



## SEISMIC VULNERABILITY ANALYSIS FOR OPTIMUM DESIGN OF MULTISTORY REINFORCED CONCRETE BUILDINGS

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### SUMMARY

A method is presented for the estimation of the optimum values of the strength and stiffness properties of reinforced concrete structural systems. For this purpose, use is made of seismic hazard and vulnerability functions for the site considered and for the types of systems of interest. The vulnerability functions are obtained by means of a probabilistic model based on a simplified reference system (SRS) associated with the detailed model under study. The method is applied to a ten story wall-frame system. The properties (strength and stiffness) that result from the optimization analysis are transformed into performance-based acceptance criteria for practical design conditions.

### INTRODUCTION

Earthquake resistant design aims at attaining an optimum balance between construction and maintenance costs, on one side, and acceptable risk levels for system failure or ill performance during the system's lifetime, on the other. In performance-based design, design acceptance criteria must be expressed in terms of the allowable damage levels and their consequences on the safety and serviceability requirements under the action of earthquakes with intensities corresponding to given return intervals at the site of interest. Hence the expected damage levels can be transformed into performance levels.

In order to apply the mentioned criteria to the practice of earthquake resistant design, the performance levels must be expressed in terms of quantitative indicators of structural response and capacity to be controlled by the designer. Lateral distortions and relative displacements are among the simplest of such indicators [1]. Their estimation under current practical design conditions is based on the availability of simplified methods for the analysis of the nonlinear response of multi-degree-of-freedom systems that account for the influence of the along-height distribution of lateral strength and stiffness [2]. The use of simplified reference systems (SRS) provides a reasonable alternative for this purpose. Their use must be complemented by the application of adequate uncertainty factors that represent the statistical errors between the responses that would be given by detailed system models and those obtained by means of SRS.

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A reliability and performance-based method is proposed in this paper that can be used to formulate optimum seismic design criteria. Two control variables are used: lateral strength and stiffness; the former is strongly dependent on the expected consequences of ultimate failure, in case it occurs, while the latter has a stronger correlation with the expected damage under survival conditions; however, both performance requirements depend on both sets of mechanical properties and their interaction is taken into account in the present study.

### OPTIMIZATION CRITERIA

The following control variables are adopted in the following:  $c$ , the base-shear ratio that corresponds to  $y_R$ , the design intensity for the ultimate-capacity performance requirement, and  $\psi_{max}$ , the allowable value of the lateral story distortion under the action of the intensity  $y_S$  specified for the serviceability limit state performance requirement. Both intensities are expressed in terms of the pseudo-acceleration linear response spectrum for the nominal value of the fundamental period of the system,  $T$ .

The optimization criterion proposed here is based on the determination of the values of  $c$  and  $T$  that minimize the following objective function [3]:

$$U(c, T) = C_o(c, T) + \frac{D(c, T)}{\gamma} \quad (1)$$

In this equation,  $C_o(c, T)$  is the initial construction cost;  $\gamma$  is a net discount rate, which transforms future benefits and costs into their present values, and  $D(c, T)$  is the expected value of the costs of damage and failure per unit time. The latter term is calculated as the sum of  $\Delta_S$  and  $\Delta_F$ , which represent the contributions of the expected damage for the survival and failure conditions, respectively [4]:

$$D = \Delta C_o = (\Delta_S + \Delta_F)C_o \quad (2)$$

Here,

$$\Delta_S = \int \left| \frac{dv_Y(y)}{dy} \right| \delta(y|S)(1 - p_F(y)) dy \quad (3)$$

$$\Delta_F = \int \left| \frac{dv_Y(y)}{dy} \right| \delta_F p_F(y) dy \quad (4)$$

In these equations,  $\delta(y|S)C_o$  is the expected cost of damage as a function of intensity, conditional to the survival of the system;  $\delta_F C_o$  is the expected cost of the consequences of failure;  $p_F(y)$  is the probability of ultimate failure under the action of an earthquake with intensity  $y$ , and  $v_Y(y)$  is the annual rate of occurrence of earthquakes with intensities greater than  $y$  at the site of interest.

### ESTIMATING NONLINEAR RESPONSES BY MEANS OF THE SRS

The seismic nonlinear response of a multi-degree-of-freedom (MDOF) system can be estimated by means of an associated SRS capable of representing the most relevant dynamic properties of the former. This implies establishing relations linking the peak values of the response amplitudes of both systems by means of uncertain transformation factors, with statistical properties determined by calibration with the results of the detailed models studied [5].

In this paper, the SRS is characterized by means of a single-degree-of-freedom (SDOF) system, similar to that used by Esteva *et al* [6]. The relations between the mechanical properties of both systems, as well as between their corresponding responses have been presented by Esteva [7].

### Response transformation factors

The following variables are used to account for the uncertainties associated with the estimation of peak response values by means of the SRS's:

$$\rho = \frac{\psi}{\psi_o}; \alpha_s = \frac{S_d(Q)}{\bar{S}_d(Q)}; \rho_i = \frac{\varepsilon_i}{\alpha_i \psi}; \alpha_i = \frac{\varepsilon_{oi}}{\psi_o} \quad (5a, b, c, d)$$

In these equations,  $\psi$  is the peak value of the global distortion of the MDOF system,  $\psi_o$  its value estimated by means of the SRS;  $\alpha_s$  the ratio between the nonlinear response of the SRS and its expected value, as a function of  $Q$ , the ratio of the linear response spectral ordinate  $S_{dL}(Q)$  to the yield displacement  $u_y$  for the SRS;  $\varepsilon_i$  is the local response of interest obtained by means of the MDOF model and  $\alpha_i$  is a deterministic value that represents the ratio between the value of that local response and that resulting from the deformed configurations given by the pushover analysis.

The following functions are used to describe the mean values and variation coefficients of the response transformation factors defined in accordance with Equations 5a-c [7]:

$$V_{\alpha_s}^2 = a(Q-1) + b(1 - \exp(-c(Q-1))) \quad (6a)$$

$$v(Q) = a + b(Q-1) + c(1 - \exp(-d(Q-1))) \quad \text{for } Q > 1 \quad (6b)$$

$$v(Q) = a \quad \text{for } Q \leq 1 \quad (6c)$$

By definition, the expected value of  $\alpha_s$  is unity and the square of its variation coefficient is given by Eq. 6a. Equations 6b and c are used to represent random factors  $\rho$  and  $\rho_i$ , with  $v(Q)$  taken as a generic variable used to represent the expected values of those variables. Given the expected values, functions  $(v/\bar{v} - 1)^2$  are represented as functions of  $Q$ , which is taken as a normalized measure of intensity. The expected values of the resulting functions, fitted in accordance with Equations 6b-c, are equal to the squares of the variation coefficients of  $\rho$  and  $\rho_i$ .

Peak values of the responses of interest, both global and local, can be estimated by means of equations of the following form:

$$x_i = \rho \rho_i \alpha_s \eta_i \quad (7)$$

Here,  $\eta_i = \alpha_i \lambda \bar{S}_d / H$ , and  $\lambda$  is the participation factor obtained for the SRS. It is assumed that  $\rho$ ,  $\rho_i$  and  $\alpha_s$  are mutually independent random variables; therefore, the expected value and the square of the variation coefficient of the local response of interest are given as follows:

$$\bar{x}_i = \bar{\rho} \bar{\rho}_i \bar{\alpha}_s \eta_i \quad (8)$$

$$V_{x_i}^2 = (1 + V_{\rho}^2)(1 + V_{\rho_i}^2)(1 + V_{\alpha_s}^2) - 1 \quad (9)$$

### Uncertainties about structural properties and excitation

These uncertainties are taken into account in the step-by-step dynamic response analyses of the MDOF model and the SRS. Uncertainties about structural properties are incorporated through Monte Carlo simulation, as proposed by Alamilla [8]. These include those associated with gravitational loads (both dead and live), with the geometrical properties of structural members, the ratio of longitudinal reinforcement and the mechanical properties of concrete and steel. The seismic excitation is represented by families of artificial ground motion time histories, simulated in accordance with an algorithm developed by Alamilla *et al* [9, 10]. The statistical properties of those families are representative of those observed on natural records obtained at soft-soil site SCT, in the Valley of Mexico [11]. For this purpose, the intensity of each artificial record ( $y$ ) is expressed by means of the maximum ordinate of the linear pseudo-acceleration response spectrum for a damping ratio of 0.05.

## DAMAGE FUNCTIONS

### Damage functions for the MDOF system

For the wall-frame systems considered in this study, the total physical damage at any story is obtained as the sum of those affecting the shear wall, the frame and the beams connecting both subsystems [4]. The following functions proposed by Esteva *et al* [3] are used for this purpose:

$$d(u) = 1 - \exp(-au^m) \quad (10)$$

In this equation,  $a$  and  $m$  are parameters to be determined, and  $u$  is the local deformation of interest, normalized with respect to its peak value at failure (total loss). Damage functions for the frame and the infill walls are obtained as functions of the corresponding story distortions. The latter are taken here as light aggregate concrete panels confined by light gage steel members, for which the initiation of damage and total loss are associated with story distortions of 0.004 and 0.008, respectively [12].

For the girders connecting the frame and the shear wall, the damage function is made to depend on the angular distortion associated with the seismic response. In this paper, this distortion is obtained in an approximate manner, making use of some response parameters obtained from the step-by-step response analysis [4].

For the shear wall, the damage function makes use of an index that is calculated in terms of two components: the shear distortion and the bending curvature. This function is given as follows:

$$d_{wi}(u) = d_V(\psi_i - \bar{\theta}_i) + d_M\left(\frac{(\theta_i - \theta_{i-1})}{h_i}\right) \quad (11)$$

In this equation,  $d_V(\cdot)$  and  $d_M(\cdot)$  are the damage functions associated with shear and bending, respectively, as given by Equation 10;  $\psi_i$  is the lateral distortion of the  $i$ -th story,  $\theta_i$  and  $\theta_{i-1}$  the bending rotations of the shear-wall cross sections at levels  $i$  and  $i - 1$ , respectively;  $\bar{\theta} = 0.5(\theta_i + \theta_{i-1})$  is the mean value of the story distortion due to bending at two consecutive floor levels and  $h_i$  is the height of the  $i$ -th story. The damage function due to wall bending depends on the local curvature; for its computation, the axial load acting on the wall is taken into account [4].

The superposition criterion used to evaluate the damage function on the shear wall,  $d_w(\cdot)$ , must comply with the conditions required for the damage indicator, that is,  $0 \leq d \leq 1.0$ . Thus, the damage function can be expressed as

$$d_w(\mu) = 1 - 0.25(2 - \mu)^2 \quad (12)$$

where  $\mu$  is the sum of the physical damage functions values for shear and bending:  $\mu = d_v(\cdot) + d_M(\cdot)$ .

### Expected damage functions in terms of the SRS

In order to determine the expected damage functions in terms of the SRS, use is made of a two-point estimator proposed by Rosenblueth [13]:

$$\bar{g}_i(x_i | y) = \frac{1}{2} [g_i(\bar{x}_i(1 + V_{x_i})) + g_i(\bar{x}_i(1 - V_{x_i}))] \quad (13)$$

According to this equation, the expected damage function depends on the first two statistical moments of the response amplitude  $x$ , given the intensity  $y$ .  $\bar{x}_i$  and  $V_{x_i}$  designate respectively the expected value and the variation coefficient of that response (Eqs. 8 and 9); subscript  $i$  designates the story for which the damage function is evaluated. For the cases of the frame and the infill walls,  $x$  is a lateral story distortion; for the linking girders,  $x$  designates a transverse distortion; for the shear walls, it corresponds to the angular distortion resulting from the superposition of both bending and shear deformations.

### Expected damage costs as functions of intensity

For any element in the system considered, the expected cost of damage as function of intensity, conditional to the event that the structure does not fail, is evaluated in accordance with the following equation, proposed by Ismael [4]:

$$\delta(y | S) = \frac{1}{C_0} \left( 1 + \frac{r_I}{c} \right) c [\delta_{LG} C_{0LG} + \delta_{FR} C_{0FR} + \delta_{SW} C_{0SW} + \delta_{IW} C_{0IW}] \quad (14)$$

Here,  $\delta_{XX}$  and  $C_{0XX}$  are respectively the expected value of the indicator of physical damage, conditional to the survival of the system, and the initial cost of the element considered. Subscripts  $XX$  identify that element:  $LG$  designates the linking girder,  $FR$  the frame,  $SW$  the shear wall and  $IW$  the infill walls. Also,  $r_I$  designates the ratio of indirect to direct costs. Its value depends on the type of construction; here it is assumed as 1.5. The value of  $c$  is given by equation 15, which depends on the values of the expected damage indicators,  $\delta_{XX}$ , and on a factor  $\alpha$ , greater than zero, which takes into account a fixed initial cost attached to repair costs, associated to the logistic arrangements that have to be implemented before the actual repair work starts. Here,  $\delta = 1.5$ .

$$c = \alpha - \frac{\alpha - 1.2}{4} [\delta_{LG} + \delta_{FR} + \delta_{SW} + \delta_{IW}] \quad (15)$$

The initial cost  $C_0$  is approximately estimated in terms of the volumes of materials and amount of labor involved in the construction of a system defined through a preliminary design [6].

## RELIABILITY FUNCTIONS

### Estimation of the reliability functions in terms of the secant-stiffness-reduction index (SSRI)

A method proposed by Esteva [14] is applied in this paper to estimate the reliability of a MDOF system with respect to collapse, with the aid of its associated SRS. For this purpose, the following damage index is defined:

$$I_D = \frac{(K_0 - K)}{K_0} \quad (16)$$

Here,  $K = V_b/\psi H$  is defined as the reduced lateral secant stiffness for a nonlinear system, evaluated at the instant where the global distortion  $\psi$  produced by a seismic excitation reaches its peak absolute value;  $V_b$  is the base shear that occurs at the same instant,  $H$  the height of the system, and  $K_0$  is the value that would be obtained for  $K$  under conditions of linear response. The latter can be obtained by pushover analysis of the MDOF system under study.

Using this information, the collapse condition is assumed to be reached when  $I_D = 1$ . For simplicity, a variable  $Z = \ln I_D$  is introduced, so that the collapse condition corresponds to  $Z = 0$ . An auxiliary variable,  $U$ , is also introduced, such that  $Z = U$  for  $Z = 0$  and  $Z = 0$  for  $U = 0$ . The probability density function of  $U$  is taken as Gaussian, with mean  $m_U$  and standard deviation  $\sigma_U$ . These parameters are dealt with as functions of the peak ductility,  $\mu_0$ , determined from the response of the SRS. These functions are expressed in the following forms:

$$m_U(\mu_0) = a + b \ln \mu_0; \quad \sigma_U(\mu_0) = c + d \ln \mu_0 \quad (17a, b)$$

In these equations,  $a$ ,  $b$ ,  $c$  and  $d$  constitute a vector  $\alpha$  of parameters to be determined in accordance with a maximum likelihood criterion. For this case, the likelihood function adopts the following form:

$$L(\alpha) = \prod_{i=1}^{n_s} \varphi\left(\frac{z_i - m_U(\mu_{0i}|\alpha)}{\sigma_U(\mu_{0i}|\alpha)}\right) \prod_{j=n_s+1}^n \left[1 - \Phi\left(-\frac{m_U(\mu_{0j}|\alpha)}{\sigma_U(\mu_{0j}|\alpha)}\right)\right] \quad (18)$$

Here,  $\varphi(\cdot)$  and  $\Phi(\cdot)$  represent, respectively, the Gaussian standard probability density and cumulative distribution functions. For the evaluation of Eq. 18, a set of  $n$  pairs of values of  $\mu_{0i}$  and  $Z = z_i$  must be available; these values can be generated through dynamic response analysis for the MDOF system. For the purpose of obtaining the value of  $\alpha$  that maximizes the second member in Eq. 18, use was made of program Genesis, version 5.0, based on the use of genetic algorithms [15].

Once the vector of parameters  $\alpha = (a, b, c, d)$  that correspond to the maximum likelihood value have been determined, the seismic reliability for a system belonging to the family used for the establishment of the likelihood function  $L(\alpha)$  can be estimated as  $\Phi(-m_U(y)/\sigma_U(y))$ , where  $m_U(y)$  and  $\sigma_U(y)$  are obtained by means of the following equations, which are based on the properties of conditional probability distributions [16]:

$$m_U(y) = a + b m_{L\mu}(y) \quad (19a)$$

$$\sigma_U^2(y) = c^2 + 2cd m_{L\mu}(y) + d^2 E[\ln^2 \mu_0(y)] + b^2 \sigma_{L\mu}^2(y) \quad (19b)$$

The operator  $E[\cdot]$  that appears in Eq. 19b denotes expectation or expected value. In order to evaluate Equations 19a and b, it is necessary to count with parameters  $m_{L\mu}(y)$  and  $\sigma_{L\mu}^2(y)$ , which are the expected value and the variance of  $\ln \mu_0$ , obtained from the analysis of the dynamic responses of the SRS's to a set of earthquakes of different intensities. These parameters are obtained by least squares fitting. The functions proposed to represent them are as follows:

$$m_{L\mu}(y) = a_1 + b_1 y - c_1 y^{-n} \quad (20a)$$

$$\sigma_{L\mu}^2(y) = a_2 + b_2 y \quad (20b)$$

In these equations,  $a_1$ ,  $a_2$ ,  $b_1$ ,  $b_2$ ,  $c_1$  and  $n$  are fitting parameters that must satisfy the following conditions:  $a_2 > 0$ ,  $b_2 > 0$ ,  $n > 1$ .

## ILLUSTRATIVE EXAMPLE

### Description of the system

A ten story reinforced concrete wall-frame building is analyzed. The structure is regular in plan and elevation, and it is assumed to stand at a soft soil site in the lake zone in the Valley of Mexico. For simplicity, the effects of torsion and soil-structure interaction are neglected. Figure 1 shows the plan and elevation of the structural arrangement, as well as the structural model adopted to study the nonlinear dynamic response.

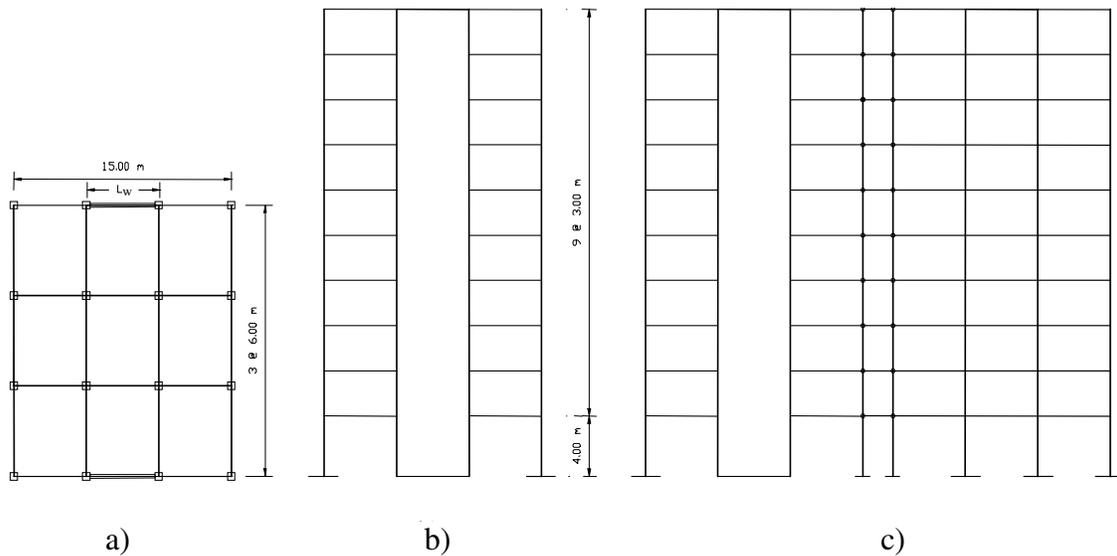


Figure 1. a) Plan distribution, b) Elevation, c) Model used for the response analysis

The starting point for the optimization analysis is a preliminary design of the structure in accordance with Mexico City Building Code [17] and its Complementary Technical Norms [18, 19]. The starting case is specified in terms of the corresponding values of  $c$  and  $T$ ; this system is taken as a reference for the family of alternatives to be considered. The starting system also serves to determine and calibrate the response transformation factors. It is also assumed that this system constitutes an approximation to the optimum; therefore, it is used to estimate the minimum cross section dimensions of its structural members needed to satisfy the stiffness and strength code requirements. In reinforced concrete members, those dimensions are determined by the maximum allowable values of the longitudinal steel reinforcement ratio. The restrictions relative to maximum allowable values of vertical deflections under the action of gravitational loads are taken into account to define the minimum acceptable cross section dimensions of beams.

A lateral force reduction factor of 4, specified in [18] for systems similar to that studied here, is used to account for nonlinear ductile behavior. A set of six alternative structures is assumed, each characterized by a combination of values  $c$ ,  $T$ . The natural period  $T$  is made to depend on the width  $L_w$  of the shear wall (see Figure 1a). The set of six systems determined in this manner include combinations of three values for

the base shear ratio  $c$  and two for the fundamental period  $T$ . The combination associated with the starting system is  $c = 0.1$  (base shear design coefficient, after reduction to account for overstrength and nonlinear behavior),  $T = 1.081$ s. Other values assumed for  $c$  and  $T$  are shown in Table 1. For their selection, it was assumed that  $T$  could not be shorter than its starting value; no limitations were imposed regarding possible values of  $c$ .

### Determination of utility functions

For each alternative structure considered, Equation 1 was applied, using the corresponding expected damage functions determined in accordance with Equation 14 and the reliability function determined as previously described. It was also necessary to use the hazard function  $v_Y(y)$  (rates of exceedance of given intensities) at the site of interest. The seismic vulnerability functions for the starting case are shown in Figures 2a, b.

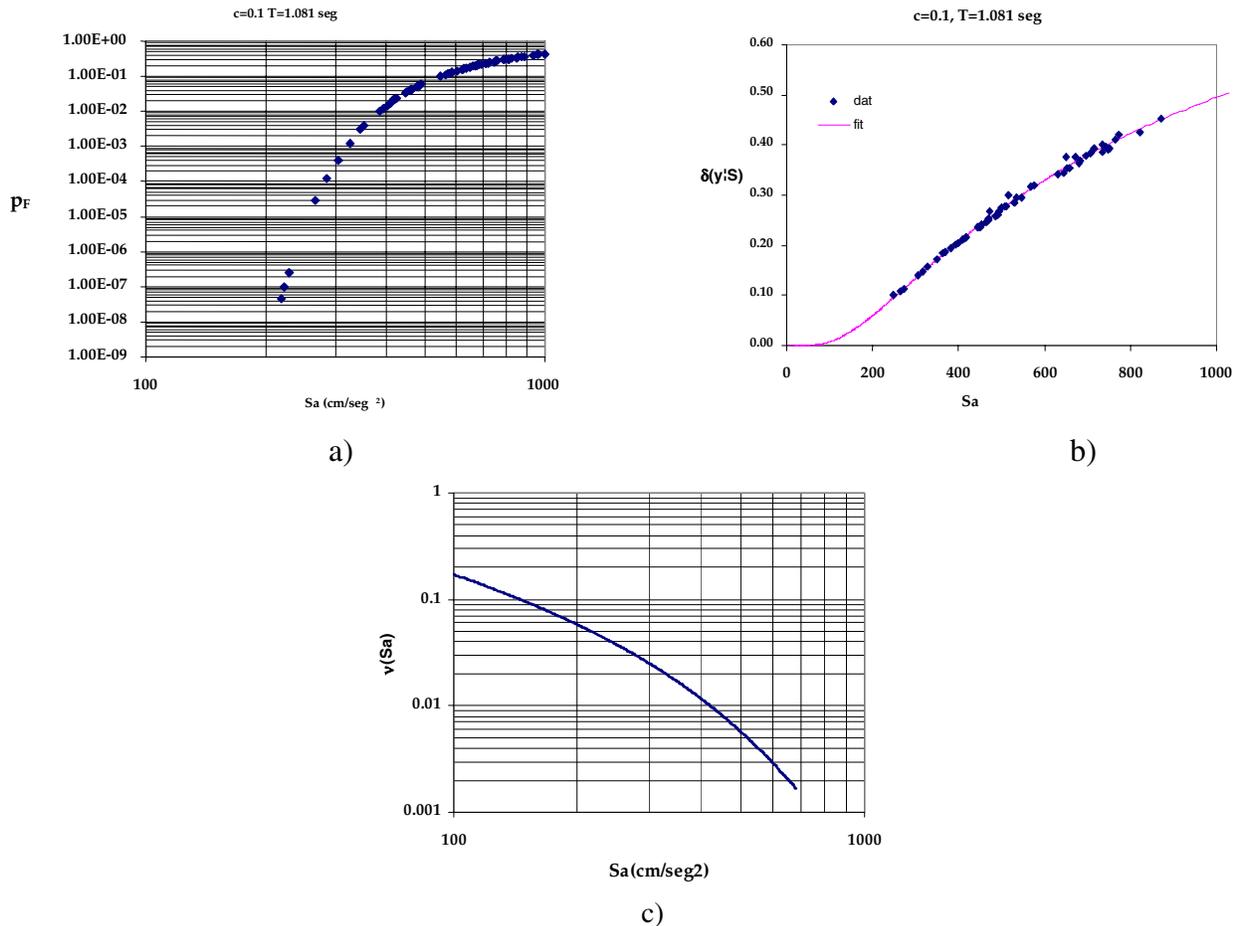


Figure 2. Vulnerability and seismic hazard functions for the starting system, a) Failure probability function, b) Expected damage cost function and c) Hazard function

The seismic hazard function presented in Figure 2c adopts as intensity measure the ordinate of the linear pseudo-acceleration response spectrum for the fundamental period of the system of interest [8]; in this case, the natural period of the SRS was used.

The values of the utility functions, and the terms that constitute them are summarized in Table 1. These values are graphically presented in Figure 3 for the sets of values of  $c$  and  $T$  considered.

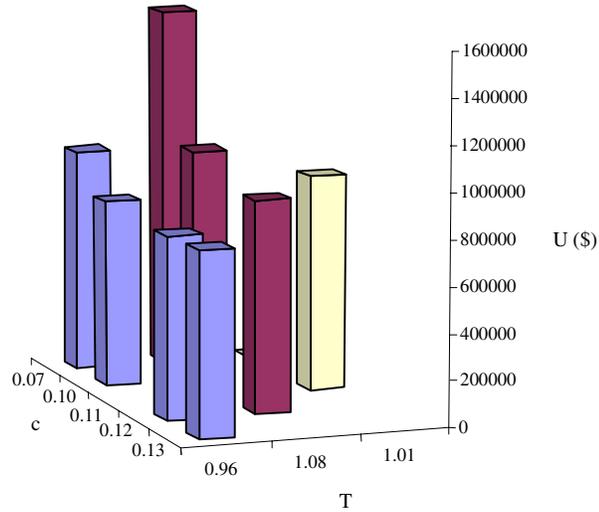


Figure 3. Values of the utility function  $U$  for the selected values of  $c$  and  $T$

Table 1. Utility function components for the cases studied

CASE	$c$	$T(\text{seg})$	$L_w(\text{cm})$	$\psi_{\max}$	$\Delta_S$	$\Delta_F$	$D=(\Delta_S+\Delta_F)*C_0$	$C_0(\$)$	$U=C_0+D/\gamma$
1	0.075	1.081	300	0.0074	0.0225392	0.0378029	428524.06	7101577.99	15672059.24
2	0.1	1.081	300	0.0098	0.0107687	0.0074180	134212.33	7379673.72	10063920.41
3	0.125	1.081	300	0.0122	0.0089106	0.0007907	74903.93	7721023.13	9219101.80
4	0.114	1.012	340	0.0076	0.0117518	0.0007194	93675.74	7511362.64	9384877.40
5	0.075	0.965	500	0.0037	0.0082972	0.0105687	132260.32	7010550.86	9655757.31
6	0.1	0.965	500	0.0050	0.0049949	0.0005419	40347.54	7287160.58	8094111.31
7	0.125	0.965	500	0.0062	0.0035985	0.0001013	27371.43	7398171.75	7945600.40
8	0.138	0.965	500	0.0069	0.0024061	0.0000358	18727.75	7669098.50	8043653.44

The results presented in Table 1 clearly show that, as expected, the initial cost  $C_0$  increases with the design base shear ratio  $c$ . In addition, a slight decrease in  $C_0$  is observed when  $L_w$  is increased from 3.00m to 5.00m. In general,  $\Delta_S$  is greater than  $\Delta_F$ ; both grow directly with  $T$  and inversely with  $c$ . For  $c = 0.075$ ,  $\Delta_F$  is greater than  $\Delta_S$ . This can be ascribed to the large reduction in the reliability function that appears when  $c$  reaches such a low value without similar reductions in the lateral stiffness of the system.

### Selection of the optimum system

Two alternative criteria were explored to obtain the optimum value of the utility function:

#### Criterion A

This is based on the adoption of a utility function of the form

$$U(c, T) = a_0 + a_1c + a_2T + a_3c^2 + a_4T^2 + a_5cT \quad (21)$$

Here,  $a_i$ ,  $i = 0, 1, \dots, 5$  are parameters to be determined by means of a nonlinear regression analysis. Using this criterion, once these values are determined on the basis of a set of values of  $U$  for a set of values of  $c$  and  $T$ , a second degree two dimensional function of  $U$  in terms of  $c$  and  $T$  will be available, which can be differentiated with respect to these two variables in order to establish the optimality conditions, which can be easily solved in closed form.

### Criterion B

This criterion makes use of an iterative approach based on Equation 1. This permits to obtain values of  $U$  for cases close to the minimum value obtained before. This follows from the consideration that the behavior of  $U$  with respect to the independent control variables can be extrapolated within a small interval in the vicinity of a previously assumed point.

For the purpose of illustration, in this paper both criteria described above are applied. In order to apply Equation 21, the six values initially obtained for  $U$  are used to determine the set of parameters  $a_0$  to  $a_5$ . These parameters are substituted in Eq. 21, in order to obtain the minimum value of  $U$  by equating to zero its partial derivatives with respect to  $c$  and  $T$ . This leads to  $c = 0.114$  and  $T = 1.012s$ , which correspond to Case 4 in Table 1. In order to assess the accuracy associated with this criterion, these values are used to make a direct evaluation of  $U$  by means of Eq.1. The results are presented in Table 1 and shown graphically in Figure 4. It is observed that, for the values of  $c$  and  $T$  used to fit Equation 21, the values of  $U$  given by the latter are in reasonable agreement with those given by Equation 1. However, this does not occur for the case given as optimum in accordance with Criterion A. Criterion B was then applied. It was observed that  $U$  is much lower for the cases in Table 1 for which  $L_w = 5.00$ . Therefore, only these cases will be considered as candidates for optimum in the sequel. The fact that  $U$  was decreasing with increasing  $c$  (cases 5-7, with  $c$  varying from 0.075 to 0.125), suggested the idea of exploring Case 8 ( $c = 0.138$ ,  $T = 0.965$ ); however, as seen in Table 1, the value of  $U$  resulted greater than for Case 7 ( $c = 0.125$ ,  $T = 0.965$ ), which was then taken to correspond to the optimum.

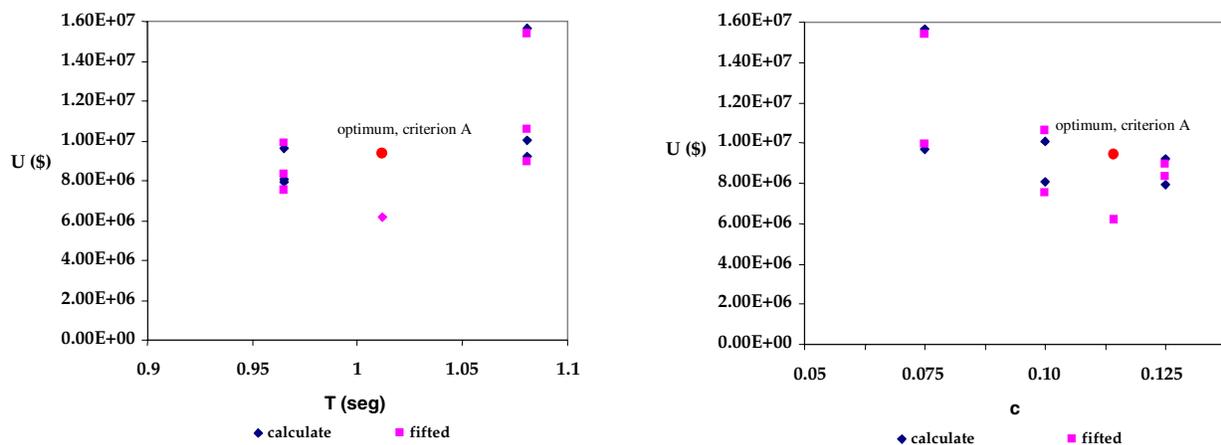


Figure 4. Influence of control variables on the utility function, according to Criterion A.

### Transforming the results of the optimization study into design parameters

Two alternative approaches to the optimization analysis are identified, according to the parameters that are taken as independent variables for the purpose of minimizing the objective function [7]. These alternatives are: a) taking the mechanical properties of the structural members as independent variables, and b) indirectly controlling those properties through the lateral strength requirements for the system and the limitations imposed on the acceptable values of lateral distortions produced by earthquake intensities associated with specified return intervals. Alternative b) is adopted here, because it leads to simpler design rules for engineering practice.

As mentioned above, for the case studied the optimum solution corresponds to the combination  $c = 0.125$ ,  $T = 0.965s$ . According to the design code adopted (Mexico City design code of 1993), for the type of system considered this corresponds to reduction factor of 4 to account for overstrength and nonlinear

behavior. The unreduced value of  $c$  would therefore equal 0.5, which corresponds to an ordinate of the linear pseudo acceleration response spectrum,  $S_a(T)$ , equal to  $490.5 \text{ cm/s}^2$ , which corresponds in turn to a return interval of 485 years. In order to obtain the design requirements for the serviceability condition that would lead to the value of  $T = 0.965\text{s}$ , determined as optimum, the lateral stiffness values corresponding to this value of  $T$  were assumed to calculate the peak values of story distortions for linear response for the intensities associated with different return intervals. This led to suggesting the adoption of acceptable values of peak story distortions equal to 0.0014, 0.0020 and 0.0025 for the intensities corresponding to return intervals of 10, 20 and 30 years, respectively.

## CONCLUSIONS

A method has been presented to obtain optimum values of the mechanical properties of reinforced concrete wall-frame systems. Two control variables are used for this purpose: the required base shear resistance ratio and the fundamental period of the system. For the case studied here, the base shear ratio was observed to show a much stronger influence on the utility function than the natural period. Use of the SRS to estimate peak values of dynamic responses of MDOF systems permits considerable reductions in the amount of computational work required for the optimization analysis. For the same case, the results show that current design regulations included in the normative documents applied lead to designs that are near optimum. Finally, the method presented here can be easily applied to other types of structural systems.

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