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STRENGTH REDUCTION FACTORS FOR THE VALLEY OF MEXICO, CONSIDERING LOW-CYCLE FATIGUE EFFECTS

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SUMMARY

The influence of low-cycle fatigue in the ductility capacity of single degree of freedom systems with elasto-plastic flexural behavior is evaluated. The systems are subjected to moderate and intense ground motions recorded in the Valley of Mexico. The ductility capacity is obtained using the concept of equivalent ductility proposed by P. Fajfar. The damage is measured by means of the Park and Ang index. Expressions for the equivalent ductility as function of the period and of the ductility under monotonic loading are obtained for four zones in the Valley of Mexico. Finally, correction factors are proposed that take into account the low-cycle fatigue effects which are applicable to strength reductions factors.

INTRODUCTION

Seismic design criteria normally used are in general based on the strength-design approach. This does not take into account explicitly the cumulative damage that takes place on structures subjected to long duration and strong ground motions, in spite of the fact that this phenomenon can be significant in structural design.

When the structures are subjected to very intense motions, their structural elements may suffer deterioration of their mechanical and dynamical properties, due to the cumulative plastic demands. This phenomenon is known as low-cycle fatigue. Due to this phenomenon, the structural ductility capacity can be reduced. One way to take into account the strength reduction in the structural design is by limiting the ductility that the structure is permitted to develop. Fajfar [1] has proposed an equivalent ductility that takes into account the cumulative damage on the structural elements. Such damage can be measured by means of different models. In the present paper the Park and Ang model [2] is used.

The objective of this study is to obtain equivalent ductility values and strength reduction factors for motions recorded in the valley of Mexico. Here the cumulative damage is considered through the

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equivalent ductility definition suggested by Fajfar[1]. The single degree of freedom systems analysed are supposed to have elastoplastic flexural behaviour.

Model based on Park and Ang Index for reinforced concrete structures

The Park and Ang Index [2] is defined as follows:

$$I_D = \frac{D_d}{D} + \beta \frac{E_H}{F_y D} = \frac{\mu_d}{\mu} + \beta \frac{E_H}{F_y D_y \mu} \quad (1)$$

where D_d represents the maximum displacement developed by the structure, D_y is the yield displacement, D is the ultimate displacement under monotonic loading, μ is the ultimate ductility under monotonic loading, μ_d is the ductility developed by the structure, F_y represents the yield force in the structure, E_H is the dissipated hysteretic energy, and β is a parameter that depends on the structural characteristics.

When the Park and Ang damage index (I_D) is less than 0.4, the damage may be considered to be repairable. Values of I_D larger than 0.4 but smaller than unity represent damage beyond repair, while values larger than or equal to unity represent total collapse of the system. Cosenza et al [3] found that a value $\beta = 0.15$ correlates closely with results of structures that have stable hysteretic behaviour. In this paper, $\beta = 0.15$ was used.

It is possible to establish a relation between the equivalent ductility μ, the ultimate monotonic ductility μ_u, and the Park and Ang Index I_D, as follows (equation 13 in Fajfar [1]):

$$\mu_{eq} = \frac{\sqrt{1 + 4 I_D \beta \gamma^2 \mu} - 1}{2\beta\gamma^2} \quad (2)$$

where:

$$\gamma = \frac{\sqrt{E_H / m}}{\omega D_d} \quad (3)$$

here m represents the mass of the system and ω its natural frequency. The dimensionless parameter γ is a relatively stable quantity in a wide range of periods in the case of broad-band motions; however, it is not the same for narrow-band motions, like those treated in this paper. This is shown later.

Equation 2 indicates that the reduction in the ductility due to low-cycle fatigue is controlled by the parameters β and γ, as well as by the ultimate ductility μ, and by the permissible damage index I_D. Equation 2 is used in this study to obtain the equivalent ductility values shown later.

STRONG GROUND MOTIONS

In this study 223 moderate an intense ground motions recorded in the Valley of Mexico were used. These motions were originated by subduction seismic events with magnitudes equal or greater than 6.9. The events occurred between 1960 and 1997 at the West Coast of Mexico.

The ground motions were grouped in four bins, which depend on the type of soil that exists in the Valley of Mexico. Each bin of motions has similar frequency content characteristics. Table 1 shows the four zones (T1, T2, T3 and T4) in which the Valley of Mexico was "divided" as well as the corresponding soil predominant periods (T_s). In what follows the bins will be called T1, T2, T3 and T4, which correspond to zones T1, T2, T3 and T4, respectively.

Table 1. Zones in the Valley of Mexico

Zone	Period (T_s)
T1	$0.5s < T_s \leq 1.5s$
T2	$1.5s < T_s \leq 2.5s$
T3	$2.5s < T_s \leq 3.5s$
T4	$T_s > 3.5s$

The motions recorded in T1 zone have much shorter durations than those in the T4 zone. This is because the T1 zone corresponds mainly rock and the latter to soft soil.

The 223 records were filtered in order to correct their baseline, and also they were rotated to get their maximum Arias' Intensity (Arias [4], Villa-Velazquez and Ruiz [5], Bojorquez [6]).

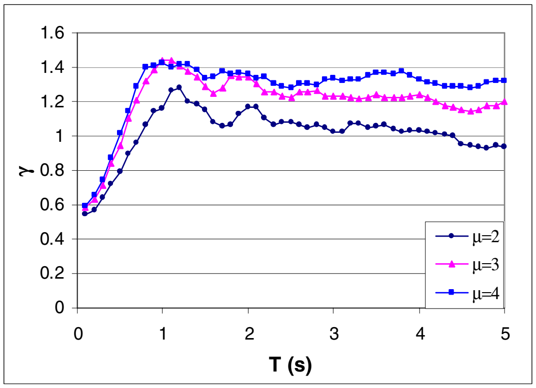
INFLUENCE OF LOW-CYCLE FATIGUE ON THE DUCTILITY CAPACITY OF SINGLE DEGREE OF FREEDOM (SDOF) SYSTEMS

Parameter γ

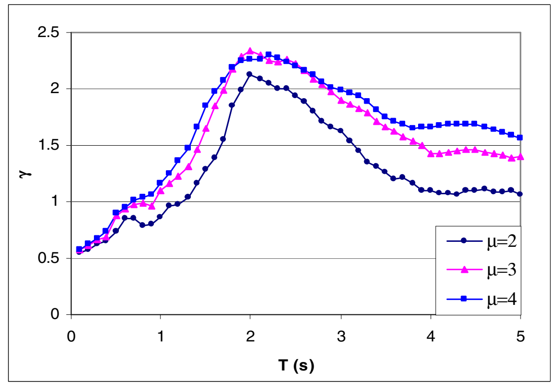
The ductility reduction due to low-cycle fatigue is controlled by the dimensionless parameter γ mentioned in equation 3. This shows that γ is a function of the hysteretic dissipated energy per unit mass, the natural frequency of the structure, and the maximum displacement. The value of γ increases as the low-fatigue effect becomes important, and γ is small when the low-fatigue effect is negligible and consequently, the usual displacement ductility controls the damage.

By means of some examples, Fajfar [1] shows that the form of variation of γ (as a function of the period, T) depends mainly on the number of large-amplitude inelastic cycles that a structure experiences during an earthquake. The author also mentions that this number depends on the duration of the ground motion, and also on other parameters like the dominant period (T_s) of the ground motion and the shape of the response spectrum.

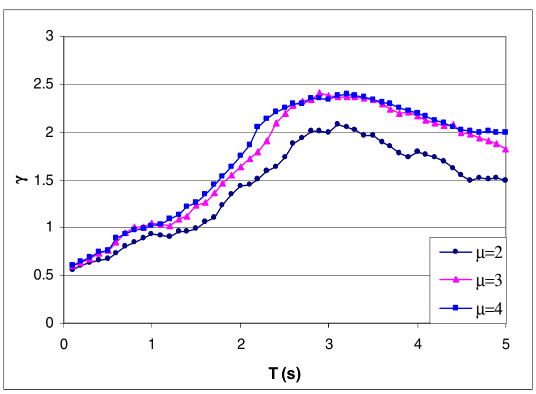
In the present study, the significant influence that the predominant period T_s and the pseudo-acceleration spectra have on the parameter γ are verified. This is shown by the γ – versus – period (T) curves shown in Figures 1a - d, based on constant target ductility (μ =2, 3 and 4). Figures 1a, b, c and d correspond to the mean values of γ obtained for Bins T1, T2, T3 and T4, respectively.



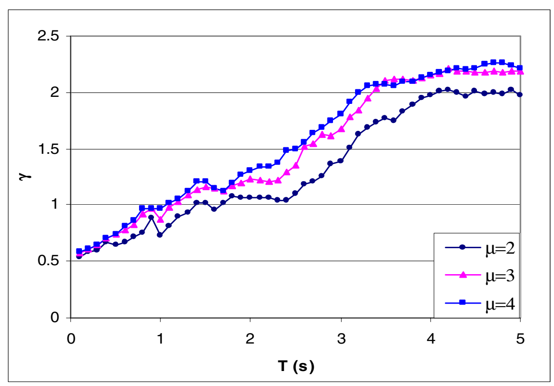
a) Zone T1



b) Zone T2



c) Zone T3



d) Zone T4

Figure 1. Values of γ corresponding to Bins T1, T2, T3 and T4

From Figures 1-4 it can be seen that as the ductility value increases, the parameter γ also grows (slightly). The figures also indicate that the parameter γ is much more significant for soft soil (Bins T2, T3 and T4) than for hard soil (Bin T1), as expected. This is due to the fact that the intensity and the duration of the seismic motions are much smaller on hard ground (T1) than on soft soil, during the same seismic event. Notice that the maximum γ value (for μ = 3, 4) for Bin T1 is of the order of 1.4, whereas for Bins T2, T3 and T4 it is about 2.3. Also notice that the maximum value of γ is very close to the dominant period T_s (see Table 1).

The maximum coefficients of variation (CV_{max}) of γ for Bins T1 – T4 are shown in Table 2. The CV_{max} varied between 0.33 and 0.48, except for Bin T4 that had larger coefficients of variation. These were around 0.6.

Table 2. Maximum coefficients of variation of γ

BINS	μ	$CV_{máx}$
T1	2	0.488
	3	0.399
	4	0.362
T2	2	0.437
	3	0.399
	4	0.359
T3	2	0.422
	3	0.366
	4	0.337
T4	2	0.607
	3	0.598
	4	0.604

Equivalent ductility

Mean values of the equivalent ductility (μ_{eq}) were obtained for elastoplastic sdof systems, having critical damping ξ = 5%, and periods of vibration between 0.01a and 5.0s. For the analysis $I_D = 1$ was assumed. Three different values were used for the target ductility obtained under increasing monotonic load (μ = 2, 3 and 4). The results are presented in figures 2, 3, 4 and 5, which correspond to Bins T1, T2, T3 and T4, respectively

Figures 3 - 5 show that for periods close to the dominant one (T_s) the equivalent ductility is smaller than for other periods. This indicates that the reduction in the maximum ductility due to low-cycle fatigue is more significant for structures with natural vibration period close to the dominant ground period. These observations are valid for the systems located on soft soils, but not for those located on rock, where no dominant period is observed.

Fitting equations to the equivalent ductility

In order to calculate the equivalent ductility (μ_{eq}) some analytical expressions were fitted. These expressions are functions of the ultimate ductility under monotonic loading (μ) and of the natural period of the system (T). The equations were fitted for the four zones T1, T2, T3 and T4.

Zone T1 (Bin T1)

In figure 2 it can be seen that for Bin T1 the equivalent ductility is practically constant for T $\geq$ 1s. This means that for that range of periods μ_{eq} depends only on μ. This is expressed by the following equation:

$$\mu_{eq} = \mu^{0.723} \tag{4}$$

From Figure 2 it is verified that expression 4 is adequate for most of the periods, except for the short period range, where it is conservative. Equation 4 is extremely simple to apply and it gives sufficiently accurate results.

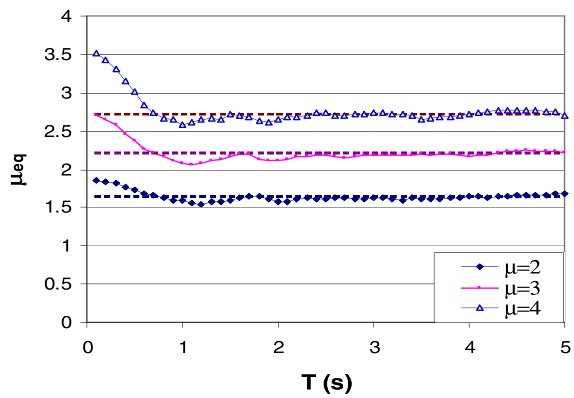


Figure 2. Mean value of the equivalent ductility and of that obtained with equation 4, for Bin T1.

Zone T2 (Bin T2)
The following expression was fitted for Bin T2:

$$\mu_{eq} = 0.296\,T(1-\mu) + 0.87\mu + 0.211, \qquad \text{for } T \le T_s$$

$$\mu_{eq} = (0.072\mu - 0.037)T + 0.131\mu + 0.878, \qquad \text{for } T > T_s \tag{5}$$

Here, T is the period of the structure, and T_s represents the dominant period of the soil.

Mean values of μ_{eq} and those obtained using equation 5, for Zone T2, are shown in Figure 3. It is evident that equation 5 is a good approximation for Bin T2, for the range of periods shown in Figure 3.

Zone T3
The equations proposed for evaluating the equivalent ductility on structures located in zone T3 are:

$$\mu_{eq} = (-0.23\mu + 0.284)T + 0.855\mu + 0.164, \qquad \text{for } T \le Ts$$

$$\mu_{eq} = (0.129\mu - 0.198)T - 0.222\mu + 1.609, \qquad \text{for } T > Ts \tag{6}$$

The results of these equations and those obtained from data are shown in the figure 4. The comments made for zone T2 are valid for zone T3.

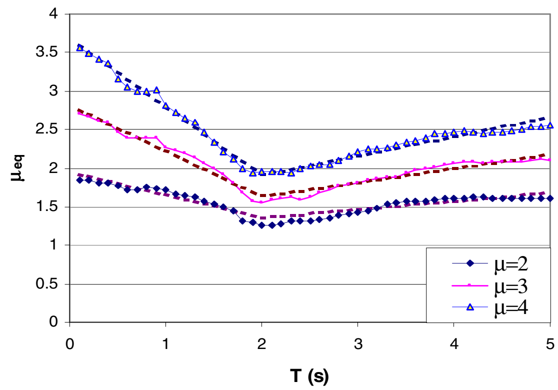


Figure 3. Mean value of the equivalent ductility and of that obtained with equation 5. Zone T2.

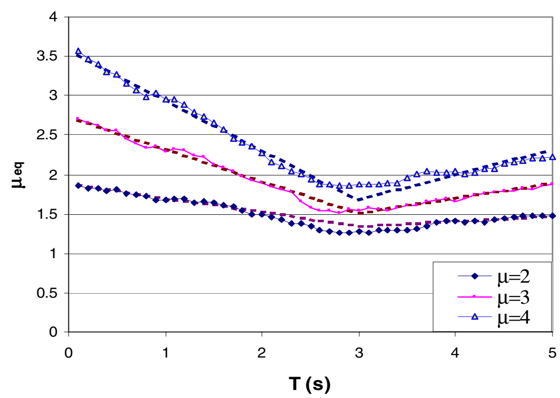


Figure 4. Mean value of the equivalent ductility and of that obtained with equation 5. Zone T3.

Zone T4

The equations fitted for zone T4 are:

$$\mu_{eq} = (-0.082\mu + 0.02)T + 0.763\mu + 0.349, \qquad \text{for T < Ts} \tag{7}$$

$$\mu_{eq} = 0.435\mu + 0.429, \qquad \text{for T} \geq \text{Ts}$$

The results obtained with equations (7), and those obtained from the data, are shown in Figure 5.

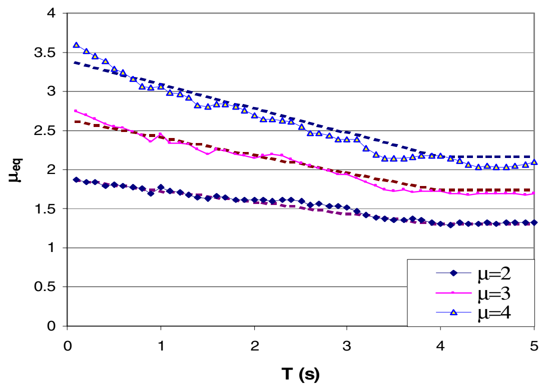


Figure 5. Mean value of the equivalent ductility and of that obtained with equation 5. Zone T4.

Notice that equations 4 to 7 depend only on the ultimate ductility under monotonic loading (μ) and on the period of the system (T), as well as on the zone where the structure is located. The application of these expressions is simple. Using them it is possible to evaluate quickly the equivalent ductility without performing time consuming dynamic analyses.

The maximum standard deviation and coefficients of variation of μ_{eq}, for each bin and for the target ductilities $\mu = 2$, 3 and 4, are shown in Table 2, Bojórquez [2].

Table 2. Maximum standard deviations and coefficients of variation of μ_{eq}

BIN	μ	$\sigma_{máx}$	CV_{max}
T1	2	0.228	0.145
	3	0.363	0.172
	4	0.508	0.190
T2	2	0.240	0.182
	3	0.377	0.220
	4	0.500	0.212
T3	2	0.243	0.188
	3	0.383	0.242
	4	0.449	0.221
T4	2	0.288	0.218
	3	0.441	0.255
	4	0.571	0.255

CORRECTION TO THE STRENGTH REDUCTION FACTORS DUE TO THE LOW-CYCLE FATIGUE EFFECT

In order to consider the effect of low-cycle fatigue effect on the structures, a correction factor (Fc) was obtained, which is applicable to the strength reduction factors that intend to consider the non-linear behaviour of the structures. This means that, in order to consider the cumulative damage on the structures, the strength reduction factors (that take into account the non-linear behaviour) should be divided by Fc.

The Fc values were calculated as the ratio of the strength reduction factors of systems without considering the low-cycle effects, to the corresponding reduction factors considering the cumulative structural damage.

Figure 6 shows the Fc values obtained for zone T1. The curves in this figure correspond to target ductility values μ =2, 3 and 4. It is clear from the figure that for short periods the relation is very close to unity, which implies that the reduction factor due to the influence of low-cycle fatigue is negligible. However, for vibration periods longer than 1.0, the Fc reaches values between 1.2 and 1.8. In figure 6, it can be seen that as the value of μ increases, the Fc factor grows as well.

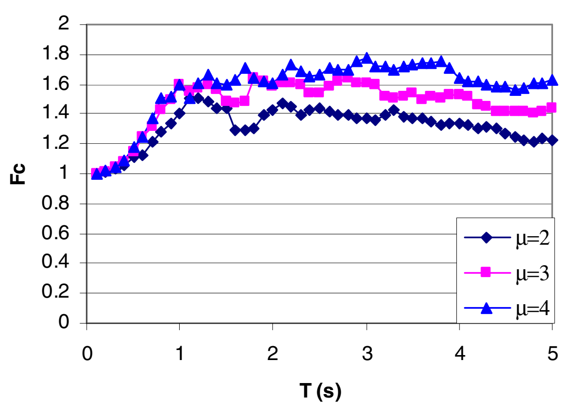


Figure 9. Correction factors for zone T1. μ= 2, 3 and 4.

The following analytical expressions were fitted to the curves shown in figure 9, which corresponds to zone T1:

$$Fc = 1, \qquad T \le 0.2 \text{ s}$$

$$Fc = \frac{5}{4}\left(Fu - 1\right)\frac{T}{T_s} - \frac{1}{4}Fu + \frac{5}{4}, \qquad 0.02 \le T \le 1 \text{ s} \tag{8}$$

$$Fc = Fu, \qquad T \ge 1 \text{ s}$$

where: $Fu = -0.0575\mu^2 + 0.5025\mu + 0.5625$

The values obtained with these expressions and those in Figure 9 are compared in Figure 10.

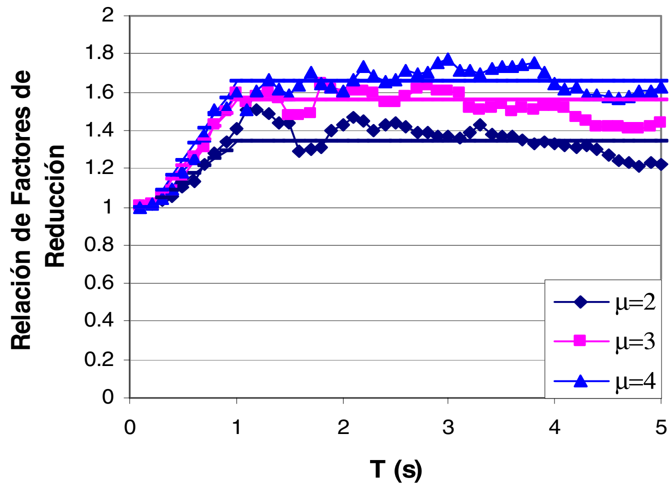


Figure 10. Comparison between curves shown in Figure 9 and those obtained with equation 8.

Factors Fc for zones T2, T3 and T4 present their maximum values at the dominant periods $\overline{T}_s$ = 2, 3 and 4s, respectively. Due to this reason, in what follows the authors decided to work with the normalized period T/$\overline{T}_s$. In this way, the maximum value of Fc is very close to T/$\overline{T}_s$=1. On the other hand, the variation of the Fc factors is very similar for the three soft soil zones, T2, T3 and T4. Based on this, the same analytical form is proposed for the three zones:

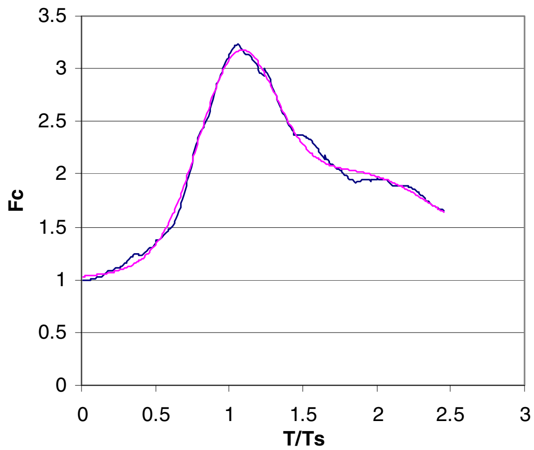
$$Fc = 1 + \alpha_1 e^{\frac{(T/Ts - \alpha_2)^2}{-\alpha_3}} + \alpha_4 e^{\frac{(T/Ts - \alpha_5)^2}{-\alpha_6}} \tag{9}$$

The parameters α_1, α_2, α_3, α_4, α_5 and α_6 values, which depend on the ductility μ of the structure, are indicated in Table 2. Figures 11a, b and c show the mean values of Fc and those obtained with equation 9, for μ = 2, 3 and 4, respectively.

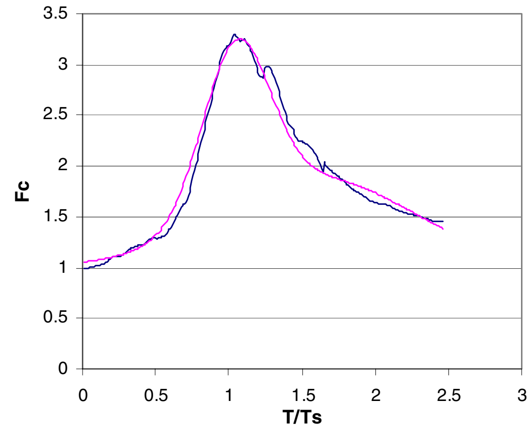
Table 2. Values of α_1, α_2, α_3, α_4, α_5 and α_6 to be substituted in equation 9

	α_1	α_2	α_3	α_4	α_5	α_6
μ = 2	1.7	1.1	0.15	0.3	2	0.7
μ = 3	1.614	1.05	0.105	0.879	1.6	0.9
μ ≥ 4	1.61	1.05	0.134	1.021	1.8	0.917

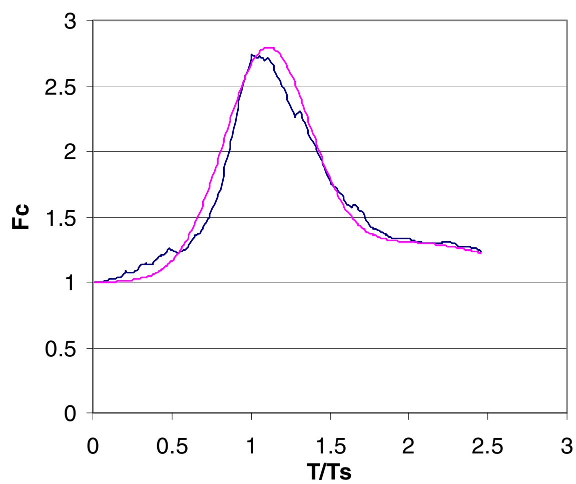
From figures 9-11 it is observed that the Fc values are close to unity for very short and for very long periods, but for periods close to $\overline{T}_s$, reaches its maximum, which means that in this period range the low-cycle-fatigue effect is more significant. The correction factor Fc increases as the value of μ grows.



a) $\mu = 2$



b) $\mu = 3$



c) $\mu = 4$

Figure 11. Comparison between the mean values of Fc obtained for zones T2, T3 and T4, and those obtained with equation 9.

From figures 11a and b it can be seen that the correction factor Fc reaches values up to 3.2. This very large value could be attributed probably to the damage index used in the study.

CONCLUSIONS

The results obtained in this study show that there is a significant influence of the cumulative damage on the parameter γ and consequently on the equivalent ductility of sdof elastoplastic systems subjected to ground motions recorded in the soft soil on the valley of Mexico.

The correction factors (Fc) which are applicable to the strength reduction factors reach values up to Fc_{max} = 1.8 (μ = 4) for hard soil, and up to 3.3 (μ = 3, 4) for soft soils. The maximum value (Fc_{max}) occurs near the dominant period of the soil. This becomes more obvious for soft soils were the motions are narrow-banded. The above implies that the low-cycle fatigue effect is more significant for structures located on soft soil, and with structural vibration period close to the dominant period of the soil.

It is desirable, in future versions of the seismic codes, to incorporate strength reduction factors (in an explicit and transparent way), that take into account the effect of low-cycle fatigue. This is particularly required for structures with periods close to the dominant period of the soil and subjected to long-duration motions like those recorded in the soft soil in the Valley of Mexico. Perhaps the biggest challenge to reach this objective is the need to establish design requirements for the structural elements, as functions of the ductility obtained under increasing monotonic displacement, (Arroyo and Terán-Gilmore [7], Terán-Gilmore [8]).

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