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BEHAVIOR OF REINFORCED CONCRETE FRAMES DESIGNED FOR DIFFERENT LEVELS OF DUCTILITY

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SUMMARY

This paper describes the seismic performance of a reinforced concrete frame structure designed and detailed according to the current Canadian practice. On this basis, designers have two options for the seismic design of reinforced concrete frames. The first option is to design a ductile frame, which involves special design and detailing provisions to ensure ductile behavior. The second option is to design a nominally ductile frame. This option involves designing for twice the seismic lateral load as that for ductile frames, but without taking all the special provisions for good detailing in the design of the frame members. By allowing such a choice, the Code implies that either type of frames will provide equivalent seismic performance under the design level earthquake. In this study, a typical 5-story frame building is designed for both conditions. Analytical investigation in the form of pushover analysis is performed to evaluate and to compare the performance of each frame. The results in terms of story displacement, ductility, drift, sequence of cracking and yielding and the damage potential are presented. It is concluded that the performance of the ductile frame is much better than that of the nominally ductile frame.

INTRODUCTION

Moment resisting frames are the most commonly used framing system for reinforced concrete structures. According to the current Canadian practice, designers have two options for the seismic design of reinforced concrete frames (CAN3.A23.3 [1]). The first option is to design a ductile frame, which involves special design and detailing provisions to ensure ductile behavior. The second option is to design a nominally ductile frame. This option involves designing for twice the seismic lateral load as that for ductile frames, but without taking all the special provisions for good detailing in the design of the frame members. By allowing such a choice, the Code implies that either type of frames will provide equivalent seismic performance under the design level earthquake. The seismic design lateral loads and the level of seismic reinforcement detailing incorporated in a reinforced concrete moment resisting framed structure depend on its available ductility capacity. In "ductile" moment resisting frames, the design lateral loads

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reduce significantly, but high ductility capacity is ensured through strict detailing requirements to avoid premature modes of brittle failure. For frames with "nominal ductility," the design loads are higher, but very little seismic reinforcement detailing is required. According to the seismic design philosophy of the Canadian Standard, both approaches should offer the same level of seismic protection against the design earthquake at the construction site.

In this study, the response of a 5-story reinforced concrete frame designed as ductile and nominally ductile subjected to pushover analysis is presented.

BUILDING CONFIGURATION AND LOADING

Description of building

The plan view and elevation of the 5-story building is shown in Fig. 1. Each frame is assumed to be part of the lateral load resisting system of a building. The story height is 4m for first floor and 3m for other stories resulting in a total building height of 16m.

Lateral loading and torsion

The NBCC 1995 [2] seismic base shear is given by:

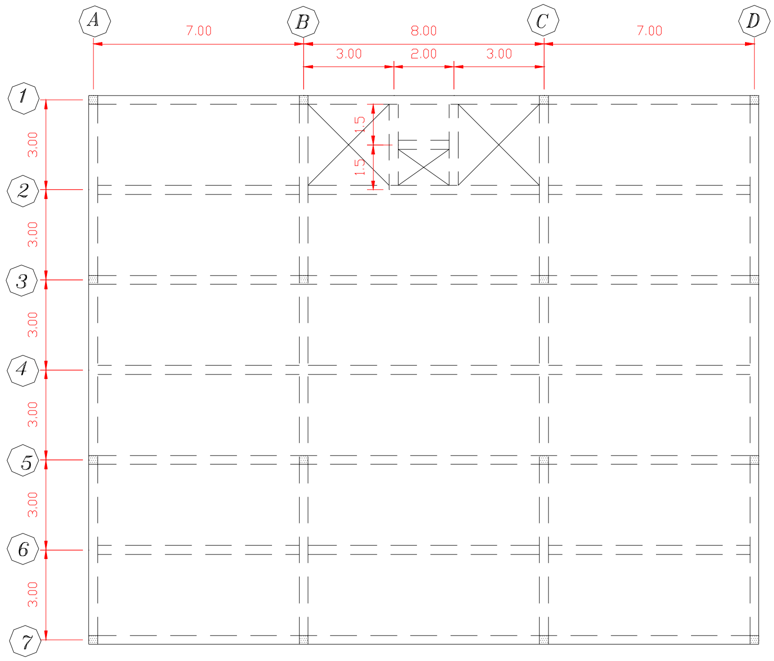
$$(V_e / R)\ U \qquad (1)$$

Where V_e is the equivalent lateral seismic force representing elastic response, R is the response modification factor (given R =2 for nominal ductile frame and R= 4.0 for ductile moment resisting frame structures), U= 0.6 is a calibration, V_e is the elastic lateral seismic force, which is given by:

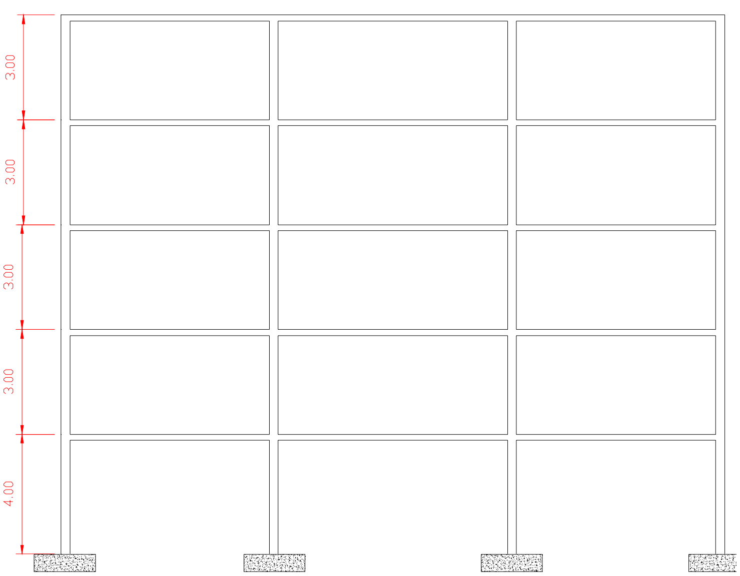
$$V_e = vSIFW \qquad (2)$$

Where:

v is zonal velocity ratio. It is assumed that the building is located in the highest seismic zone (i.e. v = 0.4), S is the seismic response factor = $1.5 / \sqrt{T}$ for $T \geq 0.5$ seconds (given T = 0.1N = 0.5 seconds, S = 2.121) where T is the fundamental period of vibration, N is the total number of stories above grade, I is the seismic importance factor assumed to be 1.0 as the building is intended for typical office occupancy, F is the foundation factor assumed to be equal to 1.3, as the structure is assumed to be built on soft base soil.



a) Plan



b) Elevation

Fig. 1 Building Details

The dead load (W) of the building is calculated as 15482 kN. The calculated base shears are 5122 kN and 2561 kN for the nominally ductile and the ductile frames respectively. NBCC 1995 requires that the lateral load to be distributed over the building height as follows:

$$F_x = (V - F_t)\, h_x\, W_x / (\Sigma h_i\, W_i) \qquad (3)$$

Where, F_x is lateral force applied at level x, F_t = additional lateral force applied to the top of building (Ft = 0.0 if T ≤ 0.7 seconds), W_i and W_x are portions of W at levels i and x respectively, h_i and h_x are the heights above the base to levels i and x respectively. NBCC 1995 requires that the effects of torsional moments be included in the design of the lateral force resisting system. Since there is no eccentricity in building, the accidental applied torsional moment is calculated using the following formula at each level (x):

$$T_x = (F_x)\, (\pm 0.1 D_{nx}) = 2.2\, F_x \qquad (4)$$

Where, D_{nx} = 22 m is plan dimension of the building in the direction of the computed eccentricity.

ANALYSIS AND DESIGN

Elastic analysis

Initial elastic analysis of both structures is performed in order to determine the structural elements seismic design forces using SAP 2000 [3]. A summary of design seismic lateral loads on frame-B as shown in Fig.1 for ductile and nominally ductile (ND) frames is shown in Table 1.

Seismic design

The structures are designed according to the CAN3-A23.3 [1]. The ductile frame is designed using the capacity design approach. Summery of designed sections and reinforcement are shown in Tables 2 and 3 for beams and columns respectively.

Table 1 Summary of design seismic lateral loads for ductile and ND frames

Floor	h_i(m) Storey Height	W_i (MN) Storey Weight	F_X(kN) Design Base Shear		T_X Torsion (kN-m)		F_{xt} Torsional Lateral Forces		Total Lateral Forces	
			ND	Ductile	ND	Ductile	ND	Ductile	ND	Ductile
Roof	16	2.75	1465.0	732.5	3223.0	1611.5	48.3	24.2	414.6	207.3
5	13	3.28	1419.0	709.5	3121.8	1560.9	46.8	23.4	401.6	200.8
4	10	3.28	1091.0	545.5	2400.0	1200.0	36.0	18.0	308.7	154.4
3	7	3.28	763.3.0	381.6	1679.3	839.5	25.2	12.6	216.0	108.0
2	4	2.89	384.7.0	192.35	846.3	423.2	12.7	6.35	108.9	54.5
1		Σ=15.48								

Table 2 Summery of designed sections and reinforcement of beams

a) ND frame

Storey	Beams	Sectional Size(mm)	Reinforcement	A_s (mm^2)	ρ%	M_r (kN.m)	M_f (kN.m)
1st	Int & Ext Ends (-ve)	600*600	5No35+1NO25@top	5700	1.75	863	869.9
	Int & Ext Ends (+ve)	600*600	6No30@bot	4200	1.287	676	637.5
	Mid Span (+ve)	600*600	2No30@bot	1400	0.43	246.3	219.6
2nd	Int & Ext Ends (-ve)	600*600	5No35+1NO25@top	5700	1.75	863	782.4
	Int & Ext Ends (+ve)	600*600	6No30@bot	4200	1.287	676	527.7
	Mid Span (+ve)	600*600	2No30@bot	1400	0.43	246.3	216
3rd	Int & Ext Ends (-ve)	500*500	5No35+1NO30@top	5900	2.668	646.2	647.4
	Int & Ext Ends (+ve)	500*500	4No30+1NO20@bot	3100	1.4	402	386.5
	Mid Span (+ve)	500*500	2No30@bot	1400	0.631	246.3	216.8
4rh and 5th	Int & Ext Ends (-ve)	500*500	4No35@TOP	4200	1.893	508.8	485.2
	Int & Ext Ends (+ve)	500*500	2No30@bot	1400	0.631	197.8	198.7
	Mid Span (+ve)	500*500	2No30@bot	1400	0.631	246.3	215.2

(b) Ductile frame

Storey	Beams	Sectional Size(mm)	Reinforcement	A_s (mm^2)	ρ %	M_r (kN.m)	M_f (kN.m)
1st	Int & Ext Ends (-ve)	600*600	5No25@top	2700	0.82	459	468.6
	Int & Ext Ends (+ve)	600*600	5No20@bot	1500	0.46	273	257.2
	Mid Span (+ve)	600*600	4No20@bot	1200	0.37	219	195.8
2nd	Int Ends (-ve)	600*600	4No25+1NO10@TOP	2300	0.7	397	400.2
	Ent Ends (-ve)	600*600	4No25+1NO15@TOP	2400	0.7	412.7	417
	Int & Ext Ends (+ve)	600*600	4No20@bot	1200	0.37	214.5	198.5
	Mid Span (+ve)	600*600	4No20@bot	1200	0.37	219	198.5
3rd	Int & Ext Ends (-ve)	500*500	5No25@top	2700	0.12	359.7	340.7
	Int & Ext Ends (+ve)	500*500	4No20@bot	1200	0.54	172.1	170.3
	Mid Span (+ve)	500*500	5No20@bot	1500	0.67	212.1	257.2
4rh and 5th	Int & Ext Ends (-ve)	500*500	3No25+1NO20@TOP	2000	0.89	276	279
	Int & Ext Ends (+ve)	500*500	2No25@bot	1000	0.45	144.8	135.2
	Mid Span (+ve)	500*500	3No25@bot	1500	0.67	212	210

Table 3 Summery of designed sections and reinforcement of columns

Storey	Ductile		ND	
	Sectional Size(mm)	Reinforcement	Sectional Size(mm)	Reinforcement
1^{st} and 2^{nd}	600*600	12 No 20	600*600	12 No 30
3^{rd}	500*500	12 No 20	500*500	8 No 35
4^{th} and 5^{th}	500*500	8 No 20	500*500	8 No 30

NONLINEAR INELASTIC MODELING

The inelastic dynamic analysis of reinforced concrete building structures program, IDARC (Reinhorn et al. [4]) is used to calculate the response of the structure to pushover loading. The moment curvature envelope describes the changes in force capacity with deformation during a nonlinear analysis. In both models, default generated moment curvature envelopes by IDARC are used for each element. The material properties are assumed to be constant throughout the height of the structure. The material properties are specified as, reinforcement steel modulus E_s=200000 MPa; Yield strength f_y = 400 MPa; concrete compressive strength f'_c=35 MPa. Default values of characteristics of steel stress-strain curve are specified as, ultimate strength (FSU=1.4*f_y), modulus of strain hardening (E_s/60) and strain at start of hardening (EPSH=3.0%). For concrete, these default values are: initial Young's Modulus (EC= 57*√(f'$_c$*1000)), strain at maximum strength of concrete (EPSO=0.2%), stress at tension cracking (FT=0.12* f'$_c$), ultimate strain in compression (EPSU) and parameter defining slope of falling branch are derived by the program and depending on section data. The nonlinear dynamic computer program, IDARC includes several types of hysteretic response models. A hysteretic model incorporates the effects of stiffness degradation, strength deterioration, and slip control (pinching). The effect of these features of reinforced concrete behavior under cyclic loading is included in the model through the selection of model hysteretic parameters (Reinhorn et al. [4]). A detailed description of hysteretic models is beyond the scope of this paper. However, in this investigation, the behavior of the structural elements is modeled by a commonly used bilinear model. The values of the IDARC parameters used in the analysis of ductile frame are HC = 2.0, HBE = HBD = 0.001, and HS = 1.0 and for nominally ductile frame these values are HC = 0.1, HBE = HBD = 0.4, and HS = 0.1 for the control of stiffness deterioration, strength degradation, and pinching behavior respectively. In ductile frame, modeling a stable loop is used and for nominally ductile frame, modeling the deterioration is considered (Filiatrault et al. [5]). Use of stable loops for ductile frame is the result of good detailing of the members.

NONLINEAR PUSHOVER ANALYSIS

The inelastic nonlinear dynamic analysis program IRDAC is used to calculate the inelastic response of both structures (Ductile and Nominally Ductile frames) subjected to pushover loading. The pushover analysis, or collapse mode analysis is a simple and efficient technique to predict the seismic response of structures. A pushover analysis can obtain the resistance of the building against lateral loads, ductility capacity of structure and sequence of component yielding. The pushover analysis may be carried out using either force control or displacement control.

In this investigation, the structure is subjected to an incremental distribution of lateral forces and the incremental displacement is obtained. With the increase in the magnitude of the loading, damages and failure modes of the structure are investigated. The pushover loading is an inverted triangular with the effects of the cyclic behavior and load reversals being estimated by using a modified monotonic force-deformation criteria.

Results of pushover analysis

A pushover analysis is performed on each structure to determine the base shear- lateral displacement envelope and the sequence of formation of plastic hinging. In such analysis, a monotonic load is applied to the ductile and the nominally ductile structure until an ultimate load is approached. In terms of base shear coefficient, the results of analysis correspond to a value of V/W = 0.278 for the ductile frame and a value of V/W = 0.377 for the nominally ductile frame. For each structure, the ultimate lateral loading is compared with seismic design lateral loading distribution of NBCC 1995. Based on the results of seismic analysis, the design base shear coefficient for ductile frame is 0.165 and for nominally ductile frame is 0.33. The displacement-base shears relationship of frames and comparison with NBCC design base shear obtained is shown in Fig. 2. This figure illustrates the base shear - lateral displacement envelope and the sequence of plastic hinging. The displacement-base shear relationship of frames determined during the pushover analysis gives an indication of the global response to lateral loading, including the over strength and deflections. The envelope for the structure with nominal ductility (R = 2) shows a descending behavior following the yield of structure. Note that the design base shear for the nominally ductile frame is almost twice as that of ductile frame.

Over strength ratios

The over strength ratio is defined as base shear divided by design base shear. The results of ductile frame, indicate that first beams and columns yield at an over strength ratio of about 1.03 and 1.18 respectively. Similarly the over strength ratio of 0.99 and 0.925 are obtained at the first beam and column yield states for the nominally ductile frame. The larger over strength ratio is observed in ductile frame than the nominally ductile structure. This is mainly the result of the higher strength of the columns related to beams to ensure plastic hinging in the beams.

Storey displacement

Fig. 3 compares the maximum base shear-story displacement due to pushover loading for ductile and nominally ductile frames. The nominally ductile frame shows a displacement of 316 mm in the first story and the rest of floors act as a rigid body. The ductile frame shows less deformation in the first floor (253 mm) as compared with the nominally ductile frame. The inelastic deformations in plastic hinges at the ends of beams in ductile frame cause relative movements in upper stories. The occurrence of plastic hinges and severe damages of columns in the first floor of the nominally ductile frame cause this large deformation due to lack of strong column-weak beam mechanism.

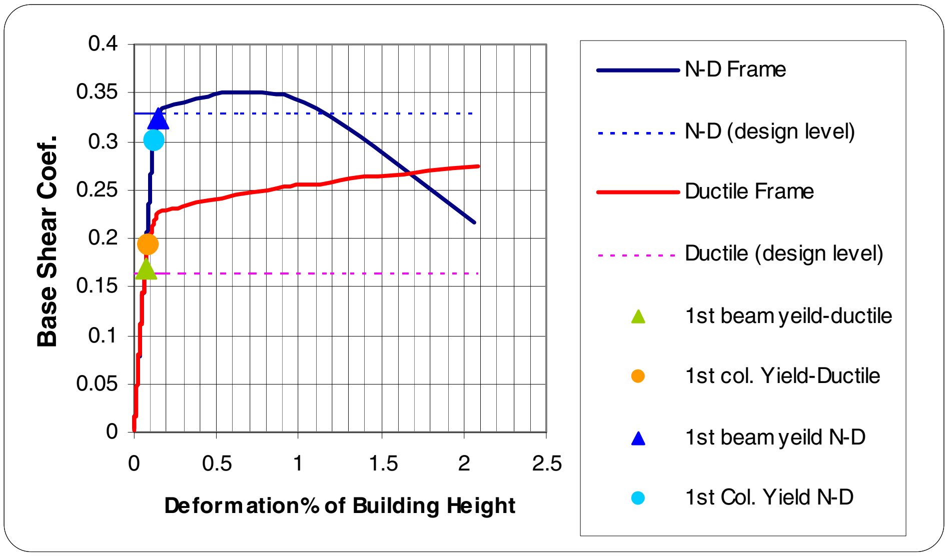


Fig. 2 Base shear coefficient versus displacement percentage of height

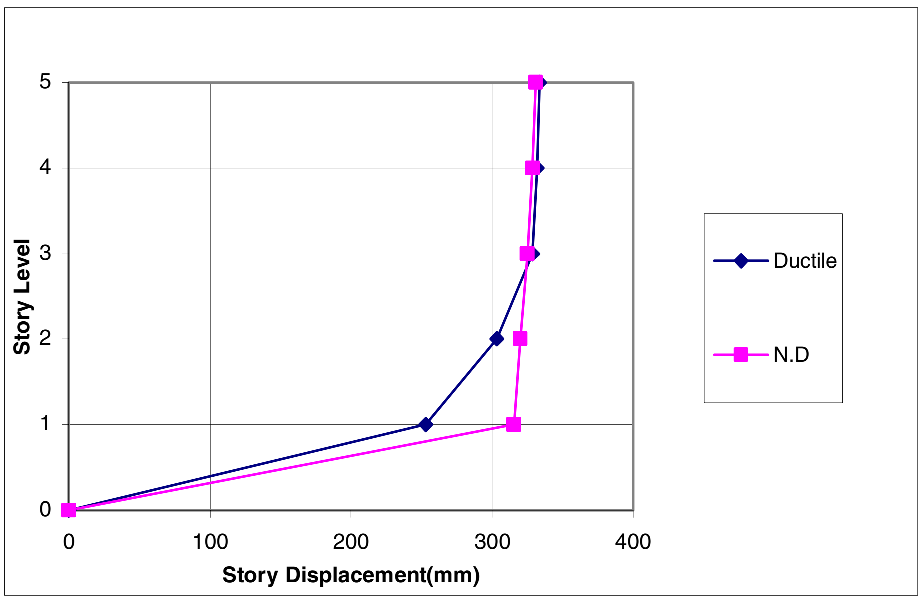


Fig. 3 Base shear story displacement

Damage analysis

The response index is used to estimate the damage in RC ductile members as developed by Park and Ang [6]. This model is used in IDARC. A global value of damage index can be used to characterize damage in the ductile members of RC frames. Damage indices can characterize damage in the members of RC frames as follow:

$$DI_{P\&A} = \delta_m / \delta_u + \beta / \delta_u P_y \int d\, E_h \qquad (5)$$

Where δ_m is the maximum experienced deformation, δ_u is the ultimate deformation of the element, P_y is the yield strength of element, $\int d\, E_h$ is the hysteretic energy absorbed by the element during response history and β is a model constant parameter. Typical range of damage index is presented in Table 4. The overall structural damages obtained are 0.224 and 0.528 for the ductile frame and the nominally ductile frame respectively. The overall damage of ductile frame is less than that of the nominally ductile frame because of better performance of ductile frames due to stronger columns and better structural detailing.

Table 4 Typical Range of Damage Index

Degree of Damage	Physical Appearance	Damage Index	State of building
Collapse	Partial or total collapse of building	>1.0	Loss of building
Severe	Extensive crushing of concrete, disclosure of buckled reinforcement	0.4 – 1.0	Beyond repair
Moderate	Extensive large cracks; spalling of concrete in weaker elements	<0.4	Repairable
Minor	Minor cracks; partial crushing of concrete in columns		
slight	Sporadic occurrence of cracking		

Sequence of yielding

Table 5 and Figs. 4 and 5 indicate the state of cracking/yielding and damage index statistics of ductile and nominally ductile frame respectively in order to show the progression of damage as the load is increased. The sequence of yielding in Table 5(a) and Fig. 4(a) indicates that beams at 1st, 2nd and 3rd floors yielded resulting in plastic hinges occurring at the beams. The hinges form in the beams followed by four others at the base of the columns and then the columns in the first story yield. This behavior confirms the successful application of strong column-week beam theory in designing the ductile frame. The sequence of plastic hinging in the ductile frame conforms to the capacity design concept. The sequence of yielding in Table 4(b) and Fig 4(b) indicate that, first plastic hinges occur at the base of the columns. A severe yielding of column at top of first story immediately follows. Finally, the plastic hinges are formed in the beams of first floor. This hinging pattern in the structure with nominal ductility is far from the requirements of capacity design and energy dissipation criteria. Note that X and 0 express crack and yield states of frame and the numbers in parentheses indicate sequence of yielding. Fig. 2 showed yielding of structural elements in nominally ductile frame where column yields at base shear coefficient of 0.3015. First beam yields at base shear coefficient of 0.3231. In ductile frame, first yielding occurs in beam at base shear coefficient of 0.1696 and first column yields at base shear coefficient of 0.1946. These results illustrate better response of ductile frame due to week beam- strong column considerations.

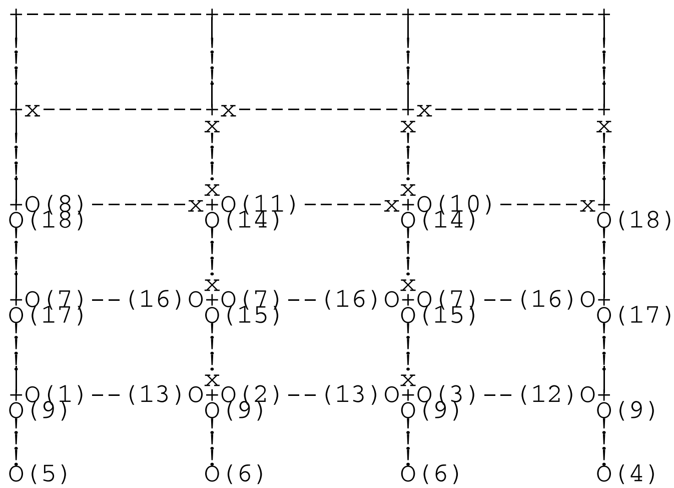
Table 5 Sequence of Component Yielding

a) Ductile frame

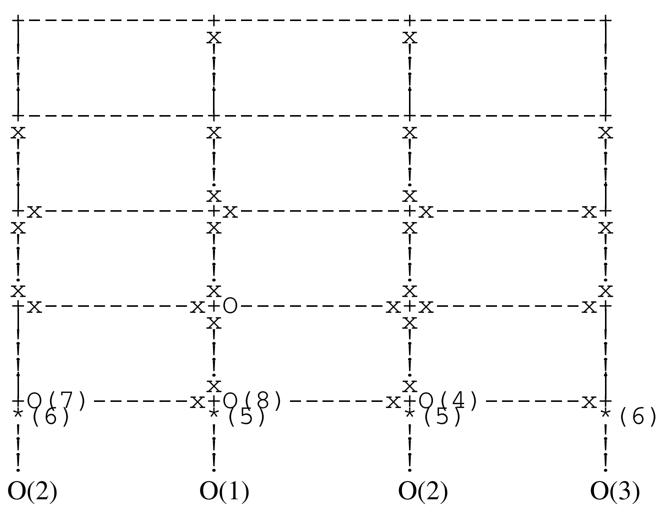
NO.	STORY LEVEL	ELEMENT	BASE SHEAR	SEQUANCE OF YIELDING
1	1^{st}	BEAM 1	0.1699	YIELDING DETECTED AT LEFT
2	1^{st}	BEAM 6	0.1751	YIELDING DETECTED AT LEFT
3	1^{st}	BEAM 11	0.1779	YIELDING DETECTED AT LEFT
4	1^{st}	COLUMN 4	0.1945	YIELDING DETECTED AT BOT
5	1^{st}	COLUMN 1	0.1973	YIELDING DETECTED AT BOT
6	1^{st}	COLUMN 2,3	0.2000	YIELDING DETECTED AT BOT
7	2^{nd}	BEAM 2,12	0.2055	YIELDING DETECTED AT LEFT
8	3^{rd}	BEAM 3	0.2261	YIELDING DETECTED AT LEFT
9	1^{st}	COL 1,2,3,4	0.2329	YIELDING DETECTED AT TOP
10	3^{rd}	BEAM 13	0.2356	YIELDING DETECTED AT LEFT
11	3^{rd}	BEAM 8	0.2384	YIELDING DETECTED AT LEFT
12	1^{st}	BEAM 11	0.2411	YIELDING DETECTED AT RGHT
13	1^{st}	BEAM 1,6	0.2439	YIELDING DETECTED AT RGHT
14	3^{rd}	COL. 10,11	0.2466	YIELDING DETECTED AT TOP
15	2^{nd}	COLUMN 6,7	0.2493	YIELDING DETECTED AT TOP
16	2^{nd}	BEAM 2,7,12	0.2548	YIELDING DETECTED AT RGHT
17	2^{nd}	COLUMN 5,8	0.2576	YIELDING DETECTED AT TOP
18	3^{rd}	COLUMN 9,12	0.2658	YIELDING DETECTED AT TOP

b) Nominally Ductile

NO.	STORY LEVEL	ELEMENT	BASE SHEAR	SEQUANCE OF YIELDING
1	1^{st}	COLUMN 2	0.3015	YIELDING DETECTED AT BOT
2	1^{st}	COLUMN 1,3	0.3053	YIELDING DETECTED AT BOT
3	1^{st}	COLUMN 4	0.3091	YIELDING DETECTED AT BOT
4	1^{st}	BEAM 11	0.3231	YIELDING DETECTED AT LEFT
5	1^{st}	COLUMN 2,3	0.3376	YIELDING DETECTED AT TOP
6	1^{st}	COLUMN 1,4	0.3411	YIELDING DETECTED AT TOP
7	1^{st}	BEAM 1	0.3473	YIELDING DETECTED AT LEFT
8	2^{nd}	BEAM 7	0.3503	YIELDING DETECTED AT LEFT



a) Ductile frame



b) Nominally Ductile frame

Fig. 4 State of Failure and Sequence of Yielding

CONCLUSIONS

The analytical investigation presented in this paper is intended to provide a better understanding of the seismic behavior of ductile and nominally ductile structures. The ductile frame (R=4) performed very well under pushover loading. This was due to weak beam- strong column considerations. The response showed that the capacity design philosophy and ductility level as applied in current Canadian standards are effective.

The nominally ductile frame (R=2) was stronger than the ductile frame due to larger member sizes, but the results showed lower ductility capacity. A single storey failure mechanism of this structure was observed.

The nominally ductile frame performed as expected under the pushover loading. Inelastic deformations were concentrated mainly in the first floor and the rest of the floors acted as a rigid body. The damage index obtained for the nominally ductile frame was much higher than that of the ductile frame. The lack of the incorporation of the week beam-strong column concept in the nominally ductile frame can be of concern on the level of seismic protection offered by this type of structures.

```
+-----------+-----------+-----------+
!    0.00   !    0.00   !    0.00   !
!   (0.09)  !   (0.09)  !   (0.06)  !
!0.00       !0.00       !0.00       !0.00
!(.10)      !(.27)      !(.27)      !(.11)
!           !           !           !
+-----------+-----------+-----------+
!    0.00   !    0.00   !    0.00   !
!   (0.05)  !   (0.10)  !   (0.06)  !
!0.00       !0.01       !0.01       !0.00
!(.01)      !(.36)      !(.38)      !(.04)
!           !           !           !
+-----------+-----------+-----------+
!    0.01   !    0.01   !    0.01   !
!   (0.04)  !   (0.03)  !   (0.03)  !
!0.13       !0.12       !0.12       !0.14
!(.19)      !(.25)      !(.25)      !(.21)
!           !           !           !
+-----------+-----------+-----------+
!    0.11   !    0.10   !    0.11   !
!   (0.21)  !   (0.17)  !   (0.21)  !
!0.20       !0.22       !0.21       !0.23
!(.08)      !(.12)      !(.12)      !(.09)
!           !           !           !
+-----------+-----------+-----------+
!    0.23   !    0.22   !    0.24   !
!   (0.07)  !   (0.06)  !   (0.07)  !
!0.27       !0.22       !0.22       !0.27
!(.18)      !(.22)      !(.22)      !(.18)
!           !           !           !
```

a) Ductile frame

```
+-----------+-----------+-----------+
!    0.00   !    0.00   !    0.00   !
!   (0.02)  !   (0.03)  !   (0.02)  !
!0.00       !0.00       !0.00       !0.00
!(.03)      !(.46)      !(.40)      !(.03)
!           !           !           !
+-----------+-----------+-----------+
!    0.00   !    0.00   !    0.00   !
!   (0.01)  !   (0.01)  !   (0.01)  !
!0.00       !0.00       !0.00       !0.00
!(.06)      !(.47)      !(.42)      !(.03)
!           !           !           !
+-----------+-----------+-----------+
!    0.00   !    0.00   !    0.00   !
!   (0.16)  !   (0.07)  !   (0.19)  !
!0.00       !0.01       !0.01       !0.00
!(.09)      !(.24)      !(.20)      !(.04)
!           !           !           !
+-----------+-----------+-----------+
!    0.00   !    0.01   !    0.01   !
!   (0.20)  !   (0.23)  !   (0.28)  !
!0.00       !0.00       !0.00       !0.00
!(.00)      !(.16)      !(.13)      !(.00)
!           !           !           !
+-----------+-----------+-----------+
!    0.01   !    0.01   !    0.02   !
!   (0.00)  !   (0.00)  !   (0.00)  !
!0.43       !0.62       !0.62       !0.43
!(.22)      !(.27)      !(.27)      !(.22)
!           !           !           !
```

b) Nominally ductile frame

Fig. 5 Damage Index Statistics

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