13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 513

AN EQUIVALENT-LINEARIZATION APPROACH TO PERFORMANCE-BASED DESIGN

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SUMMARY

An equivalent linearization method is proposed to estimate the seismic non linear response of multi-degree-of-freedom building structures. The method is based on an iterative procedure used to estimate the system's response through a conventional modal analysis for a linear equivalent system at each step in the process. For simplicity, the equivalent linearization criterion is established on the basis of a single variable, namely the roof displacement relative to the base. Two groups of analyses are performed, one for a single earthquake ground motion time history another for a sample of equal intensity simulated ground motion records. The results show that, in general, the proposed method produces reasonably accurate estimates of the roof displacement; however, the estimates of story distortions are less satisfactory.

INTRODUCTION

Accomplishing the objectives of applying performance-based seismic design criteria for a specific system is strongly dependent on our capability to obtain reasonably accurate estimates of peak dynamic response amplitudes. Our need for accuracy must be balanced with the requirements for simplicity that are advocated in usual conditions of engineering design practice. According to the latter, the accuracy between accuracy and complexity in the seismic design of multistory systems has been attained through the use of linear system models and reduced response spectra. The influence of nonlinear behavior on the global response of a system is roughly accounted for in this manner.

A frequently used approach to the production of better estimates of local responses while avoiding the application of excessively demanding methods of response analysis is based on the use of simplified "equivalent" systems, which are characterized by a base shear vs roof displacement curve and a set of amplitude dependent lateral displacement configurations, all obtained by pushover analysis of the system considered.

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A number of studies have been devoted to the estimation of the peak nonlinear displacement demand of single-degree-of-freedom (SDOF) systems through equivalent linearization (Rosenblueth [1], Gulkan [2], Iwan [3]); however, their use has been strictly limited to that kind of systems. Some recently developed methods deal with the estimation of peak response values of multi-degree-of-freedom (MDOF) systems; these include the incremental dynamic analysis method proposed by Vamvatsikos and Cornell [4] and the modal pushover criteria proposed by Chopra and Goel [5]. These methods have shown to be effective; however, their application often demands excessive computational efforts, thus limiting their adequacy for typical practical design conditions.

This study aims to develop and evaluate an equivalent linearization approach, suitable for practical applications, which is capable of leading to more accurate estimates of the peak amplitudes of the local response variables of MDOF systems. This formulation is intended to avoid the assumption of a proportionally growing mass-acceleration pattern generally applied in typical pushover studies. This is accomplished at the expense of introducing an iterative procedure. For the systems studied here, equivalent values of story stiffness and viscous damping coefficient are made to depend on the peak amplitudes of the corresponding distortions. Because equivalence transformation rules are sensitive to the shapes of the response spectra considered, the study includes a section dealing with the establishment of those rules for sets of ground motion time histories with spectral properties typical of a soft soil site in Mexico City. An extrapolation algorithm is introduced in order to reduce convergence problems and keep the number of required iterative cycles sufficiently small.

LINEAR EQUIVALENT SYSTEM

The method of analysis proposed to estimate the seismic nonlinear response of MDOF systems is based on the use of a linear equivalent system (LES). The mechanical properties of this system (lateral stiffness and damping ratio) are determined through an iterative process; they are chosen so as to represent both the stiffness reduction and the hysteretic energy dissipation that result from the nonlinear behavior of structural members. This is achieved by means of a pushover analysis in each iterative step, applying as excitation the vector of equivalent lateral forces obtained through a modal analysis and an adequate superposition criterion. For simplicity, the linearization approach is formulated in terms of a single degree of freedom: the roof displacement (Fernández-Palafox [6]).

Iterative process

Consider a nonlinear MDOF system, with mechanical properties defined by the stiffness matrix K for lateral displacements and the damping ratio ξ, subjected to a seismic excitation with intensity y. The system's response for low intensities can be estimated by means of a conventional modal analysis, and applying an adequate criterion to superimpose he contributions of all significant modes of vibration (for instance, Rosenblueth [7] SRSS criterion). In the present study it will be assumed that a similar criterion can be applied for the case of nonlinear response, provided that the force-displacement relations for the system are represented by means of the stiffness matrix of the corresponding linear equivalent system.

Consider now that K_i and ξ_i are the stiffness matrix and the damping ratio associated with the LES at the start of the i-th iteration. The results of the conventional modal analysis using these values for the properties of the linear equivalent system will lead to vectors φ_i, Q_i of floor displacements relative to the base and inertial forces, respectively, to the resulting values of the roof displacement u_i and the base shear V_i. These values are then used to perform a pushover analysis of the corresponding LES, with the objective of determining a new stiffness matrix $K'_i$and a new damping ratio ξ_i', which will serve to characterize the new LES.

With the objective of developing a practically applicable method, a simple form is adopted for the determination of K'_i and ξ'_i in each cycle. According to it, in order to determine K'_i, the deformed configuration of the system at the start of the *i*-th cycle is assumed to correspond to a shear building; this permits expressing the mentioned matrix exclusively in terms of the story-stiffness values, each of which is in turn calculated as the ratio of the story shear to the corresponding story drift (relative lateral displacement). These values are directly determined from the lateral displacement and load configurations given by the pushover analysis for a roof displacement equal to that obtained at the start of the iterative cycle considered.

The curve relating the roof displacement u with the base shear V_b resulting from the pushover analysis can be used to estimate the new value of ξ_H, the hysteretic component of damping. This value is added to the assumed "viscous friction ratio" ξ_v associated with the linear system response for low ground motion intensities in order to obtain the resulting damping ratio ξ'_i of the LES.

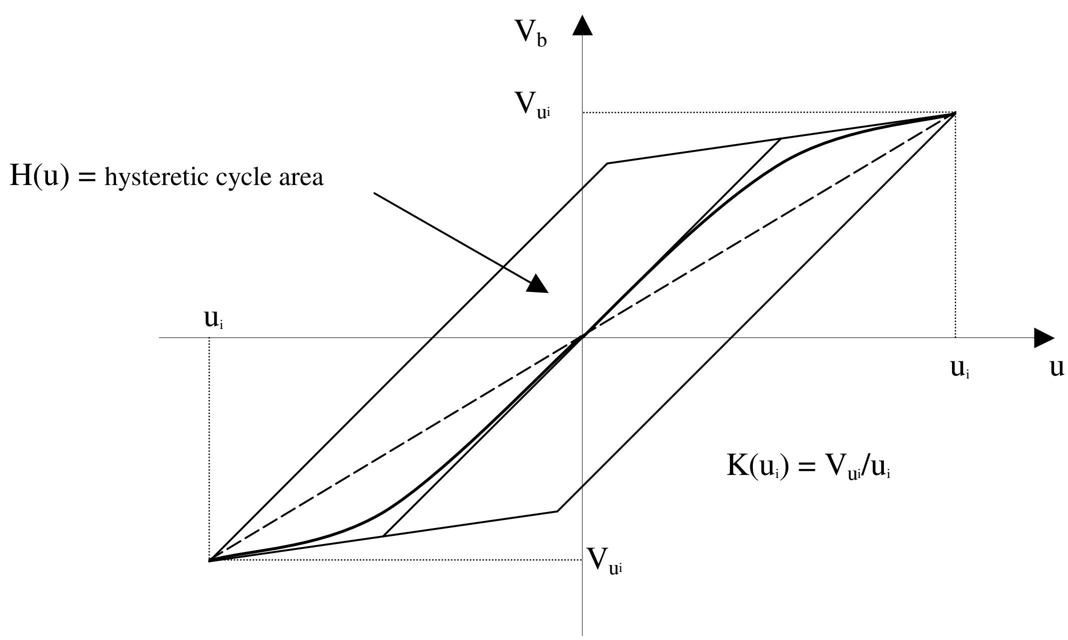


Figure 1. Hysteretic component of damping

The hysteretic component of the damping ratio is obtained from the area under the curve in a force-deformation cycle (V_b - u) with an amplitude equal to $2u_i$ (see Figure 1). For this purpose, use is made of the following equation (Newmark [8]).

$$\xi'(u) = \xi_v(u) + \frac{H(u)}{2\pi K(u)u^2} \qquad (1)$$

In this equation, $H(u)$ is the energy dissipated by hysteresis during a cycle of amplitude u in each direction; $K(u)$ is the secant stiffness corresponding to this deformation amplitude.

Modal spectral analysis with the updated properties of the LES

The variables K'_i and ξ'_i define the new properties of the LES. A modal spectral analysis for the response of this linear equivalent system leads to the new response vectors φ'_i and Q'_i, as well as to the new estimate of the roof displacement u'_i. If this value is sufficiently close to u_i, the iterative process is concluded and the last calculated values of the response vectors φ'_i and Q'_i and of the mechanical properties K'_i and ξ'_i will be assumed to characterize the LES; otherwise, a new iterative cycle must be started. In order to accelerate convergence, the following equation can be used to obtain the value of the roof displacement at the start of cycle $i + 2$ on the basis of the initial and final values of the roof displacement for the two previous cycles (u_i, u'_i ; u_{i+1}, u'_{i+1}):

$$u_{i+2} = \frac{u_{i+1}\left(\dfrac{u'_{i+1}}{u_{i+1}} - r\right)}{(1-r)} \tag{2}$$

where r is given by the following equation:

$$r = \frac{(u'_{i+1} - u'_i)}{(u_{i+1} - u_i)} \tag{3}$$

CASES STUDIED

A set of five, ten and fifteen story high building frames (Figure 2) is studied. All are assumed to be located at a soft soil site in Mexico City. The frames were designed in accordance with the Federal District Building Code [9]. Five and fifteen story systems were designed for basic seismic design coefficients, c, ranging from 0.1 to 0.6, while the fifteen story frames were designed for values of c ranging from 0.1 to 0.4. These coefficients were divided by a seismic behavior factor Q = 4, intended to account for system overstrength and ductile nonlinear behavior [10].

Two different methods were applied to estimate peak values of the nonlinear displacement demands of the system studied: one based on the linear equivalent system (LES) proposed here and another based on a step-by-step (SBS) dynamic nonlinear response analysis.

The results obtained for a single ground motion time history are presented first; the results for a set of simulated records are presented later.

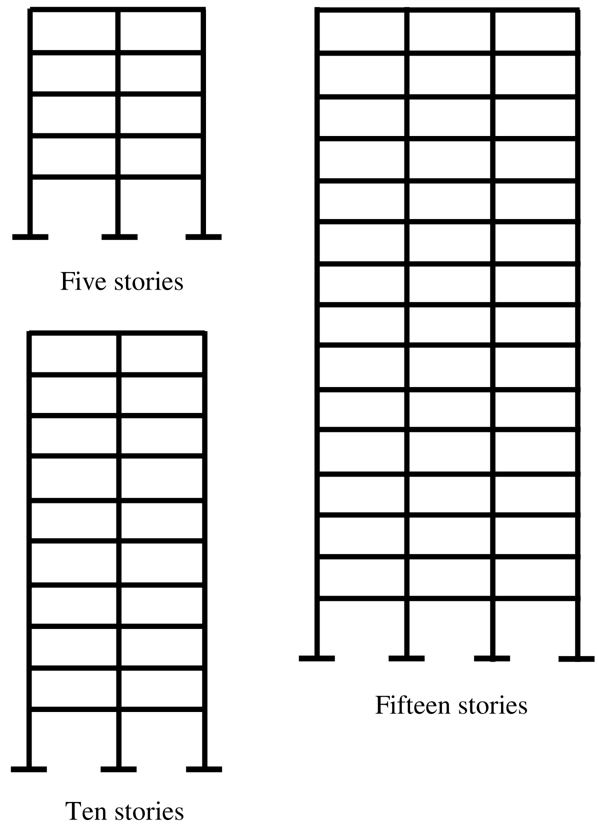


Figure 2. Cases studied

RESULTS

Roof displacement

Response to a single ground motion record (SCT)

The results of the dynamic response analyses of the frames studied are presented in the following. As mentioned before, two different methods were applied in all cases: the linear equivalent system (LES) and the step-by-step integration procedure (SBS). The mechanical properties of the structural members and the gravitational loads acting on the system were taken as deterministically equal to the expected values of their instantaneous probability density functions. A viscous damping ratio of 0.05 was adopted. The seismic excitation was taken equal to the EW component of the ground motion acceleration time history recorded at the SCT site in the soft soil area of Mexico City during the earthquake of September 19, 1985 (Figure 3). The estimation of the peak values of the corresponding story distortions by means of the LES model was achieved with the aid of a computer program that performs automatically the iteration process. That program was linked to Program DRAIN-2DX (Prakash [11]), which was used to perform the pushover studies. Table 1 shows the seismic design coefficient for each system, c (before applying reductions intended to account for the overstrength ratio or the influence of non linear ductile behavior), the initial value of the natural period of vibration of the system for low deformations, T_i, the final value of the period of the LES, T_f, the hysteretic damping ratio, ξ_H, and the viscous damping ratio of the LES, ξ. Also shown is the number of iterations, NIC, necessary to achieve convergence of the proposed iteration process. The results show that the process converged in all cases studied.

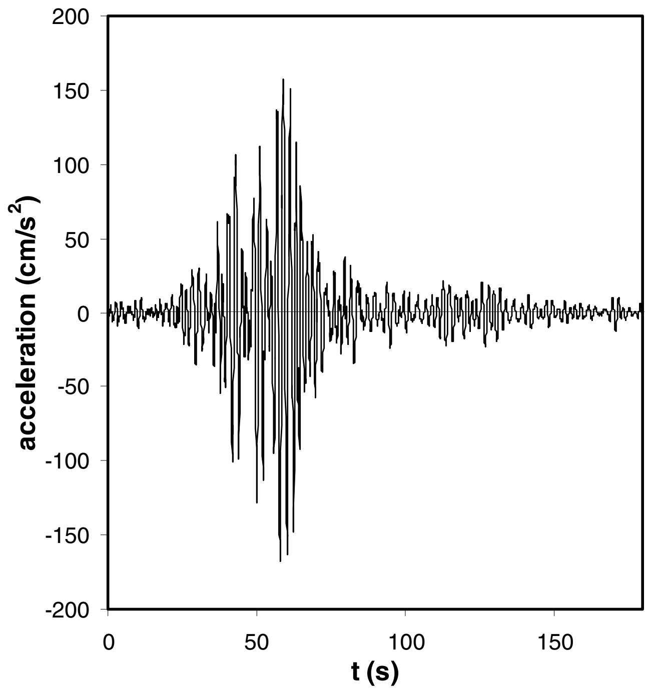


Figure 3. SCT record; EW component

Table 1. Properties of linear equivalent systems

Levels	c	T_i(s)	T_f(s)	ξ_h	ξ	NIC
5	0.1	0.584	0.837	0.230	0.280	5
	0.2	0.584	0.681	0.136	0.186	6
	0.3	0.584	0.645	0.097	0.147	3
	0.4	0.584	0.610	0.043	0.093	3
	0.5	0.584	0.589	0.010	0.060	3
	0.6	0.584	0.584	0.000	0.050	4
10	0.1	1.002	2.381	0.382	0.432	7
	0.2	1.002	2.067	0.387	0.437	10
	0.3	1.002	1.664	0.345	0.395	9
	0.4	1.002	1.010	0.009	0.059	4
	0.5	1.002	1.002	0.000	0.050	4
	0.6	1.002	1.002	0.000	0.050	2
15	0.1	1.414	3.025	0.381	0.431	4
	0.2	1.414	2.543	0.380	0.430	4
	0.3	1.414	2.173	0.339	0.389	3
	0.4	1.414	1.923	0.280	0.330	4

The computer program DRAIN-2D (Powell [12]) was used to perform the step-by-step analysis. Three different models were adopted to represent the nonlinear hysteretic behavior of the structural members (Figure 4): Bilinear (B), Takeda (T) and Strength-and-Stiffness Degrading (SSD).

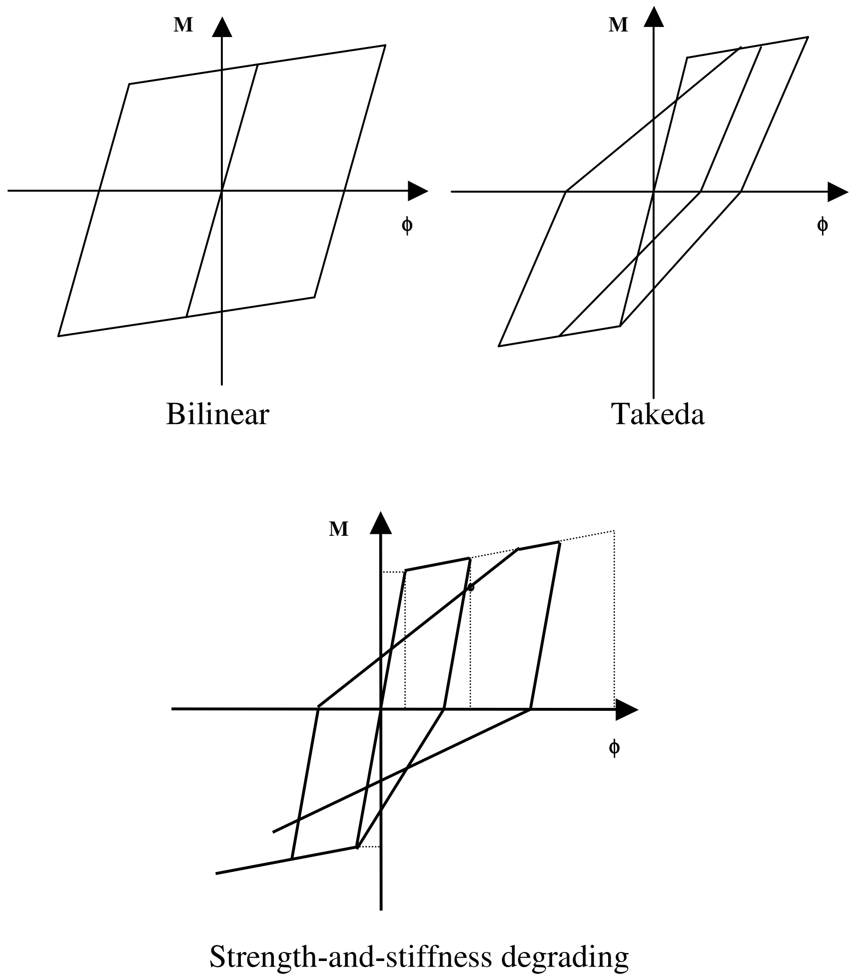


Figure 4. Moment-curvature constitutive functions

The values of the peak values of the roof displacement relative to the base, calculated with the various methods mentioned above, are shown in Table 2. Story distortions are obtained dividing the peak values of the relative story displacements (between consecutive floors) by the corresponding story heights. Figures 5-7 show the variation of the story distortions along the building height for the case of bilinear behavior according to the SBS analysis, as well as those estimated with the aid of the LES. Five, ten and fifteen story buildings are included, as well as the three values of the seismic design coefficient *c* studied.

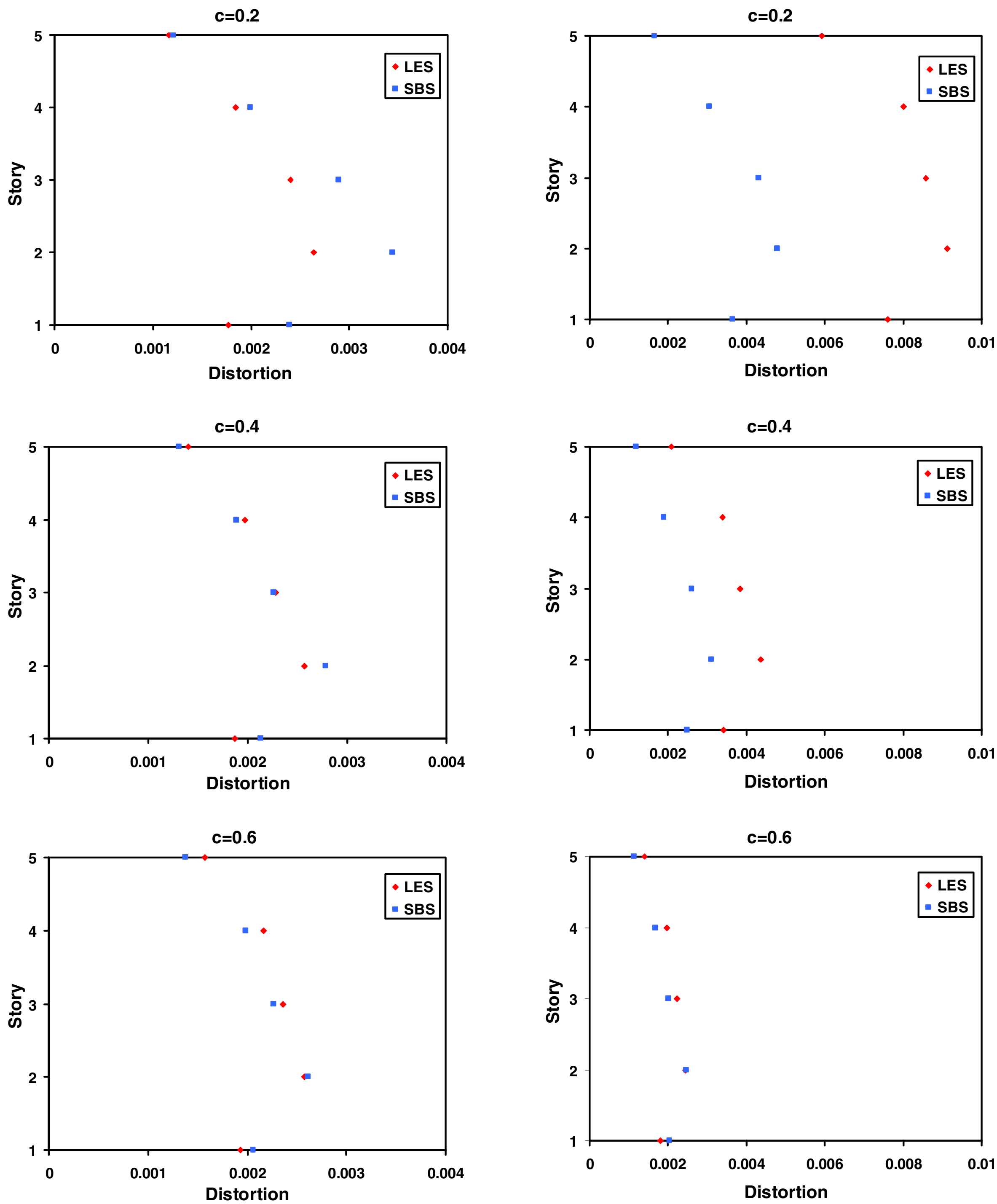
Table 2. Peak values of the roof displacement. Response to a single ground motion time history (SCT)

				SBS (cm)	
Levels	c	LES (cm)	B	T	SSD
5	0.1	4.360	3.291	3.7450	4.6527
	0.2	2.833	3.510	3.4343	3.5853
	0.3	2.875	3.177	3.1678	3.2702
	0.4	2.930	3.052	3.0527	3.1314
	0.5	3.024	3.030	3.0298	3.0758
	0.6	3.078	3.030	3.0298	3.0764
10	0.1	33.867	30.994	59.1299	45.1714
	0.2	28.532	19.924	24.6156	38.3778
	0.3	21.267	10.523	12.4543	16.3436
	0.4	8.385	8.379	8.3718	8.4153
	0.5	8.569	8.289	8.2888	8.2888
	0.6	8.569	8.289	8.2888	8.2888
15	0.1	37.356	41.508	112.0009	51.9172
	0.2	35.938	34.485	94.8466	54.9030
	0.3	34.407	31.314	72.1421	53.9159
	0.4	33.854	30.402	46.9685	49.4452

The results show that for the five story frames the roof displacement estimated by means of the simplified method is very similar to that obtained by the SBS analysis; this is true for the three alternative models assumed to represent the behavior of the structural members. The largest differences are observed for the case c = 0.1. For the ten story systems the LES model leads to a good approximation for low levels of nonlinear response (nearly linear response); that is, for high values of c. However, larger discrepancies are observed for the Takeda and SSD models. For the fifteen story systems with bilinear behavior, the LES method, again, leads to response amplitude values similar to those given by the SBS analysis. However, significant differences are observed for the cases of Takeda and SSD nonlinear behavior models.

Response to a sample of simulated ground motion time histories

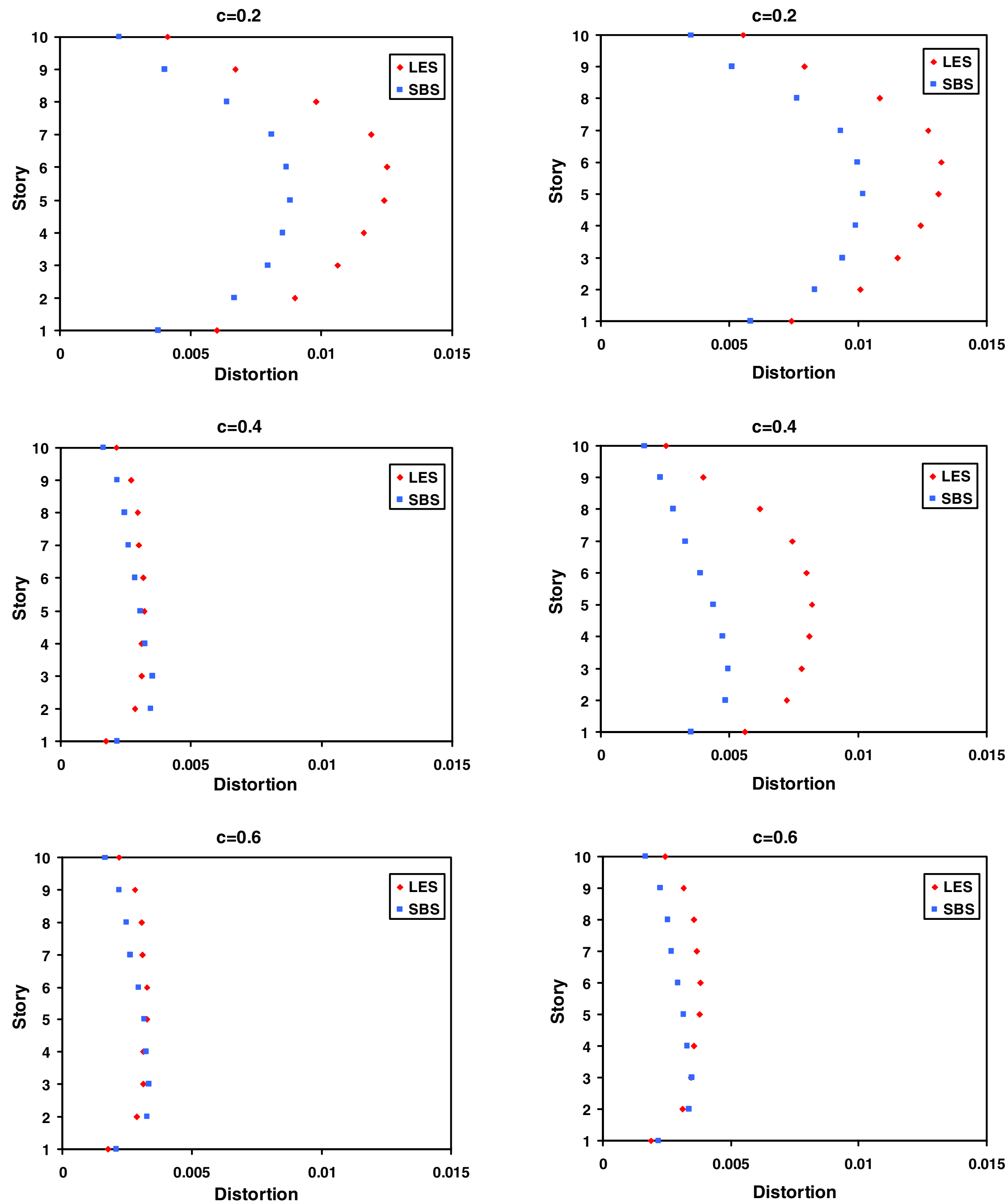
The proposed linear-equivalent-system method was applied to study the dynamic responses to a sample of simulated ground motion records with statistical properties similar to those of the SCT record used in the preceding sections (Grigoriu[13]). The sample studied included three records. As for the first part of the study, the computer program DRAIN-2D (Powell[12]) was applied. A hysteretic bilinear model was adopted to represent the behavior of the structural members.



a) SCT record

b) Sample of simulated records

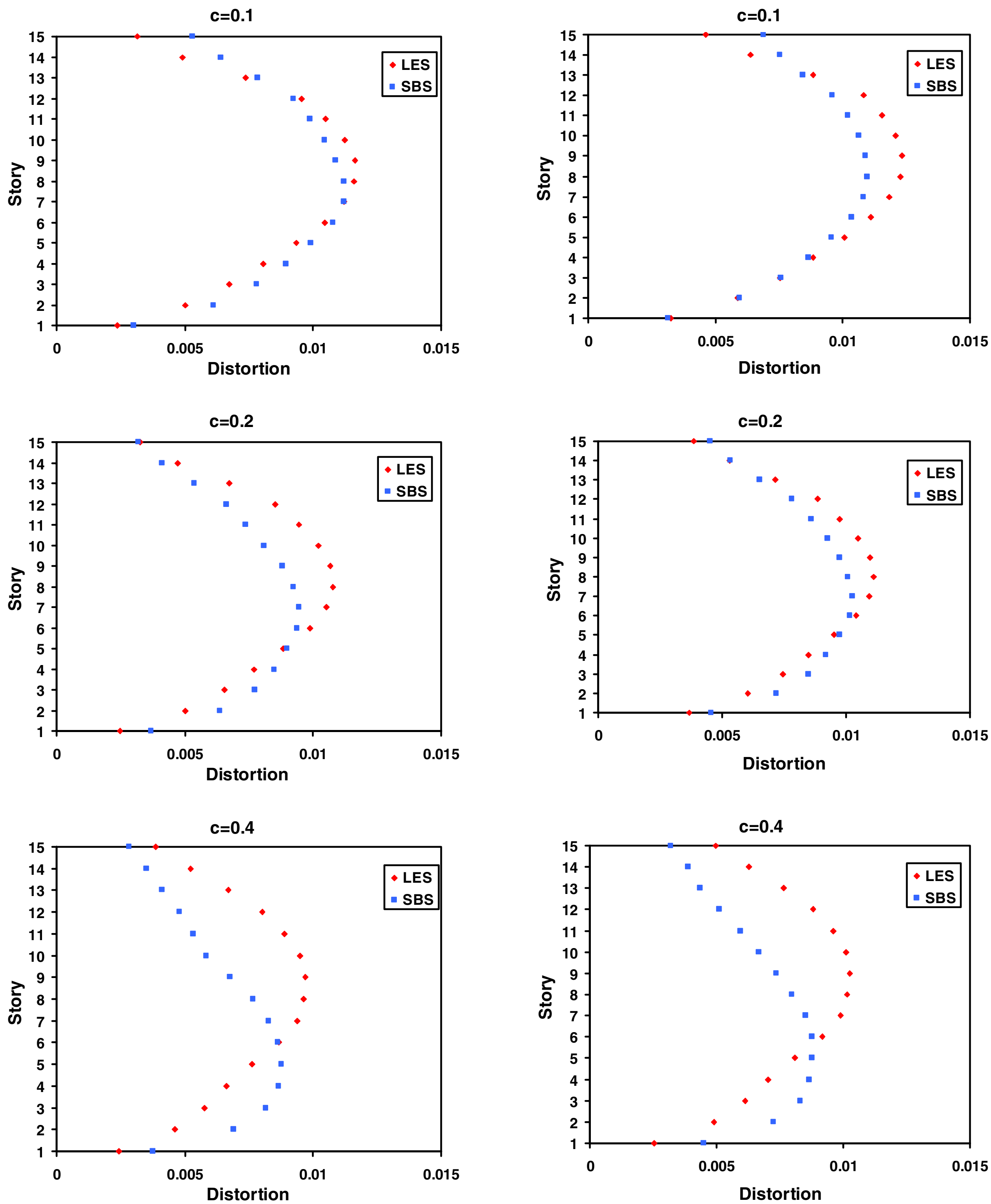
Figure 5. Story distortions; five story frames



a) SCT record

b) Sample of simulated records

Figure 6. Story distortions; ten story frames



a) SCT record

b) Sample of simulated records

Figure 7. Story distortions; fifteen story frames

Table 3 shows values of the sample mean values of the roof displacements for the systems studied. For the five story systems, the LES method leads to response amplitudes that are larger than those resulting from the SBS analysis, mainly for systems designed with low *c* values, for which the levels of nonlinearity in the response are higher. As *c* grows, the differences decrease. For ten story systems, the LES method also overestimates the response, but the differences are smaller than for the five story cases. For the fifteen story systems the discrepancies between the responses predicted by the two alternative approaches considered are much smaller. For this case, the LES method underestimates responses; this trend is reversed as the design coefficient *c* increases.

Tabla 3. Peak values of the roof displacement. Responses to a set of simulated ground motion time histories.

Levels	c	LES (cm)	SBS(cm)
5	0.1	15.630	6.838
	0.2	11.512	5.141
	0.3	7.919	3.976
	0.4	5.057	3.330
	0.5	3.092	2.897
	0.6	2.850	2.758
10	0.1	35.96788	31.41458
	0.2	31.26103	24.32401
	0.3	25.15417	16.13015
	0.4	19.75139	11.27826
	0.5	13.55119	9.101638
	0.6	9.709812	8.495631
15	0.1	40.59433	42.24205
	0.2	38.05348	39.21225
	0.3	36.8013	35.22075
	0.4	36.29494	32.0895

For the ten and fifteen story systems, the mean values of the roof displacement amplitudes predicted by the LES and SBS methods for the sample of ground motion records considered are similar to the values that correspond to the SCT acceleration time history. For the five story systems the differences are larger, with the sample mean values being greater than those obtained for the SCT record. This trend to overestimate response amplitudes shown by the LES model may be associated with the criterion used to estimate damping, which seems to have a more significant effect on the low period structures for the type of ground motion time histories used in this study.

Story distortions

The variation of the story distortions along the building height is shown in Figures 5-7 for the systems studied, for the two cases of excitations considered: the SCT record and the sample of thirteen simulated acceleration time histories.

For the five story systems (Figure 5) and low values of *c*, story distortions resulting from application of the LES method are significantly greater than those given by the SBS method; this is true for both cases: the SCT record (Figure 5a) and the sample of simulated time histories (Figure 5b). As *c* grows, the results of both methods of analysis become closer for the case of the SCT record. The discrepancies remain for the sample of simulated records, but they decrease significantly for the highest values of *c*, corresponding to linear or almost linear responses for all the simulated records.

For the ten story frames and the SCT record (Figure 6a), the LES method underestimates the story distortions. For the high values of the design coefficient, both methods (LES and SBS) give similar results. For the family of simulated records (Figure 6b) and low values of *c*, the mean values of the story distortions estimated by means of the LES method are significantly higher than those given by the SBS method. As expected, the differences between the predictions of both methods are smaller for high values of *c*.

For the fifteen story systems (Figure 7), the results are very similar both for the SCT earthquake record (Figure 7a) and the sample of simulated ground motion time histories (Figure 7b). For this set of systems, the responses obtained with the aid of the LES model are somewhat smaller than those predicted by the SBS analysis for low values of *c*. The differences increase with *c*; as this coefficient grows, the LES method overestimates the responses of the upper stories and underestimates those of the lower portion of the buildings.

For the five and ten story systems, the forms of variation and the amplitudes of the story distortions show that for low values of *c* the LES leads to response estimates greater than those resulting from the SBS analysis. This may be due to the method used to estimate the equivalent damping value; the differences in the forma of variation and amplitudes mentioned above tend to disappear as *c* grows.

In general, the mean values of the responses calculated for the sample of thirteen simulated ground motion time histories are greater than the response corresponding to the SCT record; however, the forms of variation of the story distortions look similar in practically all cases.

CONCLUSIONS

In general, the accuracy of the results given by the equivalent linearization method proposed in this study varies with the characteristics of both the systems studied and the ground motion excitations. The assumptions made regarding the method used to estimate the properties of the linear equivalent system (stiffness and damping) have a significant influence on the results. Better results might be obtained using an improved method to estimate the equivalent damping. Also, the assumption of representing the systems studied as shear buildings for the purpose of estimating the equivalent stiffness matrix at each step in the iterative process may affect the along-height variation of the story distortions obtained. These concepts should be the objective of new studies in the near future.

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