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OPTIMAL INSERTION OF VISCOUS DAMPERS INTO SHEAR-TYPE STRUCTURES: DISSIPATIVE PROPERTIES OF THE MPD SYSTEM

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SUMMARY

This paper illustrates the superior dissipative properties offered by inserting viscous dampers into shear-type structures in accordance with a special scheme (referred to as MPD system) based upon the mass proportional damping component of Rayleigh viscous damping matrices. This scheme is characterized by a innovative damper arrangement that sees dampers (a) placed so that they connect each storey to a fixed point and (b) sized proportionally to each storey mass.
In the first part of the paper, the physical basis which leads to the peculiar dissipative properties of MPD system are recalled.
In the second part of the paper, the dynamic responses (to a stochastic input) of shear-type structures equipped with MPD system are compared with those offered by other damping systems identified as optimal using numerical methods. Different sets of optimal damping systems are identified for the following three cases: (a) dampers connect adjacent storeys, (b) dampers connect each storey to a fixed point and (c) no constraint upon the damper placement is imposed. All systems considered in the comparison satisfy a mathematical constraint which imposes that the sum of the damping coefficients of all added viscous dampers is the same (equal "total cost" constraint).
The results indicate that, among all systems considered, the MPD system is capable of providing the best overall dissipative properties. This suggests a new and efficient way of inserting viscous dampers in structures to be built in seismic areas, which is alternative to the common (and less efficient) interstorey damper placement.

INTRODUCTION

In recent years various innovative technologies for protecting civil engineering structures from earthquakes have been developed and implemented [1,2]. Among these technologies, the use of added viscous dampers has proven to be quite effective in reducing the effects of seismic excitation upon building structures [1,2] and several research works have investigated the "optimal" way of inserting viscous dampers into shear-type structures [3,4,5,6,7].

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PROBLEM FORMULATION

In order to identify the system of added viscous dampers which optimizes the dissipative properties of a given shear-type structure and make meaningful comparisons, it is necessary to (address the following key issues): (a) introduce a constraint upon the total size (cost) of the system of added viscous dampers, and (b) select synthetic indexes capable of capturing the overall dissipative capacities of the different systems of added viscous dampers and their effectiveness when applied to given structures.

As far as the constraint upon the total size of the viscous dampers is concerned, it is here imposed that the sum, c_{tot}, of the damping coefficients, c_j, of all M dampers introduced into the structure, be equal to a set value, $\overline{c}$, as also adopted in other research works [4,5,6,7]. The above constraint (in the following referred to as equal "total cost" constraint) mathematically translates in the following formula:

$$c_{tot} = \sum_{j=1}^{M} c_j = \overline{c} \tag{1}$$

As far as the identification of indexes capable of capturing the dissipative effectiveness of various damping systems are concerned, indexes based upon the response of the dynamic systems to given stochastic inputs have proven to be effective and versatile [5,6,7,8,9]. In the analyses here presented, internal damping is neglected and linear modeling for the force-velocity relationship of each damper is adopted:

$$F_d = c \cdot v \tag{2}$$

where F_d is the force provided by the damper, c is its damping coefficient and v is the relative velocity between the two damper ends.

PROBLEM SOLUTION STRATEGIES FOR THE IDENTIFICATION OF OPTIMAL DAMPING SYSTEMS

In 1997 [4], Takewaki proposed a systematic algorithm based upon an inverse problem approach to identify the damping coefficients of added viscous dampers which minimize the sum of amplitudes of the transfer functions of interstorey drifts evaluated at the undamped fundamental natural frequency. In 2000 [5], the same author applied a steepest descent method to find the optimal damper arrangement in structures subjected to the critical excitation.

In 2001 [6] and 2002 [7], Singh & Moreschi used the Rosen's gradient projection method and genetic algorithms to identify the damping coefficients of added viscous dampers which minimize a number of performance functions based on the system response to a design earthquake ground motion defined by a Kanai-Tajimi spectral density function.

All the above analyses were carried out for a restricted class of structural systems characterized by dampers placed between adjacent storeys (which leads to a banded damping matrix).

Since 2001 [8,9,10,11,12,13,14,15,16], the authors have been studying the problem of optimal damper arrangement in an innovative, physically-based manner which led to the identification of a system of added viscous dampers (the "MPD system") which has proven to provide very good overall dissipative performances.

THE MPD SYSTEM

For Rayleigh damped multi-degree-of-freedom systems [17], the damping matrix $[C]$ becomes:

$$[C]^R = \alpha[M] + \beta[K] \tag{3}$$

where $[M]$ and $[K]$ are, respectively, the mass matrix and the stiffness matrix and α and β are two proportionality constants having units of sec^{-1} and sec, respectively. Eq. (3) allows to define the two following damping matrices:

- mass proportional damping (MPD) matrix:

$$[C]^{MPD} = \alpha[M] \tag{4}$$

- stiffness proportional damping (SPD) matrix:

$$[C]^{SPD} = \beta[K] \tag{5}$$

which correspond, respectively, to the MPD and SPD limiting cases of Rayleigh damping.

For the sake of clarity, the added-damper system that allows an MPD matrix to be obtained is defined herein as "MPD system" and, likewise, that which allows an SPD matrix to be obtained is referred to as "SPD system".

PHYSICAL DISSIPATIVE PROPERTIES OF THE MPD AND SPD SYSTEMS

In previous research works carried out by the authors [10], it is proven that, for the class of shear-type structures characterized by constant values of storey lateral stiffness ($k_j = k$, $\forall j$) and floor mass ($m_j = m, \forall j$) and under the equal "total cost" constraint, the first modal damping ratio of the MPD system, ξ_1^{MPD}, is always larger than the first modal damping ratio of the SPD system, ξ_1^{SPD}, and other Rayleigh damping systems, ξ_1^{R}. The analytical demonstration [10] is based upon the modal damping ratios and the properties of the eigenproblem governing the modal responses of the above-defined class of shear-type structures, and, moreover, identifies also an upper bound and an approximation for the ratio $\xi_1^{SPD} / \xi_1^{MPD}$, as follows:

$$\xi_1^{SPD} \Big/ \xi_1^{MPD} < \frac{1}{N} \tag{6}$$

$$\xi_1^{SPD} / \xi_1^{MPD} \cong \frac{2}{N^2 + N} \tag{7}$$

where N is the total number of storeys of the structure.

Given that, in most cases, the first mode of vibration controls the dynamic response of shear-type structures subjected to base excitations, these physically-based results clearly indicate that the MPD system provides an overall damping efficiency which is higher than those provided by the SPD system and other Rayleigh damping systems.

Numerical verifications [8,9,11,12,13,14,15,16] have been then carried out upon a wide range of shear-type structures with reference to both stochastic and seismic inputs, and have confirmed the higher dissipative efficiency of the MPD system with respect to that of the SPD system.

In the following, the dynamic responses (to a stochastic input) of shear-type structures equipped with MPD system are compared with those offered by other damping systems identified as optimal using numerical methods.

DAMPER PLACEMENT VS. DAMPER SIZING

The introduction of a system of added viscous dampers in a shear-type structure involves the identification of (a) the damper placement (where should dampers be placed) and (b) the damper sizing (which size, in terms of c_j, should they have).

Whilst remaining in fairly general terms, let us consider the specific case of a 3-storey shear-type structure, as represented in Fig. 1a. Fig. 1b shows the structure in question equipped with a system of

added viscous dampers that lead to a Rayleigh damping matrix. Figures 1c and 1d provide physical representation (physical counterpart) of the structure in question with the MPD and SPD systems of added viscous dampers, respectively.
These representations allow to formulate the following alternative definitions, in terms of damper placement and sizing, for the MPD and SPD systems:

- MPD system: dampers are placed in such a way as to connect each storey to a fixed point (fixed point placement: FP-placement) and sized so that each damping coefficient c_j is proportional to the corresponding storey mass m_j (mass proportional sizing: MP-sizing);
- SPD system: dampers are placed in such a way as to connect two adjacent storeys (interstorey placement: IS placement) and sized so that each damping coefficient c_j is proportional to the lateral stiffness k_j of the vertical elements connecting these two storeys (stiffness proportional sizing: SP-sizing).

As previously mentioned, in the research works of Takewaki [4,5] and Singh & Moreschi [6,7], the search for the "optimal" damping system was carried out for systems characterised by an interstorey damper placement (as per the SPD system), and therefore the results presented in their works identify the "optimal" sizing for this particular placement.
Given the above considerations, the analyses presented in the following compare the dissipative performances of the MPD systems with those of offered by:

- systems characterized by an interstorey (IS) placement and "optimal" sizing;
- systems characterized by a fixed point (FP) placement and "optimal" sizing;
- systems characterized by a "free" (FREE) placement and "optimal" sizing.

With FREE-placement it is meant any type of damper placement (i.e. dampers may connect adjacent storeys, non adjacent storeys and storeys to a fixed point).

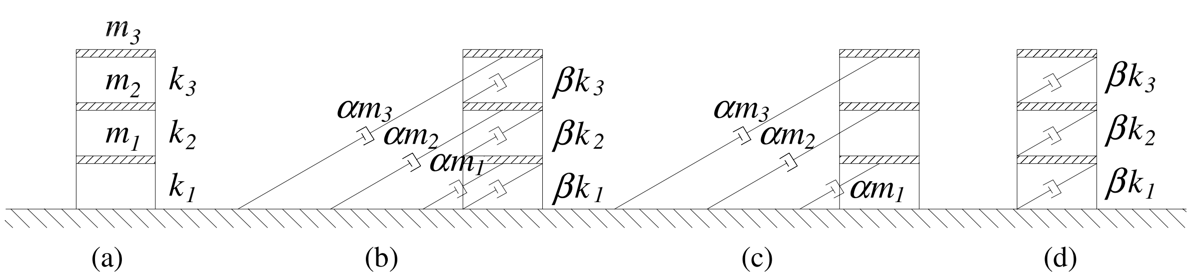

Fig. 1. 3-storey shear-type structure: (a) undamped, (b) equipped with Rayleigh damping system, (c) equipped with MPD system and (d) equipped with SPD system.

THE TWO "REFERENCE" STRUCTURES

The analyses here presented are developed with reference to two shear type structures.
The first one is a 5-storey r.c. building structure with a rectangular layout of $30m \times 18m$ and an interstorey height of $h = 3.3m$. The structure consists of four frames arranged lengthways along the building plan ($30m$).In the analyses carried out herein, infinitely stiff beams (with respect to vertical columns) are assumed so that use of the two-dimensional shear-type schematisation of Fig. 2 is permitted [16]. The five stiffness values, the five storey mass values and the five resultant periods of vibration are set out hereafter:

$k_1 = 1.2174 \cdot 10^9$ N/m	$m_1 = 5.4 \cdot 10^5$ kg	$T_1 = 0.578$ sec
$k_2 = 0.7987 \cdot 10^9$ N/m	$m_2 = 5.4 \cdot 10^5$ kg	$T_2 = 0.252$ sec
$k_3 = 0.4986 \cdot 10^9$ N/m	$m_3 = 5.4 \cdot 10^5$ kg	$T_3 = 0.180$ sec
$k_4 = 0.2923 \cdot 10^9$ N/m	$m_4 = 5.4 \cdot 10^5$ kg	$T_4 = 0.131$ sec
$k_5 = 0.1578 \cdot 10^9$ N/m	$m_5 = 2.7 \cdot 10^5$ kg	$T_5 = 0.091$ sec

The second structure is a 6-storey building model characterized by values of mass and lateral stiffness which do not vary along the building height (see Fig. 3). The lateral stiffness k_j of the vertical elements connecting each j-th storey to the one below is equal to $k = 4 \cdot 10^7$ N/m and the floor mass m_j of each j-th storey is equal to $m = 0.8 \cdot 10^5$ kg, with the first undamped circular frequency ω_I = 5.39 Hz (first period: $T_1 = 1.17$ sec). Interstorey height is h = 3 m and total height is h_{tot} = 18m. This structure has been selected for the sake of comparison with other research results regarding the optimal placement of added viscous dampers that are available in literature [4].

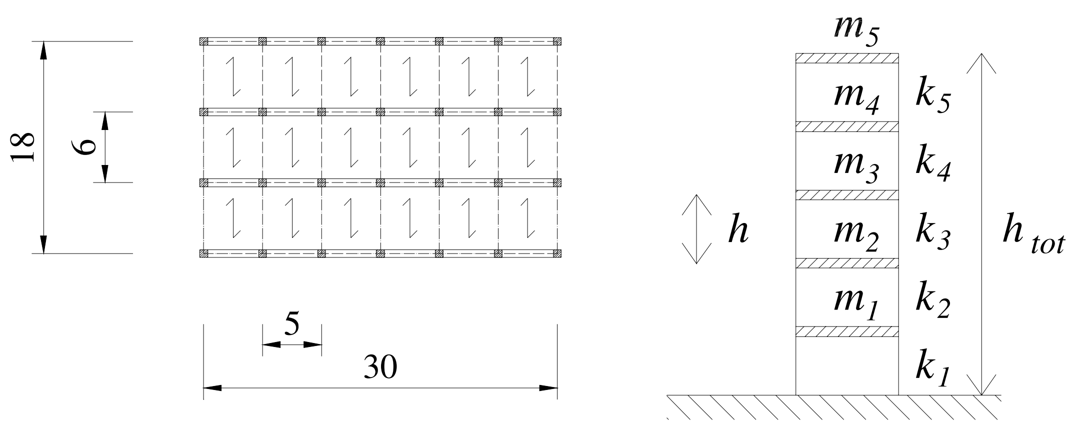

Fig. 2. Plan and shear-type schematization of the 5-storey r.c. building structure.

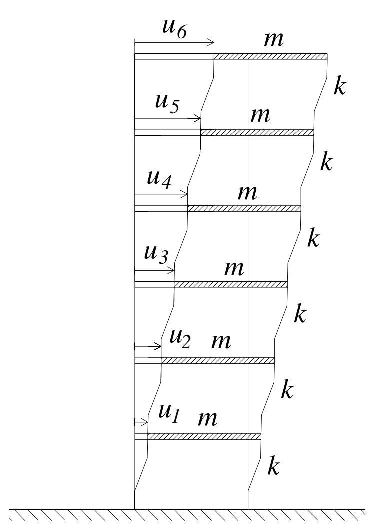

Fig. 3. The 6-storey structure.

THE STOCHASTIC RESPONSE INDEX *I*

In previous research works [4,5,8,9], it has been seen that performance indexes based upon the system response to a stochastic input are capable of capturing the overall dissipative performances of damping systems. In the results here presented, use is made of the *average (over all storeys) of the standard deviations of the interstorey drift angles* (index *I*) of the system response to the following white-noise stochastic input:

- band-limited between 0 and $\bar{\omega}=60\,rad/\sec$, stationary, Gaussian with zero mean, and
- with constant power spectral density of amplitude $A^2=0.144\,m^2/\sec^3$

(these values have been chosen so that standard deviation of acceleration at the base of the structure supplied by this stochastic process is equal to $0.3g$, being g the gravity acceleration).
The mean square response [18] (that coincides with variance for stochastic inputs with zero mean value), $\sigma^2_{ID\,j}$, of the *j*-th interstorey drift of a structure subjected to the white-noise base input of above is calculated as:

$$\sigma^2_{ID\,j}=A^2\int_0^{\bar{\omega}}\left|H_{ID\,j}(\omega)\right|^2 d\omega \tag{8}$$

where $H_{ID\,j}(\omega)$ is the *j*-th component (*j* corresponding to the coordinate of the *j*-th storey) of the transfer function vector, $\{H_{ID}(\omega)\}$, of the interstorey drifts, defined as:

$$\{H_{ID}(\omega)\}=[T]\{H(\omega)\} \tag{9}$$

with $[T]$ being a $N\times N$ constant matrix consisting of 1,-1, and 0 of this kind:

$$[T]=\begin{bmatrix} 1 & 0 & \ldots & & & 0 \\ -1 & 1 & 0 & & & \ldots \\ 0 & -1 & 1 & 0 & & \\ \ldots & & & \ldots & & \\ & & & & \ldots & 0 \\ 0 & \ldots & & 0 & -1 & 1 \end{bmatrix} \tag{10}$$

and

$$\{H(\omega)\}=-\left(-\omega^2[M]+i\omega[C]+[K]\right)^{-1}[M]\{1\} \tag{11}$$

where ω represents the natural circular frequency, $i=\sqrt{-1}$ and $\{1\}$ is a vector whose elements are all unity for shear-type structures. According to the usual notation of probabilistic theory, $\sigma_{ID\,j}$ denotes the standard deviation of the *j*-th interstorey drift.
Eq. (8) allows the definition of an index *I*, equal to the *"average of the standard deviations of the interstorey drift angles"*, as:

$$I=\frac{1}{h}\frac{1}{N}\sum_{j=1}^{N}\sigma_{ID\,j}=\frac{1}{h}\frac{1}{N}\sum_{j=1}^{N}A\sqrt{\int_0^{\bar{\omega}}\left|H_{ID\,j}(\omega)\right|^2 d\omega} \tag{12}$$

where *h* is the interstorey height and *N* is the total number of storeys of the structure. For a given dynamic system (structure + dampers), a small value of index *I* means a small dynamic response to stochastic input. When comparing the given structure equipped with different types of damper systems, a small value of index *I* means a high dissipative effectiveness of the damper system.

THE SYSTEMS OF ADDED VISCOUS DAMPERS

To obtain the "optimal" damper sizing for the IS-, FP- and FREE-placements, a search has been carried out for the damper sizing (damping coefficients of dampers) which minimize response index *I* for the two reference structures. For this search, use is made of genetic algorithms (GA) [7]. The basic characteristics of the GA here adopted can be summarized as follows:

- population: 30 individuals;
- mutation choice: 18%;
- elitism choice: 18%;
- number of iteration: 150.

The "genetically identified optimal" (GIO) systems which minimise index *I* in the cases of IS-, FP- and FREE-placements are herein referred to as GIOIS, GIOFP and GIOFREE systems, respectively.

For the 5-storey structure, the equal "total cost" constraint is imposed with $\overline{c}$ equal to $2.729 \cdot 10^7 \ \mathrm{N \cdot sec/m}$, so that the first modal damping ratio of the structure equipped with the SPD system is equal to $\xi_1^{SPD} = 0.05$. With reference to Fig. 4, the values of the damping coefficients of the GIOIS, GIOFP and GIOFREE systems are given in Table 1. Notice that the GIOFREE system presents no interstorey dampers. For comparison purposes, Table 1 also gives the values of the damping coefficients of the MPD and SPD systems.

For the 6-storey structure, the equal "total cost" constraint is imposed with $\overline{c}$ equal to $9 \cdot 10^6 \ \mathrm{N \cdot sec/m}$, (for sake of comparison with results available in literature [4]). With reference to Fig. 5, the values of the damping coefficients of the GIOIS, GIOFP and GIOFREE systems are given in Table 2. Notice that the GIOFP and the GIOFREE systems coincide. In addition to the three "genetically identified optimal" systems, the damping scheme identified in the recent works by Izuru Takewaki [4] as "optimal" (for the 6-storey structure here considered) is also taken into account and will be referred hereafter to as TAK system. The specific values of the damping coefficients of the TAK system minimise the sum of amplitudes of the transfer functions of interstorey drifts evaluated at the undamped fundamental natural frequency ω_1:

$$\sum_{j=1}^{6} \left| H_{ID\,j}(\omega_1) \right| \tag{13}$$

within the restricted class of dampers placed between adjacent storeys (IS placement) and satisfy the "equal total cost" constraint. This system was identified by Takewaki using an algorithm based upon an inverse problem approach, proposed by the same author [4]. For comparison purposes, Table 2 also gives the specific values of the damping coefficients of the MPD, SPD and TAK systems. Notice that the TAK system is very similar to the GIOIS system which minimises performance index *I* within the same class of IS placement.

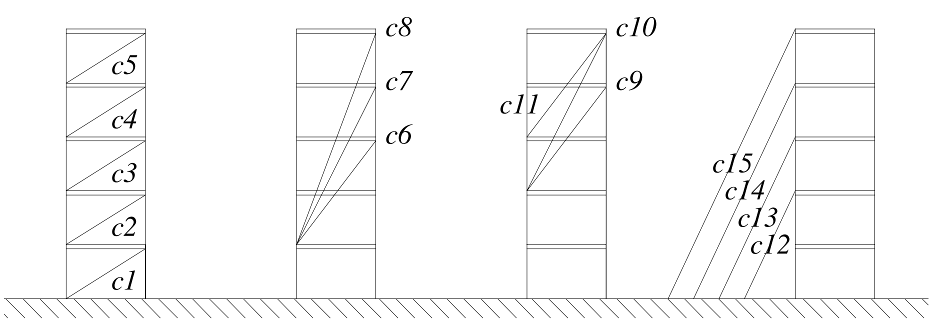

Fig. 4. All possible damper placements for the 5-storey shear-type structure.

Table 1. Damping coefficients [$\times 10^6\ \mathrm{N \cdot sec/m}$] and index *I* [$\times 10^{-3}$] of the SPD, MPD, GIOIS, GIOFP and GIOFREE systems for the 5-storey structure.

	SPD	MPD	GIOIS	GIOFP	GIOFREE	
c1	11.206	6.064	0	0	0	*c1*
c2	7.352	0	0	0	0	*c2*
c3	4.589	0	10.916	0	0	*c3*
c4	2.691	0	12.281	0	0	*c4*
c5	1.452	0	4.093	0	0	*c5*
c6	0	0	0	0	0	*c6*
c7	0	0	0	0	0	*c7*
c8	0	0	0	0	0	*c8*
c9	0	0	0	0	2.823	*c9*
c10	0	0	0	0	1.882	*c10*
c11	0	0	0	0	0	*c11*
c12	0	6.064	0	5.248	1.882	*c12*
c13	0	6.064	0	8.397	9.410	*c13*
c14	0	6.064	0	9.447	8.469	*c14*
c15	0	3.032	0	4.198	2.823	*c15*

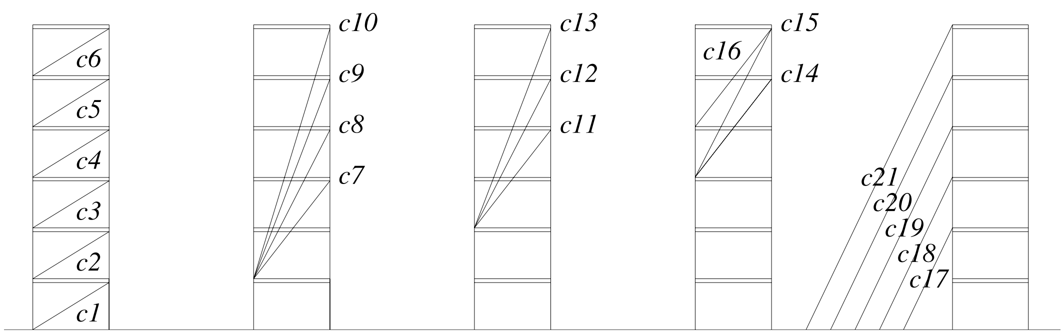

Fig. 5. All possible damper placements for the *6*-storey shear-type structure.

Table 2. Damping coefficients [$\times 10^6\ \mathrm{N \cdot sec/m}$] and index *I* [$\times 10^{-3}$] of the SPD, MPD, GIOIS, GIOFP, GIOFREE and TAK systems for the 6-storey structure.

	SPD	MPD	GIOIS	GIOFP	GIOFREE	TAK	
c1	1.50	1.50	3.91	1.02	1.02	4.80	*c1*
c2	1.50	0	3.13	0	0	4.20	*c2*
c3	1.50	0	1.96	0	0	0	*c3*
c4	1.50	0	0	0	0	0	*c4*
c5	1.50	0	0	0	0	0	*c5*
c6	1.50	0	0	0	0	0	*c6*
c7	0	0	0	0	0	0	*c7*
c8	0	0	0	0	0	0	*c8*
c9	0	0	0	0	0	0	*c9*
c10	0	0	0	0	0	0	*c10*
c11	0	0	0	0	0	0	*c11*
c12	0	0	0	0	0	0	*c12*
c13	0	0	0	0	0	0	*c13*
c14	0	0	0	0	0	0	*c14*
c15	0	0	0	0	0	0	*c15*

c16	0	0	0	0	0	0	*c16*
c17	0	1.50	0	1.53	1.53	0	*c17*
c18	0	1.50	0	1.70	1.70	0	*c18*
c19	0	1.50	0	1.70	1.70	0	*c19*
c20	0	1.50	0	1.53	1.53	0	*c20*
c21	0	1.50	0	1.53	1.53	0	*c21*

THE SYSTEMS RESPONSE TO STOCHASTIC INPUT

The standard deviation, σ_j, of each (*j*-th) storey displacement of a structure subjected to the white-noise acceleration input above-described, can be computed as:

$$\sigma_j = A\sqrt{\int_0^{\bar{\omega}} \left|H_j(\omega)\right|^2 d\omega} \tag{14}$$

where $H_j(\omega)$ is the *j*-th component of the system transfer function vector, $\{H(\omega)\}$.

Figures 6 and 7 show σ_j at all storeys for the 5-storey and the 6-storey structures under the different damping configurations, respectively. It can be seen how an IS-placement leads to storey responses which are much larger (up to 3 times at the top-storey) than those offered by FP- and FREE-placements. Moreover, the MPD system leads to storey responses which are very close to those of the GIOFP and GIOFREE systems. The importance of damper placement with respect to damper sizing comes out from these results.

Furthermore, Table 3 and Table 4 provide the values of index *I* and of other meaningful stochastic performance indexes, for the two reference structures equipped with the damping systems above described.

For both reference structures, it can be seen that the MPD and the GIOFP systems (characterised by FP-placement) provide values of index *I* which are very close to the minimum ones (provided by the GIOFREE system), while the SPD and the GIOIS systems (characterised by IS-placement) provide values of index *I* which are much higher than the others. For the 6-storey structure, the TAK system provides a value of index *I* which is very similar to the one of the GIOIS system.

As far as other stochastic indexes are concerned, this trend is confirmed: FP placement (i.e. MPD and GIOFP systems) always provides smaller values of such indexes than IS placement (i.e. SPD and GIOIS systems). Moreover, notice that, for the 6-storey structure, the MPD system provides the smallest value of the standard deviation of the base shear, even better than the GIOFREE system (which is numerically identified as optimal with reference to interstorey drift angles and not to base shear). This result attests the “robustness” of the MPD system with respect to any changes in the choice of the meaningful performance index. Actually, a specific performance index let us identify the damping system which optimizes that specific index only, without giving any guarantee of good global behavior either in terms of different types of input or in terms of different types of response parameters. For this reason, it becomes fundamental the identification of damping systems characterized by good dissipative performances which are linked and referable to physical properties (such as the MPD system) and not obtained from numerical investigations only.

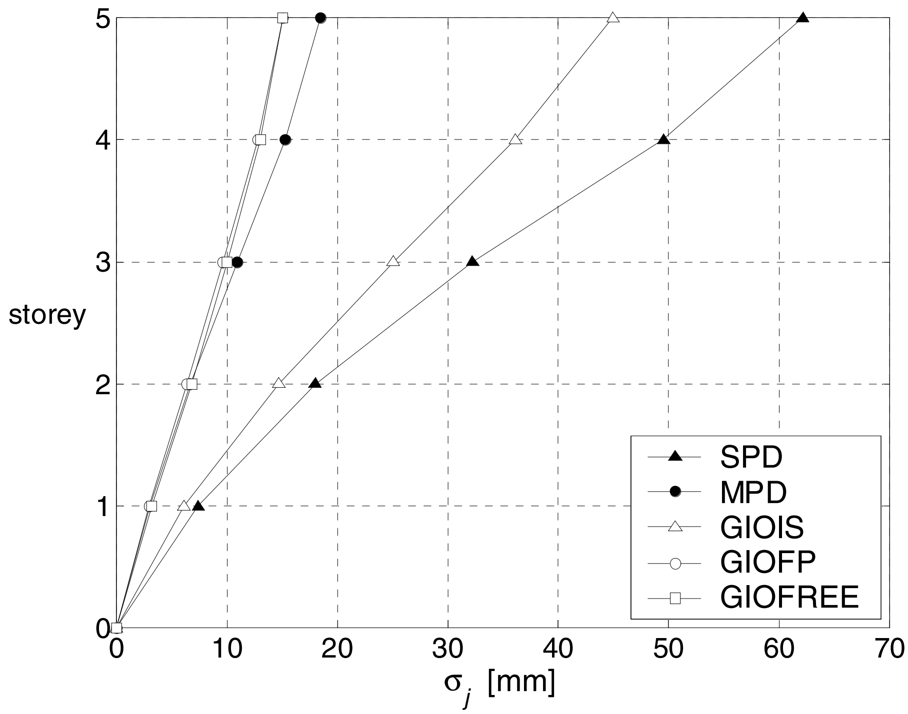

Fig. 6. σ_j **for the 5-storey structure equipped with the five damping systems considered.**

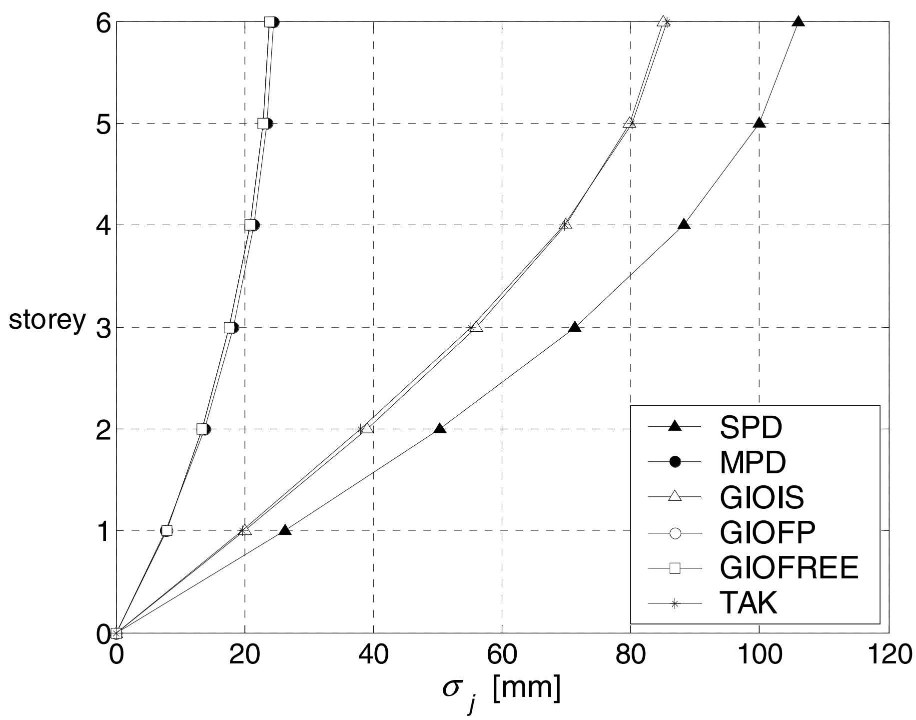

Fig. 7. σ_j **for the 6-storey structure equipped with the six damping systems considered.**

Table 3. Values of several stochastic performances indexes offered by the SPD, MPD, GIOIS, GIOFP and GIOFREE systems for the 5-storey structure.

	SPD	MPD	GIOIS	GIOFP	GIOFREE
average of the standard deviations of the interstorey drift angles (index I) $[\times 10^{-3}]$	3.88	1.33	2.88	1.18	1.16
sum of amplitudes of the transfer functions of interstorey drifts evaluated at the undamped fundamental natural frequency ω_1 (index used by Takewaki [4]) [sec^2]	5.67 $\times 10^{-5}$	0.97 $\times 10^{-5}$	3.81 $\times 10^{-5}$	0.93 $\times 10^{-5}$	1.05 $\times 10^{-5}$
standard deviation of the top-storey displacement [mm]	62	18	45	15	15
standard deviation of the base shear [kN]	9164	3785	7513	3718	3953

Table 4. Values of several stochastic performances indexes offered by the SPD, MPD, GIOIS, GIOFP, GIOFREE and TAK systems for the 6-storey structure.

	SPD	MPD	GIOIS	GIOFP	GIOFREE	TAK
average of the standard deviations of the interstorey drift angles (index I) $[\times 10^{-3}]$	5.92	1.46	4.92	1.45	1.45	5.13
sum of amplitudes of the transfer functions of interstorey drifts evaluated at the undamped fundamental natural frequency ω_1 (index used by Takewaki [4]) [sec^2]	0.2139	0.0130	0.1369	0.0124	0.0124	0.1351
standard deviation of the top-storey displacement [mm]	106	25	85	24	24	86
standard deviation of the base shear [kN]	1047	314	802	317	317	785

CONCLUSIONS

In this paper, the problem of optimal damper insertion in shear-type structures for maximum efficiency in mitigation of the seismic effects has been faced in a innovative and across-the-board manner using a physically based approach.
First, the physically-identified optimal dissipative properties of the mass proportional damping (MPD) system are recalled.
Second, the dissipative performances offered by the MPD system (as applied to two reference shear-type structures) are compared with those offered by numerically identified optimal systems.
Genetic algorithms are here used to identify damping systems characterized by an interstorey (IS) damper placement and "optimal" damper sizing, systems characterized by a fixed point (FP) damper placement

and “optimal” damper sizing, and systems characterized by a “free” (FREE) damper placement and “optimal” damper sizing.
The results indicate that the MPD system and systems characterised by fixed point (FP) placement provide the largest dissipative effectiveness.

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