13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 504

APPROXIMATE METHODS OF ACCOUNTING FOR BEAM GROWTH EFFECTS

Jubum Kim[1], John F. Stanton[2], Gregory A. MacRae[3]

SUMMARY

In reinforced concrete frames subjected to seismic loading, flexural yielding and cracking occur at beam ends. As the cracks form, the horizontal distance between column centerlines increases and this "beam growth" leads to an increase in member demands. This paper describes a method of analysis that accounts approximately for the effects of beam growth. The total displacement was assumed to consist of two components: one due to sway, and the other due to beam growth. The method predicted the column displacements well. The column moments obtained from the approximate method were also comparable to the results of an independent, nonlinear FEM analysis.

INTRODUCTION

In seismic regions of the US, reinforced concrete moment frames are often used to resist the lateral loads on buildings. Such frames are usually analyzed using models in which the beam is represented by a flexural line element. The computed axial forces in these beams are often low, while their axial stiffnesses are high, so the change in beam length is negligible. However, in a real frame, the beams start to crack as the frame moves laterally, causing the distance between column centerlines to increase as shown in Fig. 1. There, flexural cracking is modeled as being concentrated at the beam ends and the distance between column centerlines increases by the sum of the crack widths at mid-height of the beam. This phenomenon is known as "beam growth" or "beam elongation", and verified experimentally (e.g. Zerbe and Durrani [1,2], Sakata and Wada [3], Fenwick and Davidson [4]). Greater beam growth occurs with greater beam depths and story drift ratios. In a frame subject to large lateral displacements, beam growth effects push the exterior columns outwards, causing an increase in column curvature demand on one side of the frame, but a reduction on the other, as shown in Fig. 2.

This paper describes a method of analysis that accounts approximately for the effects of beam growth in a reinforced concrete frame, if the response of the same frame, without beam growth, is already known. The method can be programmed in a matrix analysis language such as MATLAB. The advantage of such an approach is that it allows the response of the frame, including beam growth, to be evaluated approximately without having to resort to use of a nonlinear model that uses a large number of contact elements.

---

[1] Assistant Professor, Dept. of Civil and Env. Eng., Clarkson Univ., Potsdam, NY 13699, USA.
[2] Professor, Dept. of Civil and Env. Eng., U. of Washington, Seattle, WA 98195-2700, USA.
[3] Associate Professor, Dept. of Civil and Env. Eng., U. of Washington, Seattle, WA 98195-2700, USA.

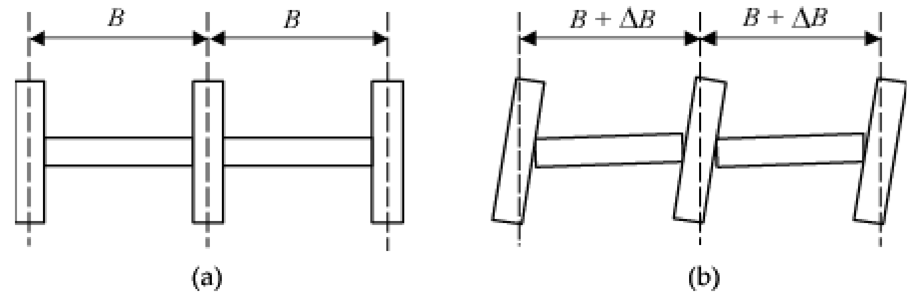

Figure 1. Effects of beam growth; (a) initial condition (b) idealized displaced shape

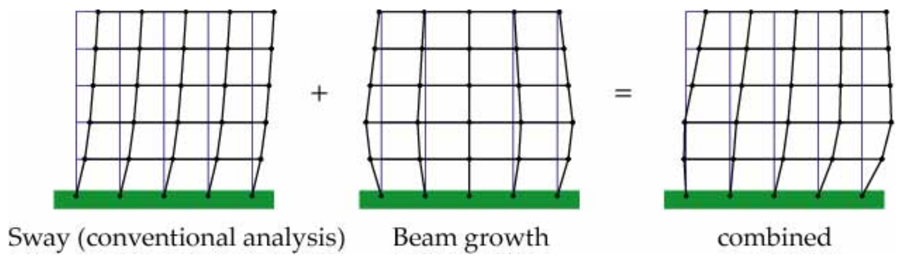

Figure 2. Frame sway including beam growth

BACKGROUND

The authors (Kim et al. [5], Kim [6]) proposed an analytical FEM model that takes into account of beam growth. In the RFIBG (*R*einforced concrete *F*rame *I*ncluding *B*eam *G*rowth) model, the opening at each beam-column interface represents the opening from all of the nearby cracks in the beam. A second, conventional model was also developed to provide comparable response but without the beam growth. It was called the RFEBG (*R*einforced concrete *F*rame *E*xcluding *B*eam *G*rowth) model. Details of the models and their verification can be found in Kim et al. [5]. A series of prototype frames were designed and analyzed in the study and it was found that the responses differed between the models that included (RFIBG) and ignored (RFEBG) beam growth effects. The frame using the RFIBG model was slightly stiffer and stronger than that using the RFEBG model and they also had different distributions of moment and shear among individual members. In particular, the moments and shears at the outermost columns in the RFIBG model were significantly higher than those in the RFEBG model. Also it was noted that beam growth increases with greater beam depth, number of bays in the frame and story drift ratio. It is larger at the lower floors, where the story drift ratios are larger. It becomes important after the drift reaches some threshold value, which was a roof drift ratio of approximately 0.5%.

The RFIBG model represented the behavior of the frames including beam growth, but it contained a large number of contact elements (9 contact elements per beam-column interface in [5]). It required considerable development effort and was complex enough that it cannot be considered suitable for use in a design office today. There is thus a need for a simpler method, approximate if necessary, that allows prediction of response including the effects of beam growth.

Such an approximate method is developed and discussed in this paper. It is based on the concept that the total displacement of the frame consists of two components, the effects of which can be superimposed. One is due to sway, without beam growth, and the other consists of a lateral expansion of the frame, due purely to beam growth, as seen in Fig. 2.

DEVELOPMENT OF ANALYTICAL PROCEDURE

Frames Designed

A suite of 15 five-story buildings was designed and analyzed in order to calibrate and verify the analytical procedures presented here. The number of bays was 2, 4, 6, 8, or 10 and beam depths were 914 mm, 1219 mm and 1524 mm (36", 48" and 60") as summarized in Table 1. Each frame was named according to the number of stories and bays, and the beam depth. For example, the reference frame, RF0504-48, was a 5-story, 4-bay frame, with 48" (1219 mm) deep beams over the height. The bay width and story height were assumed to be the same for all the frames.

Table 1. Naming Convention of Reinforced Concrete Frames Analyzed

Number of bays	2	4	6	8	10
h_b = 36" (ξ = 0.136)	RF0502-36	RF0504-36	RF0506-36	RF0508-36	RF0510-36
h_b = 48" (ξ = 0.182)	RF0502-48	RF0504-48	RF0506-48	RF0508-48	RF0510-48
h_b = 60" (ξ = 0.227)	RF0502-60	RF0504-60	RF0506-60	RF0508-60	RF0510-60

Overview

The growth of a beam along its centerline is given by the sum of the widths of all the cracks that cross that centerline. The cracks are for convenience lumped together as a single crack at the end of the beam. Thus the beams are treated as undergoing uncracked elastic bending in the body of the member, plus a concentrated rotation at the end. That end rotation is accompanied by beam elongation, because the center of rotation of the cross-section is not at the mid-height of the beam. The gap at the end of the beam is thus a function of the relative rotation between the beam and column. The conceptual procedure for predicting the frame response consists of the following steps:

1. Assume the displaced shape of the central column. (This is the "sway" component of the total displaced shape).
2. Find the displaced shapes of all the columns in the frame as follows:
   2.1. Assume, temporarily, that all columns have the same displaced shape as the central one.
   2.2. Knowing the column rotations at each floor level from the displaced shapes of the columns, compute the beam elongations due to the gaps that open between the beams and columns.
   2.3. Re-compute the displaced shapes of the outer columns, adding the assumed displaced shape due to sway of the central column and the displacement due to beam growth.
   2.4. Iterate on steps 2.2-2.3 until the displaced shapes converge.
3. Compute member forces from the displaced shapes.

In developing the analytical method, several modeling assumptions were made:

- At design drift level, the column remains elastic, except at the base where yielding has taken place.
- The column joint rotations consist of two parts: one caused by the lateral displacements of the columns, which can be obtained by geometry alone, and the other caused by the beam moments.
- The center of rotation, or neutral axis, at the beam end changes as the load increases. This, in turn, affects the magnitude of the gap opening at the beam-column interface. The center of rotation at design drift level was approximated here by an empirical approach.
- The displaced shape at the central column in a frame including beam growth effects (RFIBG) must be known in the procedure. It was derived from the displaced shape of the frame from conventional analysis (RFEBG), using an empirical modification procedure.

- The RFIBG model proved stiffer and stronger than the RFEBG model. Relationships between them therefore had to be derived either at the same base shear or at the same drift ratio. Drift ratio was used here, because of the geometric nature of the beam growth effects.

Step 1: Displaced Shape of Central Column

The procedure requires that the displaced shape of one column be known. Figure 3 shows the ratio of the displacement at the first story in the RFIBG model, Δ_{RFIBG}, divided by the displacement at the same story in the RFEBG model, Δ_{RFEBG}, when the roof drift ratio of the central column was 2%. The ratio becomes smaller for buildings with more bays or with deeper beams, because the accumulation of beam growth across the building is larger in them and thus increases the stiffness of the frame. Equation (1) was obtained by curve fitting and gives the approximate ratio between first story displacements.

$$\frac{\Delta_{RFIBG}}{\Delta_{RFEBG}} = 1.2 - \xi(\frac{n_b}{10} + 1.15) \qquad (< 1.0) \tag{1}$$

where ξ is the clear span-to-depth ratio and n_b is the number of bays. The displacement ratios at the first story using Eq. (1) are shown as solid lines in Fig. 3. The relationship between Δ_{RFIBG} and Δ_{RFEBG} at other stories was then approximated by assuming that the ratio varied linearly between the first floor and top floor, where the displacements are equal.

Step 2: Displaced Shape of Frame

After the deformed shape of the central column is assumed, the deformations of the other columns must be found. The analytical procedure is explained here in terms of a one bay frame. If the shape of the left column is known, the procedure can be used to find the shape of the right column. In a multi-bay frame the procedure can be used successively to find the shape of each column, starting at the center column.

The joint rotation of the column caused by its lateral displacement can be found by assuming that the beams are pin-connected to the column and that the elastic column is pinned at the base. The joint rotation of the column is then:

$$[\theta] = [A][\Delta] \tag{2}$$

where $[\theta]$ and $[\Delta]$ are matrices containing the nodal rotations and displacements in a column. $[A]$ is kinematic matrix which can be derived purely by geometric information. The size of both $[\theta] = [\theta_L\ \theta_R]$ and $[\Delta] = [\Delta_L\ \Delta_R]$, are $n_f \times 2$, where n_f is the number of floors. The subscripts L and R indicate the left and right columns. Matrix $[A]$ has dimensions $n_f \times n_f$.

The additional column rotation induced by the beam moments, which can be treated as external

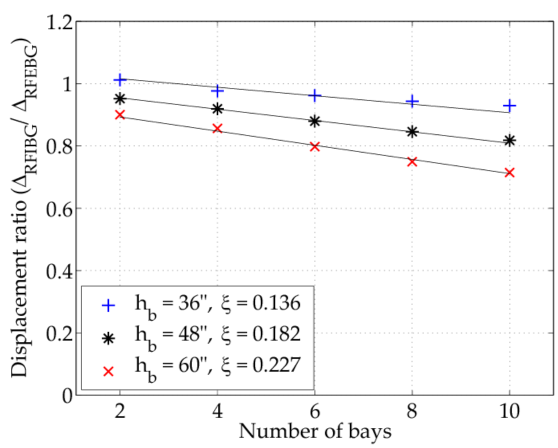

Figure 3. Ratios of first story displacements at central column between RFIBG and RFEBG models at 2% roof drift ratio

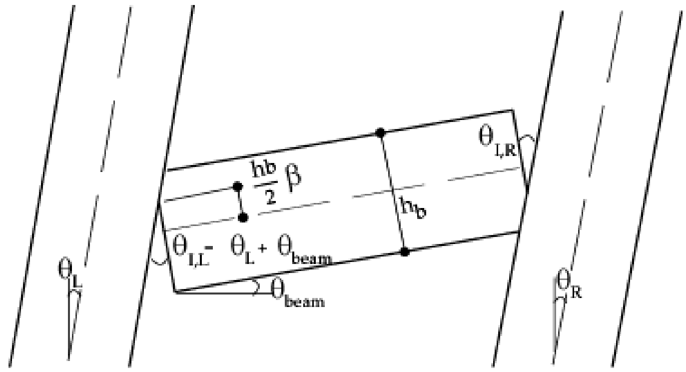

Figure 4. Rotation at beam-column interface

moments applied to the column joints, can be added to Eq. (2). If this additional rotation is denoted as θ', Eq. (2) becomes:

$$[\theta]=[A][\Delta]+[\theta'] \tag{3}$$

Once the rotations of the two columns, $[\theta]$, are obtained, the lift-off angles between columns and beams, $[\theta_I]$, in Fig. 4 can be derived as follows:

$$[\theta_I]=[\theta][D] \tag{4}$$

where the transformation matrix, $[D]$, is used to take into account the beam rotations. The matrix $[D]$ takes the following form:

$$[D]=\begin{bmatrix} 1+\dfrac{h_c}{2L'} & \dfrac{h_c}{2L'} \\ \dfrac{h_c}{2L'} & 1+\dfrac{h_c}{2L'} \end{bmatrix} \tag{5}$$

where $L' = L_b - h_c$. L_b is the center-to-center column distance and h_c is the column width.

Equation (4) implies that the gaps at the beam-column interfaces open up as soon as the load is applied. The beam growth, however, starts only after inelastic action at the interface has occurred. The lift-off is therefore assumed to start only after the column rotation reaches a threshold value, denoted here as θ_o. Equation (4) can be then modified as:

$$[\theta_I]=[\theta^*][D], \tag{6}$$

where $\theta^*_{ij}=\left|\theta_{ij}\right|-\theta_o$, if $\left|\theta_{ij}\right|>\theta_o$; otherwise 0.

The absolute value is used to ensure that the lift-off angles at the interface and the beam growth are always positive.

The beam growth in the one-bay frame can be obtained by finding the gap opening associated with the lift-off angle at the beam-column interface. If the beam is assumed to rotate with respect to the top (or bottom) of the beam ends, the opening at mid-height of the beam ends can be expressed as the product of rotation angle at the interface, and half of the beam depth, h_b. The total beam growth, $\{\delta\}$, on the one-bay frame is then:

$$\{\delta\}=\frac{h_b}{2}[\theta_I]\{e\} \tag{7}$$

where $\{e\} = \{1\ 1\}^T$. Equation (7) can be rearranged by substituting Eq. (6) into it:

$$\{\delta\}=\frac{h_b}{2}[\theta^*][D]\{e\} \tag{8}$$

As shown in Fig. 4, however, the center of rotation lies between the center and the outer face of the beam section, not at the top. It is taken to be a height $\beta h_b/2$ from the mid-depth of the cross section, where β is non-dimensional variable ranging from 0 to 1. If different β values are used at each level, but the same value is used at each column at a single level, they form a diagonal matrix $[\beta]$ for a frame. The beam growth in Eq. (8) can be then modified as:

$$\{\delta\} = \frac{h_b}{2}[\beta][\theta^*][D]\{e\} \quad (9)$$

Here the centers of rotation at the second floor up to the top floor are assumed to be the same. The matrix $[\beta]$ then has two constants, one for the first story (β_1), and the other for the second floor (β_2). This simplification can be justified since the effect of beam growth is important at the first two stories, where the magnitude of column moment is greater than that at upper stories. The constants, β_1 and β_2, can be obtained using an empirical relationship from a series of analyses as shown in Fig. 5. Note that, due to the absolute value sign and the presence of θ_o in Eq. (6), the matrix algebra is not linear, so the displaced shape of the frame may need to be obtained iteratively. The procedure can be summarized below:

Step 2-1: Assume a starting value for the beam growth, $\{\delta\}$.

Step 2-2: Find $[\Delta]$. The displaced shape of the central column is assumed using Eq. (1). For columns to the right of the central column, use Eq. (10) successively. For columns to the left of the central column, use Eq. (11).

$$[\Delta] = \{\Delta_L\}\{1 \quad 1\} + \{\delta\}\{0 \quad 1\} \quad (10)$$

$$[\Delta] = \{\Delta_R\}\{1 \quad 1\} - \{\delta\}\{0 \quad 1\} \quad (11)$$

Step 2-3: Find $[\theta]$ and $[\theta^*]$ from Eqs. (3) and (6).

Step 2-4: Find $\{\delta\}$ from Eq. (9).

Step 2-5: Compare $\{\delta\}$ from step 2-1 and step 2-4. Iterate until they converge.

VALIDATION OF MODEL

The validity of the model was examined using the results from the FEM analysis of the five-story, four-bay frame with 48" deep beams (RF0504-48). The deformed shape of the frame is shown in Fig. 6, where the displacements from both the analytical (RFIBG) model and the approximate method are shown in a bracket and a square bracket, respectively. The lateral displacements vary across the building as a result of beam growth. The results from the approximate method agree well with the analytical results.

The column moments at 2% roof drift ratio based on the RFEBG and RFIBG model are shown in Fig. 7. The moments based on the approximate method are also shown in the figure. The approximate method, though not perfect, simulated the frame behavior much better than the model ignoring beam growth altogether (RFEBG). The column moment was largest at the east column, especially at the lower stories both in Fig. 7(b) and Fig. 7(c). The top moment at the first story of the east column was 5,026 k-ft

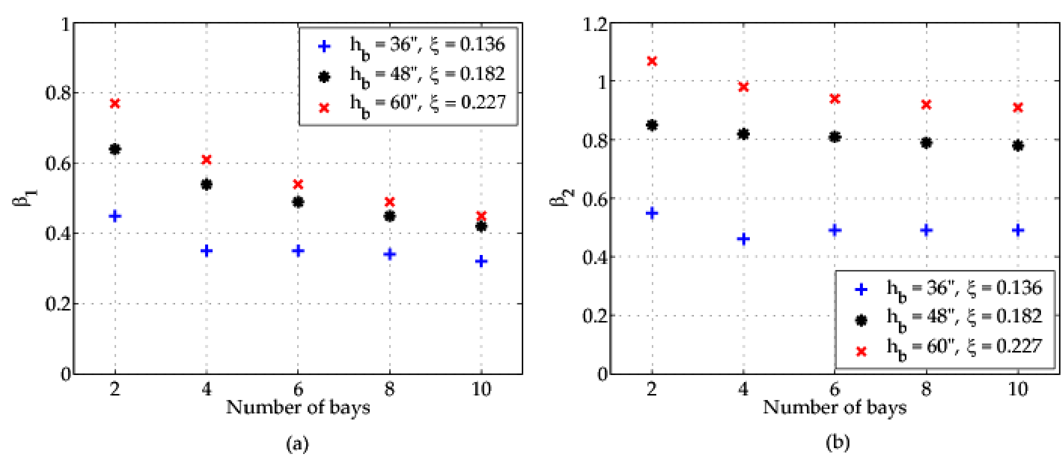

Figure 5. $[\beta]$ values; (a) β_1 (b) β_2

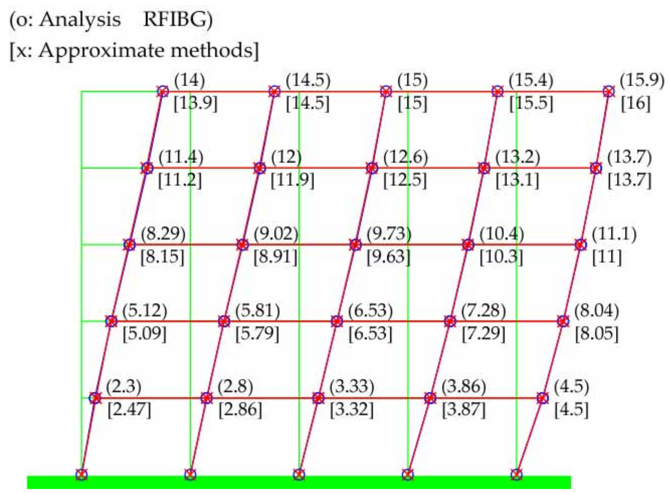

Figure 6. Deformed shape of RF0504-48 at 2% roof drift ratio (unit: inches)

in the analysis, and 4,234 k-ft in the approximate method. The error was 16%.

Some error was expected since the βs were calibrated to match the displacement at the east column, but the column rotations were only approximate because the beam moments were assumed to be constant at the yield moment. The moment at the first story of the center column in the approximate method was, for example, 31% less than the moment obtained in the FEM analysis. The main reason for this discrepancy was attributed to the overstrength of the framing beams due to axial forces, which were accounted for more accurately in the FEM analysis; the beam moment in the RFIBG model was much higher than the constant yield moment assumed in the approximate method. In addition, the moment distribution of the beams could play a role, since it was not constant across the building.

CONCLUSIONS

An approximate method of analysis that accounts for the effects of beam growth was presented. The method was based on the concept that the total displacements of the frame consisted of two components: one due to sway, and the other due to the lateral expansion by beam growth.

The displacement due to sway was the same for all the columns. It was developed using conventional analytical methods (RFEBG model), in which the effects of beam growth were not accounted for. The displacement due to beam growth was obtained from kinematic relationships. The beam growth was lumped as a single crack at the beam-column interface, and was obtained by assuming that the deformed shape of the center column can be derived from the value computed in the absence of beam growth, that the beam end moments have their yield values, and that the center of rotation at the beam-column interface is known.

Nonlinear FEM analyses (referred to here as the RFIBG model), using a large number of contact elements to simulate the lift-off of the beams from the column faces, were used to calibrate empirical relationships for obtaining the deformed shape of the column and the location of the center of rotation at the beam end. The static analyses were performed for various frame configurations.

The major findings are as follows:

- The approximate method of analysis presented here provides a simple way of accounting for the effects of beam growth. The alternative “exact” method required a nonlinear analysis that contained up to 900 contact elements and required a significant amount of development time.

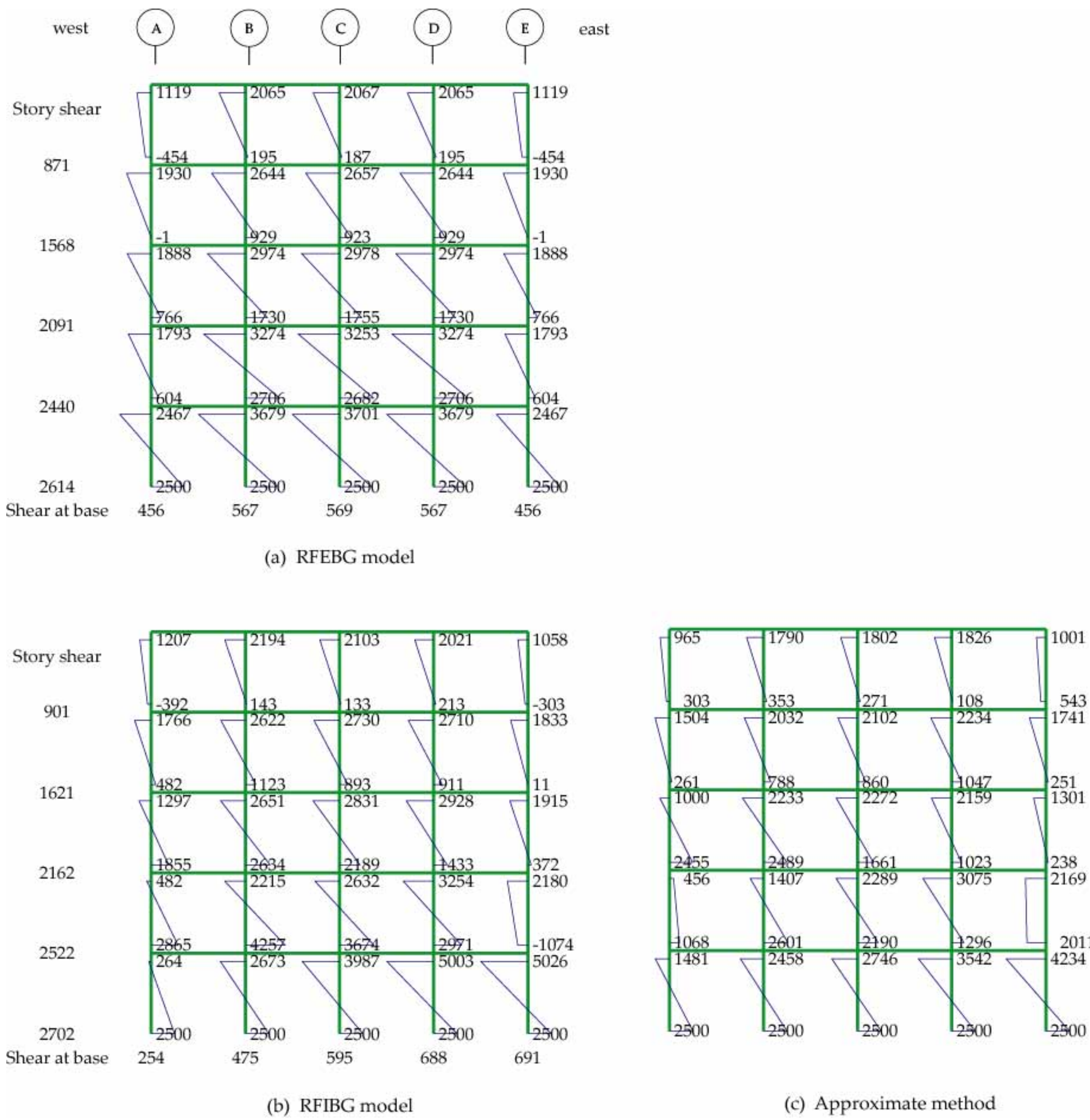

Figure 7. Column moment for RF0504-48 at 2% roof drift ratio

- As the frame sways to the east, beam growth causes the displacements to vary across the frame, with the largest values occurring at the east end column. The approximate method was able to predict the displacements at all column lines very accurately.
- Beam growth also causes the moments to vary significantly across the frame. For sway to the east, the moment in the east end column at bottom floor governs the column design.
- The column moments obtained from the approximate method were close to the FEM results from the RFIBG model, but not as accurately predicted as were the displacements. In the extreme case (of the widest building with the deepest beams) the largest error was 30%.
- The primary cause of the errors in the column moments was that the beam moments obtained in the nonlinear FEM model were significantly larger than the computed yield moments, due to the axial forces induced in the beams by the restraint of beam growth, whereas the yield moments were assumed in the approximate analysis. Nonetheless, the moments obtained from the

approximate method were much closer to those from the RFIBG model than the moments obtained from the conventional analysis (the RFEBG model).

REFERENCES

1. Zerbe, H. E. and Durrani, A. J. (1989). "Seismic Response of Connections in Two-Bay Reinforced Concrete Frame Subassemblies." *Jo. Struct. Engrg.*, 115(11), 2829-2844.
2. Zerbe, H. E. and Durrani, A. J. (1990). "Seismic Response of Connections in Two-Bay Reinforced Concrete Frame Subassemblies with a Floor Slab." *ACI Struct. Jo.*, 87(4), 406-415.
3. Sakata, H. and Wada, A. (1992). "Elasto-Plastic Behavior of One-Twentieth Scale RC Frame." *Proc. of the 10th World Conf. on Earthquake Eng.*, Vol. 6, Balkema, Rotterdam, 3335-3340.
4. Fenwick, R. and Davidson, B. (1995). "Elongation in Ductile Seismic-Resistant Reinforced Concrete Frames." *Proc. of the Tom Paulay Symposium*, Farmington Hills, MI, American Concrete Institute, SP 157-7, 143-170.
5. Kim, J., Stanton, J. F., and MacRae, G. A. (*in press*). "Effect of Beam Growth on Reinforced Concrete Frames." *ASCE, Jo. Struct. Engrg*.
6. Kim, J. (2002). "Behavior of Hybrid Frames under Seismic Loading." PhD Thesis, Dept. of Civil Engineering, Univ. of Washington, Washington.
7. Kim, J., Stanton, J. F., and MacRae, G. A. (2002). "Effects of Beam Growth on the Behavior of Reinforced Concrete Frames." *7th U.S. Nat'l Conf. Earthquake. Engrg*. July 2002, Boston, MA