

ESTIMATING SEISMIC DEMANDS FOR PERFORMANCE-BASED ENGINEERING OF BUILDINGS

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SUMMARY

Several research projects are underway worldwide to develop effective methods for estimating seismic demands for performance-based engineering of buildings. After a brief review of alternative approaches, and evaluation of methods currently standard in engineering practice, this paper emphasizes one possible approach. Based on modal pushover analysis and improved estimation of target roof displacement, this approach is shown to provide considerably improved estimates of demands, while retaining the conceptual simplicity and computational attractiveness of current nonlinear static pushover procedures. Rooted in structural dynamics theory, this procedure is ready for practical application to symmetric-plan buildings.

INTRODUCTION

A major challenge for performance-based seismic engineering is to develop simple, yet sufficiently accurate methods for analyzing designed structures and evaluating existing buildings to meet selected performance objectives. As reflected in post-1995 guidelines for evaluating existing buildings, such as FEMA-273 [1], its successor FEMA-356 [2], and ATC-40 [3] documents, the profession has shifted away from the traditional practice of elastic analysis of the structure subjected to seismic forces reduced to recognize indirectly inelastic response, instead, inelastic behavior of structures is considered explicitly in estimating seismic demands at low performance levels—such as life safety and collapse prevention.

Current Practice

Currently, the structural engineering profession uses the nonlinear static procedure (NSP) or pushover analysis described in FEMA-273/356 [1, 2] and ATC-40 [3] documents to estimate seismic demands, which are computed by nonlinear static analysis of the structure subjected to monotonically increasing lateral forces with an invariant height-wise distribution until a predetermined target displacement is reached. The target displacement is estimated from the deformation of an inelastic SDF system derived from the pushover curve by either an iterative procedure, requiring analysis of a sequence of equivalent linear SDF systems [3], or by empirical equations based on response history analysis (RHA) of a large number of inelastic SDF systems [1, 2]. In the past several years, many researchers have discussed the underlying assumptions and limitations of pushover analysis [e.g., Refs. 4-10]. Based on the approximation that the response is controlled by the fundamental mode even after the structure yields, the NSP procedure, has led to good estimates of seismic demands but such predictions are mostly restricted to

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low-rise and medium-rise structures in which inelastic action is distributed throughout the height of the structure [e.g., 5, 9].

Improved Procedures

Consequently, the development of improved procedures for estimating seismic demands has been a hot topic for research over the past decade. Improved procedures may be classified into three categories: probabilistic approach based on incremental dynamic analyses, nonlinear RHA, and improved pushover analysis procedures.

Probabilistic Approach Based on Incremental Dynamic Analysis

For several years, the Pacific Earthquake Engineering Research Center (PEER) has focused on developing a rigorous comprehensive approach for seismic performance assessment of buildings [11]. Seismic demands are computed by the incremental dynamic analysis (IDA) procedure [12], and curves showing demand against ground motion intensity are developed for a sufficiently large number of ground motions to perform statistical evaluation of the results. This implies that for a given ground motion intensity, the median and a measure of dispersion (e.g., the 84th percentile) of the response values are computed. Such analysis of generic structures leads to hazard curves showing mean annual probability of exceedance against the demand parameter. When fully developed, this rigorous approach would represent a major advancement in estimating seismic demands for performance-based earthquake engineering.

Nonlinear Response History Analysis

A rigorous procedure to estimate seismic demands, nonlinear response history analysis (RHA), is permitted as an alternative procedure in present building evaluation guidelines, but it is not implemented prudently. The FEMA-356 specifications for Nonlinear Dynamic Procedure (NDP) state that the seismic demand may be estimated as (1) the maximum of demands due to three ground motions, or (2) the mean value of demands due to seven ground motions. These estimates can vary widely, as demonstrated next for the SAC-Los Angeles 9-story building subjected to an ensemble of 20 SAC ground motions; nonlinear RHA predicted collapse of the building during three of these excitations. The nonlinear RHA results for the first story drift led to a mean value of 20.4 cm over 17 excitations (excluding three that caused collapse of the building)². The results, shown in Fig. 1, demonstrate large variation in the drift estimated by three implementations of both versions of the FEMA-356 criteria. Such wide variability obviously implies that different engineers following the same criteria could arrive at contradictory conclusions about seismic safety and rehabilitation requirements for an existing building.

Seismic demands computed by nonlinear RHA may be affected profoundly by the assumptions in preparing an inelastic model of the building and software used in implementing the computation. To demonstrate the first possibility, Fig. 2 shows the peak values of story drifts for the SAC-Los Angeles 20story building due to the LA30 ground motion for three different idealizations of the structure: (1) Model M1, a basic centerline model in which the panel zone size, strength and stiffness are not represented; (2) Model M2, a model that explicitly incorporates the strength and stiffness properties of panel zones; and (3) Model M2A, an enhanced version of model M2, which considers the interior gravity columns, shear connections, and floor slabs [9]. No results are shown for Model M1 because it predicted collapse of the building would not be acceptable. However, the most realistic model (M2A) predicts much smaller story drifts, with the largest drift among all stories near 5%. Secondly, the first story drift in the SAC-Los Angeles 9-story building due to one of the SAC ground motions computed by three widely used computer programs differed by 30%. Such variability implies that engineers using different computer programs or

 $^{^{2}}$ Ignoring the three collapses in computing the mean is strictly incorrect. Working with the median value would be better, but the mean of the data for 17 excitations was used to remain consistent with FEMA273/356 guidelines



Figure 1. First story drift: (a) maximum of demands due to three excitations; and (b) average of demands due to seven excitations. The excitations were selected randomly three times from an ensemble of seventeen excitations



Figure 2. Influence of modeling assumptions on the story drift demands for the SAC-Los Angeles 20-story building due to LA30 ground motion; results are shown for M2 and M2A models, but model M1 predicted collapse of the building. (Adapted from Gupta & Krawinkler [9].)

inelastic modeling assumptions could arrive at contradictory conclusions about the seismic performance of the same building.

Nonlinear RHA is an onerous requirement for several reasons. First, an ensemble of site-specific ground motions compatible with the seismic hazard spectrum for the site must be simulated. Second, in spite of increasing computing power, nonlinear RHA remains computationally demanding, especially for unsymmetric-plan buildings—which require three-dimensional analysis to account for coupling between lateral and torsional motions—subjected to two horizontal components of motion. Third, such analyses must be repeated for many excitations because of the wide variability in the demand due to plausible ground motions and the statistics of response must be considered. Fourth, commercial software is so far not robust, reliable, or convenient enough for structural modeling and interpretation of response results. Fifth, an independent peer review of nonlinear-RHA results is required by FEMA-356, adding to the project duration and cost.

Opinions within both the research and professional communities differ on whether nonlinear RHA and the implementing software is ready for practical application. Even if nonlinear RHA is ripe for application, it

is unreasonable to require this onerous procedure for every building—no matter how simple—and of every structural engineering office—no matter how small. Therefore, simplified methods are expected to remain important in structural engineering practice. Simplified methods must be rooted in structural dynamics theory, and their underlying assumptions and range of applicability identified. Nonlinear RHA can be employed for final evaluation of those combinations of buildings and ground motions where a simplified procedure begins to loose its accuracy.

Improved Pushover Procedures

Pushover analyses procedures have been improved to account for the contributions of higher modes to response, redistribution of inertia forces because of structural yielding and the associated changes in the vibration properties of the structure. Adaptive force distributions that attempt to follow more closely the time-variant distributions of inertia forces have been proposed [8, 13, 14]. Recently an incremental response spectrum analysis procedure (IRSA) has been developed that requires at each incremental loading step in the pushover procedure a response spectrum analysis of the structure in its current yielded state, treating it as linearly elastic until the next step [15]. Although these procedures provide improved estimates of seismic demands for the examples considered, their accuracy remains to be evaluated for a wide range of buildings and ground motion ensembles. While these sophisticated procedures are notable research contributions, they may be too complicated conceptually for implementation in structural engineering practice.

Attempts have been made to consider more than the first mode in pushover analysis [16, 17, 18]. Based on structural dynamics theory, a modal pushover analysis procedure (MPA) has been developed that includes higher mode contributions to determine the total seismic demand [19].

Objectives and Scope

The objectives of the rest of this paper are to evaluate procedures used in current structural engineering practice to estimate seismic demands for buildings, and to present improved procedures that retain the conceptual simplicity and computational attractiveness of current methods. Both aspects of the analysis, estimating the target roof displacement and pushover analysis, are improved. This paper is organized in three parts: modal pushover analysis; SDF-system estimate of roof displacement; and SDF-system deformation.

SAC BUILDINGS, GROUND MOTIONS AND RESPONSE STATISTICS

SAC Buildings and Ground Motions

SAC commissioned three consulting firms to design 3-, 9-, and 20-story model buildings with symmetric plan according to the local code requirements of three cities: Los Angeles, Seattle, and Boston. Described elsewhere in detail [9], the structural systems of these model buildings consisted of perimeter steel moment-resisting frames (SMRF). Structural systems, defined by the N-S perimeter frames of 9- and 20-story buildings, are used as examples in this paper.

For all three locations, sets of 20 ground motion records were assembled representing probabilities of exceedance of 2% and 10% in 50 years (return periods of 2475 and 475 years, respectively) [20]. Results presented in this paper are for response of the 9-story and 20-story buildings to the 2/50 set of records. This set of ground motions enables testing of the MPA procedure—an approximate method—under the most severe conditions; many of these ground motions drive the Los Angeles and Seattle buildings far into the region of inelastic behavior and strength deterioration.

Response Statistics

The dynamic response of each structural system to each of the 20 ground motions was determined by two procedures: nonlinear RHA and modal pushover analysis (MPA). The "exact" peak value of structural response or demand, r, determined by nonlinear RHA is denoted by $r_{\text{NL-RHA}}$, the approximate value from MPA by r_{MPA} , and from FEMA-356 analyses by r_{FEMA} . From these data for each ground motion, the ratio $r_{\text{MPA}}^* = r_{\text{MPA}} \div r_{\text{NL-RHA}}$ is defined. An approximate method is invariably biased in the sense that the median of this ratio differs from one, underestimates the median response if the ratio is less than one, and provides an overestimate if the ratio exceeds one.

The response of each building was also computed assuming linear elastic behavior. For elastic systems, the nonlinear RHA procedure specializes to linear RHA and the MPA procedure to standard response spectrum analysis (RSA); thus, the responses are denoted as r_{RHA} and r_{RSA} and the response ratio is written as $r_{\text{RSA}}^* = r_{\text{RSA}} \div r_{\text{RHA}}$.

Presented in this paper are the median values, defined as the geometric mean [21] of 20 observed values, of r_{FEMA} , r_{MPA} , $r_{\text{NL-RHA}}$, r_{MPA}^* , and r_{RSA}^* . In the case where one or more excitations caused collapse of the building or its first-"mode" SDF system, the median was estimated by a counting method. The 20 data values were sorted in ascending order, the median was estimated as the average of the 10th and 11th values starting from the lowest value.

NONLINEAR STATIC PROCEDURE: CURRENT PRACTICE

The nonlinear static procedure in FEMA-356 requires development of a pushover curve, a plot of base shear versus roof displacement, by nonlinear static analysis of the structure subjected first to gravity loads, followed by monotonically increasing lateral forces with a specified, invariant height-wise distribution. At least two force distributions must be considered. The first is to be selected from among the following: first mode distribution, Equivalent Lateral Force (ELF) distribution, and SRSS distribution. The second distribution is either the "Uniform" distribution or an adaptive distribution; the latter varies with change in deflected shape of the structure as it yields [13, 14, 22]. The other four force-distributions are defined next and shown in Fig. 3.:



Figure 3. FEMA-356 force distributions for Los Angeles 9-story building: (a) 1st Mode, (b) ELF, (c) SRSS, and (d) "Uniform" [25].

- 1. First mode distribution: $s_j^* = m_j \phi_{j1}$ where m_j is the mass and ϕ_{j1} is the mode shape value at the *j*th floor;
- 2. Equivalent lateral force (ELF) distribution: $s_j^* = m_j h_j^k$ where h_j is the height of the *j*th floor above the base, and the exponent k = 1 for fundamental period $T_1 \le 0.5 \text{ sec}$, k = 2 for $T_1 \ge 2.5 \text{ sec}$; and varies linearly in between;
- 3. SRSS distribution: s^* is defined by the lateral forces back-calculated from the story shears determined by response spectrum analysis of the structure, assumed to be linearly elastic; and
- 4. "Uniform" distribution: $s_j^* = m_j$.

Each of these four force distributions pushes the building in the same lateral direction over the entire height of the building (Fig. 3).

The limitations of the FEMA-356 force distributions are demonstrated in Figs. 4 and 5 where the resulting estimates of the median story drift and plastic hinge rotation demands imposed on the SAC buildings by the ensemble of 20 SAC ground motions are compared with the "exact" median value determined by nonlinear RHA of the buildings. The target displacement was not determined by the FEMA method, but was calculated accurately to ensure a meaningful comparison of the two sets of results. The first-mode force distribution grossly underestimates the story drifts, especially in the upper stories, showing that higher-mode contributions are especially significant in the seismic demands for upper stories. Although the ELF and SRSS force distributions are intended to account for higher mode responses, they do not provide satisfactory estimates of seismic demands for buildings that remain essentially elastic (Boston buildings) or buildings that are deformed far into the inelastic range (Los Angeles buildings). The "uniform" force distribution seems unnecessary because it grossly underestimates drifts in upper stories and grossly overestimates them in lower stories of four buildings; the other two (Boston buildings) remain essentially elastic. Because FEMA-356 requires that seismic demands be estimated as the larger of results from at least two lateral force distributions, it is useful to examine the upper bound of results from the four force distributions considered. This upper bound also significantly underestimates drifts in upper stories, but grossly overestimates them in lower stories. The FEMA-356 lateral force distributions either fail to identify, or significantly underestimate, plastic hinge rotations in beams at upper floors.

IMPROVED NONLINEAR STATIC PROCEDURE: MODAL PUSHOVER ANALYSIS

It is clear from the preceding discussion that the seismic demand estimated by NSP using the first-mode force distribution (or others in FEMA-356) should be improved. One approach to reduce the discrepancy in this approximate procedure relative to nonlinear RHA is to include the contributions of higher modes of vibration to seismic demands. Just as for elastic systems, including higher-mode responses improves the seismic demand estimate for buildings responding in their inelastic range.

Basic Concept

The equations of motion for a symmetric-plan multistory building subjected to earthquake ground acceleration $\ddot{u}_g(t)$ are the same as if the excitation were external forces, known as the effective earthquake forces:

$$\mathbf{p}_{\rm eff}\left(t\right) = -\mathbf{m}\mathbf{1}\ddot{u}_g\left(t\right) \tag{1}$$



Figure 4. Median story drifts determined by nonlinear RHA and four FEMA-356 force distributions: 1st Mode, ELF, SRSS, and "Uniform" [25].



Figure 5. Median plastic rotations in interior beams determined by nonlinear RHA and four FEMA-356 force distributions: 1st Mode, ELF, SRSS, and "Uniform" [25].

where **m** is the mass matrix and **1** is a vector with all elements equal to unity. This spatial (height-wise) distribution of the effective earthquake forces over the building is defined by the vector $\mathbf{s} = \mathbf{m1}$ and their time variation by $\ddot{u}_g(t)$. This force distribution can be expanded as a summation of modal inertia force distributions \mathbf{s}_n [23; Section 13.2]:

$$\mathbf{s} = \sum_{n=1}^{N} \mathbf{s}_n \qquad \mathbf{s}_n \equiv \Gamma_n \mathbf{m} \boldsymbol{\phi}_n \tag{2}$$

where ϕ_n is the *n*th-mode of natural vibration and $\Gamma_n = \phi_n^T \mathbf{m} \mathbf{1} / \phi_n^T \mathbf{m} \phi_n$. Thus,

$$\mathbf{p}_{\text{eff},n}(t) = -\mathbf{s}_n \ddot{u}_g(t) \tag{3}$$

is the *n*th-mode component of effective earthquake forces.

In the MPA procedure, the peak response of the building to $\mathbf{p}_{\text{eff},n}(t)$ — or the peak "modal" demand r_n — is determined by a nonlinear static or pushover analysis using the modal force distribution $\mathbf{s}_n^* = \mathbf{m}\boldsymbol{\phi}_n$ [based on Eq. (2b)]. The peak modal demands r_n are then combined by an appropriate modal combination rule to estimate the total demand. This procedure is directly applicable to the estimation of deformation demands (e.g., floor displacements and story drifts) but computation of plastic hinge rotations and member forces require additional consideration, as will be elaborated later.

Although modal analysis theory is strictly not valid for inelastic systems, the fact that elastic modes are coupled only weakly in the response of inelastic systems [19] permitted development of the MPA procedure.

Summary of Procedure

The MPA procedure is implemented in a sequence of steps:

- 1. Compute the natural frequencies, ω_n and modes, ϕ_n , for linearly elastic vibration of the building (Fig. 6a).
- 2. For the nth-mode, develop the base shear-roof displacement, $V_{bn} u_{rn}$, pushover curve for lateral force distribution, $\mathbf{s}_n^* = \mathbf{m}\phi_n$. These force distributions for the first three modes are shown in Fig. 6b and the pushover curves in Fig. 7. Gravity loads, including those on the interior (gravity) frames, are applied before the lateral forces, causing roof lateral displacement u_{rg} .
- 3. Idealize the pushover curve, which may exhibit negative post-yield stiffness because of P- Δ effects, as a bilinear curve (Fig. 8a).
- 4. Convert the idealized $V_{bn} u_{rn}$ pushover curve to the force-displacement, $F_{sn}/L_n D_n$, relation (Fig.8b) for the nth-"mode" inelastic SDF system by utilizing $F_{sny}/L_n = V_{bny}/M_n^*$ and $D_{ny} = u_{rny}/\Gamma_n\phi_{rn}$ in which M_n^* is the effective modal mass, and ϕ_{rn} is the value of ϕ_n at the roof.



Figure 6. (a) First three natural vibration periods and modes; and (b) Force distributions $s_n^* = m\phi_n$, n = 1, 2, and 3 for the SAC-Los Angeles 9-story building [25].



Figure 7. "Modal" pushover curves for first three "modes" of six SAC buildings [25].



Figure 8. Properties of the *n*th-"mode" inelastic SDF system from the pushover curve [19].

- 5. Compute the peak deformation D_n of the nth-"mode" inelastic SDF system defined by the forcedeformation relation developed in Step 4 and damping ratio ζ_n . The elastic vibration period of the system is $T_n = 2\pi (L_n D_{ny}/F_{sny})^{1/2}$. For an SDF system with known T_n and ζ_n , D_n can be computed using nonlinear RHA, inelastic design spectrum, or empirical equations for the ratio of deformations of inelastic and elastic systems presented later in this paper. The nonlinear RHA is appropriate for research investigations and was adopted here, but the latter two methods, which will be discussed later, are intended for practical application.
- 6. Calculate the peak roof displacement u_{rn} associated with the *n*th-"mode" inelastic SDF system from $u_{rn} = \Gamma_n \phi_{rn} D_n$.
- 7. From the pushover database (Step 2), extract values of desired responses r_{n+g} due to the combined effects of gravity and lateral loads at roof displacement equal to $u_{rg} + u_{rn}$.
- Repeat Steps 3-7 for as many "modes" as required for sufficient accuracy. In this investigation, up to three "modes" were included for 9-story buildings and up to five "modes" were included for 20-story buildings.
- 9. Compute the dynamic response due to nth-"mode": $r_n = r_{n+g} r_g$, where r_g is the contribution of gravity loads alone.
- 10. Determine the total response (demand) by combining gravity load response and the peak "modal" responses using the SRSS rule:

$$r \approx \left[r_g \pm \left(\sum_n r_n^2 \right)^{1/2} \right]$$
(4)

Plastic Hinge Rotations and Member Forces

Although the total floor displacements and story drifts are computed by combining the values obtained from gravity load and "modal" pushover analyses (Step 10), the plastic hinge rotations are not computed by this procedure. The rotations of plastic hinges can be estimated from the story drifts by a procedure presented earlier by Gupta [9], which (1) estimates the story plastic drift, defined as the total story drift minus the story yield drift; and (2) relates the story plastic drift to the beam plastic rotation. The following

simplifying assumptions were used in estimating the story yield deformation: (1) inflection points are at mid-heights of columns and mid-spans of beams; (2) story elevation has regular geometry and uniform section properties; (3) yielding occurs only in beams, i.e., columns do not yield, and panel zone effects are ignored; (4) effects of gravity loads on yielding in beams are neglected; (5) second-order effects and lateral deflections due to column axial deformation are neglected; and (6) dynamic interaction between adjacent stories has little effect on story yield drift.

The MPA procedure as described above can be used to also compute internal forces in structural members that remain elastic, but not if they deform into the inelastic range. In the latter, case, the member forces are computed from the total member deformations—determined by Step 10 in the MPA procedure—using the member force-deformation (or moment rotation) relationship, recognizing *P-M* interaction in columns. These procedures to compute member forces are described in [24].

EVALUATION OF MPA

Higher Mode Contributions in Seismic Demands

Figures 9 and 10 show the median values of story drift and beam plastic rotation demands, respectively, including a variable number of "modes" in MPA superimposed with the "exact" result from nonlinear RHA. The first "mode" alone is inadequate in estimating story drifts, but with a few "modes" included, story drifts estimated by MPA are generally similar to the nonlinear RHA results.

The first "mode" alone fails to identify the plastic hinging in the upper floors of all buildings and also in the lower floors of the Seattle 20-story building. Including higher-"mode" contributions also improves significantly the estimate of plastic hinge rotations. In particular, plastic hinging in upper stories is now identified, and the MPA estimate of plastic rotation is much closer—compared to the first-"mode" result—to the "exact" results of nonlinear RHA.

Accuracy of MPA

For each of the six SAC buildings, Fig. 11 shows the median of the story drift ratios r_{MPA}^* for two cases: gravity loads (and P- Δ effects) excluded or included; median values of r_{RSA}^* from elastic analyses are also shown. The median value of r_{RSA}^* being less than one implies that the standard RSA procedure underestimates the median response of elastic systems. Because the approximation in the RSA procedure for elastic systems is entirely due to modal combination rules, the resulting bias serves as a baseline for evaluating additional approximations in MPA for inelastic systems. Although the profession tacitly accepts the modal combination approximation by using commercial software based on this approximation, perhaps such significant underestimation of response has not been recognized fully. The additional bias introduced by neglecting "modal" coupling in the MPA procedure is small to modest if P- Δ effects are neglected unless the building responds far into the inelastic range, as in the case of the Los Angeles 20-story building [25]. The first-"mode" pushover curves with and without P- Δ effects are presented in Fig. 12, also noted are the peak values of roof displacement due to 20 ground motions except those that caused collapse in the presence of P- Δ effects: one, three, and six excitations in the case of Seattle 9-story, and Los Angeles 9-story and 20-story buildings. P- Δ effects have little influence on the MPA bias for both Boston buildings because they remain essentially elastic (Fig. 12); however, they increase the bias slightly for Seattle buildings because they are deformed moderately into the inelastic range (Fig. 12); in addition, they increase significantly the bias for Los Angeles buildings, especially for the Los Angeles 20-story building because it is deformed into the region of rapid deterioration of lateral capacity (Fig. 12), leading to collapse of its first-"mode" SDF system during six excitations. Because beam plastic rotations are directly related to story drifts, the MPA procedure is similarly accurate in estimating both demand quantities [25].



Figure 9. Median story drifts determined by nonlinear RHA and MPA with variable number of "modes"; P-∆ effects due to gravity loads are included [25].



Figure 10. Median plastic rotations in interior beams determined by nonlinear RHA and MPA with variable number of "modes"; P-∆ effects due to gravity loads are included [25].



Figure 11. Median story drift ratios Δ^*_{MPA} for two cases: P- Δ effects due to gravity loads excluded or included and Δ^*_{RSA} for SAC buildings [25].



Figure 12. First-"mode" pushover curves for SAC buildings for two cases: P-∆ effects due to gravity loads excluded or included. Identified is the drift at onset of rapid deterioration of the lateral capacity and the peak values of roof displacement due to each excitation (except for those that caused collapse of the system) [25].



Figure 13. Median response ratios r_{MPA}^* for column axial forces, P_{MPA}^* , and story drifts, Δ_{MPA}^* [24].



Figure 14: Median response ratios r_{MPA}^* for column bending moments, M_{MPA}^* , and story drifts, Δ_{MPA}^* [24].

The MPA procedure estimates member forces to similar or better accuracy compared to story drifts. This is demonstrated in Figs. 13 and 14 where the median response ratios r_{MPA}^* for member forces and story drifts are compared. Such comparative results are presented for bending moments and axial forces in columns; similar results for bending moments and shear forces in beams and shear forces in columns are available elsewhere [24].

Overall Comments

Based on structural dynamics theory, the MPA procedure retains the conceptual simplicity and computational attractiveness of the standard pushover procedures with invariant lateral force distribution. Because higher-mode pushover analyses are similar to the first-mode analysis, MPA is conceptually no more difficult than procedures now standard in structural engineering practice. Because pushover analyses for the first two or three modal force distributions are typically sufficient in MPA, it requires computational effort that is comparable to the FEMA-356 procedure, which requires pushover analysis for at least two force distributions.

Without additional conceptual complexity or computational effort, MPA estimates seismic demands much more accurately than FEMA-356 procedures, as demonstrated by a comparison of Figs. 4 and 9; however, MPA is an approximate method that cannot be expected to always provide seismic demand estimates close to the "exact" results from nonlinear RHA. The total bias in the MPA estimate of seismic demands (including P- Δ effects) for Boston and Seattle buildings is about the same as the largest errors observed in the RSA procedure—which are tacitly accepted by the profession by using commercial software based on RSA. While MPA is sufficiently accurate to be useful in seismic evaluation of many buildings for many ground motions—and much more accurate than FEMA-356 procedures—its errors may be unacceptably large for buildings that are deformed far into the region of negative post-yield stiffness, with significant deterioration in lateral capacity, e.g., Los Angeles 20-story building subjected to the SAC 2/50 ensemble of ground motions. For such cases, MPA and most other pushover procedures cannot be expected to provide accurate estimates of seismic demands and they should be abandoned in favor of nonlinear RHA. To establish the range of applicability of MPA, its bias and dispersion have been documented for 60 height-wise regular generic frames and 48 irregular frames, in addition to the six SAC buildings presented above [26, 27].

The computational effort in MPA can be further reduced by simplifying computation of the demands associated with higher vibration modes by assuming the building to be linearly elastic [28]. Such a modified MPA leads to a larger estimate of seismic demand, thus reducing the unconservatism of MPA results (relative to nonlinear RHA) in some cases and increasing their conservatism in others. While this increase in demand is modest and acceptable for systems with moderate damping, at least 5%, it is unacceptably large for lightly damped systems.

In practical application of MPA, the roof displacement for each modal pushover analyses can be estimated from the elastic spectrum defining the seismic hazard multiplied by the inelastic deformation ratio. These topics are the subject of the remainder of this paper.

SDF-SYSTEM ESTIMATE OF ROOF DISPLACEMENT

As mentioned earlier, the target value of roof displacement is determined in current NSP from the earthquake-induced deformation of an inelastic SDF system. To examine the accuracy of this procedure, we compare this SDF-system estimate of roof displacement $(u_r)_{SDF}$ —given by u_{r1} for the first mode in Step 6 of the MPA summary—with the "exact" value $(u_r)_{MDF}$ determined by nonlinear RHA of the multistory building treated as an MDF system. Calculating u_{r1} requires D_1 , which is determined by

nonlinear RHA of the SDF system, thus avoiding any of the approximations underlying the simplified methods for estimating its deformation [1, 2, 3]. The response of each SAC building to each of 20 SAC ground motions is computed and the displacement ratio is determined: $(u_r^*)_{SDF} = (u_r)_{SDF} \div (u_r)_{MDF}$. The difference between the median of this displacement ratio and unity indicates the bias in the SDF-system estimate of roof displacement.

Although our principal interest is inelastic seismic demands, for better understanding we first examine this displacement ratio for elastic systems. Under this assumption, Fig. 15 presents histograms of the 20 values of the displacement ratio together with its range of values and median value for each of the six SAC buildings. The median ratio is always less than 1.0, indicating that the SDF-system estimate is biased toward underestimating the median roof displacement. This underestimation, caused by neglecting higher mode contributions is not negligible—it varies between 9 and 26%. The SDF system underestimates the roof displacement of 9-story buildings due to 19, 18, and 17 of the 20 ground motions for Boston, Seattle, and Los Angeles locations, respectively; for 20-story buildings it is underestimated by all excitations except one for the Boston structure. Surprisingly, the displacement due to individual excitations is underestimated by as much as 40% to 50% for these buildings.

To investigate this large underestimation of roof displacement, the response history of modal contributions and of the combined value of roof displacement for the Los Angeles 9-story building due to two of the 20 SAC ground motions is presented in Fig. 16. Consistent with the prevailing view, the first mode is strongly dominant in the building response to one of these excitations (Fig. 16a) and the SDF-system estimate of roof displacement is essentially exact (192 cm versus 191 cm). For another excitation, however, the SDF-system estimate (48.6 cm) is 40% less than the "exact" value (80.8 cm) because the second mode response is too large to be ignored (Fig. 16b).

Returning to inelastic buildings, Fig 17 shows histograms of the 20-values of the displacement ratio together with its range of values and median value for each of the six SAC buildings³. The SDF-system underestimates the median roof displacement by 14% and 18% for the Boston 9- and 20-story buildings, respectively, underestimates by about 5% for Seattle buildings, and overestimates by 19% for Los Angeles buildings. The range of values for the displacement ratio is now much wider, implying that the SDF-system estimate of roof displacement due to individual ground motions may be much worse for inelastic systems. This estimate can be alarmingly small (as low as 31% to 82% of the exact value for the six buildings) or surprisingly large (as large as 145% to 215% of the exact value for Seattle and Los Angeles buildings). The errors are actually worse than indicated by Fig.17 because it does not include those cases where nonlinear RHA predicted collapse of the first-"mode" SDF system but not of the building as a whole³. This large discrepancy arises because for individual ground motions the SDF system may significantly underestimate or overestimate the yielding-induced permanent drift in the response of the building [29].

The preceding results lead to two principal conclusions. First, the first-"mode" inelastic SDF system provides a biased estimate of the roof displacement of a building. A correction factor to overcome this bias should be developed based on available research data [e.g., Ref. 29]. Second, an SDF-system should not be used to estimate the roof displacement due to an individual ground motion; however, with a bias correction factor it is a reasonable approach if the seismic hazard is defined by a smooth spectrum or an ensemble of ground motions.

³ Data for excitations that caused collapse of the SDF system are excluded, reducing the number of data to 19 for the Seattle 9-story building, 17 for the Los Angeles 9-story building, and 14 for the Los Angeles 20-story building; the median values for these buildings are computed by the counting method.



Figure 15. Histograms of ratio $(u_r^*)_{SDF}$ for SAC buildings analyzed as elastic systems; range of values and median value of this ratio are noted [29].



Figure 16. Modal contributions to roof displacement of SAC-Los Angles 9-Story building analyzed as an elastic system to two SAC ground motions: (a) Record No. 38; (b) Record No. 31; RSA estimate of roof displacement is also noted [29].



Figure 17. Histograms of ratio $(u_r^*)_{SDF}$ for SAC buildings including P- Δ effects due to gravity loads; range of values and median value of this ratio are noted [29].

DEFORMATION OF INELASTIC SDF SYSTEMS: CURRENT PRACTICE

As mentioned earlier, seismic demands are estimated in current engineering practice by pushover analysis of the building up to a target roof displacement, estimated from the deformation D of an inelastic SDF system. The methods described in ATC-40 and FEMA-356 guidelines are commonly used to determine D.

ATC-40 Method

The deformation of an inelastic SDF system is estimated by the capacity-spectrum method, an iterative method requiring analysis of a sequence of equivalent linear system; the method is typically implemented graphically. Unfortunately, the ATC-40 iterative procedure does not always converge; when it does converge it does not lead to the exact deformation. Because convergence traditionally implies accuracy, the user could be left with the impression that the calculated deformation is accurate, but the ATC-40 estimate errs considerably. This is demonstrated in Fig.18a, where the deformation estimated by the ATC-40 method is compared with the value determined from inelastic design spectrum theory and the well-established equations relating the peak deformation u_m of an inelastic SDF system to the peak deformation u_o of the corresponding linear system:



Figure 18. Deformations computed by ATC-40 and from inelastic design spectrum using three different $R_y - \mu - T_n$ equations: (i) Newmark-Hall (NH) [30]; (ii) Krawinkler and Nassar (KN) [32]; and (iii) Vidic et al. (VFF) [31]; part (a) compares deformations and part (b) shows discrepancy in ATC-40 method.

$$u_m = \frac{\mu}{R_y} u_o \qquad u_o = \left(\frac{T_n}{2\pi}\right)^2 A \tag{5}$$

where T_n is the initial elastic period of the system, R_y its yield-strength reduction factor, μ the ductility factor, and A the pseudo-acceleration ordinate of the elastic design spectrum. Presented in Fig. 18a are the deformations determined by using three different $R_y - \mu - T_n$ equations [30, 31, 32]). Both the approximate and theoretical results are presented for systems covering a wide range of period values and ductility factors subjected to ground motions characterized by an elastic design spectrum. The discrepancy in the approximate result presented in Fig. 18b shows that the ATC-40 method underestimates by 40-50% the deformation over a wide range of periods [33].

The two flaws in the ATC-40 capacity spectrum method—lack of convergence in some cases and large errors in many cases—appear to have been rectified in recent research [34]. The ATC-55 project, now nearing completion, has also led to improved procedures for equivalent linearization of inelastic systems. Both of these investigations derive the optimal vibration period and damping ratio parameters for the equivalent linear system by minimizing the differences between its response and that of the actual inelastic system. Such an equivalent linear method would obviously give essentially the correct deformation. However, the benefit in making the equivalent linearization detour is unclear when the deformation of an inelastic system can be readily determined using available equations for the inelastic deformation ratio (e.g., Refs. 35 and 36] or by using the inelastic design spectrum [e.g., 23 (Chapter 7), 37, 38].

The attractive graphical feature of the ATC-40 capacity spectrum method can be retained without the equivalent linearization detour. This is achieved in the capacity-demand-diagram method by using the well-known inelastic design spectrum to define the demand [39]. When both capacity and demand curves are plotted in the pseudo acceleration-deformation format, the yielding branch of the capacity diagram intersects the demand curves for several ductility factor values. The deformation is given by the one

intersection point where the ductility factor calculated from the capacity diagram matches the value associated with the intersecting demand curve. This deformation is identical to the value determined by Eq. (5).

FEMA-356 Method

The deformation of an inelastic SDF system is estimated by

$$D = C_1 C_2 C_3 \left(\frac{T_n}{2\pi}\right)^2 A \tag{6}$$

Multiplying the deformation of the elastic system are three coefficients, C_1 , C_2 , and C_3 . The coefficient C_1 represents the inelastic deformation ratio, u_m/u_o , for inelastic systems without pinching, stiffness degradation, or strength deterioration of their hysteresis loop. Coefficient C_2 accounts for the increase in deformation of the inelastic system due to these effects not considered in C_1 , and C_3 accounts for P- Δ effects.

Equations and numerical values for these coefficients specified in FEMA-356 guidelines are based, on research results and on judgment. However, some of the numerical values are not supported by research results; e.g., C_1 is limited to 1.5, which is much smaller than the inelastic deformation ratio (determined from dynamic response analyses) for systems in the acceleration-sensitive region of the spectrum; however, the value of $C_1 = 1.0$ at longer periods is theoretically correct. As part of the ATC-55 project, now nearing completion, coefficients C_1 , C_2 , and C_3 were investigated comprehensively and improved specifications were developed for these coefficients.

DEFORMATION OF INELASTIC SDF SYSTEMS: IMPROVED METHODS

The inelastic deformation ratio, $C_{\mu} = u_m/u_o$, if expressed as a function of elastic vibration period T_n and ductility factor μ , can be used to determine the inelastic deformation of a new or rehabilitated structure where global ductility capacity can be estimated. The inelastic deformation ratio, $C_R = u_m/u_o$, if expressed as a function of T_n and yield-strength reduction factor R_y , can be used to determine the deformation of an existing structure with known lateral strength.

Figures 19(a) and 19(b) present the median values of C_{μ} and C_R , respectively, as a function of T_n , for elastoplastic systems subjected to the LMSR⁴ ensemble of 20 ground motions; the spectral regions are noted in the plots. In the acceleration-sensitive region C_{μ} and $C_R \approx 1$ at $T_n = T_c$ but they exceed unity increasingly for shorter periods and larger μ or R_y , indicating greater inelastic action. For these shortperiod systems, the C_{μ} and C_R are very sensitive to the yield strength, increasing as the yield strength is reduced. Both C_{μ} and C_R for very short-period systems ($T_n < T_a$), even if their strength is only slightly smaller than the minimum strength required for the structure to remain elastic (e.g., $R_v = 1.5$) are much

⁴ Seven ensembles of far-fault ground motions, each with 20 records, are mentioned. The first group of ensembles, denoted by LMSR, LMLR, SMSR, and SMLR represent four combinations of large (M=6.6-6.9) or small (M=5.8-6.5) magnitude and small (R=13-30 km) or large (R=30-60 km) distance.



Figure 19. Median inelastic deformation ratios C_{μ} and C_R for elastoplastic systems subjected to the LMSR ensemble of ground motions [36].

larger than unity. In the velocity sensitive region, C_{μ} and $C_R \approx 1$ and are essentially independent of the ductility factor or yield strength. In the displacement-sensitive region, C_{μ} and $C_R < 1$ for systems in the period range T_d to T_f , where these ratios decrease as the ductility factor is increased or strength is reduced; however, for systems with periods longer than T_f , C_{μ} and $C_R \approx 1$ are essentially independent of ductility factor or strength, and both C_{μ} and $C_R = 1$ for very long-period systems, independent of μ or R_y . Results such as these are the basis for the widely used (e.g., FEMA-356 method) equal deformation rule, i.e., $u_m = u_o$, which is reasonable for systems in the velocity- and displacement-sensitive regions of the spectrum, but not for the acceleration-sensitive region. However, the limiting value of 1.5 for the coefficient C_1 in FEMA-356 is not supported by Fig. 19 for systems with T_n in the acceleration sensitive region of the spectrum.

What is the influence of earthquake magnitude and distance on the inelastic deformation ratio? To answer this question, the median C_{μ} is plotted against T_n in Fig. 20a for the LMSR, LMLR, SMSR, and SMLR ground motion ensembles⁴. These results indicate that the inelastic deformation ratio is essentially independent of earthquake magnitude and distance; however, it is different for near-fault ground motions as will be shown later.

What is the influence of soil conditions at the recording sites? To answer this question, the median inelastic deformation ratio is presented in Fig. 20b for three ensembles of ground motions recorded on firm sites: NEHRP site classes B, C, and D⁵, all of which are firm soil sites. The median C_{μ} versus T_n curves (Fig. 20b) for the three site classes are very similar; to each other and to the LMSR result. Thus, the inelastic deformation ratio is essentially independent of local soil conditions so long as they are firm soil sites, but it may be affected by soft soil conditions.

⁵ The second group of three ensembles is categorized by NEHRP site classes B, C, or D. These ground motions were recorded during earthquakes with magnitudes ranging from 6.0 to 7.4 at distances ranging from 11 to 118 km.



Figure 20. Comparison of C_{μ} for elastoplastic systems with $\mu = 4$ subjected to (a) LMSR, LMLR, SMSR, and SMLR ground motion ensembles; and (b) site class B, C, and D ensembles [36].



Figure 21. Comparison of C_{μ} for far-fault (LMSR) and near-fault ground motion ensembles plotted versus (a) elastic vibration period T_n and (b) normalized period T_n/T_c ; both plots are for elastoplastic systems with $\mu = 4$ [36].

The median inelastic deformation ratios C_{μ} and C_R for the fault-normal (FN) and fault-parallel (FP) components of near-fault (NF) ground motions differ significantly from those for far-fault (FF) motions⁶ (Figs. 21a and 22a). This systematic difference between the values of C_{μ} (and C_R) for NF and FF ground motions, especially in the acceleration-sensitive region of the spectrum, is primarily due to the difference between the values of T_c for the two sets of excitations; T_c is the period separating the

⁶ Six ensembles of NF ground motions are mentioned in this paper. The first two ensembles of NF ground motions, denoted by NF-FN and NF-FF, are the two horizontal components (FN and FP) of 15 NF ground motions, recorded during earthquakes of magnitudes ranging from 6.2 to 6.9 at distances ranging from 0 to 9 km. These ground motions were all recorded on firm soil (NEHRP site class D) or rock; the rock motions have been modified by P. Somerville for soil conditions. The next two ensembles, denoted by NF-FN (soil33) and NF-FP (soil 33) are the FN and FP components of 33 motions recorded on soil during earthquakes of magnitudes 6.0 to 7.6 at distances of 0.2 to 16.4 km. The last two ensembles, denoted by NF-FN (rock12) and NF-FP (rock12), are the FN and FP components of 12 motions recorded on rock during earthquakes of magnitudes 5.6 to 7.4 at distances of 0.1 to 14 km.



Figure 22: Comparison of C_R for far-fault (LMSR) and near-fault ground motion ensembles plotted versus (a) elastic vibration period T_n and (b) normalized period T_n/T_c ; both plots are for elastoplastic systems with $R_v = 4$ [36].

acceleration- and velocity-sensitive regions of the median response spectrum. This assertion is demonstrated by plotting the ensemble median of individual ground motion data for C_{μ} (and C_R) expressed as a function of the normalized vibration period T_n/T_c ; T_c varies with the excitation (Figs. 21b and 22b). Now the inelastic deformation ratio plots for FF ground motions and both—FN and FP—components of NF ground motions have become very similar in all spectral regions.

Estimating Deformation of Inelastic Systems

Simplified equations for inelastic deformation ratios C_R and C_{μ} would obviously facilitate estimation of the deformation of an inelastic SDF system because the deformation of the corresponding linear system is readily known from the elastic design spectrum. Developing such equations, a problem first studied by Veletsos and Newmark [40], has been the subject of many publications; the more recent ones are found in Refs. 35 and 36.

Presented next is an equation that fits the median C_R data for any ensemble of ground motions (but ignores the data showing $C_R < 1$ over the period range T_d to T_f) and satisfies the limiting values of $C_R = L_R$ at $T_n = 0$ and $C_R = 1$ at $T_n = \infty$, where

$$L_R = \frac{1}{R_y} \left(1 + \frac{R_y - 1}{\alpha} \right) \tag{7}$$

and α is the post-yield to initial stiffness ratio of bilinear systems. Such a function for C_R has been derived [36] in terms of the yield-strength reduction factor R_v and the normalized period T_n/T_c :

$$C_{R} = 1 + \left[\left(L_{R} - 1 \right)^{-1} + \left(\frac{a}{R_{y}^{b}} + c \right) \left(\frac{T_{n}}{T_{c}} \right)^{d} \right]^{-1}$$
(8)

Nonlinear regression analysis of the data for four (LMSR, LMLR, SMSR, and SMLR) ensembles of farfault (FF) ground motions led to a = 61, b = 2.4, c = 1.5, and d = 2.4.

As expected, Eq. (8) using these values for *a*, *b*, *c*, and *d* provides a good, generally modestly conservative estimate of the median C_R for LMSR, LMLR, SMSR, and SMLR ground motion ensembles (Fig. 23). Interestingly, the same equation and parameters also provide a good fit to the C_R data for NEHRP site class B, C, and D ensembles (Fig. 24). Most impressive is the fact that the same equation and parameters provide a satisfactory fit to the data for ensembles of near fault (NF) motions recorded on soil (Fig. 25) and on rock (Fig. 26). Thus, Eq. (8) and these parameter values are applicable for a wide range of conditions, except for soft soil sites, e.g., Mexico City lake bed and San Francisco Bay margins.



Figure 23. Comparison of C_R estimated by proposed equation with computed data for LMSR, LMLR, SMSR, and SMLR far-fault ground motion ensembles for elastoplastic ($\alpha = 0$) and bilinear ($\alpha = 10\%$) systems; and $R_v = 6$ [36].



Figure 24. Comparison of C_R estimated by proposed equation with computed data for far-fault ground motions recorded on site classes B, C, and D for elastoplastic ($\alpha = 0$) and bilinear ($\alpha = 10\%$) systems; and $R_v = 6$ [36].



Figure 25. Comparison of C_R estimated by proposed equation with computed data for near-fault ground motion ensembles: NF-FN (soil33) and NF-FP (soil33) recorded on soil [36].



Figure 26. Comparison of C_R estimated by proposed equation with computed data for near-fault ground motion ensembles: NF-FN (rock12) and NF-FP (rock12) recorded on rock [36].

Presented finally is an improved equation that fits the median C_{μ} data for any ensemble of ground motions (but ignores the data showing $C_{\mu} < 1$ over the period range T_d to T_f) and satisfies the limiting values of $C_{\mu} = L_{\mu}$ at $T_n = 0$ and $C_{\mu} = 1$ at $T_n = \infty$, where

$$L_{\mu} = \frac{\mu}{1 + (\mu - 1)\alpha} \tag{9}$$

Such a function for C_{μ} has been derived [36] in terms of ductility factor μ and the normalized period T_n/T_c :

$$C_{\mu} = 1 + \left[\left(L_{\mu} - 1 \right)^{-1} + \left(\frac{a}{\mu^{b}} + c \right) \left(\frac{T_{n}}{T_{c}} \right)^{d} \right]^{-1}$$
(10)

Observe that the form of this equation is the same as Eq. (8) for C_R where L_R is replaced by L_{μ} and R_y by μ . Nonlinear regression analysis of the data for four (LMSR, LMLR, SMSR, and SMLR) FF ground motion ensembles led to a = 105, b = 2.3, c = 1.9, d = 1.7.

Equation (10) with these parameter values provides a satisfactory estimate of the median C_{μ} for LMSR, LMLR, SMSR, and SMLR ground motion ensembles, NEHRP site class B, C, and D ensembles, and NF ensembles on soil and on rock; plots similar to Figs. 23-26 are available in Chopra and Chintanapakdee [36]. Thus, Eq. (10) and these parameter values are generally applicable for a wide range of conditions, except for soft soil sites. A more complicated equation would be necessary to match the $C_{\mu} < 1$ data in the T_d to T_f period range; however, these data should be re-examined with P- Δ effects included before developing such an equation.

CLOSURE

The profession has come a long way in estimating seismic demands for buildings by abandoning traditional elastic analysis of the structure for reduced seismic forces and developing instead procedures that explicitly consider inelastic behavior of the structure. However, these more recent methods, now standard in structural engineering practice to estimate seismic demands should be improved. Towards this goal, much research has been accomplished worldwide in the past five years, and several different approaches are in various stages of development. This paper has emphasized one possible approach that provides considerably improved estimate of demands, while retaining the conceptual simplicity and computational attractiveness of the procedures currently popular in professional practice. Based on modal pushover analysis (MPA) and improved methods to estimate the target roof displacement, this approach is ready for practical application to symmetric-plan buildings.

The MPA procedure for estimating seismic demands has been extended to unsymmetric-plan buildings and, based on a preliminary evaluation, appears to be promising [41]. It remains to be evaluated further considering a range of buildings subjected to ensembles of ground motions.

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