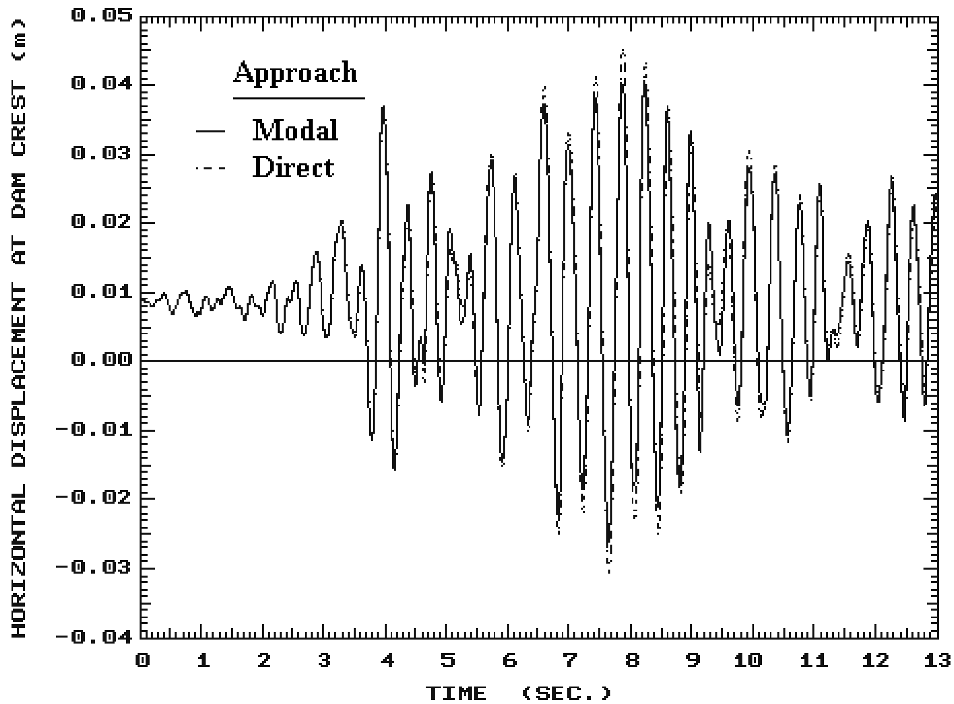


Figure 5. Comparison of displacement history at dam crest between the two approaches.

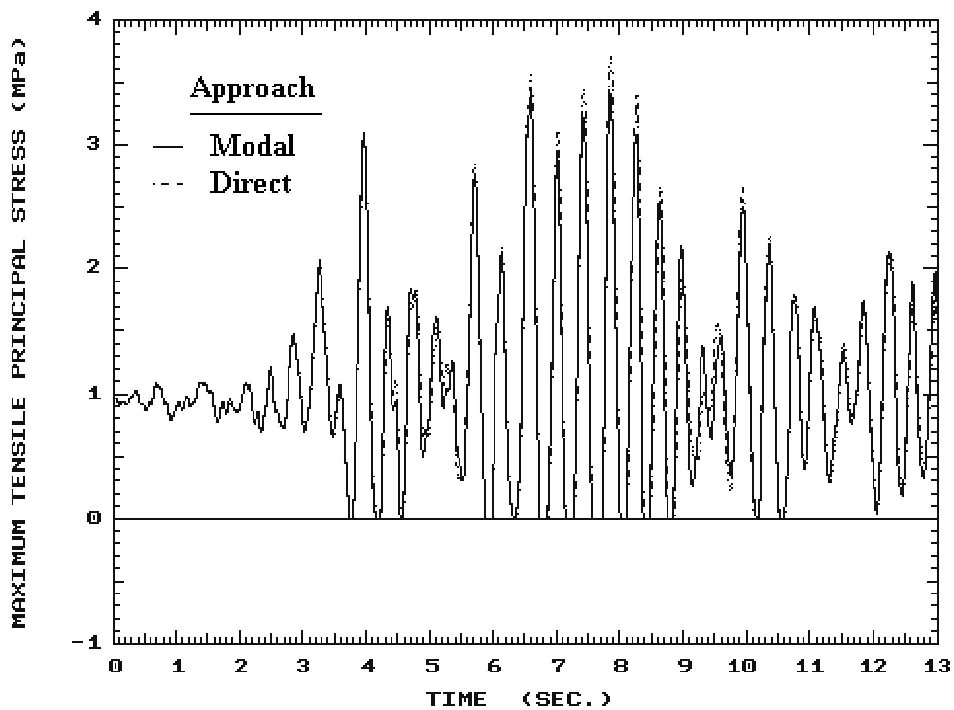


Figure 6. Comparison of maximum tensile stress history at dam heel between the two approaches.

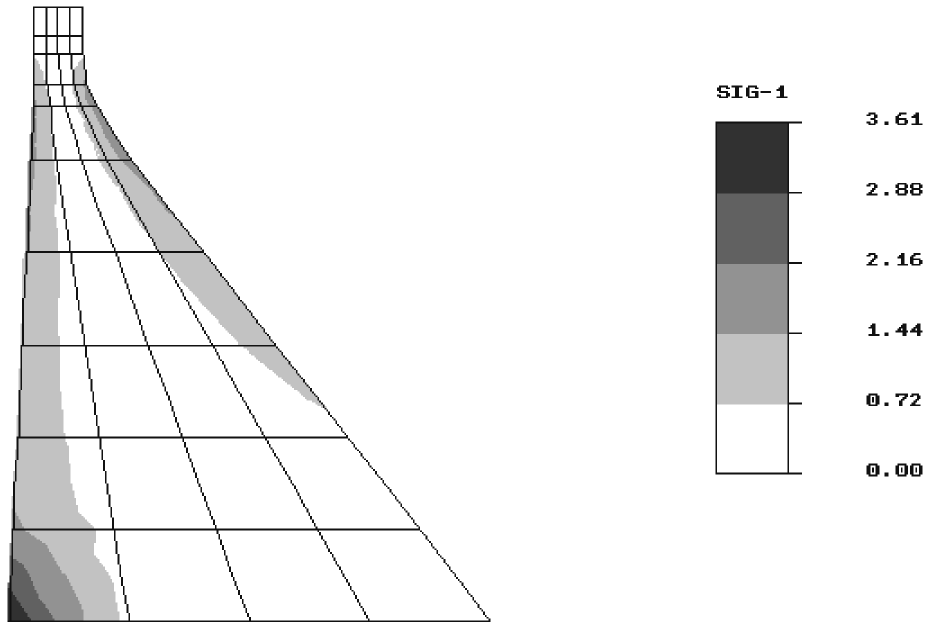


Figure 7. Envelope of maximum tensile principal stresses (MPa) when the number of modes utilized are doubled over the original case.

REFERENCES

1. Zienkiewicz, O.C. and Taylor, R.L., "The Finite Element Method", 5th Edition, Butterworth-Heinemann, Oxford, UK, 2000.
2. Lotfi, V., "Seismic analysis of concrete dams by Pseudo-Symmetric technique", Journal of Dam Engineering, Vol. XIII, Issue 2, 2002.
3. Ohayon, R., "Symmetric variational formulations for harmonic vibration problems coupling primal and dual variables – applications to fluid-structure coupled systems", La Rechereche Aerospatiale, 3, 69-77, 1979
4. Fellipa, C.A., "Symmetrization of coupled eigenproblems by eigenvector augmentation", Commun. Appl. Num. Meth., 4, 561-63, 1988.
5. Lotfi, V., "MAP-76: A program for analysis of concrete dams", Amirkabir University of Technology, Tehran, Iran, 2001.
6. Lotfi, V., "Significance of rigorous fluid-foundation interaction in dynamic analysis of concrete gravity dams", Submitted to the Journal of Structural Engineering and Mechanics, 2003.
7. Lotfi, V., "Seismic analysis of concrete gravity dams by decoupled modal approach in time domain", Submitted to the Journal of European Earthquake Engineering, 2003.