13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 525

MULTIFRACTAL ANALYSIS OF A SPRING-BLOCK SEISMIC FAULT

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SUMMARY

We carried out the multifractal analysis of time series of synthetic earthquakes obtained with the Olami, Feder and Christensen (OFC) model. The OFC model is a non-conservative two-dimensional version of the Burridge and Knopoff (BK) model used to simulate the behavior of a seismic fault. The model is solved using cellular automaton, and every time the automaton is calculated a synthetic earthquake is obtained with its magnitude and duration time. We obtained a catalogue of synthetic earthquake magnitude, which we can represent as a time series. Such series exhibit power law behavior so much for the magnitudes (Gutenberg-Richter law) as for the duration times, and they can be considered as a singular measure so we can do multifractal analysis to obtain more information. We used the method proposed by Chhabra and Jensen to obtain the multifractal spectra and the width of the spectra. If the width is small we have a monofractal behavior, this means that we require only a global Hurst exponent or a fractal dimension to characterize the series. If the spectrum is wide the series is more complex because we need a set of fractal dimensions to describe it. We found a monofractal behavior for small conservation levels but when we increase the conservation level the behavior becomes multifractal. For low conservation levels the multifractal spectra are not symmetrical but they become symmetrical as we increase the conservation level. In fact, these symmetrical spectra resemble those that are generated with binary multiplicative processes. We obtained the same behavior for the spectra of the duration time series. We also investigated the relationship between the multifractal spectrum width and the linear dimension of the grid that represents the seismic fault, but we did not find an explicit relationship as in the previous case.

INTRODUCTION

The OFC model is a non-conservative cellular automaton model introduced by Olami et al. [1, 2] for describing the dynamics of a 2-D array of rigid blocks on a frictional surface. It consists of an *LxL* array of individual blocks identified by *(i, j)*, where *i*, *j* are integers between 1 and *L*. Each block is connected to its four nearest neighbors by springs with elastic constants K_1 and K_2 and it is connected on its top to a moving driving plate by means of a spring with stiffness K_L (see reference 1). The displacement of each block from its relaxed position on the lattice is $X_{i,j}$ and the total force exerted by the springs on a block *(i, j)* is given by

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$$F_{i,j} = K_1\left[2x_{i,j} - x_{i-1,j} - x_{i+1,j}\right] + K_2\left[2x_{i,j} - x_{i,j-1} - x_{i,j+1}\right] + K_L x_{i,j} \quad (1)$$

When the two rigid plates move relatively among them the total force in each block it is increased uniformly (with a rate proportional to $K_L V$, where V is the relative speed among the plates), until a site reaches a value limit and the relaxation process begins). The redistribution of forces after local slip at position *(i, j)* due to the force on one of the blocks is larger than the maximal static friction and is given by

$$\begin{aligned} F_{i\pm1,j} &\to F_{i\pm1,j} + \delta F_{i\pm1,j} \\ F_{i,j\pm1} &\to F_{i,j\pm1} + \delta F_{i,j\pm1} \\ F_{i,j} &\to 0 \end{aligned} \quad (2)$$

where the increments in the force on the nearest-neighbor block are

$$\begin{aligned} \delta F_{i\pm1,j} &= \frac{K_1}{2K_1 + 2K_2 + K_L} F_{i,j} = \gamma_1 F_{i,j}, \\ \delta F_{i,j\pm1} &= \frac{K_2}{2K_1 + 2K_2 + K_L} F_{i,j} = \gamma_2 F_{i,j}, \end{aligned} \quad (3)$$

γ_1 and γ_2 are called the elastic ratios, and for the case $K_L > 0$ the redistribution of the force is non-conservative, as is expected to occur in actual earthquakes. This redistribution redefines the forces in the nearest-neighbor blocks, and further slips can occur, causing a chain reaction (synthetic earthquake).

The spring-block model is mapped into a continuous, non-conservative cellular automaton modeling earthquakes which is described by an algorithm described in references [1] and [2]. We repeat the algorithm many times and we obtain that the magnitude of the synthetic earthquakes follows the Gutenberg-Richter law. Although the OFC model is a great simplification for studying the dynamics of a real fault, it has other properties that are related to real seismicity [3, 4]. The catalog of synthetic earthquakes is a time series as the one showed in Fig. 1. To these time series we applied the multifractal formalism, if the multifractal spectrum is very narrow this would indicate a monofractal behavior and we can describe the series only with one Hurst exponent or one fractal dimension [5].

THE MULTIFRACTAL FORMALISM

The behavior of nonlinear dynamical systems can be often characterized by fractal or multifractal measures which correspond, for example, to the invariant probability distribution of a strange attractor [6], the distribution of voltage drops across a random resistor network [7], or the spatial distribution of dissipative regions in a turbulent flow [8]. Various multifractal formalisms have been developed to describe the statistical properties of these measures in terms of their singularity spectrum, which provides a description of the multifractal measure in terms of interwoven sets, with singularity strength α (the Lipschitz-Hölder exponent), whose Hausdorff dimension is $f(\alpha)$ [9, 10, 11]. If we cover the support of the measure with boxes of size L an define $P_i(L)$ as the probability in the i-th box, then we can define an exponent α_i by

$$P_i(L) \approx L^{\alpha_i} \quad (4)$$

and if we count the number of boxes $N(\alpha)$ where the probability P_i has a singularity strength between α and $\alpha + d\alpha$, the $f(\alpha)$ can be defined as the fractal dimension of the set of boxes with singularity strength α by

$$N(\alpha) \approx L^{-f(\alpha)} \tag{5}$$

We used the method developed by Chhabra and Jensen [10, 11] for the calculation of the $f(\alpha)$ spectrum of multifractal structures. First, a 1-parameter manifold of normalized measures $\mu_i(q)$ where the probabilities in the boxes of size L are

$$\mu_i(q,L) = {[P_i(L)]^q} \Big/ {\sum_j [P_j(L)]^q} \tag{6}$$

The parameter q can be imagined as some sort of microscope, which enlarges different areas of the multifractal. For $q>1$ the strongly singular structures are enhanced, for values of $q<1$ the less singular areas are more emphasized, and for $q=1$ the original measure $\mu(q)$ is replicated. The Hausdorff dimension of the support of $\mu(q)$ is

$$f(q) = \lim_{L\to 0} \frac{\sum_i \mu_i(q,L)\log[\mu_i(q,L)]}{\log L} \tag{7}$$

and the mean strength of the singularity $\alpha_i = \ln(P_i)/\ln L$ with respect to $\mu(q)$ is obtained by evaluating

$$\alpha(q) = \lim_{L\to 0} \frac{\sum_i \mu_i(q,L)\log[P_i(L)]}{\log L} \tag{8}$$

These equations provide a relationship between a Hausdorff dimension f and an average singularity strength α as implicit functions of the parameter q. The f versus α curves are the multifractal spectra.

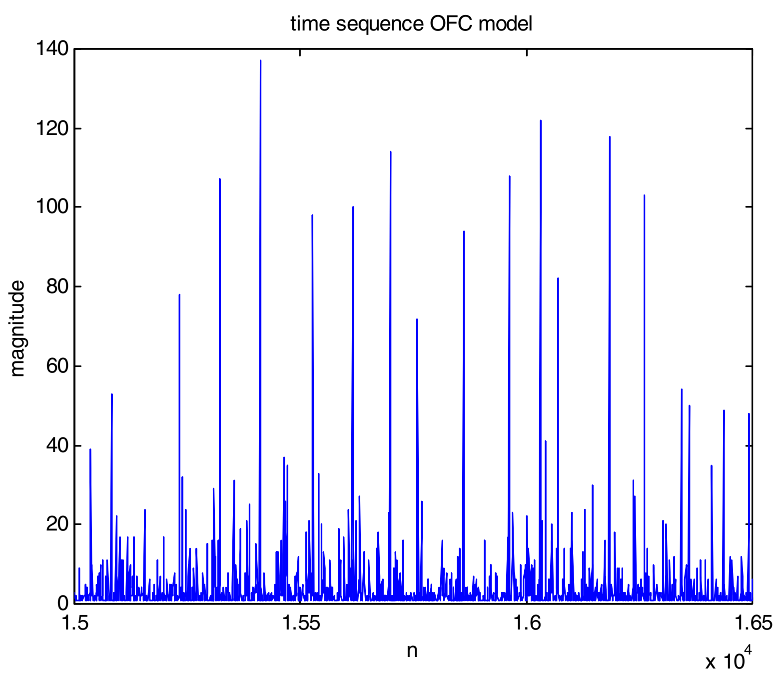
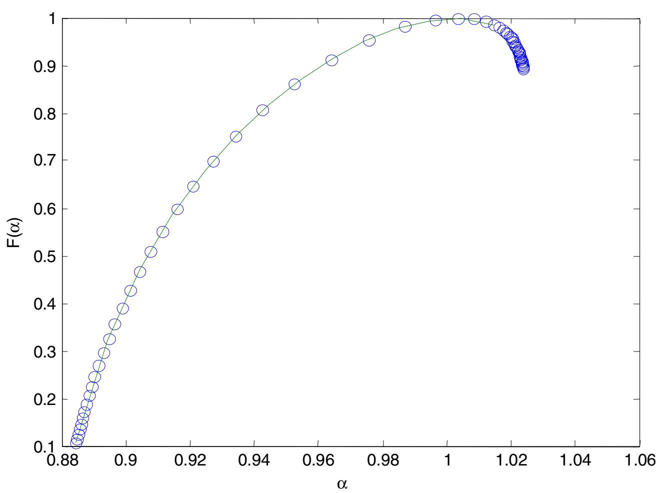


Figure 1. Time series of synthetic earthquakes (16384 events).

RESULTS

First, we fixed $L = 100$, it means that we considered grids with $100x100 = 10000$ blocks representing the seismic fault, and we obtained time series with the magnitude of 16384 synthetic earthquakes. We applied the Chhabra and Jensen algorithm to the series and the spectrum multifractal was obtained for different values of the conservation level γ. The multifractal spectra were obtained for q from -30 up to 30. The α_{min}, α_{max} and α_0 (the α-value which corresponds to the spectrum maximum). Then the degree of multifractality or spectrum width was calculated as $\Delta\alpha = \alpha_{max} - \alpha_{min}$. The spectra for conservation levels $\gamma < 0.24$ tend to be asymmetric, just as it is shown in Figure 2, and when γ approximates to the conservative case ($\gamma = 0.25$) the spectrum becomes almost symmetrical. In Figure 3, we show α_{min}, α_{max} and α_0 values versus γ, there is a small tendency of α_0 to move to the right, and the other α's tend to separate as the conservation level increases, in this figure we can observe with clarity the mentioned asymmetry. The most interesting graph is depicted in Figure 4, in this figure we show the width of the multifractal spectrum against the conservation level, just as it is observed, the width grows as the conservation level grows.



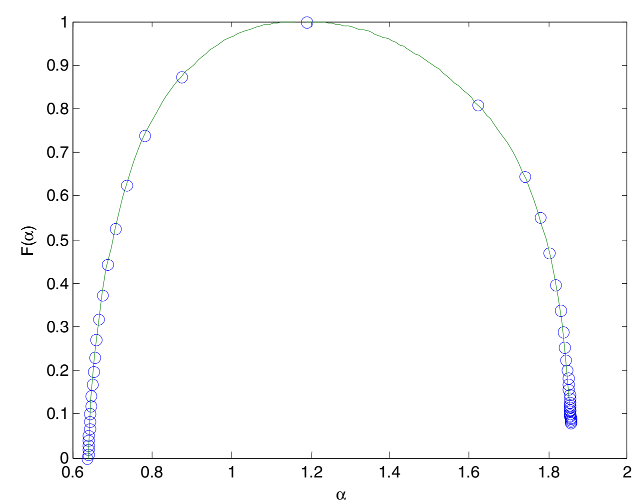


Figure 2. Multifractal spectra $\gamma = 0.1$ (above) and $\gamma = 0.2499$ (bottom). Note the widening of the spectrum as γ approximates to the conservative case. If γ is small the spectra is asymmetrical. $L = 100$.

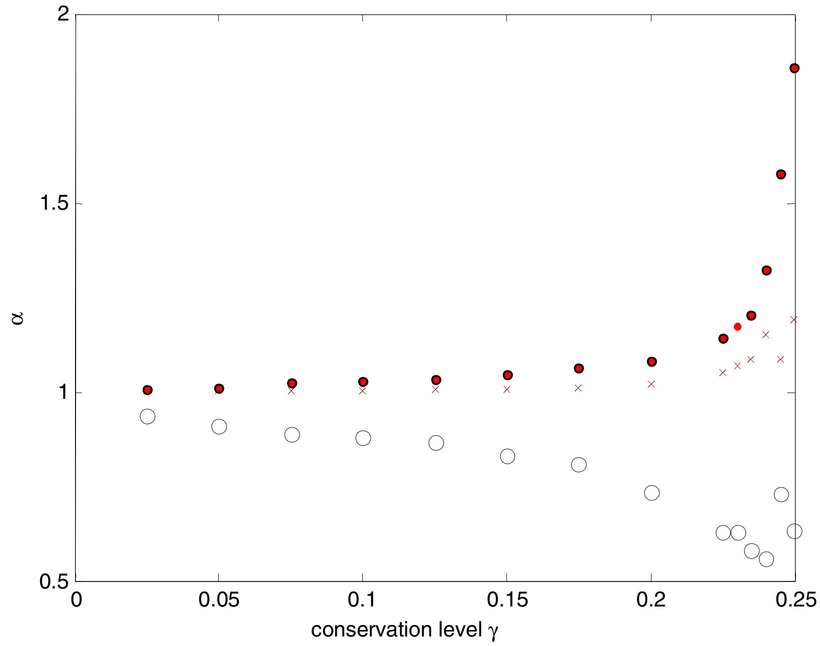


Figure 3. α_{min},(circles) α_{max} (points)and α_0 (crosses) versus the level of conservation. $L = 100$.

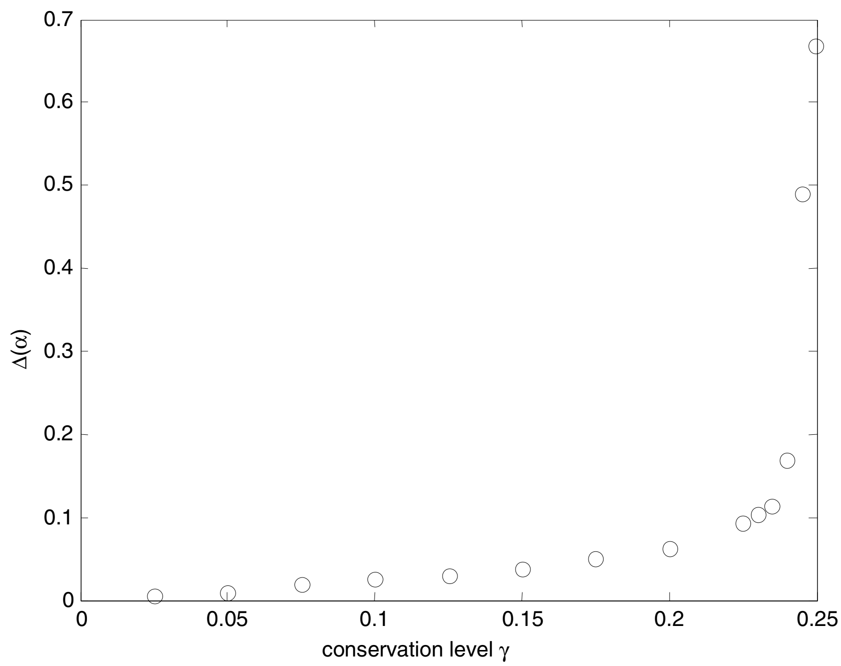


Figure 4. $\Delta\alpha$ versus the level of conservation. $L = 100$.

Similar graphs are shown in Figures 5 and 6. First, in figure 5, we plot α_{min}, α_{max} and α_0 versus L, the linear dimension of the grid that represents the seismic fault. In figure 6, we plot $\Delta\alpha$ versus L. We did not observe tendencies like those observed in Figures 5 and 6; we only observe in Figure 7 that when L grows the multifractal spectrum becomes symmetrical.

The fact that a multifractal spectrum is wider than another is important because it indicates that we need more fractal dimensions in order to describe the time series, if the width is small the series comes closer to a monofractal behavior. Although we cannot say that they are totally monofractals, we can conclude that the multifractal spectra for conservation levels close to zero tend to monofractality, and it means that we can describe the time series with only a fractal dimension. As the conservation level increases we need a set of fractal dimensions (probably infinite) to describe such time series. This seems to make physical sense because at small conservation levels only earthquakes of small magnitude occur, energy cannot be stored to produce earthquakes of medium or great magnitude. On the other hand, as γ comes closer to 0.25 the magnitude of the earthquake grows because there is not energy dissipation in other processes (for instance friction), so practically the whole energy is spent in producing earthquakes of medium and great magnitude; that does not mean that the Gutenberg-Richter law is no more valid (the small earthquakes are much more abundant that the medium ones and these more abundant than the big ones) but there exists a tendency that privileges the appearance of earthquakes of greater magnitude when γ is close to 0.25. It is also outstanding the symmetry observation, because we can approximate these real multifractals by means of theoretic multifractals generated by multiplicative processes, the most symmetrical spectra could be estimated by using binary multiplicative processes. On the other hand, for the asymmetric ones, it would be necessary to use so much binary as ternary in a first approach and of other orders if we want a better approximation.

Other observation is pertinent: The time series of the earthquake duration also have multifractal behavior and there is a very similar tendency than the series of magnitude: the spectrum becomes wider and more symmetrical for γ close to 0.25.

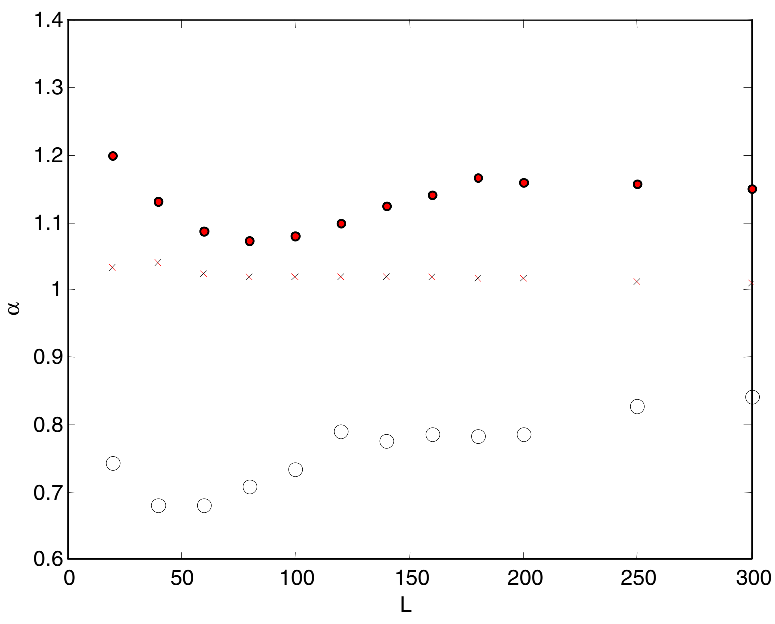


Figure 5. α_{min},(circles) α_{max} (points)and α_0 (crosses) versus L, $\gamma = 0.2$.

CONCLUSIONS

We have shown that a monofractal analysis is not always suitable for the analysis of the time series obtained from the OFC model, we think that a multifractal analysis is adequate when the time series were generated with conservation levels $\gamma > 0.175$, because the width of the multifractal spectrum is not close to cero. Many researchers [5, 12, 13] have stated that a time series with a large width is more complex that a time series with a small width because we need more fractal dimensions to describe it, this fact seems to be related with the dynamics of the system (in this case the spring-block), so big values of γ imply a more complex dynamics. We need to do more research to fully establish this fact. The analysis of time series of synthetic earthquakes by using multifractals can open a new perspective, because if we can approximate real multifractal spectra with theoretic spectra generated with multiplicative processes then we can use the concepts an terms developed for these multiplicative processes, this probably would help to understand the dynamics of these models that have been used intensively to mimic the dynamics of seismic faults.

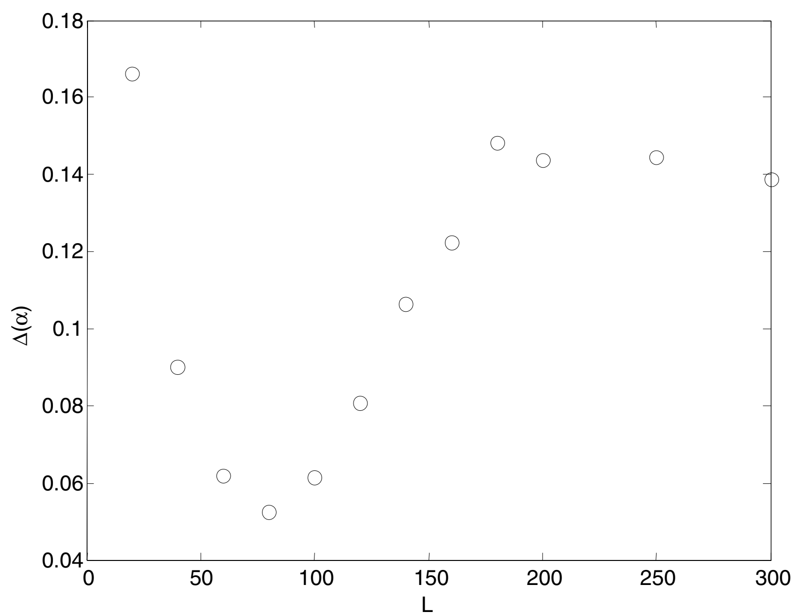


Figure 6. $\Delta\alpha$ versus versus L, $\gamma = 0.2$.

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