13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 480

STATISTICAL CHARACTERISATION OF THE SEISMIC ACTION (PGA AND PGV) FOR PERFORMANCE BASED SEISMIC DESIGN

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SUMMARY

In countries directly affected by seismic phenomena, the assessment of vulnerability and seismic hazard throughout the territory plays a very important role, in order to minimize seismic effects on the population. Furthermore to effectively apply the performance based design approaches to seismic design it is necessary to accurately identify appropriate Intensity Measures (IM).

This paper aims at illustrating a method for determining the probability functions (PDF and/or CDF) of the peak ground acceleration (PGA) and peak ground velocity (PGV) due to the seismic action at a specific site, over a given observation time, which, for Performance Based Seismic Design purposes can be used as vectorial Intensity Measures. The method is based upon the Cornell widely upheld methodology (1968), specifically updated to take into account the latest contributions in geophysics developed for the Italian territory. This methodology assimilates the occurrence of seismic events to Poisson processes and characterizes the magnitude return period of each seismogenetic area using the Gutenberg-Richter law (1954). At this regard, the authors propose the innovative use of the completeness analysis suggested by Mulargia, Tinti and Gasperini (1985) for the correct interpretation of the seismic catalogues. The method makes use of the Sabetta-Pugliese attenuation law (1987) which is developed for the Italian territory for both PGA and PGV. The method treats the distance of the site from the epicenter as a random variable, by subdividing the territory in circular sectors, annuluses and annulus portions. This treatment allows an exact determination of the PDF and CDF of PGA and PGV at the site.

The statistical characterizations of PGA and PGV thus obtained feature the same accuracy. This allows to use the couples PGA-PGV (characterised by given probabilities of occurrence) as vectorial Intensity Measures in multi-input incremental dynamic analysis IDA (D. Vamvatsikos and C. Cornell, 2002) for probabilistic design approach in performance based seismic engineering. The paper presents also illustrative examples of how PGA and PGV can be used as an effective Intensity Measure.

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INTRODUCTION

This paper aims at illustrating a method for determining the probability functions (CDF: cumulative distributive function and PDF: probability density function) of the Peak Ground Acceleration (PGA) and of the Peak Ground Velocity (PGV) due to the seismic action at a specific site, over a given observation time.

Most of the analysis for the determination of the PDF of the PGA (and PGV) are based upon Cornell's widely upheld method [1]; this methodology allows an easy identification of the CDF and PDF of seismic magnitude MS of a seismogenetic area, starting from the assimilation of seismic events to Poisson processes and the magnitude-return period characterization as given by the Gutenberg-Richter law.

Still, in order to obtain a statistical characterization of the PGA (or PGV) of the site is necessary to make use of an attenuation law which provides a relationship between the PGA (PGV), the magnitude MS and the distance R of the site from the epicenter; this step may often present difficulties in its application.

Most programmes and procedures for solving this problem make use of small discretization of the spatial domain, so that the distance R can be treated, for each discrete contribute, as a constant, thus solving the problem in a numerical way. This approximation, which considers the distance R as a constant value, may affect the results, which become dependent on the spatial discretization of the domain in the neighborhood of the site of interest.

A specific feature of the method illustrated in this paper lies in the treatment of the distance R as a random variable just like the magnitude MS, thus considering the PGA (or PGV) as a random variable obtained as a function of the two aforementioned random variables (magnitude MS and distance R).

GLOBAL METHODOLOGY

The global methodology here applied is based upon that suggested by Cornell in 1968 [1] for the determination of the seismic hazard. This methodology consists of the following basic steps:

- Choice of an appropriate seismic catalogue and identification of the areas of homogeneous seismic activity (also referred to as Seismo-Genetic Zones).
- Definition of an analytical relationship between magnitude and return period for each Seismo-Genetic Zone.
- Choice of a specific attenuation law for the territory.
- Computation of the overall seismic hazard for the specific site under investigation.

THE RECURRENCE LAW

For an area of uniform seismic activity (Seismo-Genetic Zone), the Gutenberg-Richter law provides the following relationship between the average number of seismic events of a given magnitude that occur in a year (occurrence rate $\overline{\lambda}$) and the seismic magnitude MS:

$$Log\,\overline{\lambda}\left(MS\right)=\overline{aa}-\overline{bb}\cdot MS \tag{1a}$$

where $Logx=\log x/\log 10$, being $Logx$ the base 10 logarithm and $\log x$ the natural logarithm.

Given the hypothesis of uniform seismic activity within each Seismo-Genetic Zone, it is possible, for each area, to perform a linear regression of the cumulative annual average number (NC in Fig. 1) of seismic events strictly larger than MS (corrected to agree with a "completeness analysis", such that described later in the section regarding the application to the Italian territory) according to the Gutenberg-Richter law considered to be valid. Notice that, for the purpose of this paper, in this case, specific values of $\overline{aa}$ and

$\overline{bb}$ (denoted aa and bb) parameters have been found, and that $\overline{\lambda}$ (MS) is consequently defined as λ(MS), where:
λ(MS) = number of seismic events characterized by a magnitude > MS that occurs in the seismic zone under investigation within a unit time period (one year).
In this case, Eq. (1a) becomes:

$$Log\,\lambda(MS) = aa - bb \cdot MS \tag{1b}$$

The least square fit of the linear regression of the historical data (one example developed with data referring to the Italian territory [2] is given in Fig. 1) allows one to determine, for each Seismo-Genetic Zone, the *aa* and *bb* coefficients of the equation above and consequently to determine the return period of seismic events as a function of the event magnitude MS, as given by:

$$T(MS) = \frac{1}{\lambda(MS)}$$

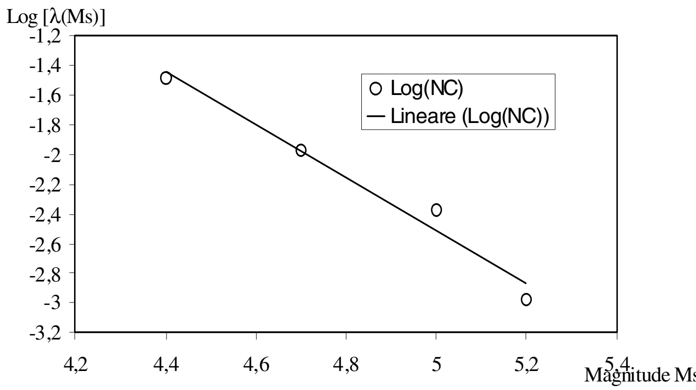

Fig. 1. Linear regression as obtained for the Marche-Abruzzo area – Italy (Seismo-Genetic Zone 53 of the italian territory w.r.t. the ZS4 subidivision)

ATTENUATION LAW

In order to evaluate the seismic effect at a given site, it is not sufficient to know the law of occurrence of seismic events within a given Seismo-Genetic Zone. It is, in fact, necessary also to know how a seismic event propagates from its epicenter to the site under investigation. In order to do so, an attenuation law must be used. Generally, attenuation laws provide the Peak Ground Acceleration (PGA) as a function of the seismic magnitude MS and of the distance R of the site from the epicenter (other, deterministic, parameters are also often used to account for local soil effects).
Without loss of generality, in the investigation described in this paper use is made of the attenuation law proposed by Sabetta and Pugliese [3] for the Italian territory. This law has the unique feature of being developed for the whole Italian territory as opposed to other attenuation laws that were developed for specific Italian sites. According to the Sabetta Pugliese law, the PGA at a particular site is given by:

$$Log\left(A\right) = -1.845 + 0.363 \cdot MS - Log(R^2 + 5.0^2)^{\frac{1}{2}} + 0.195s \tag{2a}$$

$$Log\left(V\right) = -0.828 + 0.489 \cdot MS - Log(R^2 + 15.21)^{\frac{1}{2}} + 0.116s \tag{2b}$$

where:

- *A* represents the peak ground horizontal acceleration (PGA) expressed in terms of g (gravity),
- *V* is the peak horizontal velocity in centimeters/ second,

- *MS* is the event magnitude in Richter scale,
- *R* is the distance of the site from the event epicenter expressed in kilometers, and
- *s* is a deterministic coefficient taking values zero for bedrock soil condition or one for soft soil condition.

THE PDF OF THE DISTANCE *R* FOR CIRCULAR (ANNULAR AND CIRCULAR SECTOR) SEISMO-GENETIC ZONES

Given the fact that the spatial probability of occurrence of seismic event within each seismic zone is uniform, the distance *R* can be easily treated as a random variable. Considering a Seismo-Genetic Zone of circular shape (radius *r*) centered at the site under investigation, the probability density function PDF of the distance *R* of epicenter from site can be expressed as:

$$f_R(r)dr = P[r < R \le r + dr] = \frac{2r}{r'^2} dr \tag{3}$$

It can be easily shown that for a seismo-genetic area shaped as a circular sector centered at the site of interest, identified by the angle at the center (θ), the PDF of the distance *R* has the same expression obtained for the circular area, as given in Eq. (3). In fact:

$$f_R(r)dr = P[r < R \le r + dr] = \left(\theta \cdot r \cdot dr\right) / \left(\theta \cdot r'^2 / 2\right) = \frac{2r}{r'^2} dr$$

Similarly, it can be easily shown that for a seismo-genetic area shaped as a circular annulus (or sector of a circular annulus) centered at the site of interest, of internal radius $r_{\min}$ and external radius $r_{\max}$, the PDF of the distance R has the following expression:

$$f_R(r) = \left(2 \cdot r\right) / \left(r_{\max}^2 - r_{\min}^2\right) \tag{4}$$

Notice that (4) reduces to (3) in the case of $r_{\min} = 0$.

THE PDF OF THE MAGNITUDE MS, $f_{MS}(ms)$, FOR EVENTS OCCURRING IN A GIVEN SEISMO-GENETIC ZONE

From the Gutenberg-Richter relationship (1b), it is possible, for each Seismo-Genetic Zone, to express the occurrence rate of seismic events as a function of the event magnitude:

$$\lambda(MS) = \hat{a} \cdot \exp\left(-\hat{b} \cdot MS\right) \tag{5}$$

where $\hat{a}$ = exp(*aa* log10), and $\hat{b}$ = *bb* log10.

Assuming that seismic events occur as Poisson processes, the probability that exactly *x* events characterized by magnitude > MS occur within a given observation time *t*, $P[X = x]$, is given by the following expression:

$$P[X = x] = \frac{(\lambda t)^x}{x!} e^{-\lambda t} \tag{6}$$

Introducing in (6) the expression of $\lambda(MS)$ provided by (5), it is possible to obtain:

$$P[X = x] = \frac{\left[\hat{a} \cdot t \cdot \exp\left(-\hat{b} \cdot MS\right)\right]^x}{x!} e^{-t \cdot \hat{a} \cdot \exp\left(-\hat{b} \cdot Ms\right)} \tag{7}$$

From the basics of probability theory [11], the probability that all seismic events characterized by a magnitude $\leq$ MS occur within an observation time *t*, $P\left[MS \leq ms\right]$, corresponds to the probability that zero events of magnitude > MS occur within an observation time *t*, $P\left[X = 0\right]$, i.e.:

$$P\left[MS \leq ms\right] = P\left[X = 0\right] \tag{8}$$

Substituting in (8) the value of $P\left[X = 0\right]$ given by (7), one can obtain:

$$P\left[MS \leq ms\right] = e^{-t\cdot\hat{a}\cdot\exp\left(-\hat{b}\cdot Ms\right)} \tag{9}$$

The above expression then corresponds, by definition, to the cumulative distribution function CDF ($F_{MS}\left(ms\right)$) of seismic events characterized by magnitude MS, thus:

$$F_{MS}\left(ms\right) = e^{-t\cdot\hat{a}\cdot\exp\left(-\hat{b}\cdot ms\right)} \tag{10}$$

And, by derivation, one can obtain the PDF of the magnitude MS for a given Seismo-Genetic Zone:

$$f_{MS}(ms) = \hat{a}\cdot\hat{b}\cdot t\cdot e^{-\hat{b}\cdot ms - \hat{a}\cdot t\cdot\exp(-\hat{b}\cdot ms)} \tag{11}$$

If, instead of a whole Seismo-Genetic Zone, only a portion of the zone (sub-zone *i*) is considered, given that the spatial distribution of the seismic events within a Seismo-Genetic Zone is uniform, the scaled PDF of the magnitude MS is given by:

$$f_{MS-i}\left(ms\right) = \alpha_i\cdot\hat{a}\cdot\hat{b}\cdot t\cdot e^{-\hat{b}\cdot ms - \hat{a}\cdot t\cdot\exp\left(-\hat{b}\cdot ms\right)} \tag{12}$$

where:

$$\alpha_i = \frac{Area_{sub-zone-i}}{Area_{Seismo-Genetic-Zone}}.$$

THE PDF OF THE PEAK GROUND ACCELERATION, $f_A\left(a\right)$, FOR A SINGLE CIRCULAR (ANNULAR AND CIRCULAR SECTOR) SEISMO-GENETIC ZONE

The Sabetta-Pugliese attenuation law allows one to obtain the estimated PGA at a given site as a function of the magnitude *MS* and of the kilometric distance *R* from the epicenter. Given that both *MS* and *R* are random variables, the peak ground acceleration, herein referred to as *A*, is also a random variable. From the theory of probability it is known [11] that if an m-dimensional variable, say *Y*, is an analytic function *g* of several random variables (i.e. $Y = g\left(X_1, X_2, ..., X_n\right)$), one can determine the probability density function (PDF) of the derived random variable *Y* starting from the joint probability density function (joint PDF) of the basic variables X_i.

Considering that in this specific case $m = 1$ (*Y* is a one-dimensional variable) and $n = 2$, one can consider the following statement:

$$\begin{cases} Y_1 = A = g_1\left(MS, R\right) \\ Y_2 = g_2\left(R\right) = R \end{cases} \tag{13}$$

In order to obtain the PDF of *A*, it is useful to consider the following relation:

$$f_A\left(a\right) = \int_{-\infty}^{+\infty} f_{(A,R)}\left(a, r\right) dr \tag{14}$$

The joint PDF of the random variables *A* and *R*, named $f_{A,R}\left(a, r\right)$, can be obtained from the analytical expressions of the PDF's of *MS* and *R* as follows:

$$f_{A,R}\left(a, r\right) = f_{MS(A,R)}\left(a, r\right) f_R\left(r\right)\left|J\right| \tag{15}$$

where *J*, the Jacobian of the transformation (13), for this specific case reduces to $J = \partial ms / \partial a$.
Then Eq. (14) can be specialized using:

- for $f_R(r)$ the expression (4);
- for $f_{MS}(ms)$ the expression (11);
- for the transformation g the Sabetta-Pugliese law (2).

This leads to the determination of the Jacobian *J* as follows:

$$J = \frac{1}{0.363} \cdot \frac{1}{a \cdot \log 10} \tag{16}$$

The PDF of the random variable A, $f_A(a)$, for a Seismo-Genetic Zone of annular circular shape, can be obtained integrating out the PDF function of the radial distance from the site of interest to the epicenter between the values of $r_{\min}$ and $r_{\max}$.

$$f_A(a) = \frac{2 \cdot \hat{a} \cdot \hat{b} \cdot t}{r_{\max}^2 - r_{\min}^2} \cdot \int_{r_{\min}}^{r_{\max}} r \cdot \exp\left[-V_1 - \hat{a} \cdot t \cdot \exp(-V_1)\right] \cdot dr \tag{17}$$

where: $V_1 = \frac{\hat{b}}{\hat{c}} \cdot \left(\log a + 1.845 \cdot \log 10 \cdot \sqrt{r^2 + 25} - 0.195 \cdot s \cdot \log 10\right)$

and $\hat{c}_1 = 0.363 \cdot \log 10$.

Eq. (17) can also be rewritten as:

$$f_A(a) = \frac{2 \cdot K_1}{r_{\max}^2 - r_{\min}^2} \cdot \int_{r_{\min}}^{r_{\max}} K_2 \cdot r \cdot a^{-(K_1+1)} \cdot \exp\left(-K_2 \cdot a^{K_1}\right) dr \tag{17a}$$

with:

$K_1 = \hat{b} / \hat{c}_1$ and $K_2 = a \cdot t \cdot \exp(-V_1)$.

THE PDF OF THE PEAK GROUND VELOCITY, $f_V(v)$, FOR A SINGLE CIRCULAR (ANNULAR AND CIRCULAR SECTOR) SEISMO-GENETIC ZONE

The Sabetta-Pugliese attenuation law allows also to obtain the estimated PGV at a given site as a function of the magnitude *MS* and of the kilometric distance *R* from the epicenter. Following a precedure similar to that described above for the determination of the PDF of the PGA it is possible to obtain:

$$f_V(v) = \frac{2 \cdot H_1}{r_{\max}^2 - r_{\min}^2} \cdot \int_{r_{\min}}^{r_{\max}} H_2 \cdot r \cdot a^{-(H_1+1)} \cdot \exp\left(-H_2 \cdot a^{H_1}\right) dr \tag{17b}$$

where:

$H_1 = \hat{b} / \hat{c}_2$ and $H_2 = v \cdot t \cdot \exp(-V_2)$

with:

$V_2 = \frac{\hat{b}}{\hat{c}} \cdot \left(\log a + 1.845 \cdot \log 10 \cdot \sqrt{r^2 + 25} - 0.195 \cdot s \cdot \log 10\right)$

and $\hat{c}_1 = 0.489 \cdot \log 10$.

THE CDF OF THE PGA AND PGV, FOR A SINGLE, GENERICALLY SHAPED, SEISMO-GENETIC ZONE

In the previous sections, we determined the PDF of the peak ground acceleration and peak ground velocity due to the seismic activity (over a period of time *t*) of a single Seismo-Genetic Zone of circular (or annular, or annular sector) shape centered at the site under investigation. The corresponding CDF's can be obtained by integration as follows:

$$F_A(a) = \int_0^a f_A(a) \cdot da \tag{18a}$$

$$F_V(v) = \int_0^a f_V(v) \cdot dv \tag{18b}$$

Given that, in general, Seismo-Genetic Zones are characterized by irregular shapes, one can divide each zone of interest into "I" sub-zones for which Eq. (4) holds. Under this hypothesis, Eq. (14) can be specialized using for $f_{MS}(ms)$ the scaled expression (12). This leads to the determination of the scaled PDF of the random variables *A* and *V*:

$$f_{A-i}(a) = \alpha_i \frac{2 \cdot K_1}{r_{\max-i}^2 - r_{\min-i}^2} \cdot \int_{r_{\min-i}}^{r_{\max-i}} K_2 \cdot r \cdot a^{-(K_1+1)} \cdot \exp\left(-K_2 \cdot a^{K_1}\right) dr \tag{19a}$$

$$f_{V-i}(v) = \alpha_i \cdot \frac{2 \cdot H_1}{r_{\max-i}^2 - r_{\min-i}^2} \cdot \int_{r_{\min-i}}^{r_{\max-i}} H_2 \cdot r \cdot v^{-(H_1+1)} \cdot \exp\left(-H_2 \cdot v^{H_1}\right) dr \tag{19b}$$

Let us call "*event E_i*" the occurrence, at the site under investigation and over a period of observation *t*, of a peak ground acceleration "A_i" (or a peak ground velocity "V_i") smaller than or equal to a given acceleration value "*a*" (or smaller than or equal to a given velocity value "*v*") due to the seismic action of the sub-zone "*i*" only.

The probability of occurrence of event E_i can then be expressed as follows:

$$P[E_i] = P[A_i \le a] = \alpha_i \frac{2 \cdot K_1}{r_{\max-i}^2 - r_{\min}^2 - i} \cdot \int_0^a \int_{r_{\min-i}}^{r_{\max-i}} K_2 \cdot r \cdot a^{-(K_1+1)} \cdot \exp\left(-K_2 \cdot a^{K_1}\right) \cdot dr \cdot da \tag{20a}$$

$$P[E_i] = P[V_i \le v] = \alpha_i \frac{2 \cdot H_1}{r_{\max-i}^2 - r_{\min}^2 - i} \cdot \int_0^v \int_{r_{\min-i}}^{r_{\max-i}} H_2 \cdot r \cdot v^{-(H_1+1)} \cdot \exp\left(-H_2 \cdot v^{H_1}\right) \cdot dr \cdot dv \tag{20b}$$

which correspond to scaled CDF's.

The group of the "*I*" Events E_i, which are mutually exclusive, is collectively exhaustive, regarding the activity of the Seismo-Genetic Zone of interest.

The total probability theorem [11] allows then the computation of the CDF of the PGA and PGV due to the seismic action of a generically shaped Seismo-Genetic Zone as:

$$F_A(a) = P[A < a] = \sum_{i=1}^{n} \alpha_i \cdot \frac{2 \cdot K_1}{r_{\max-i}^2 - r_{\min-i}^2} \int_0^a \int_{r_{\min-i}}^{r_{\max-i}} K_2 \cdot r \cdot a^{-(K_1+1)} \cdot \exp\left(-K_2 \cdot a^{-K_1}\right) \cdot dr \cdot da \tag{21a}$$

$$F_V(v) = P[V < v] = \sum_{i=1}^{n} \alpha_i \cdot \frac{2 \cdot H_1}{r_{\max-i}^2 - r_{\min-i}^2} \int_0^v \int_{r_{\min-i}}^{r_{\max-i}} H_2 \cdot r \cdot v^{-(H_1+1)} \cdot \exp\left(-H_2 \cdot a^{-H_1}\right) \cdot dr \cdot dv \tag{21b}$$

Evaluation of the CDF of the PGA (and PGV) due to the seismic action of more than one Seismo-Genetic Zone

Equation (21a) gives the probability that a peak ground acceleration $A \le a$ is developed at a given site as the result of the seismic action in a single Seismo-Genetic Zone *j* of generic shape. We shall refer to this as Event $X_j(a)$ and to the corresponding probability as $P\left[X_j(a)\right]$. One can extend this result in order to compute the probability that a peak ground acceleration $A \le a$ is developed at a given site as the result of the seismic action of all Italian Seismo-Genetic Zones. This corresponds to the probability that the event $X(a)$, intersection of all events $X_j(a)$, occurs. $X(a)$, being defined as:

$$X(a) = \cap X_j(a) \tag{22}$$

Assuming that the occurrence of seismic events is assimilated to a Poisson process (a common assumption in seismic hazard analysis) each Event is statistically independent. From statistics [11], it is known that the probability of occurrence of the Event "*intersection of k statistically independent events*" is given by the product of the probability of occurrence of each single event, and therefore:

$$P\left[X(a)\right] = P\left[\cap X_j(a)\right] = \prod_{j=1}^{k} F_A^j(a) \tag{23}$$

where each $F_A^j(a)$ is given by (21a).

Finally, the probability of occurrence of the Event $X(a)$ (occurrence at the site of a peak ground acceleration $A \le a$ due to the seismic action of all Seismo-Genetic Zones) corresponds to the CDF of the PGA, $F_A^{tot}(a)$, at a site due to the combined action of all Seismo-Genetic Zones, and therefore:

$$F_A^{tot}(a) = \prod_{j=1}^{k} F_A^j(a) \tag{24}$$

The PDF of the PGA can then be obtained by derivation as follows:

$$f_A^{tot}(a) = \frac{\partial \prod_{j=1}^{k} F_A^j(a)}{\partial a} \tag{25}$$

Developing the above procedure also for the peak ground velocity, it is possible to obtain:

$$F_A^{tot}(a) = \prod_{j=1}^{k} F_A^j(a) \tag{26}$$

and

$$F_V^{tot}(v) = \prod_{j=1}^{k} F_V^j(v) \tag{27}$$

where each $F_V^j(v)$ is given by (21b).

APPLICATION TO THE ITALIAN TERRITORY

The seismic catalogue

Several seismic catalogues are available for the Italian territory. Each one being characterized by a specific nature and purpose. For several years, the PFG catalogue has been used as reference [4]. Nonetheless, in this analysis, use is made of the more recent catalogue NT4.1.1 developed in 1997 by the GNDT (Gruppo Nazionale Difesa Terremoti – national group for seismic protection) with the specific purpose of being used as input file for seismic hazard evaluations to be carried out using the Cornell methodology. One of the characterizing features of the NT4.1.1 catalogue is the absence of fore-shocks and aftershocks (the catalogue accounts, independently of the intensity (Io) and magnitude (*MS*), only for the largest event within time-space frames of +/- 90 days and radius of 30 km of each seismic event). This catalogue is parametric, i.e. it is made of a sequence of information strings, one for each seismic event, containing information regarding the seismic event such as intensity, magnitude, epicenter position, date, etc. Of particular importance, for our analysis, is the association of each event to a particular Seismo-Genetic Zone according to the ZS4 Subdivision of the Italian Territory (GNDT). The NT4.1.1 catalogue covers the time window between the year 1000 and the year 1992 and encompasses all events characterized by an MCS Intensity (Mercalli Cancani Sieberg) ≥ 5/6 or a Magnitude MS ≥ 4.0. Overall the NT4.1.1 catalogue is characterized by the following two special features:

- the quality and richness of information upon which the earthquake events parameters are determined;
- the procedures used to determine the event parameters.

Regarding this last matter it is worth to recall that the magnitudes *MS* have been computed either from observed (recorded) field data according to the procedures proposed by Karnik [5], Gasperini et al. [6], and Margottini [7] or obtained from the Intensity values according to the procedure proposed by Rebez e Stucchi [8].

Completeness Analysis

As previously mentioned, the seismic catalogue used for the study presented in this paper (NT4.1.1) encompasses all data relative to seismic events occurred in Italy dating from year 1000 to 1992. Recent studies have shown how generally seismic catalogues obtained from historical data may be affected by errors due to the fact that the information contained in historical archives used as basis for the catalogue may not be complete.

Overall, it is reasonable to say that, due to historical errors of various nature, data encompassed in seismic catalogues do not reflect the actual seismic activity of most areas. In order to compensate for such errors, Mulargia, Gasperini and Tinti [2] have proposed to perform a “completeness analysis” in order to filter the historical data. The proposed completeness analysis (here adopted) is based upon the following assumptions:

- The completeness analysis is performed separately for each class of event magnitude as the completeness of information is a function of the event magnitude.
- The cumulative number of events for each class of magnitude is plotted as a function of time.
- The time interval that is characterised by the largest seismic activity (as visually identified) is considered to be complete and therefore used to determine the seismic characteristics of the class of magnitude under investigation. In general this coincides with the most recent data, as shown in Figure 2.

The completeness analysis performed by the authors upon the NT4.1.1 catalogue considered the following three classes of magnitude:

- Class I. $4.0 \leq MS < 4.5$
- Class II. $4.5 \leq MS < 5.0$
- Class III. $MS \geq 5.0$

Figure 1 show that for magnitude Class II the catalogue can be considered to be complete starting from year 1856. Such completeness analysis has been performed both for the whole Italian territory and separately for Northern, Central and Southern Italy. It has been observed that the “complete” time windows determined for the whole Italian territory were very similar to those identified for each part, and therefore the completeness characteristics of the catalogue are assumed to be homogeneous over the whole Italian territory.

The seismo-genetic subdivision ZS4

The sub-division of the Italian territory into areas of homogeneous seismic activity has been performed according to the so called ZS4 zoning (Catalogazione ZS4 GNDT 1996). The seismic activity within each zone is supposed to be spatially uniform. This means that seismic events have the same probability of occurrence over the whole area of each Seismo-Genetic Zone. Furthermore, within each zone, seismic events follow the same law of occurrence. So far the seismo-genetic model of the Italian territory has not yet been completely defined (excluding the model proposed by Doglioni [9]) in terms of a detailed reconstruction of the seismo genetic structures responsible for all seismic events documented over the last thousand years. This lack of knowledge is mainly due to the extreme geological complexity of the peninsula. The recent seismo genetic subdivision ZS4 is mainly based upon the cinematic model (by Scandone et al. [10]) of the principal tectonic units that have been active in recent times. The determination of homogeneous area based upon the seismic history of the territory is of crucial importance given also the fact that paleo-seismicity investigations have brought results (in terms of seismic activity) that are comparable with those obtained from the analysis of historical seismic catalogues. Nonetheless, due to the above-mentioned lack of complete modeling of the geo-dynamic activity, different seismic structures (characterized by different release rate) may coexist within the same area. The ZS4 subdivision of the Italian territory consists of 80 Seismo-Genetic Zones.

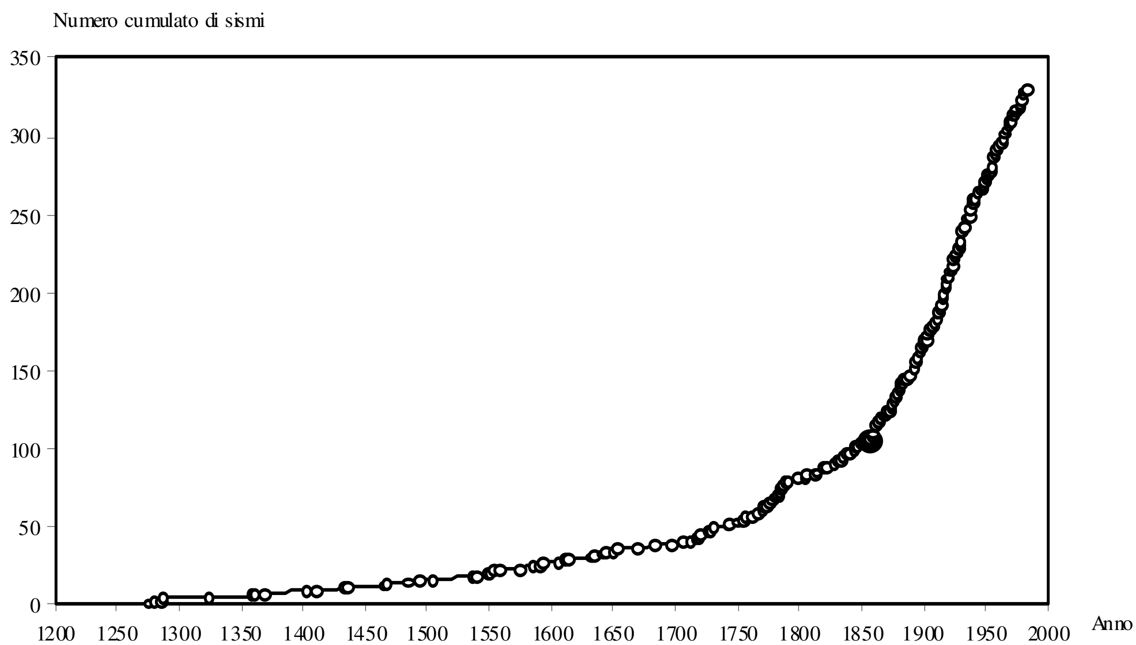

Fig. 2 Cumulative Curve, as obtained for 4.5 ≤ *MS* < 5.0 for the Italian territory.

AN APPLICATIVE EXAMPLE

Using Eq. (23), the Cumulative Distribution Function CDF of the peak ground acceleration and peak ground velocity for the city of Bologna (P.za Maggiore) have been numerically computed for observation time $t = 50$ years. The results obtained are given in the following Figures 3 and 4. The plots in the figures can be readily used in order to know directly the probability of having ground accelerations (or velocities) in excess of given values, or to identify PGA and PGV values characterized by given probability of exceedence.

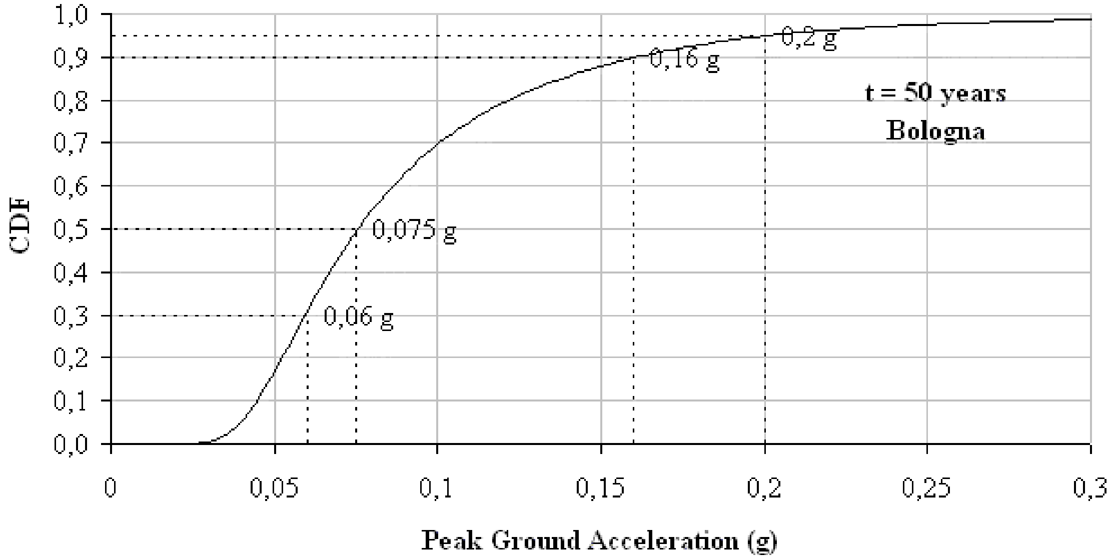

Fig. 3. Cumulative Distribution Function (CDF) of the Peak Ground Acceleration for the city of Bologna (P.za Maggiore) for an observation time t = 50 years.

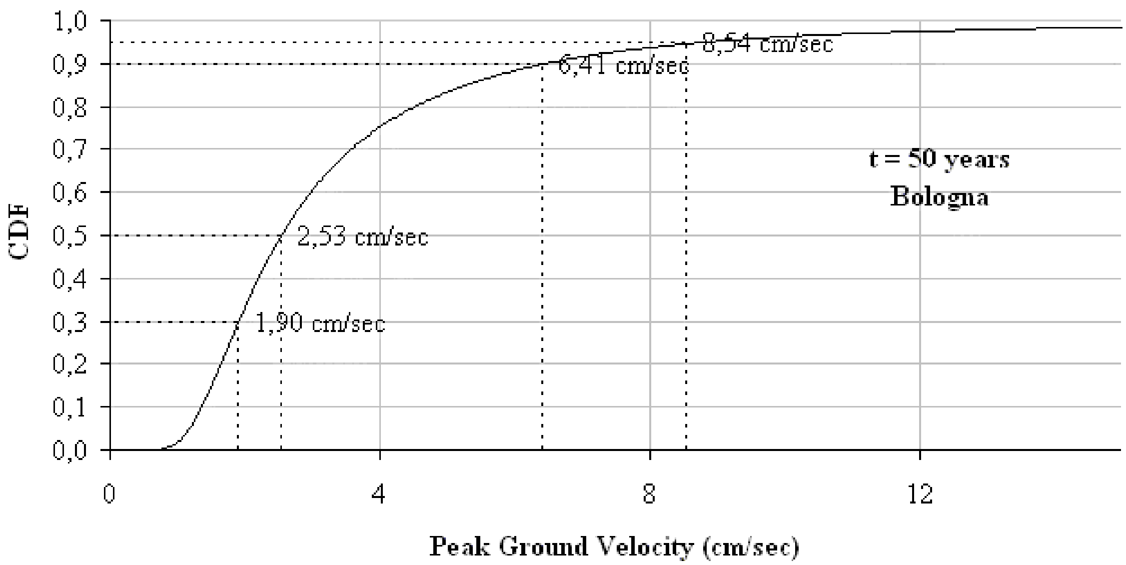

Fig. 4. Cumulative Distribution Function (CDF) of the Peak Ground Velocity for the city of Bologna (P.za Maggiore) for an observation time t = 50 years.

THE USE OF CDF OF THE PGA AND PGV FOR THE IDENTIFICATION OF AN EFFECTIVE INTENSITY MEASURE FOR PERFORMANCE BASED SEISMIC DESIGN

The cumulative distribution function of PGA and PGV thus obtained feature the same accuracy. This allows to use the couples PGA-PGV (characterised by given probabilities of occurrence) as vectorial

Intensity Measures [15] in multi-input incremental dynamic analysis IDA [16] for probabilistic design approach in performance based seismic engineering.
Figures 5 and 6 compare the response spectra obtained for two "bouquet" of 25 earthquake inputs characterized by a probability of occurrence of 10 % in 50 for the city of Bologna (Italy).
The first "bouquet", for sake of simplicity hereafter referred to as "bouquet A" is characterised by the use of the sole PGA as Intensity Measure (IM). More specifically all earthquake inputs considered in this bouquet (composed of historically recorded data) are characterised by a PGA equal to 0.16 g. For practical reasons all quakes were scaled in order to have this "exact" value of PGA. It is worth to mention that all records considered (retrieved from the NISEE data base) are characterized (before scaling) by values of PGA within the range 0,158 – 1,62 g. All records were selected considering the PGA only and discarding near field records, as well as records obtained from sites characterised by a soft soil (the peak ground velocities, before scaling, of these records lie in the range between 2,5 and 48.40 cm/sec).
The second "bouquet", for sake of simplicity hereafter referred to as "bouquet B" is characterised by the use of the vectorial Intensity Measure (IM) composed of PGA and PGV. In detail, all earthquake inputs considered in "bouquet B" (also this one composed of historically recorded data) are characterised by a PGA equal to 0.16 g and a PGV around to 6.4 cm/sec). For practical reasons all quakes were scaled in order to have this "exact" value of PGA and a value of PGV around the target one. It is worth to mention that all records considered (retrieved from the NISEE data base) are characterized (before scaling) by values of PGA in the range between 0,140 and 0,176 g, and by values of PGV in the range between 5,10 and 6,60 cm/sec. Only 1 record (out of 25) of "bouquet A" is comprised also in "bouquet B".
As expected, the response spectra of the bouquet of earthquake records identified by the vectorial IM (bouquet B) are characterised by a much smaller coefficient of variation than those identified by the scalar IM (bouquet A). Nonetheless the magnitude reduction (about – 50 %) of the c.o.v. of the response spectra is quite astonishing and give a clear indication of how this vectorial IM can be effective for incremental dynamic analysis so crucial in Performance Based Seismic Engineering.

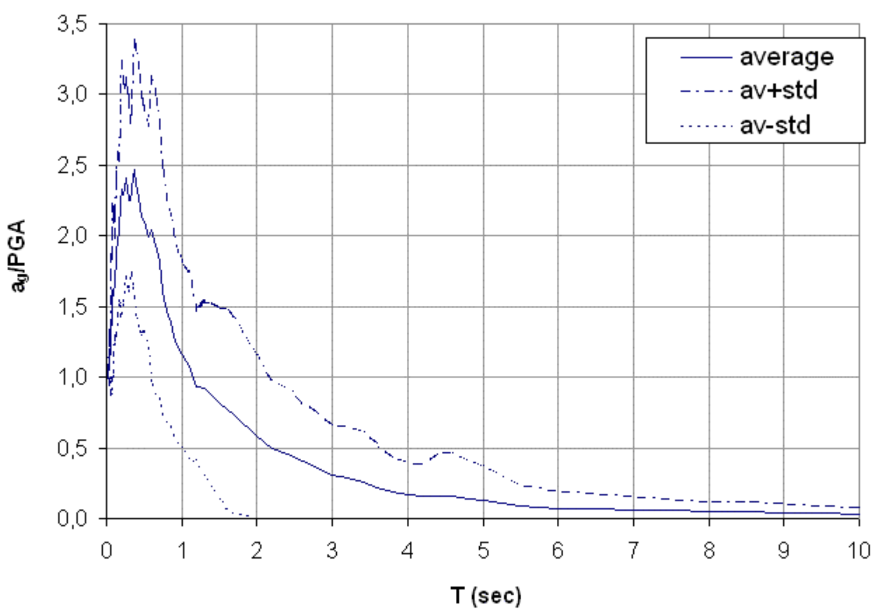

Fig. 5: Response spectra (average, average +/- one stardard deviation) for "bouquet A" of earthquake records (scalar Intensity Measure)

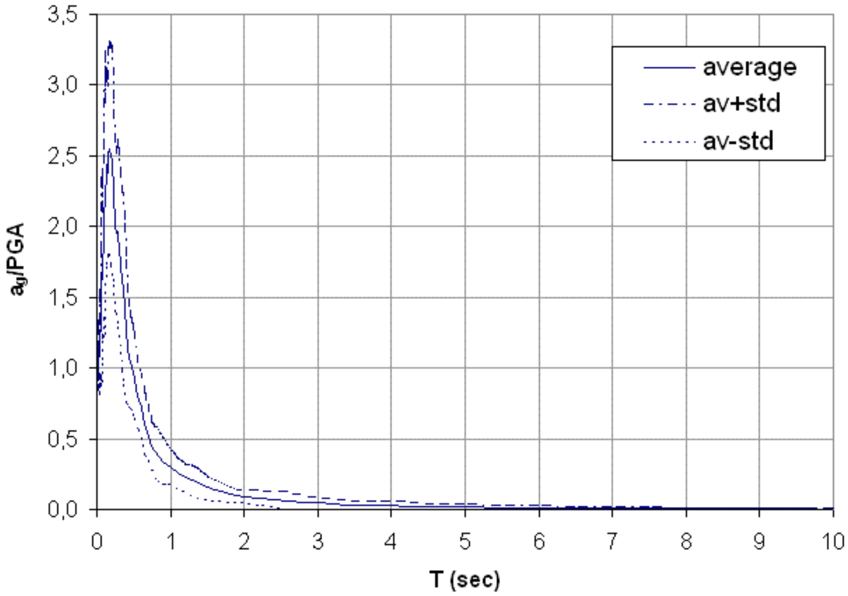

Fig. 6: response spectra (average, average +/- one stardard deviation) for "bouquet B" of earthquake records (vectorial Intensity Measure)

CONCLUSIONS

The determination of the Probability Density Functions (PDF and CDF) of the Peak Ground Acceleration (PGA) and Peak Ground Velocity (PGV) due to the seismic activity is here developed for the Italian territory according to an original procedure based upon Cornell's methodology.
The procedure is characterized by the treatment of the distance *R* from the epicenter to the site as a continuous random variable. This leads to the identification of the Cumulative Distributive Functions (CDF's) of both the Peak Ground Acceleration and Peak Ground Velocity that can readily be used to create an effective Intensity Measure to be used in the incremental dynamic analysis (IDA) for Performance Based Seismic Design procedures.

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